Learning about academic ability and the college drop-out decision

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Abstract: Much empirical research examining the (relatively) low levels of educational attainment of students from low income families has focused on the role of financial factors such as credit constraints. In this paper we use unique longitudinal data to provide some of the first direct evidence about one of the most prominent alternative explanations - that departures from school arise as students learn about their academic ability or grade performance. Examining college drop-out, we find that learning about grade performance/ability plays a very prominent role; our simulations suggest that drop-out between the first and second years of college would be reduced by approximately 40% if no learning occurred about these factors. The paper also contributes directly to the understanding of gender differences in educational attainment.

I. Introduction

It has been widely recognized that issues related to college graduation are of central policy importance, with students from low socio-economic backgrounds garnering particular attention due to their high drop-out rates relative to other students (Bowen et al., 2009). Nonetheless, a comprehensive understanding of the relative importance of various possible explanations for drop-out and other post-secondary outcomes of this group has remained elusive.¹ This is the case, in large part, because general longitudinal surveys, which are typically designed to allow researchers from a variety of disciplines to study a broad range of topics, often have difficulty providing data which are ideal for studying specific issues at their most fundamental levels.

We provide direct, new evidence about the underlying reasons for the college drop-out of students from low income families by taking advantage of our ongoing longitudinal survey project, The Berea Panel Study (BPS). The design of survey instruments in the BPS is guided closely by the specific economic models that are of relevance given specific issues of interest. This focused approach to survey design, combined with our ability to survey students at great frequency during school, results in a longitudinal dataset that is unique in the depth and detail it provides about a student's time in college. Of particular interest here, the survey provides detailed information about a comprehensive set of factors that theory suggests could influence the drop-out decision. Further, motivated by the recognition that outcomes in education are often best viewed as the end result of a learning process in which a student resolves important uncertainty after entering school, the detailed information about factors is collected multiple times each year, starting at the time of entrance.

The project takes place at a school, Berea College, that is valuable for an in-depth case study of this sort because it operates under a mission of providing access to students from low income families. Similar to what is found for low income students elsewhere, forty percent of students at Berea do not graduate (Stinebrickner & Stinebrickner, hereafter, S&S, 2003).

Much previous research on the educational attainment of low income students has focused on the role

¹Describing the traditional difficulties of understanding the underlying reasons for college drop-out, Bowen and Bok (1998) write, "One large question is the extent to which low national graduation rates are due to the inability of students and their families to meet college costs, rather than to academic difficulties or other factors." Tinto (1975) suggests that drop-out is related to academic and social integration, but direct tests of this are scarce (Draper, 2005).

of credit constraints. Berea College offers a full tuition (and a large room and board) subsidy to all entering students so credit constraints cannot arise due to difficulties covering the direct costs of schooling. Further, S&S (2008a) found that, in the aggregate, difficulties borrowing money to pay for consumption during school also do not have an important effect on drop-out at Berea. Thus, explanations unrelated to short-term credit constraints play an important role in determining drop-out at Berea. This is consistent with recent literature which, while perhaps somewhat unsure about the exact extent to which credit constraints are important, agrees that the large majority of variation in drop-out remains unexplained.² This paper provides some of the first direct evidence about the importance of the most prominent alternative explanations.

We pay particular attention to perhaps the most prominent alternative explanation - that college drop-out arises as students learn about their academic ability or grade performance after matriculation (Manski, 1989; Altonji, 1993; Carneiro et al., 2005; Cunha et al., 2005). That little direct empirical evidence exists about this explanation stems from the reality that identifying beliefs (expectations) about a factor such as academic ability is difficult using standard choice data because a particular behavior may be consistent with multiple characterizations of preferences and expectations (Manski, 2002, 2004). In response to this identification issue, economists have recently paid closer attention to the virtues of eliciting self-reports of subjective probabilities using carefully worded survey questions (Dominitz, 1998; Dominitz and Manski, 1996, 1997). However, when one wishes to characterize learning, difficulties related to the timing of surveys may also exist. In the drop-out context, providing relevant information about how beliefs change requires the elicitation of beliefs both at the time of entrance and at a time close to when the drop-out decision is made, with this task being difficult in standard longitudinal survey designs which typically contact students at most once a year.

The BPS was perhaps the first sustained longitudinal survey to have a central focus on the elicitation of beliefs. This allows us to overcome the standard difficulties described above because, motivated by learning models of behavior, we elicited beliefs at multiple times each year, starting at the time of college entrance. As such, in the process of providing a new understanding of the drop-out decision, we make a second contribution

²See Lochner and Monge (2011) for a very recent survey of the large literature on credit constraints in education.

- providing new evidence about how agents update subjective beliefs in response to the arrival of new information. Manski (2004) writes that there exists a "critical need for basic research on expectations formation."³

Section II describes the BPS sample, a simple theory of drop-out, the survey questions motivated by this theory, and a set of equations that utilize the survey questions in our analysis. Section III examines beliefs about grade performance and academic ability. At entrance students are considerably too optimistic about grade performance, with this due mostly to overoptimism about ability rather than, for example, overoptimism about the amount of time that will be spent studying. Subsequently, students update beliefs about grade performance significantly. We find that updates depend on both initial beliefs and new information arriving in the form of grades, with individual heterogeneity in the weights assigned to initial beliefs and grades depending on individual-specific views about the underlying reasons for grade performance that are suggested to be of importance by a Bayesian model. Of interest for policymakers concerned that at-risk students may leave school prematurely, we find that, while students with the worst grade performance update dramatically, they tend to remain overoptimistic because they understate the importance of permanent factors in determining their performance. Section IV examines drop-out. Our main finding is that learning about grade performance/ability plays a very prominent role; our predictions suggest that drop-out would be reduced by 41% if no learning occurred about these factors.

This paper also contributes to the understanding of gender differences in educational attainment. Goldin et al. (2006) find a current college completion advantage for females, with this advantage being greatest for low income students. NCES (2007) shows that much of this advantage is due to college drop-out; the gender graduation gap is 25% at moderately selective undergraduate institutions with large numbers of low income students. We find that the substantial gender difference in drop-out in our sample is predicted almost entirely by academic differences (1st year grades and beliefs about future grades), and we find direct evidence of gender differences in effort and gender differences in the non-pecuniary cost of effort suggested to be of importance by

³Work measuring revisions to expectations includes Dominitz (1998); Dominitz and Manski (2003); Dominitz and Hung (2003); Delevande (2006); Lochner (2007); Madeira (2007); Zafar (2011); Arcidiacono et al. (forthcoming A). Wolpin (1999), van der Klaauw (2000), and van der Klaauw and Wolpin (2008) use this data in structural models.

Goldin et al. (2006). As to why poorly performing males decided to enter college, males are substantially more overoptimistic than females at entrance. Thus, our work contributes to a literature examining the role that gender differences in overconfidence play in determining behavior (Barber and Odean, 2001; Niederle and Vesterlund, 2007), with a primary benefit of our data being that we directly observe a measure of overconfidence as well as a behavior of interest.

In terms of contributions to the survey literature, while interactions with respondents make us confident that students were comfortable with the survey questions we used to elicit expectations, it is worth noting that it will never be possible to directly examine how accurately self-reported expectations data represent a person's true beliefs. Instead, confidence in the usefulness of this sort of data is best accumulated by examining, as in Manski (2004), its performance across a variety of substantive contexts. As such, we take as our starting point that useful information about subjective beliefs can be elicited from carefully worded survey questions. Nonetheless, our findings provide perhaps the strongest evidence to-date for this starting point. Simple theory related to the drop-out decision suggests that both a person's actual grade point average in the first year and the person's beliefs about future grade performance at the end of the first year should be important predictors of whether a person returns to college after the first year. In Section IV we find that this theoretical implication is satisfied when we measure beliefs about future grade performance directly using self-reported expectations data. However, this theoretical implication is not satisfied when we construct beliefs about future grade performance directly using self-reported expectations data. However, this theoretical implication is not satisfied when we construct beliefs about future grade performance using the types of assumptions often employed in empirical work when it is necessary to explicitly characterize beliefs.

II. Model, BPS survey, and the equations for analysis that use survey information

II.A. A simple model of dropout and basic data needs

Let $d(t_0)$ be an indicator variable that is equal to one if a student decides at the time of (potential) college entrance, t_0 , that he will not attend college. Then, defining $E(V_S^{t0})$ to be the expected present value of lifetime utility at time t_0 of entering college and $E(V_N^{t0})$ to be the expected present value at time t_0 of not entering,

(1) $d(t_0) = 1$ iff $d^*(t_0) = E(V_N^{t_0}) - E(V_S^{t_0}) > 0$.

Earnings play a central role in determining lifetime post-college utility. Workers without college degrees work

in jobs that require unskilled human capital. Workers with college degrees work in jobs that require skilled human capital. A person may be uncertain about how much skilled human capital he will accumulate during college because he may be unsure about his inherent ability to do college level work, the amount he will study, or the effectiveness of studying. Beliefs about skilled human capital may be related to other factors that influence $V_{N}^{t0}-V_{N}^{t0}$. For example, studying may impact current utility through the effect of leisure on the enjoyability of school and impact future utility through the effect of human capital on post-college earnings.

Defining $d(t_1)$ to be an indicator variable that is equal to one if the student decides at a particular time after entrance, t_1 , that he will leave school, the drop-out decision can be written analogously to Eq. (1),

(2) $d(t_1) = 1$ iff $d^*(t_1) = E(V^{t_1}_N) - E(V^{t_1}_S) > 0$.

Differences between the expectations in Eqs. (1) and (2) reflect learning that takes place between entrance and t_1 . However, because Eq. (2) shows that a person's state at time t_1 is sufficient for him to make the drop-out decision, learning does not explicitly enter Eq. (2). Thus, roughly speaking, in order to use Eq. (2) to understand the importance of learning, one must know the difference between the probability that $d(t_1)=1$ given actual beliefs at t_1 and the probability that $d(t_1)=1$ under the counterfactual that beliefs had stayed the same as they were at t_0 .

Alternatively, one could rewrite Eq. (2) so that learning enters explicitly:

(3) $d(t_1) = 1$ iff $d^*(t_1) = [E(V_N^{t_0}) - E(V_S^{t_0})] + [E(V_N^{t_1}) - E(V_N^{t_0}) - E(V_N^{t_0})] > 0.$

Eq. (3) indicates that drop-out will be present if the amount that the person learns about the expected benefits of being in school relative to being out of school, $[E(V_N^{t1})-E(V_N^{t1})-\{E(V_N^{t0})-E(V_N^{t0})\}]$, is sufficient to push the student out given how close to the margin of indifference he was at the time of entrance, $[E(V_N^{t0})-E_0(V_N^{t0})]$.⁴ We note that first period realizations of factors of interest can influence the term $[E(V_N^{t1})-E(V_N^{t1})-\{E(V_N^{t0})-E(V_N^{t1})-\{E(V_N^{t0})-E(V_N^{t1})-\{E(V_N^{t0})-E(V_N^{t1})-E(V_N^{t1})-\{E(V_N^{t0})-E(V_N^{t1})-E(V_N^{t1})-E(V_N^{t1})-E(V_N^{t1})-E(V_N^{t1})-E(V_N^{t1})-E(V_N^{t1})-E(V_N^{t1})-E(V_N^{t1})-E(V_N^{t1})-E(V_N^{t1})-E(V_N^{t1})-E(V_N^{t1})-E(V_N^{t1})-E(V_N^{t1})-E(V_N^{t1})-E(V_N^{t1})-E(V_N^{t1})-E(V_N^{t1})-E(V_N^{t1})-E(V_N^{t1})-E(V_N^{t1})-E(V_N^{t1})-E(V_N^{t1})-E(V_N^{t1})-E(V_N^{t1})-E(V_N^{t1})-E(V_N^{t1})-E(V_N^{t1})-E(V_N^{t1})-E(V_N^{t1})-E(V_N^{t1})-E(V_N^{t1})-E(V_N^{t1})-E(V_N^{t1})-E(V_N^{t1})-E(V_N^{t1})-E(V_N^{t1})-E(V_N^{t1})-E(V_N^{t1})-E(V_N^{t1})-E(V_N^{t1})-E(V_N^{t1})-E(V_N^{t1})-E(V_N^{t1})-E(V_N^{t1})-E(V_N^{t1})-E(V_N^{t1})-E(V_N^{t1})-E(V_N^{t1})-E(V_N^{t1})-E(V_N^{t1})-E(V_N^{t1})-E(V_N^{t1})-E(V_N^{t1})-E(V_N^{t1})-E(V_N^{t1})-E(V_N^{t1})-E(V_N^{t1})-E(V_N^{t1})-E(V_N^{t1})-E(V_N^{t1})-E(V_N^{t1})-E(V_N^{t1})-E(V_N^{t1})-E(V_N^{t1})-E(V_N^{t1})-E(V_N^{t1})-E(V_N^{t1})-E(V_N^{t1})-E(V_N^{t1})-E(V_N^{t1})-E(V_N^{t1})-E(V_N^{t1})-E(V_N^{t1})-E(V_N^{t1})-E(V_N^{t1})-E(V_N^{t1})-E(V_N^{t1})-E(V_N^{t1})-E(V_N^{t1})-E(V_N^{t1})-E(V_N^{t1})-E(V_N^{t1})-E(V_N^{t1})-E(V_N^{t1})-E(V_N^{t1})-E(V_N^{t1})-E(V_N^{t1})-E(V_N^{t1})-E(V_N^{t1})-E(V_N^{t1})-E(V_N^{t1})-E(V_N^{t1})-E(V_N^{t1})-E(V_N^{t1})-E(V_N^{t1})-E(V_N^{t1})-E(V_N^{t1})-E(V_N^{t1})-E(V_N^{t1})-E(V_N^{t1})-E(V_N^{t1})-E(V_N^{t1})-E(V_N^{t1})-E(V_N^{t1})-E(V_N^{t1})-E(V_N^{t1})-E(V_N^{t1})-E(V_N^{t1})-E(V_N^{t1})-E(V_N^{t1})-E(V_N^{t1})-E(V_N^{t1})-E(V_N^{t1})-E(V_N^{t1})-E(V_N^{t1})-E(V_N^{t1})-E(V_N^{t1})-E(V_N^{t1})-E(V_N^{t1})-E(V_N^{t1})-$

⁴Eq. (3) highlights the direct connection between entrance and drop-out (Manski,1989). If students who choose Berea are closer to the margin at entrance than students elsewhere, then learning about ability might have a bigger effect at Berea. Mitigating the concern that marginal students might be attracted to Berea because of the tuition subsidy is that low income students also tend to pay low costs elsewhere. In addition, we find no direct evidence that students are particularly close to the margin at entrance. The college entrance exam scores at Berea described in Section II.B. are similar to those at the University of Kentucky (S&S, 2008a), and Question D (Appendix A, introduced in Section II.C.4) reveals that 88% of students believe at entrance that being in college will be either somewhat more enjoyable or much more enjoyable than not being in college. Finally, summarizing more information that is typically unobserved, we find that, on average, students believe at entrance that there is an 86% chance of graduation from Berea.

that directly affect future utility. For example, first year grade realizations may have a direct effect on future earnings and may influence a student's beliefs about future grade realizations.

Then, examining the importance of learning using either Eq. (2) or Eq. (3) requires beliefs about a factor of interest at both t_0 and t_1 . Eq. (3) suggests the additional value of data related to a student's distance from the margin at the time of entrance. We discuss this issue in more detail later.

II.B. The BPS sample

That traditional longitudinal surveys have difficulty providing direct information about beliefs at the relevant times t_0 and t_1 motivated our initiation of the BPS. In addition to the complete flexibility of survey content, the key features of the BPS are the flexibility of survey timing and the frequency of contact;

students were surveyed 10-12 times each year while in school with the baseline survey being administered immediately before the start of first year classes in 2001. The survey data are linked to administrative data.

We examine learning during the first year of college. To keep the sample as large as possible and to take advantage of unique one-time questions on the survey that took place before the second semester, the analyses of belief formation in Section III take t_1 to be the beginning of the second semester and examine the 325 students who answered the survey at t_1 and answered the baseline survey at t_0 . To study drop-out during its modal period, the analyses in Section IV take t_1 to be the end of the first year and examine the 268 students (of 325) who also answered the last survey of the year.⁵ In the sample of 325 students, .56 of students are female and .17 are black. The low income nature of students is evident; family income has a mean (std. dev.) of \$26,627 (\$17,133). The mean (std. dev.) of high school grade point average (HSGPA) is 3.38 (.467), and the mean (std. dev.) of combined American Achievement Test (ACT) scores is 23.31 (3.64). The means (std. devs.) of HSGPA and ACT are 3.24 (.48) and 22.66 (3.86) for male students, and 3.49 (.42) and 23.85 (3.37) for female students.

II.C. BPS survey questions and the equations for analysis that utilize survey information

⁵The participation rate on the baseline survey was 89% with 375 students responding. The 325 person middle-of-the-year sample arises because 25 (of the 375) students left school before the second semester and an additional 25 (of the 375) students were still in school, but did not answer the survey before the beginning of the second semester. The 268 person end-of-the-year sample arises because fourteen of the 325 students have missing or zero values for second semester grade point average (and, therefore, almost certainly left school officially or unofficially by t_1) and an additional 43 students did not complete the final survey of the year.

Motivated by previous theoretical literature, our primary focus is on beliefs related to the accumulation of skilled human capital. We consider two potential proxies for these beliefs: a measure describing beliefs about grade performance and an adjusted measure that, by taking into account study effort and course difficulty, more closely reflects beliefs about academic ability. While our decision to study both proxies comes, in part, because theory does not tell us which is more relevant for determining lifetime earnings,⁶ the two measures are also complementary. The former measure can be compared easily to actual grade performance. The latter measure may be closer to satisfying standard assumptions of learning models and is useful for understanding why students have particular beliefs about grade performance. We study beliefs about absolute grade performance and ability. While one might also be interested in beliefs about grade performance and ability relative to other students, the distinction between absolute and relative is likely least important for students of particular interest to policymakers concerned about drop-out; students who perform very poorly see grades that are very low in absolute terms and also likely realize that these grades are near the bottom of the overall grade distribution.⁷

Sections II.C.1-II.C.3 describes the two belief measures and the survey questions used to construct these measures at t_0 and t_1 . II.C.4 describes questions related to beliefs about other factors that might influence V_{N}^{t0} . V^{t0}₈. Section II.C.5 describes questions for understanding why beliefs change between t_0 and t_1 .

II.C.1 Survey: Beliefs about grade performance at t₀

Letting $t_0:t_1$ denote the period between t_0 and t_1 , and letting $\varepsilon_{(t0:t1),i}$ represent mean-zero, period-specific transitory variation in student i's grade point average (GPA),

(4) GPA_{(t0:t1),i} = $\theta_i + \epsilon_{(t0:t1),i}$.

The constant θ_i represents i's person-specific average GPA, the average if i were to complete the period $t_0:t_1$ many

⁶The former measure might be most relevant if skilled human capital is primarily accumulated while in college and grades are a good measure of what is learned, or if potential employers use college grades as the most important signal of a worker's skill. The latter measure might be relevant if much skilled human capital accumulation takes place after college.

⁷One general concern is that, if students are misinformed about the level of grades that are given in college, a student may be seen revising beliefs about grade performance downward even if he has not learned anything about either his absolute ability or his ability relative to other students. However, this issue does not seem to be of particular concern for students who are performing especially badly. The roles that relative and absolute grade performance play in determining earnings is of relevance to a wide range of literature, including literature examining: 1) the issue of mismatch in education (see, for example, Arcidiacono et al., forthcoming A); 2) the question of the extent to which degrees/grades are valuable for the signal they provide (Spence, 1973); and 3) the difficulty of identifying the effect of college quality on earnings (see, for example, Dale and Krueger, 2002; Black and Smith, 2006).

times. Our focus in this paper on θ_i is natural; if student i knew his average GPA in each period he would have a good sense of his cumulative GPA at graduation since the average value of ε tends to zero over multiple periods. A student may be uncertain about the constant θ_i at time t_0 . This uncertainty is captured by a personspecific random variable θ^{t0}_i whose distribution characterizes i's beliefs about θ_i at time t_0 . Then, letting GPA^{t0}_{(t0:t1),i} and $\varepsilon^{t0}_{(t0:t1),i}$ be random variables which represent i's beliefs at t_0 about GPA_{(t0:t1),i} and $\varepsilon_{(t0:t1),i}$, respectively,

(5) $\text{GPA}^{t0}_{(t0:t1),i} = \theta^{t0}_{i} + \varepsilon^{t0}_{(t0:t1),i}$.

We focus primarily on the mean of θ^{t0}_{i} , $E(\theta^{t0}_{i})$. This focus is motivated primarily by practical issues of identification. Our evidence about beliefs at t_0 for the period $t_0:t_1$ comes from Question A.2 of the baseline survey (note: all survey questions appear in Appendix A) which asks each student to report the "percent chance" that his GPA_{(t0:t1),i} will fall in each of a set of mutually exclusive and collectively exhaustive categories.⁸

Question A.2 does not identify the entire distribution of θ^{t_0} since GPA^{t0}_{(t0:t1),i} is a mixture of θ^{t_0} and $\epsilon^{t_0}_{(t0:t1),i}$. However, under the assumption that $E(\epsilon^{t_0}_{(t0:t1),i})=0$, taking expectations in Eq. (5) yields (6) $E(\theta^{t_0})=E(GPA^{t_0}_{(t0:t1),i})$.

Thus, under this assumption, which is discussed more in footnote (12), A.2 does identify $E(\theta^{t0}_{i})$ as long as A.2 delivers $E(GPA^{t0}_{(t0:t1),i})$. To understand whether A.2 delivers $E(GPA^{t0}_{(t0:t1),i})$ requires recognizing that i will be uncertain at t_0 about $S_{(t0:t1),i}$, the average amount that he will study per day during $t_0:t_1$. We let $S^{t0}_{(t0:t1),i}$ denote the random variable representing i's beliefs about $S_{(t0:t1),i}$ at t_0 . At issue is how students interpret the wording in A.2 which asks for beliefs about grade performance "given the amount of study-time indicated in Question A.1." The **first interpretation** possible is that, although A.1 technically elicits $E(S^{t0}_{(t0:t1),i})$, students view the wording in A.2 as a prompt to think generally about how much they might study as given by the entire distribution of $S^{t0}_{(t0:t1),i}$. In this interpretation, A.2 provides the unconditional distribution of $GPA^{t0}_{(t0:t1),i}$ and the mean of the reported distribution is $E(GPA^{t0}_{(t0:t1),i})$. The alternative **second intepretation** is that students view the wording

⁸Even for eliciting information about means this is our preferred question, in part, because our pre-survey classroom training focused directly on this question format and, in part, because we find it appealing to compute expected values ourselves (rather than relying on students to use an appropriate definition of "expected" grade performance).

II.C.2 Survey: Beliefs about ability at t₀

Conceptually, we think of person i being of higher ability than person k if i would have a higher average GPA across a large number of semesters in which the two students studied the same amount and took classes of the same difficulty. With respect to study effort, in addition to collecting $E(S^{i0}_{((0:11),i}))$, we also observe a noisy proxy for $S_{((0:11),i)}$. This proxy, denoted $\hat{S}_{((0:11),i)}$, is the average number of hours that student i studied on (up to) four particular days during the first semester on which we collected 24-hour time diaries. With respect to course difficulty, the majority of first year courses are mandated under a General Studies curriculum, so that variation in course difficulty arises from the small number of other elective courses which are determined largely by a person's intended major.⁹ We characterize a person's intended major by defining the seven dummy variables Major¹_{((0:11),i}, such that Majorⁱ_{((0:11),i} is equal to one if at t₀ person i indicates on Question C that he is most likely to end up with a major in major group j.¹⁰

⁹Arcidiacono (2004) analyzes major choice and the drop-out decision in a structural, learning model.

¹⁰The simplifying assumption that it is reasonable to assign each student one particular major at a particular point in time is consistent with previous work on college major (Zafar, 2011; Arcidiacono et al., forthcoming B). Moving away from this assumption, S&S (2011) find that students tend to have much uncertainty about their college major at early stages of

To hold study effort and course difficulty constant, we define $\text{GPA}_{(t0:t1),i}^*$ to be i's GPA if his study effort was equal to the sample average of $\text{E}(\text{S}^{t0}_{(t0:t1),i})$, denoted S*, and if he had the average course difficulty in the sample, denoted D*.¹¹ As discussed in Appendix B, D* is the sample average of $\sum_{j=1}^{7} \tau^{j}_{(t0:t1)}$ Major^j_{(t0:t1),i} where $\tau^{j}_{(t0:t1)}$ is the effect on grades in t₀:t₁ of having major group j (relative to having the omitted major group). Then, assuming that the true effect of studying on GPA is linear/homogenous and denoted by α , the analog to Eq. (4) is

(7)
$$GPA_{(t0:t1),i} = GPA_{(t0:t1),i} - \alpha(S_{(t0:t1),i} - S^*) + \sum_{j=1}^{7} (\tau^{j}_{(t0:t1)} - D^*) \text{ Major }^{j}_{(t0:t1),i}$$

= $[\theta_i - \alpha(S_{(t0:t1),i} - S^*) + \sum_{j=1}^{7} (\tau^{j}_{(t0:t1)} - D^*) \text{ Major }^{j}_{(t0:t1),i}] + \varepsilon_{(t0:t1),i}$
= $\theta_i^* + \varepsilon_{(t0:t1),i}$.

Thus our measure of ability θ_i^* is a constant which represents the average GPA of student i if he had study effort equal to the sample average S*and had course difficulty equal to the sample average D*.

In terms of beliefs, while student i may be uncertain at t_0 about $S_{(t0:t1),i}$, we assume that he knows $Major_{(t0:t1),i}^{j}$, j=1,...,7. Then, again letting the superscript t_0 indicate a random variable describing beliefs, if we assume that students know α and $\tau_{(t0:t1)}^{j}$, j=1,...,7 and continue to assume that $E(\epsilon_{(t0:t1),i}^{t0})=0$, the analogues to Eqs. (5) and (6) are¹²

college. In the presence of uncertainty, the simplifying assumption is appealing to the extent that students tend to resolve uncertainty by experimenting with majors sequentially. Regardless, due, in large part, to the presence of required General Studies courses in the first year, Appendix B does not find overly large differences in difficulty across majors. As a result, the general findings in this paper are not sensitive to assumptions about major, with this being exhibited most clearly by comparisons with earlier versions of the paper that did not include any information about major (S&S, 2009). In the case where there were two majors in the first semester that are most likely (i.e., there is a tie), we chose the major that had the higher probability in the second semester. In the case where there were two majors in the first semester.

¹¹Given assumptions discussed below about α , the choice of whether to normalize by S* or by some other constant (e.g., average actual study effort) has no effect on the measurement of changes in beliefs across time. As will be seen below in Eq. (9), choosing to normalize by S* may facilitate certain comparisons in the first semester by implying that, on average in the sample, the mean of the distribution describing beliefs about ability, θ_i^* , is equal to the mean of the distribution describing beliefs about ability, θ_i^* , is equal to the mean of the distribution describing beliefs about ability.

 $^{{}^{12}}E(\epsilon^{t0}_{(t0:t1),i})=0$ implies that, when reporting beliefs, students do not know, for example, how their match will be with a particular teacher or whether they might get sick at an important time. While the assumption seems reasonable for the purposes of this paper, it is unlikely that nothing is observed about $\epsilon^{t0}_{(t0:t1),i}$ when reporting beliefs. Observed study effort would contain information that could potentially help relax this assumption in a model where a belief that ability is high and a belief that $E(\epsilon)$ is high have different effects on decisions about how much to study. This might be the case if, for example, the disutility (leisure costs) of studying varies with ability. However, we find no evidence of this; answers to Question I regarding the disutility of studying do not vary with ACT scores (study amounts also do not vary with ACT).

(8)
$$\operatorname{GPA}_{(t0:t1),i}^{t0} *= \operatorname{GPA}_{(t0:t1),i}^{t0} - \alpha(\operatorname{S}_{(t0:t1),i}^{t0} - \operatorname{S}^*) + \sum_{j=1,7}^{7} (\tau_{(t0:t1)}^j - \operatorname{D}^*) \operatorname{Major}_{(t0:t1),i}^j = \theta_{i}^{t0,*} + \varepsilon_{(t0:t1),i}^{t0} \operatorname{and}_{i}^{t0}$$

(9)
$$E(\theta_{i}^{t_{0}})=E(GPA_{(t_{0}:t_{1}),i}^{t_{0}})-\alpha(E(S_{(t_{0}:t_{1}),i}^{t_{0}})-S^{*}))+\sum_{j=1}^{i}(\tau_{(t_{0}:t_{1})}^{j}-D^{*})$$
 Major $j_{(t_{0}:t_{1},i)}$.

We use an estimate of α =.36 from S&S (2008b) which achieves identification of the causal effect of studying on academic performance by taking advantage of exogenous variation in study effort created by whether a student's randomly assigned roommates brought a video game to school. The Local Average Treatment Effect estimated in S&S (2008b) is appropriate for use in Eq. (9) under the assumptions that the true effect of studying is homogenous across students, is linear in the number of hours studied, and is known by students. However, because each of these assumptions can be questioned, we also briefly discuss results which use Questions B.1 (second column) and B.2 to relax these assumptions. Throughout, we maintain the assumption that the $\tau^{i}_{(t0:t1)}$'s are known, with estimation discussed in Appendix B.

II.C.3 Survey: Beliefs about grade performance and ability at t₁

For t_1 , equations are identical to equations (4)-(9) with t_1 replacing t_0 and t_2 replacing t_1 everywhere. For example Eqs. (4-6) become

(10)
$$\text{GPA}_{(t1:t2),i} = \theta_i + \varepsilon_{(t1:t2),i}$$
 (11) $\text{GPA}_{(t1:t2),i}^{t1} = \theta_i^{t1} + \varepsilon_{(t1:t2),i}^{t1}$ (12) $\text{E}(\theta_i^{t1}) = \text{E}(\text{GPA}_{(t1:t2),i}^{t1})$

Using the same parameter θ_i to represent average GPA for both $t_0:t_1$ and $t_1:t_2$ is an abuse of notation if course difficulty or study effort varies systematically between periods. The measure of ability, θ_i^* , was introduced, in part, to deal with this issue. In order to compare ability across people, Section II.C.2 held study effort and course difficulty constant at S* and D*. Similarly, to compare ability across time for the same person, Eqs. (13-14) hold study effort and course difficulty constant at these same values with Eqs. (7) and (9) becoming:

(13)
$$GPA_{(t1:t2),i} = GPA_{(t1:t2),i} - \alpha(S_{(t1:t2),i} - S^*) + \sum_{j=1}^{7} (\tau^j_{(t1:t2)} - D^*) \text{ Major }^j_{(t1:t2),i} = \theta_i^* + \varepsilon_{(t1:t2),i}$$

(14) $E(\theta^{t1}) = E(GPA^{t1}) - \sigma(E(S^{t1})) - S^*) + \sum_{j=1}^{7} (\tau^j) - D^*) \text{ Major }^j$

(14)
$$E(\theta^{t_1}*) = E(GPA^{t_1}_{(t_1:t_2),i}) - \alpha(E(S^{t_1}_{(t_1:t_2),i}) - S^*) + \sum_{j=1}^{t} (\tau^{j}_{(t_1:t_2)} - D^*) Major^{j}_{(t_1:t_2,i)}.$$

As discussed earlier, in Section III, where we examine beliefs and updating of beliefs, t_1 represents the beginning of the second semester. In this case, we characterize $E(S^{t1}_{(t1:t2),i})$ and $E(GPA^{t1}_{(t1:t2),i})$ using Questions A.3 and A.4 and characterize Major $j_{(t1:t2,i)}$ using the answers to Question C from the beginning of the second semester. In

Section IV, where we examine drop-out, t_1 represents the end of the first year. In this case, we characterize $E(S^{t1}_{(t1:t2),i})$ and $E(GPA^{t1}_{(t1:t2),i})$ using Questions A.5 and A.6. These questions are identical to A.3 and A.4 except that A.6 asks students to "assume that the courses that you take next year are of equal difficulty to those you took this semester." Holding difficulty constant means that the term $\sum_{j=1}^{7} (\tau^{j}_{(t1:t2)} - D*)$ Major $j_{(t1:t2,i)}$ remains the same as in the second semester as characterized by answers to Question C from the beginning of the second semester. Appendix B discusses estimation of the $\tau^{j}_{(t1:t2)}$'s.

II.C.4 Survey: Other Views/Information

Question D measures i's views about how enjoyable being in school will be relative to being out of school at t_0 (enjoyability^{t0}_i) and at t_1 (enjoyability^{t1}_i). Question E measures i's health on a four point scale at t_0 (health^{t0}_i) and at t_1 (health^{t1}_i). As described in more detail in Section IV, other BPS questions are used to measure: 1) views at t_0 and t_1 about the financial returns to schooling conditional on a level of college grade performance (financial_returns^{t0}_i and financial_returns^{t1}_i) and 2) whether a parent lost a job during the period (parental_job_loss_{(t0:t1),i}).

Our proxy for how far a person is from the margin of indifference at entrance, prob_grad_i, comes from question F which elicits beliefs about the probability that the student will eventually graduate from Berea College.

II.C.5. Survey: Understanding revisions

While characterizing beliefs (II.C.1-II.C.3) may help identify who will drop out, understanding what information is used to revise beliefs may help identify contexts where misperceptions, which could influence drop-out, may exist. The Bayesian model provides guidance about types of information that might matter in the updating process. Here, we focus on the ability model because holding study effort and course difficulty constant is desirable given that the textbook version of the Bayesian model assumes that ε_{ti} represents randomness that is idiosyncratic and not observed directly by agents. Nonetheless, we stress that the textbook assumptions are too strong to be taken literally in any specification.

Because our survey questions do not identify the entire distribution describing beliefs about ability (II.C.1, II.C.2), we focus on revisions to the mean of the ability belief distribution. If θ_{i}^{t0} and $\varepsilon_{(t0:t1),i}^{t0}$ are normal,

a convenient form exists for $E(\theta_{i}^{t_{1}})$, often called the posterior mean:

(15a)
$$E(\theta_{i}^{t1}) = W_{i}^{1} \cdot E(\theta_{i}^{t0}) + W_{i}^{2} \cdot GPA_{(t0:t1),i}$$

(15b) = $E(\theta_{i}^{t0}) + W_{i}^{2} \cdot [GPA_{(t0:t1),i} - \alpha_{i}(S_{t0:t1,i} - E(S_{(t0:t1),i}^{t0})) - E(\theta_{i}^{t0})]$ where

(15c)
$$W_{i}^{1} * = \frac{Var(\varepsilon_{(t0:t1),i}^{t0})}{Var(\varepsilon_{(t0:t1),i}^{t0}) + Var(\theta_{i}^{t0})}, \quad W_{i}^{2} * = \frac{Var(\theta_{i}^{t0})}{Var(\varepsilon_{(t0:t1),i}^{t0}) + Var(\theta_{i}^{t0})}.$$

Earlier we discussed how we construct $E(\theta_{i}^{t_{0}}*), E(\theta_{i}^{t_{1}}*)$, and $GPA_{(t_{0}:t_{1}),i}*$. Additional survey questions, motivated by the following two implications of Eq. (15), are needed to explore heterogeneity in updating conditional on $E(\theta_{i}^{t_{0}}*)$ and $GPA_{(t_{0}:t_{1}),i}*$.

- Implication 1: A student should put more weight on $\text{GPA}_{(t0:t1),i}^*$ and less weight on $\text{E}(\theta_i^{t0})^*$ if he believes that his better or worse than expected performance is due to permanent factors.
- Implication 2: A student should put more weight on $\text{GPA}_{(t0:t1),i}^*$ and less weight on $\text{E}(\theta^{t0}_i^*)$ if $\text{Var}(\epsilon^{t0}_{(t0:t1),i})$ is small.

Implication 1 follows from Eq. (15b) which shows that W_i^2 *can be interpreted as the proportion of the gap between actual and expected performance that the student believes will persist into the future, or, equivalently, is due to permanent factors, after adjusting the portion that is due to more/less than expected study effort. This motivated Question G which, before the second semester, elicited perceptions about the percentage of the (GPA_{((0:11),i}-E(θ^{10}_i)) gap that should be attributed to a variety of factors, with some of these factors likely to be viewed as permanent and others likely to be viewed as transitory. Implication 2 follows immediately from Eq. (15c). This motivated question H which, at the beginning of the first semester, asked about the importance of "luck" in the determinations of grades, where we have attempted to define luck to include a wide range of transitory factors that would be contained in ε .

III. Analysis: Beliefs about grade performance and ability

In Section III we begin by describing beliefs at t_0 (III.A), and then both describe the amount of updating between t_0 and t_1 (III.B) and provide some evidence about the updating process (III.C). In Section III.D we examine the quality of updates by focusing on a group - poorly performing students - of particular interest to policymakers. As a reminder, in all of Section III we take $t_0 t_1$ to be the first semester and $t_1 t_2$ to be the second semester.

III.A. Beliefs at the time of entrance, t₀

The first panel in Column 1 of Table 1 shows the subjective probabilities from Question A.2 averaged over the 325 students in our sample. Column 2 of Table 1 shows the proportion of students in the sample whose $\text{GPA}_{(t0:t1),i}$ falls in each category. Our measure of $\text{E}(\theta^{t0}_{i})$, the mean of the distribution describing i's beliefs about his average GPA, is the person-specific mean constructed from i's response to A.2 under the assumption that the grade density is uniform within each of the grade categories. Averaging over the 325 students in the sample, we find that students are too optimistic; average $\text{E}(\theta^{t0}_{i})$ – average $\text{GPA}_{(t0:t1),i} = 3.220 - 2.879 = .34$ (Col. 1, Table 1) and the null hypothesis of no population difference is rejected (test statistic 8.05).¹³

Table 2 shows that overoptimism is focused in certain subgroups. Average $E(\theta^{10}_{i})$ – average $GPA_{(10:11),i}$ =3.124-2.464=.66 for students whose high school grade point average (HSGPA) is in the bottom third of the sample while students in the top third have an average difference of only 3.278-3.289= -.01. Male (MALE) students have average $E(\theta^{10}_{i})$ – average $GPA_{(10:11),i}$ =3.176-2.692=.48 while female students have average $E(\theta^{10}_{i})$ – average $GPA_{(10:11),i}$ =3.256-3.032=.22. Regressing $[E(\theta^{10}_{i}) - GPA_{(10:11),i}]$ on HSGPA and MALE reveals that each variable is significant with estimates (std. errors) of -.133 (.058) and .205 (.089).¹⁴ The distributions of $E(\theta^{10}_{i})$ – $GPA_{(10:11),i}$ in the bottom panel of Table 2 reveal substantial variation in $E(\theta^{10}_{i}) - GPA_{(10:11),i}$ even after stratification by HSGPA and gender.

In terms of beliefs about ability at t_0 , students are assumed to know their major that influences course difficulty for $t_0:t_1$, but are found to be somewhat overoptimistic about study effort with average $E(S^{10}_{(t0:t1),i})$ – average $\hat{S}_{(t0:t1),i} = 3.710-3.421=.28$ (Table 1). Using Eqs. (7) and (9) reveals that average $E(\theta^{t0}_{i}*)$ – average $GPA_{(t0:t1),i}*= 3.220$ – 2.983 = .24 (Table 1). Then, roughly 70% of the overoptimism about GPA at t_0 (i.e.,

¹³Under the **first interpretation** in Section II.C.1 one could also compare the full distributions shown in the two columns of Table 1. A chi square goodness-of-fit test rejects the null hypothesis that the distribution in Column 2 is obtained by sampling from the distribution in Column 1 at all traditional levels of significance (chi square statistic = 139.8 with 5 d.f.).

¹⁴There is no evidence that family income is related to initial beliefs. Arcidiacono et al. (forthcoming A) find racial differences in overoptimism.

.24/.34=.71 of the average $E(\theta^{t0}_{i})$ -average $GPA_{(t0:t1),i}$ gap of .34) is due to overoptimism about ability.¹⁵

In terms of gender differences in beliefs about ability, Table 1 (Columns 3 and 4) shows that, on average, males and females take courses of the same difficulty (-.077 vs. -.072), males expect to study half an hour less per day more than females (3.442 vs. 3.928), and males actually do study half an hour less per day than females (3.127 vs. 3.666). Because males and females are similarly overoptimistic about how much they will study, the gender difference in overoptimism about ability is similar to the earlier gender difference in overoptimism about ability is similar to the earlier gender difference in overoptimism about ability is similar to the earlier gender difference in overoptimism about grade performance, with average $E(\theta^{10}*)$ -average $GPA_{(10:11),i}*=3.272-2.901=.371$ for males and average $E(\theta^{10}*)$ -average $GPA_{(10:11),i}*=3.178-3.050=.128$ for females. Because males study about half an hour less per day than females, the average ability gap by gender (2.901-3.050=-.149) is smaller in magnitude than the average GPA gap by gender (2.692-3.032=-.340).

Question I, which four times during the year elicited the compensation that would be required to induce an additional hour of study "under the counterfactual that the additional hour of studying would not affect grades," indicates that gender differences in study amounts are related to gender differences in the disutility (leisure costs) of studying; despite the fact that males study less than females, they require, on average, 15% more compensation to study the additional hour (\$8.47 vs. \$7.40), with this difference being significant at .05. A similar conclusion comes from Question J which elicits why students did not study more in the first semester; on average, males assign 20% more weight to the categories "studying is unenjoyable" and "leisure activities are particularly enjoyable" (54.96% vs. 45.45%), with this difference being significant at .01.

Overoptimism about GPA and ability seem to translate into overoptimism about completion. While approximately 60% of entering students will graduate from Berea, Question F finds that, on average, students believe that there is an 86% chance of graduating.

III.B. The amount of updating between t_0 and t_1

Table 3 shows the subjective probabilities from Question A.4 averaged over the sample. Again focusing on

¹⁵Eq. (9) uses the value α =.36 to compute the counterfactual change in grades that would occur if a student planned to study S* instead of E(S^{t0}_{(t0:t1),i}). An alternative is to relax assumptions related to beliefs about the causal effect of studying by computing the counterfactual change using the second column of Question B.1. This does not change conclusions with average E(θ^{t0}_{i} *)=3.210.

means, we find evidence of learning; average $E(\theta^{t1}_{i})$ -average $E(\theta^{t0}_{i})=3.140-3.220=-.08$, and a test of the null hypothesis that the population difference is zero has a p-value of .0017. In terms of beliefs about ability, Column 1 of Tables 1 and 3 reveal that, on average, students have very similar views at t_0 and t_1 about how much they will study (3.710 vs. 3.664) and, on average, have very similar course difficulty the first and second semesters (-.074 vs. -.041). Note also that 65% of students do not change majors and course difficulty tends to vary insignificantly with major (Appendix B).¹⁶ Then differencing Eq. (9) from Eq. (14) shows that learning about GPA should arise primarily from learning about ability, and we find that average $E(\theta^{t1}_{i}*)$ -average $E(\theta^{t0}_{i}*) =$ 3.123 -3.220=-.09 is indeed similar to average $E(\theta^{t1}_{i})$ -average $E(\theta^{t0}_{i})$ =-.08.¹⁷

Further, these numbers, while statistically significant, mask some of the importance of updating at the individual level. For example, average $|E(\theta^{t1}_i) - E(\theta^{t0}_i)| = .288$ and the standard deviation of $E(\theta^{t1}_i) - E(\theta^{t0}_i) = .359$. We also find that average updating is larger for certain subgroups of interest. For example, students in the bottom third in a measure of grade performance that we discuss later have average $E(\theta^{t1}_i)$ -average $E(\theta^{t0}_i) = .34$, and, not surprisingly given the overoptimism of males at entrance, males have average $E(\theta^{t1}_i)$ -average $E(\theta^{t0}_i^*)$ = 3.086-3.272=-.18 while females have average $E(\theta^{t1}_i^*)$ - ave

III.C Evidence about the updating process

The first sentence of Section II.C.5 described a motivation for examining the updating process. We begin with a specification motivated by Eq. (15a) which ignores the heterogeneity in weights in Eq. (15c),

(16) $E(\theta_{i}^{t1}) = \beta_0 + \beta_1 E(\theta_{i}^{t0}) + \beta_2 GPA_{(t0:t1),i} + u_i.$

The construction of the variable $\text{GPA}_{(t0:t1),i}^*$ (Eq. 7) requires $S_{(t0:t1),i}$ which is not fully observed. Using $\hat{S}_{(t0:t1),i}$ from Section II.C.2 directly in the construction of $\text{GPA}_{(t0:t1),i}^*$ leads to an errors-in-variables problem which we address using a Maximum Likelihood (MLE) approach suggested by S&S(2004) and described in Appendix C. Column 1 of Table 4 shows MLE estimates (std. errors) for β_1^* and β_2^* of .431 (.054) and .314 (.030). The conclusion that both the "prior mean" and "noisy signal" influence the "posterior mean" remains

¹⁶The fact that 35% of people do change major groups is consistent with the large amount of uncertainty that students have about their final major at the time of entrance (S&S, 2011).

¹⁷The value of average $E(\theta_{i}^{t_{i}})$ remains largely unchanged, 3.113, when in Eq. (14) we compute the grade effect associated with the counterfactual change in study effort $E(S_{(1+2)i}^{t_{i}})$ -S* using Question B.2 instead of using α =.36.

when we examine the specification for the grade belief model that results by removing all of the *'s in Eq. (16). In this case, Column 1 of Table 5 shows OLS estimates (std. errors) for β_1 and β_2 of .396 (.051) and .245 (.019). Although not shown, results are very similar by gender; the Column 1, Table 5 estimates (std. errors) for β_1 and β_2 are .380 (.078) and .223 (.029) for males and .414 (.068) and .266 (.025) for females. For reasons related to sample size, in the remainder of Section III we do not examine gender differences.

Next, we examine why heterogeneity exists in updating conditional on $E(\theta^{t0}_{i}^{*})$ and $GPA_{(t0:t1),i}^{*}$. We look for empirical evidence of Implications 1 and 2 (Section II.C.5) which describe why heterogeneity might exist in the weights assigned to $E(\theta^{t0}_{i}^{*})$ and $GPA_{(t0:t1),i}^{*}$.

Implication 1 Eq. (15b) shows that W_i^2 *can be interpreted as the proportion of the GPA_{(10:11),i}- E(θ^{10}_i) gap that the student believes will persist into the future, or equivalently, is due to permanent factors, *after* removing the portion of the gap that arises because a person studies a different amount than expected in the first semester. Our measure, which we refer to as \hat{W}_i^2 *, is created under the assumption that Line A (ability) and Line B (preparation) in Question G tend to be viewed as persistent and Line D (luck) tends to be viewed as transitory.

(17)
$$\hat{W}_{i}^{2} * = \frac{\% \text{lineA} + \% \text{lineB}}{\% \text{lineA} + \% \text{lineB}}, \qquad \hat{W}_{i}^{1} * = 1 - \hat{W}_{i}^{2} *$$

Line C plays no explicit role because Eq. (15b) takes into account the role of different than expected study effort. For the subsample of students who did not have a percentage of 100 on line C and correctly recognized in Question G.1 whether they had performed better or worse than expected, the mean and standard deviation of \hat{W}_{i}^{2} are .688 and .408.¹⁸ For this subsample, we estimate by Maximum Likelihood

$$(18) E(\theta_{i}^{t1}) = \beta_0 * + \beta_1 * E(\theta_{i}^{t0}) + \beta_2 * GPA_{(t0:t1),i} * + \beta_3 * E(\theta_{i}^{t0}) \times \hat{W}_{i}^{t} * + \beta_4 * GPA_{(t0:t1),i} \times \hat{W}_{i}^{t} * + u_i *.$$

Consistent with Implication 1, β_3 * and β_4 * in Table 4, Column 2 are statistically significant with t-statistics of 2.46 and 2.17. They are also quantitatively important. For example, a student believing that his better (or worse)

¹⁸Two hundred forty-six of 325 students correctly identified whether they did better or worse than expected with the majority of the incorrect responses coming from individuals whose performance was very close to what they expected. Thirty-five individuals attributed 100% of the gap to study effort. These students come disproportionately (32 out of 35) from the group that performed worse than expected.

than expected performance is caused entirely by persistent factors $(\hat{W}_{i}^{2}*=1)$ would have a coefficient on $GPA_{((t0:t1),i}*$ that is 2.4 times as large as a student believing that his better (or worse) than expected performance is caused entirely by transitory factors $(\hat{W}_{i}^{2}*=0)$. Table 5, Column 2 shows similar conclusions using an OLS estimator for the grade performance belief model that results from removing all *'s from Eq. (18).

Implication 2 As discussed in Section II.C.5, we use Question H to provide the measure of VAR($\varepsilon^{t0}_{(t0:t1),i}$) needed to test Implication 2. If "luck" is roughly synonymous with ε (and under the previous assumption that $E(\varepsilon^{t0}_{(t0:t1),i})=0$), the sum of lines H.2 and H.3 would represent a person's beliefs about $Pr(\varepsilon_{(t0:t1),i}>.25|\varepsilon_{(t0:t1),i}>0)$ which should be strongly correlated with VAR($\varepsilon^{t0}_{(t0:t1),i}$). This measure has a mean (std. dev.) of .511 (.250). Referring to this measure as $\hat{\sigma}^2_{\varepsilon^i}$, the last column of Table 4 shows results from Maximum Likelihood estimation of

(19)
$$E(\theta_{i}^{t1}) = \beta_0 * + \beta_1 * E(\theta_{i}^{t0}) + \beta_2 * GPA_{(t0:t1),i} * + \beta_3 * E(\theta_{i}^{t0}) * x \hat{\sigma}_{\epsilon i}^2 * + \beta_4 * GPA_{(t0:t1),i} * x \hat{\sigma}_{\epsilon i}^2 + u_i^*.$$

Consistent with Implication 2, the estimates of β_3 * and β_4 * in Column 3 of Table 4 are significant with t-statistics of 1.78 and -2.14, respectively. Column 3 of Table 5 shows even stronger results (t-stats of 2.38 and -2.90) using an OLS estimator for the grade belief model that results from removing all *'s from Eq. (19).

We find that the null hypothesis of no relationship between $\hat{W}_{i}^{1} * and \hat{\sigma}_{\epsilon i}^{2}$ is rejected at levels greater than .007. Thus, a student attributes more of his performance to permanent factors if VAR($\epsilon_{(10:t1),i}^{t0}$) is small.

III.D Evidence about the quality of updates: an examination of poorly performing students

We now examine the quality of updates, paying close attention to the accuracy of beliefs about factors found to be of importance when examining Implication 1 in III.C. We focus on poorly performing students who have the highest risk of dropping out, and, as seen in Section III.A, tend to be particularly overoptimistic. Specifically, to take advantage of Question G, we focus on students in the bottom third of the $[GPA_{(t0:t1),i} - E(\theta^{t0}_i)]$ distribution. Nine of the 109 students in this group did not recognize in Question G.1 that their $GPA_{(t0:t1),i}$ was lower than expected, and ten students did not stay in school until the end of the year. The remaining 90 students have average $GPA_{(t0:t1),i}$ -average $E(\theta^{t0}_i)=2.06-3.26=-1.20$ and average $GPA_{(t1:t2),i}$ -average $E(\theta^{t1}_i)=2.43-2.93=-.50$. Thus, while students update substantially, a non-trivial gap remains.

In the grade performance analogs to Eqs. (15a-15b), W_{i}^{2} is the proportion of the [GPA_{(10:11),i}-E(θ_{i}^{t0})] gap

due to permanent factors. Then, the finding that average $[GPA_{(t1:t2),i}-E(\theta^{t1})]$ remains negative suggests that at t_1 students tend to understate the importance of permanent factors in determining [GPA_{(t0 t1) i}-E(θ^{t0}_i)]. In an attempt to pinpoint the source of this understatement, we examine the accuracy of the interpretations from Question G.3. Table 6 shows, on average, the percentage of the $[GPA_{(0,t1)}, -E(\theta^{t0})]$ gap students in this group believe should be attributed to worse than expected study effort, luck, and ability/preparation. We now discuss how we obtain the numbers in Table 7 which show the percentage of the average $[GPA_{(10,11)i}-E(\theta^{10})]$ gap of -1.20 that should actually be attributed to each of these three possibilities. With respect to study effort, average $E(S^{t0}_{(t0:t1)i})$ -average $\hat{S}_{(10:11),i} = 4.01 - 3.07 = .94$. Then, using the estimate of $\alpha = .36$, (.94*.36/1.20)% = 28% of the average gap should be attributed to less than expected study effort. With respect to luck, the intuition is that, under the assumption that the transitory portion of grades (ε) is independent across semesters, a group of students who, on average, have bad luck in the first semester should, on average, see a grade rebound in the second semester (after adjusting for study effort and course difficulty in the two semesters). For students in the bottom third, $\text{GPA}_{(10;11),i}$ and $\text{GPA}_{(11;12),i}$ have averages of 2.06 and 2.43, $\hat{S}_{(10;11),i}$ and $\hat{S}_{(11;12),i}$ have averages of 3.07 and 3.06, and the course difficulty measures $\sum_{i=1}^{7} \tau^{j}_{(t0:t1)}$ Major $^{j}_{(t0:t1),i}$ and $\sum_{i=1}^{7} \tau^{j}_{(t1:t2)}$ Major $^{j}_{(t1:t2),i}$ have averages of -.11 and -.05. Then, $\{[(2.43-2.06)+(3.07-3.06)*.36+(-.11+.05)]/1.20\}$ %=26% of the average gap should be attributed to worse than expected luck. Finally, the portion of the average gap that should be attributed to worse than expected ability/preparation is the residual 100%-28%-26%=46%.

The results indicate that poorly performing students take even more personal responsibility for their poor grade performance than they should; there is no evidence that students overstate the importance of bad luck (18% perceived, 26% actual) and students do overstate the importance of less than expected study effort (55% perceived, 28% actual). Put another way, the average [GPA_{(t1:t2),i}-E(θ ^{t1}_i)] gap is negative, in part, because students understate the importance of permanent ability/preparation (27% perceived, 46% actual).¹⁹ We also find that, while students recognize that worse than expected performance in t₀:t₁ is due to lower than expected study

¹⁹A conclusion that students do not tend to overstate the importance of permanent factors is quite robust. Using α =.52, the upper limit of the narrowest 95% confidence interval in S&S (2008b), the actual importance of ability/preparation is 33%, still higher than the perceived amount. Using Question B.2 at the beginning of second semester to characterize the grade effect of studying S_{(10:t1),i} instead of E(S¹⁰_{(10:t1),i}) (i.e., assuming that beliefs are correct), the actual importance is 53%.

effort, they tend to understate the extent to which low effort will be permanent; average $E(S^{t1}_{(t1:t2),i})=3.89$, but average $\hat{S}_{(t1:t2),i}=3.06$. Interestingly, these misperceptions about the extent to which poor grade performance will be permanent explain the entire average [GPA_{(t1:t2),i}- $E(\theta^{t1}_{i})$] gap of -.50.²⁰

Recent evidence that families of potential college students tend to substantially overestimate the direct costs of college has led to a concern that inaccurate information may lead to a situation where too few students choose to enter college (NCES, 2003). The results here highlight that misperceptions can also potentially increase educational attainment. From a policy standpoint, that students tend to not fully understand the role that ability/preparation plays in determining poor grade performance (i.e., they tend to understate the extent to which poor performance will be permanent) is of relevance to policymakers worried that students leave school prematurely when things go badly. It is also of relevance for policymakers concerned that scarce public resources could be consumed inefficiently if misperceptions lead students to remain in school longer than they otherwise would.

IV. Analysis: The role of learning in determining drop-out

As a reminder, for this section, t₁ represents the end of the first year so our outcome variable, dropout_i, indicates whether students enrolled at the end of the first year leave Berea before the beginning of the second year. To interpret what it means to leave Berea, we turn to post-college surveys that were sent to BPS participants and non-participants. Three years after college entrance, we observe the activity status for 81 of the 108 students who left Berea sometime during the first three semesters. At that point, only fourteen students were enrolled in a four-year school and two students were enrolled in a two-year school. Thus, while some departing students do likely eventually graduate from another school, for the large majority leaving means not obtaining a four year degree.

From the 268 person subsample described in Section II.B., we remove six students who have $\text{GPA}_{(t0:t1),i}$ <1.5, and, therefore, did not have the option of returning for the second year. For the resulting 262 person

²⁰Under the assumption that α =.36, GPA_{(t1:t2),i} would increase by (3.89-3.06)*.36=.298 if S_{(t0:t1),i} =E(S^{t1}_{(t1:t2),i}). Similarly, if the true effect of ability on average [GPA_{(t0:t1),i}-E(θ^{t0}_i)] was as in Table 6 rather than as in Table 7, GPA_{(t1:t2),i} would increase by (.457-.271)*1.20=.228. Then, under these two counterfactuals, average [GPA_{(t1:t2),i}-E(θ^{t1}_i)] would be only .02.

subsample, .126 have dropout_i=1.²¹

Eq. (2) motivates specifications in which variables related to a person's state at t_i enter as independent variables. In terms of state variables related to the accumulation of skilled human capital, theory suggests that GPA_{(10:11),i} influences E(V^{t1}_S) because it represents a stock of skilled human capital, while beliefs about grade performance, E(θ^{t1}_{i}), and beliefs about ability, E(θ^{t1}_{i} *), influence E(V^{t1}_S) because they represent beliefs about the future accumulation of skilled human capital. Consistent with the suggestion of theory, the probit estimates in Column 1 of Table 8 show that both GPA_{(10:11),i} and E(θ^{t1}_{i}) are significant predictors of drop- out (the t-statistics of the underlying coefficients are -2.55 and -2.36).²² As discussed in Section II, theory does not indicate whether E(θ^{t1}_{i}) or E(θ^{t1}_{i} *) is more relevant. Then, given the strong correlation between these two measures and given that accounting for beliefs about study effort and major may make E(θ^{t1}_{i} *) more susceptible to measurement error, we simplify the discussion by focusing, to a large extent, on E(θ^{t1}_{i}). However, consistent with the theory, Column 2 of Table 8 shows that E(θ^{t1}_{i} *) is significant when included with GPA_{((0:11),i}.²³

From the standpoint of understanding the usefulness of the expectations data employed here, one might be interested in whether the suggestion of theory above is satisfied when $E(\theta^{t1})$ is replaced with a belief variable, traditional_mean_i, constructed under the types of assumptions that are used when expectations data are not available. A common approach, often referred to as Rational Expectations (RE) in the empirical microeconomic literature, is to assume that the distribution describing a person's beliefs corresponds to the distribution of actual outcomes for people deemed by the econometrician to be "like" or "similar" to the person (typically) in observable ways (Manski, 2004; Das, Marcel and van Soest, 2000).

²¹Under the assumption that $E(\theta^{t1}_{i})$ is missing at random, our MLE estimator could be used to add in the 43 students who were in school at the end of the year but did not answer the final survey. However, because dropout_i, GPA_{(t0:t1),i} and Corr(dropout_i, GPA_{(t0:t1),i}) are similar between this group and the current 262 person subsample, doing so would not lead to much change in results. Of the 325 students, .176 left school before the start of the second year. The difference between .176 and .126 is due primarily to our removal of the six poorly performing students and the fact that some students in the full sample left school before the end of the second semester.

 $^{^{22}}$ GPA_{((0:11),i} has a mean (std. dev) of. 2.98 (.62). E(θ^{t1}_{i}), described in II.C.3, has a mean (std. dev.) of 3.13 (.34).

²³When a linear probability model is used, we find evidence of non-linear effects. However, in the non-linear Probit model there is no evidence that quadratic terms are needed. For the ability results, results are very similar when Question B.2 is used (instead of the assumption α =.36) to compute the grade effect associated with E(S^{t1}_{(11:2),i})-S*in Eq. (14).

To construct traditional_mean_i in the spirit of Rational Expectations, we start by making the assumption that individuals update in a Bayesian manner,

(18) traditional_mean_i =
$$W_i^1 \cdot E(\theta_i^0) + W_i^2 \cdot GPA_{(t0:t1),i}$$
.

Here, t_i is the end of the first year, so the noisy signal GPA_{(0,11),i} is the grade point average for the first full year. Then, what is needed to construct traditional_mean_i are values of $E(\theta^{10}_{i})$, W^1_{i} , and W^2_{i} . Under the RE assumption, $E(\theta^{00}_{i})$ can be viewed as the average first year GPA for students who have the same observable characteristics as i. Under Bayesian updating, the weights in Eq. 18 will be functions of $Var(\theta^{00}_{i})$ and $Var(\varepsilon^{10}_{(0,11),i})$.²⁴ Under the RE assumption, $Var(\theta^{00}_{i})$ can be viewed as the amount of permanent heterogeneity that exists in GPA (i.e., the amount of variation in average GPA) across students who have the same observable characteristics as i. Similarly, $Var(\varepsilon^{00}_{(0,11),i})$ can be thought of as the amount of transitory variation in GPA for students who have the same observable characteristics as i. Then, it is natural to compute $E(\theta^{00}_{i})$, W^1_{i} , and W^2_{i} , using estimates from a Random Effects model of GPA which incorporates a permanent/transitory error structure and controls for the observable characteristics that the econometrician deems relevant for determining which students are "similar." Taking advantage of semester variation in grades between the first semester and the second semester, the estimates associated with this model are shown in Table 10. $E(\theta^{00}_{i})$ is determined for person i as the predicted value from Table 10. The estimate for $Var(\theta^{00}_{i})$ of .131 and the estimate for 2·Var($\varepsilon^{10}_{((0,11),i}$) of

The estimates in Column 3 of Table 8, which use traditional_mean_i in place of the elicited mean, $E(\theta^{t1}_{i})$, provide strong evidence of the usefulness of the self-reported measures; the effects of traditional-mean_i and $GPA_{(t0:t1),i}$ are imprecisely estimated, neither is statistically significant at .10, and the estimated effect of traditional-mean_i is extremely close to zero, -.008. The lack of precision occurs because $GPA_{(t0:t1),i}$ both enters

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$$W_{i}^{1} = \frac{Var(\varepsilon_{(t0:t1),i}^{t0})}{Var(\varepsilon_{(t0:t1),i}^{t0}) + Var(\theta_{i}^{t0})}, \quad W_{i}^{2} = \frac{Var(\theta_{i}^{t0})}{Var(\varepsilon_{(t0:t1),i}^{t0}) + Var(\theta_{i}^{t0})}$$

 $^{^{25}}$ Var($\epsilon_{(t0:t1),i}^{t0}$) is the variance for the entire year. It is half as large as the variance of one semester, which is the variance estimated when the random effects estimator is used with semester data. The standard Random Effects model imposes a simplifying assumption that Var(θ_i^{t0}) and Var($\epsilon_{(t0:t1),i}^{t0}$) do not vary with i's observable characteristics.

the drop-out specification and is forced to play a central role in the construction of traditional_mean_i. The estimated effect of traditional_mean_i is extremely close to zero because, in the data, the observable variables (e.g., HSGPA) other than $\text{GPA}_{(t0:t1),i}$ that help determine traditional_mean_i (Appendix D) are found not to affect drop-out after conditioning on $\text{GPA}_{(t0:t1),i}$. Then, while estimating a model that differentiates between actual performance and expectations about future performance is potentially important for understanding drop-out, it is very difficult using standard data.

It is possible that $\text{GPA}_{((0,11),i}$ or $\text{E}(\theta^{11}_{i})$ (or $\text{E}(\theta^{11}_{i}^{*})$) are correlated with other factors that might influence drop-out. Perhaps the most prominent non-academic explanation for drop-out is that students find school to be unenjoyable because they fail to establish a strong social attachment (Tinto, 1975). For simplicity, we ignore the qualitative nature of survey Question D and treat enjoyability¹⁰_i and enjoyability¹¹_i as quantitative, continuous variables with five possible values.²⁶ Column 4 of Table 8 shows that students who find school to be unenjoyable are much more likely to drop out, with the estimated effect of enjoyability¹¹_i being significant at .003. However, enjoyability¹¹_i is strongly correlated with $\text{GPA}_{(t0:t1),i}$ (p-value .0002) and $\text{E}(\theta^{11}_{i})$ (p-value < .0001). Including enjoyability¹¹_i, $\text{GPA}_{(t0:t1),i}$ and $\text{E}(\theta^{11}_{i})$ together in Column 5 leads to a 60% decrease in the estimated marginal effect of enjoyability¹¹_i from Column 4 while leaving the estimated marginal effects of $\text{E}(\theta^{11}_{i})$ and $\text{GPA}_{(t0:t1),i}$ roughly unchanged from Column 1.²⁷ Thus, much of the drop-out of unhappy students appears to arise because these students have also received poor grades in the past and expect low future grades.²⁸

As discussed in II.C.4, the BPS also contains information about other factors that could influence a student's state at the end of the first year. In Table 8, Column 6 we add, to the specification in Column 5,

 $^{^{26}}$ At the end of the first year, the sample proportions associated with the five categories in Question D are .544 (much more enjoyable), .266 (somewhat more enjoyable), .104 (about the same enjoyability), .054 (somewhat less enjoyable), and .030 (much less enjoyable).

²⁷This conclusion is not sensitive to the manner in which we construct the enjoyability^{t1} variable. Replacing enjoyability^{t1} with a dummy variable that has a value of one if the student believes that college is much more enjoyable or somewhat more enjoyable than not being in college, the estimate (std. error) in Column 4 becomes -.140 (.063) and the estimate (std. error) in Column 5 becomes -.058 (.050).

²⁸One might wonder whether, to some extent, bad grade performance causes school to be unenjoyable or vice-versa. If unhappiness at school causes bad grade performance, one might expect this to be revealed primarily in substantial decreases in effort (relative to what was expected at the time of entrance) for those who find school to be unenjoyable. We do not find evidence of a quantitatively important decline of this type.

financial_returns^{t1}, parental_job_loss_{(10:t1),i}, and health^{t1}.²⁹ The message from Column 6 of Table 8 - that previous academic performance and beliefs about future grade performance/ability play a central role in the drop-out decision - remains strong with the estimated effects of $E(\theta^{t1})$ and $GPA_{(10:t1),i}$ being roughly unchanged.

Computing predicted probabilities using Column 6 of Table 8 can provide a sense of the prominent role of learning about grade performance. The average predicted probability of drop-out decreases by 41% (from .127 to .075) under the counterfactual assumption that no learning takes place about grade performance (i.e., that $E(\theta^{t1}_{i})=E(\theta^{t0}_{i})$ and $GPA_{(t0:t1),i}=E(\theta^{t0}_{i})$). The decrease is quite similar using the ability results in Column 7 (from .127 to .081) with the counterfactual assumptions that $E(\theta^{t1}_{i}*)=E(\theta^{t0}_{i}*)$ and $GPA_{(t0:t1),i}=E(\theta^{t0}_{i})$.

We also explore specifications that incorporate learning directly. The univariate Probit results in Column 1 of Table 9 indicate that the effect of learning about grade performance is highly related to drop-out; a test of the null hypothesis that $E(\theta^{t1})$ - $E(\theta^{t0})$ has no effect yields a t-statistic of -3.80.³⁰ As discussed in Section II.A, when using a specification in which learning enters directly, one must consider the possibility that $E(\theta^{t1})$ - $E(\theta^{t0})$ might be correlated with both the amount that a person has learned about other factors of relevance and how far the student was from the margin of indifference at the time of college entrance. Column 2 adds variables representing learning about other factors and adds our proxy for distance from the margin at entrance, prob_grad_i, which is found to be statistically significant. The estimated importance of $E(\theta^{t1})$ - $E(\theta^{t0})$ changes little from Column 1. Column 3 includes the additional learning term $GPA_{(t0:t1),i} - E(\theta^{t0})$ which is meant to take into account that a student may have learned during the first year that his GPA at the end of the year will be lower than he expected at entrance.³¹ Using Column 3, the average predicted probability of drop-out decreases by 44% (from .129 to .072) under the counterfactual assumption that no learning takes place about grade performance (i.e.,

²⁹financial_returns^{t1}_i is the difference, at the end of the first year, between a student's beliefs about the median earnings he would receive if he graduated from college with a 3.0 grade point average and the student's beliefs about the median earnings he would receive if he left school immediately. Attanasio and Kaufmann (2009) pay close attention to the relationship between returns to schooling and school attendance decisions.

 $^{{}^{30}}E(\theta^{t1}_{i})-E(\theta^{t0}_{i})$ has a mean (std. dev.) of -.103 (.342).

³¹The point is that two people with identical values of $E(\theta^{t1}_{i})-E(\theta^{t0}_{i})$ may be in different situations if one has a lower GPA at the end of the year. Roughly, the additional term is meant to capture the fact that, during the first year, students may learn about both the flow of grades after the first year and stock of grades that will be present at the end of the first year.

setting both $E(\theta^{t1}_{i})-E(\theta^{t0}_{i})$ and $GPA_{(t0:t1),i} - E(\theta^{t0}_{i})$ to zero).

In terms of gender differences, average dropout, is .172 for males and .092 for females in the sample, and the null hypothesis that there is no gender difference in drop-out is rejected at significance levels greater than .05. Of interest is how much of the gender gap of .08 can be predicted by the end-of-the-year academic variables $E(\theta^{t1})$ and $GPA_{(0,t1),i}$. Given the insignificance found for the other variables in Column 6, Table 8, we avoid complications associated with holding these other variables constant across gender by using the estimates in Column 1, Table 8. We predict an average gender difference in dropout, of .168-.095=.073, so that almost the entire gender gap is predicted by gender differences in the academic variables $E(\theta^{t1})$ and $GPA_{(0,t1),i}$ at t_1 . Under the counterfactual assumption that no learning takes place about grade performance (i.e., that $E(\theta^{t1})=E(\theta^{t0})$ and $GPA_{(0,t1),i}=E(\theta^{t0})$), the average predicted drop-out gap declines by 73% to .083-.063=.020. The fact that the drop-out rate of males is much lower under the no learning scenario suggests that overoptimism of males may be an important explanation for why poorly performing males are willing to enter. To give a sense of the importance of gender differences in effort, a rough calculation suggests that about 23% of the gender gap in drop-out would disappear if each male studied an extra .40 of an hour per day (the average gender difference in the 262 person sample) and the decrease in leisure did not directly influence drop-out.

V. Conclusion

From a survey methodological standpoint, the paper provides support for the value of directly eliciting individual beliefs. It is generally not possible to determine the quality of beliefs elicited directly using expectations questions relative to the quality of beliefs constructed under the arbitrary assumptions that become necessary when expectations data are not available. However, we find that an intuitively appealing suggestion of simple theory – that the drop-out decision at a point in time should depend on both a student's cumulative GPA and the student's beliefs about future GPA – is satisfied when we elicit beliefs directly, but is not satisfied when we use beliefs constructed under a version of Rational Expectations. More generally, the paper highlights some of the benefits of using specific economic/statistical models to closely guide the construction of survey questions. For example, by taking advantage of the elicited belief data

along with additional survey questions designed to examine why there might exist unobserved heterogeneity in updating, we are able to provide new evidence about how agents update in a real-world setting. Broadly speaking, we find that updating is broadly consistent with implications of a Bayesian framework.

The differences found between beliefs elicited directly and beliefs constructed under Rational Expectations have important implications for a variety of issues that have received attention in the higher education literature. As one example, our finding that students are, on average, substantially overoptimistic about grade performance at the time of entrance has a direct bearing on conclusions about the option value of schooling (Manski, 1989; Altonji, 1993; Cunha et al. 2005; Stange, forthcoming). The option value arises because, when uncertainty exists about academic performance at the time of entrance, students benefit from a system in which they decide sequentially (e.g., on a semester-by-semester basis) whether or not to stay in college as uncertainty is resolved.³² As such, the option value will be lower if the student is more certain at the time of college entrance that he will graduate from college. Our finding that students tend to assign very little probability to bad grade outcomes suggests that students may be too certain about graduation at the time of college entrance, a suggestion that we confirm at the end of Section III.A by looking at an independent survey question. In this case students at entrance are likely to perceive the option value to be substantially lower than what would be suggested by a Rational Expectations assumption. However, many students learn that they will have poor academic performance if they remain in school and benefit greatly from being able to make decisions sequentially. This dichotomy would be assumed away by a Rational Expectations assumption that students know the true probabilities of performing poorly.

More directly, the results contribute to a literature which highlights the importance of information, with a portion of this literature recently recognizing that outcomes in education are often best viewed as the

³²One possible definition of the option value of college attendance is the difference between the discounted expected utility of entering college under the current system in which a student decides sequentially (e.g., on a semester-by-semester basis) whether or not to remain in college and the discounted expected utility of entering college under the counterfactual in which a student who enters college must precommit to remaining in school until graduation. This definition of the option value focuses on uncertainty about ability (Stange, forthcoming). One could also include in this definition the continuation value of starting school which is present even if no uncertainty exists about ability (Heckman, Lochner, and Todd, 2006; Heckman and Navarro, 2007).

end result of a learning process. We find that approximately 40% of all drop-out should be attributed to people learning about their academic ability/performance. Policymakers are often concerned that not enough students are finishing college. However, we find that, on average, poorly performing students tend to remain too optimistic during school. Thus, correcting misperceptions during college might tend to cause students to leave earlier. We stress that earlier departures are not inherently bad in this context since students who remain in school longer than they should consume valuable (non-private) resources. Regardless, our results suggest that changing academic performance is of first order importance if the goal is to reduce drop-out. Attempting to increase effort during college is an obvious possibility. However, using time diaries we find that, while poorly performing students do somewhat overstate how much they will study, the primary source of learning during college is about academic ability (i.e., grade performance at a given level of effort). This suggests the central importance of improving preparation by the end of high school. Improvements in the quality of elementary and secondary schools would seemingly be helpful, but ensuring that pre-college students have correct perceptions about what level of preparation is necessary to succeed in college may also be important.

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	Full Sample	Full Sample	Male	Female
	n=325	n=325	n=146	n=179
	(1)	(2)	(3)	(4)
l st semester	subjective probability from A.2 ¹	sample		
GPA interval	sample avg. (std. dev.)	proportion ²		
3.5,4.0]	.401 (.256)	.230		
3.0,3.5)	.329 (.175)	.302		
2.5,3.0)	.160 (.132)	.200		
2.0,2.5)	.072 (.073)	.123		
1.0,2.0)	.025 (.035)	.108		
0.0,1.0)	.012 (.022)	.033		
	sample avg.		sample avg.	sample avg.
	(std. dev.)		(std. dev.)	(std. dev.)
$\Xi(\theta^{t0}_{i})$	3.220 (.292)		3.176 (.306)	3.256 (.276)
GPA _{(t0:t1),i}	2.879 (.784)		2.692 (.801)	3.032 (.737)
$E(S^{t0}_{(t0:t1)i})$	3.710 (1.823)		3.442 (1.911)	3.928 (1.723)
Ŝ (t0:t1),i	3.421 (1.618)		3.127 (1.669)	3.666 (1.537)
$\sum_{i=1}^{7} \tau^{j}_{(t0:t1)} Major^{j}_{(t0:t1)}$	074(.155)		077 (.152)	072 (.157)
$E(\theta_{i}^{t0}*)$	3.220 (.706)		3.272 (.756)	3.178 (.662)
GPA _{(t0:t1),i} *	2.983 (.882)		2.901 (.892)	3.050 (.870)

Note: As in all of Section III, t_1 is the beginning of the second semester so $t_0:t_1$ is the first semester.

¹Sample average subjective probability (standard deviation) at t_0 of having GPA_{(t0:t1),i} in each category from Question A.2. ² Sample proportion with GPA_{(t0:t1),i} in each category.

Table	1
-------	---

		FIRST SE	MESTER DES	CRIPTIVE ST	ATISTICS		
	Stratified by HSGPA and gender						
	Full	HSGPA	HSGPA	HSGPA	MALE	FEMALE	
	Sample	bottom	middle	top			
		third	third	third			
	(1)	(2)	(3)	(4)	(5)	(6)	
	sample	sample	sample	sample	sample	sample	
	average	average	average	average	average	average	
	(std. dev.)	(std. dev.)	(std. dev.)	(std. dev.)	(std. dev.)	(std. dev.)	
$E(\theta_{i}^{t0})$	3.220	3.124	3.246	3.278	3.176	3.256	
	(.292)	(.308)	(.260)	(.281)	(.306)	(.276)	
$\text{GPA}_{(t0:t1),i}$	2.879	2.464	2.835	3.289	2.692	3.032	
(t0:t1),1	(.784)	(.760)	(.787)	(.590)	(.801)	(.737)	
	s a m p l e average	s a m p l e average	sample average	sample average	sample average	sample average	
	(std. err.)	(std. err.)	(std. err.)	(std. err.)	(std. err.)	(std. err.)	
$E(\theta^{t0}_{i})$ - $GPA_{(t0:t1),i}$.341	.661	.410	011	.484	.224	
	(.044)	(.066)	(.078)	(.058)	(.067)	(.056)	
$E(\theta^{t0}_{i})$ - $GPA_{(t0:t1),i}$	s a m p l e proportion	sample proportion	sample proportion	sample proportion	sample proportion	sample proportion	
interval	proportion	proportion	proportion	proportion	proportion	proportion	
[1,∞)	.203	.363	.201	.066	.280	.139	
[.5,1)	.156	.202	.174	.084	.157	.156	
[0,.5)	.233	.151	.275	.273	.239	.229	
[5,0)	.280	.242	.220	.386	.232	.318	
[-1,5)	.116	.040	.128	.160	.075	.150	
(-∞,5)	.009	.000	.000	.028	.013	.005	

Note: As in all of Section III, t_1 is the beginning of the second semester so $t_0:t_1$ is the first semester. High School Grade Point Average (HSGPA) not observed for 11 of 325 students in sample.

Table 3

GPA interval A.4 ¹ sample avg. (std. dev.) [3.5,4.0] .311 (.278) [3.0,3.5) .365 (.205) [2.5,3.0) .200 (.174) [2.0,2.5) .084 (.101) [1.0,2.0) .031 (.058) [0.0,1.0) .009 (.025) sample avg. sample avg. sample avg. (std. dev.) (std. dev.) $[0.0,1.0)$.009 (.025) sample avg. sample avg. $[0.0,1.0)$.009 (.025) sample avg. (std. dev.) (std. dev.) $[0.0,1.0)$.009 (.025) Sample avg. (std. dev.) (std. dev.) $[0.0,1.0)$.009 (.025) $E(\theta^{t1}_i)$ 3.140 (.357) 3.076 (.363) 3.191 (.345) $GPA_{(t1:12),i}^2$ 2.929 (.771) 2.719 (.815) 3.098 (.694) $E(S^{t1}_{(t1:2),i})$ 3.664 (1.841) 3.615 (2.178) 3.704 (1.519) $\hat{S}_{(t1:12),i}$ 3.424 (1.578) 3.185 (1.622) 3.591 (1.467) $\hat{\Sigma}_{j=1}^{T} \vec{r}_{(t1:12),i}$ 041 (.10		Full Sample	Male	Female
2nd semester subjective probability from A.4 ¹ GPA interval A.4 ¹ sample avg. (std. dev.) [3.5,4.0] .311 (.278) [3.0,3.5) .365 (.205) [2.5,3.0) .200 (.174) [2.0,2.5) .084 (.101) [1.0,2.0) .031 (.058) [0.0,1.0) .009 (.025) sample avg. sample avg. (std. dev.) (std. dev.) $E(\theta^{t1})$ 3.140 (.357) 3.076 (.363) 3.191 (.345) $GPA_{(11:2),i}^2$ 2.929 (.771) 2.719 (.815) 3.098 (.694) $E(S^{t1}_{(11:2),i})^2$ 3.664 (1.841) 3.615 (2.178) 3.704 (1.519) $\hat{S}_{(11:2),i}$ 3.424 (1.578) 3.185 (1.622) 3.591 (1.467) $\hat{S}_{(11:2),i}$ 041 (.100) 050 (.100) 034 (.099) $E(\theta^{t1}_i *)$ 3.123 (.766) 3.086 (.862) 3.153 (.678)		n=325	n=146	n=179
GPA interval A.4 ¹ sample avg. (std. dev.) [3.5,4.0] .311 (.278) [3.0,3.5) .365 (.205) [2.5,3.0) .200 (.174) [2.0,2.5) .084 (.101) [1.0,2.0) .031 (.058) [0.0,1.0) .009 (.025) sample avg. sample avg. sample avg. (std. dev.) (std. dev.) $[0.0,1.0)$.009 (.025) sample avg. sample avg. $[0.0,1.0)$.009 (.025) sample avg. (std. dev.) (std. dev.) $[0.0,1.0)$.009 (.025) Sample avg. (std. dev.) (std. dev.) $[0.0,1.0)$.009 (.025) $E(\theta^{t1}_i)$ 3.140 (.357) 3.076 (.363) 3.191 (.345) $GPA_{(t1:12),i}^2$ 2.929 (.771) 2.719 (.815) 3.098 (.694) $E(S^{t1}_{(t1:2),i})$ 3.664 (1.841) 3.615 (2.178) 3.704 (1.519) $\hat{S}_{(t1:12),i}$ 3.424 (1.578) 3.185 (1.622) 3.591 (1.467) $\hat{\Sigma}_{j=1}^{T} \vec{r}_{(t1:12),i}$ 041 (.10		(1)	(2)	(3)
sample avg. (std. dev.) [3.5,4.0] .311 (.278) [3.0,3.5) .365 (.205) [2.5,3.0) .200 (.174) [2.0,2.5) .084 (.101) [1.0,2.0) .031 (.058) [0.0,1.0) .009 (.025) sample avg. sample avg. (std. dev.) (std. dev.) (std. dev.) $E(\theta^{t1}_i)$ 3.140 (.357) 3.076 (.363) 3.191 (.345) $GPA_{(t1:2),i}^2$ 2.929 (.771) 2.719 (.815) 3.098 (.694) $E(S_{(t1:2),i}^{t1})$ 3.664 (1.841) 3.615 (2.178) 3.704 (1.519) $\hat{S}_{(t1:2),i}$ 3.664 (1.841) 3.615 (2.178) 3.704 (1.519) $\hat{S}_{(t1:2),i}$ 041 (.100) 050 (.100) 034 (.099) $\sum_{j=1}^{7} \vec{r}_{(t1:2)}Major i_{(t1:2),i}$ 041 (.100) 050 (.100) 034 (.099) $E(\theta^{t1}_{i}^*)$ 3.123 (.766) 3.086 (.862) 3.153 (.678)	2nd semester			
$\begin{array}{cccccccccccccccccccccccccccccccccccc$	GPA interval	$A.4^1$		
$\begin{array}{cccccccccccccccccccccccccccccccccccc$		sample avg. (std. dev.)		
$\begin{array}{cccccccccccccccccccccccccccccccccccc$	[3.5,4.0]	.311 (.278)		
$ \begin{array}{cccccccccccccccccccccccccccccccccccc$	[3.0,3.5)	.365 (.205)		
$ \begin{array}{cccccccccccccccccccccccccccccccccccc$	[2.5,3.0)	.200 (.174)		
$ \begin{array}{cccccccccccccccccccccccccccccccccccc$	[2.0,2.5)	.084 (.101)		
$\begin{array}{cccccccccccccccccccccccccccccccccccc$	[1.0,2.0)	.031 (.058)		
$(std. dev.)$ $(std. dev.)$ $(std. dev.)$ $(std. dev.)$ $E(\theta^{t1}_i)$ $3.140 (.357)$ $3.076 (.363)$ $3.191 (.345)$ $GPA_{(t1:t2),i}^2$ $2.929 (.771)$ $2.719 (.815)$ $3.098 (.694)$ $E(S^{t1}_{(t1:t2),i})$ $3.664 (1.841)$ $3.615 (2.178)$ $3.704 (1.519)$ $\hat{S}_{(t1:2),i}$ $3.424 (1.578)$ $3.185 (1.622)$ $3.591 (1.467)$ $\sum_{j=1}^7 \tau^j_{(t1:t2)}$ Major $^j_{(t1:t2),i}$ $041 (.100)$ $050 (.100)$ $034 (.099)$ $E(\theta^{t1}_{i}^*)$ $3.123 (.766)$ $3.086 (.862)$ $3.153 (.678)$ $2.902 (.810)$ $2.850 (.856)$ $2.000 (.756)$	[0.0,1.0)	.009 (.025)		
E(θ^{t1}_{i})3.140 (.357)3.076 (.363)3.191 (.345)GPA_{(t1:t2),i}^{2}2.929 (.771)2.719 (.815)3.098 (.694)E($S^{t1}_{(t1:t2),i}$)3.664 (1.841)3.615 (2.178)3.704 (1.519) $\hat{S}_{(t1:t2),i}$ 3.424 (1.578)3.185 (1.622)3.591 (1.467) $\sum_{j=1}^{7} \tau^{j}_{(t1:t2)}$ Major $^{j}_{(t1:t2),i}$ 041 (.100)050 (.100)034 (.099)E($\theta^{t1}_{i}^{*}$)3.123 (.766)3.086 (.862)3.153 (.678)2.002 (.810)2.850 (.856)2.000 (.756)		sample avg.	sample avg.	
$GPA_{(t1:t2),i}^{2}$ 2.929 (.771)2.719 (.815)3.098 (.694) $E(S^{t1}_{(t1:t2),i})$ 3.664 (1.841)3.615 (2.178)3.704 (1.519) $\hat{S}_{(t1:t2),i}$ 3.424 (1.578)3.185 (1.622)3.591 (1.467) $\sum_{j=1}^{7} \tau^{j}_{(t1:t2)}$ Major $^{j}_{(t1:t2),i}$ 041 (.100)050 (.100)034 (.099) $E(\theta^{t1}_{i}^{*})$ 3.123 (.766)3.086 (.862)3.153 (.678) 2.092 (.810)2.859 (.856)2.009 (.756)		(std. dev.)	(std. dev.)	(std. dev.)
$\begin{array}{cccccccccccccccccccccccccccccccccccc$	$E(\theta^{t1}_{i})$	3.140 (.357)		
$\sum_{j=1}^{7} \tau^{j}_{(t1:t2)} \text{Major }^{j}_{(t1:t2),i} =041 (.100) =050 (.100) =034 (.099)$ $E(\theta^{t1}_{i}^{*}) = 3.123 (.766) = 3.086 (.862) = 3.153 (.678) = 2.000 (.756) = 2.000 (.756) = 2.000 (.756) = 2.000 (.756) = 2.000 (.756) = 2.000 (.756) = 2.000 (.756) = 2.000 (.756) = 2.000 (.756) = 2.000 (.756) = 2.000 (.756) = 2.000 (.756) = 2.000 (.756) = 2.000 (.756) = 2.000 (.756) = 2.000 (.756) = 2.000 (.756) = 2.000 (.756) = 2.000 (.756) = 2.000 (.756) = 2.000 (.756) = 2.000 (.756) = 2.000 (.756) = 2.000 (.756) = 2.000 (.756) = 2.000 (.756) = 2.000 (.756) = 2.000 (.756) = 2.000 (.756) = 2.000 (.756) = 2.000 (.756) = 2.000 (.756) = 2.000 (.756) = 2.000 (.756) = 2.000 (.756) = 2.000 (.756) = 2.000 (.756) = 2.000 (.756) = 2.000 (.756) = 2.000 (.756) = 2.000 (.756) = 2.000 (.756) = 2.000 (.756) = 2.000 (.756) = 2.000 (.756) = 2.000 (.756) = 2.000 (.756) = 2.000 (.756) = 2.000 (.756) = 2.000 (.756) = 2.000 (.756) = 2.000 (.756) = 2.000 (.756) = 2.000 (.756) = 2.000 (.756) = 2.000 (.756) = 2.000 (.756) = 2.000 (.756) = 2.000 (.756) = 2.000 (.756) = 2.000 (.756) = 2.000 (.756) = 2.000 (.756) = 2.000 (.756) = 2.000 (.756) = 2.000 (.756) = 2.000 (.756) = 2.000 (.756) = 2.000 (.756) = 2.000 (.756) = 2.000 (.756) = 2.000 (.756) = 2.000 (.756) = 2.000 (.756) = 2.000 (.756) = 2.000 (.756) = 2.000 (.756) = 2.000 (.756) = 2.000 (.756) = 2.000 (.756) = 2.000 (.756) = 2.000 (.756) = 2.000 (.756) = 2.000 (.756) = 2.000 (.756) = 2.000 (.756) = 2.000 (.756) = 2.000 (.756) = 2.000 (.756) = 2.000 (.756) = 2.000 (.756) = 2.000 (.756) = 2.000 (.756) = 2.000 (.756) = 2.000 (.756) = 2.000 (.756) = 2.000 (.756) = 2.000 (.756) = 2.000 (.756) = 2.000 (.756) = 2.000 (.756) = 2.000 (.756) = 2.000 (.756) = 2.000 (.756) = 2.000 (.756) = 2.000 (.756) = 2.000 (.756) = 2.000 (.756) = 2.000 (.756) = 2.000 (.756) = 2.000 (.756) = 2.000 (.756) = 2.000 (.756) = 2.000 (.756) = 2.000 (.756) = 2.000 (.756) = 2.000 (.756) = 2.000 (.756) = 2.000 (.756) = 2.000 (.756) = 2.000 (.756) = 2.000 (.756) = 2.000 (.756) = 2.000 (.756) = 2.000 (.756) = 2.0$	GPA _{(t1:t2),i} ²	2.929 (.771)	2.719 (.815)	3.098 (.694)
$\sum_{j=1}^{7} \tau^{j}_{(t1:t2)} \text{Major }^{j}_{(t1:t2),i} \qquad041 (.100) \qquad050 (.100) \qquad034 (.099)$ $E(\theta^{t1}_{i}^{*}) \qquad 3.123 (.766) \qquad 3.086 (.862) \qquad 3.153 (.678) \\ 2.092 (.810) \qquad 2.850 (.856) \qquad 2.000 (.756)$	$E(S^{t1}_{(t1:t2)};)$	3.664 (1.841)	3.615 (2.178)	3.704 (1.519)
$E(\theta_{i}^{t1}*) \qquad 3.123 (.766) \qquad 3.086 (.862) \qquad 3.153 (.678) \\ 2.092 (.810) \qquad 2.859 (.856) \qquad 2.009 (.756) \\ \end{array}$	$\hat{\mathbf{S}}_{(t1:t2),i}$	`		3.591 (1.467)
2,002(810) $2,850(856)$ $2,000(756)$	$\sum_{j=1}^{7} \tau^{j}_{(t1:t2)} \text{Major }^{j}_{(t1:t2),i}$	041 (.100)	050 (.100)	034 (.099)
2,002(810) $2,850(856)$ $2,000(756)$	$\Gamma(0^{tl})$	2 102 (7(()	2.09(.09(2))	2152(770)
	$E(\theta_{i}^{*})$ GPA _{(t1:t2),i} * ²	2.992 (.810)	2.859 (.856)	3.099 (.756)

Note: As in all of Section III, t_1 is beginning of the second semester so $t_1:t_2$ is the second semester. ¹Sample average subjective probability (standard deviation) at t_1 of having GPA_{(t1:t2),i} in each category from Question A.4.

²Actual GPA is not observed for individuals in the sample who left school during the second semester.

DETERMINANTS OF $E(\theta^{t1})$ estimate estimate estimate **Independent Variable** (std. error) (std. error) (std. error) (1)(2) (3) Constant .876 (.190)** .528 (.232)** .615 (.253)** $E(\theta_{i}^{t0} *)$.431 (.054)** .483 (.070)** .436 (.119)** GPA(t0:t1),i .314 (.030)** .150 (.087)* .412 (.104)** $E(\theta_{i}^{t0}) \hat{W}^{1}$.195 (.079)** $GPA_{(t0:t1),i} * x \hat{W}_{i}^{2} *$.215 (.098)** $E(\theta^{t0}_{i}^{*})x\hat{\sigma}^{2}_{\epsilon i}$ $GPA_{(t0:t1),i}^{*}x\hat{\sigma}^{2}_{\epsilon i}$.198 (.111)* -.265 (.124)** Study equation (Appendix C.2) С 3.408 (.102)** 3.526 (.128)** 3.530 (.126)** $\overset{\bar{\sigma}^2}{\sigma^2_{\nu}}$ 1.325 (.073)** 1.273 (.1022)** 1.264 (.102)** 1.569 (.035)** 1.609 (.044)** 1.611 (.044)** -2034.560 -1331.560 -1321.005 Log Likelihood

As in all of Section III, t_1 is the beginning of the second semester.

Estimation of Eqs. (16), (18), and (19) by Maximum Likelihood as described in Appendix C with dependent variable $E(\theta^{tl}_{i}^{*})$. Column (1) uses all observations. Columns (2) and (3) use students who correctly recognized on Question G (Appendix A) whether they performed better or worse than expected in the first semester and did not have a percentage of 100 on line C of Question G.2 (or G.3).

*significant at .10

**significant at .05

DETERMINANTS OF $E(\theta^{t1})$ estimate estimate estimate **Independent Variable** (std. error) (std. error) (std. error) n=325 n=211 n=211 (2)(1) (3) 1.158 (.166)** 1.181 (.223)** 1.352 (.223)** Constant $E(\theta^{t0}_{i})$.396 (.051)** .362 (.069)** .221 (.089)** GPA_{(t0:t1),i} .245 (.019)** .387 (.057)** .169 (.045)** $E(\theta^{t0}) \times \hat{W}$.083 (.048)* GPA(t0:t1),ix W .101 (.055)* $E(\theta_{i}^{t0}) x \hat{\sigma}_{\epsilon i}^{2}$.225 (.094)** $GPA_{(t0:t1),i} \overset{\delta}{x} \hat{\sigma}^{2}_{ci}$ -.298 (.102)** $R^2 = .445$ $R^2 = .445$ $R^2 = .466$

As in all of Section III, t_1 is the beginning of the second semester.

Estimation of the grade performance belief models that result from removing all *'s in Eqs. (16),

(18), and (19) by OLS with dependent variable $E(\theta^{t1})$. Column (1) uses all observations.

Columns (2) and (3) use students who correctly recognized on Question G (Appendix A)

whether they performed better or worse than expected in the first semester and did not have a

percentage of 100 on line C of Question G.2 (or G.3).

*significant at .10

**significant at .05

	sample average	
	(standard deviation)	
a+b) Worse than expected ability/preparation	27.1% (30.5)	
c) Lower than expected study effort	55.2% (35.1)	
d) Worse than expected luck	17.7% (27.2)	

Self-reported beliefs (percentages) about the importance of factors in Question G.3

Students who have $\text{GPA}_{(t0:t1),i}$ -E(θ^{t0}_{i}) in bottom third, recognized on Question G.1 that grades were lower than expected and had legitimate values of $\text{GPA}_{(t1:t2),i}$ (n=90).

Table 7

Estimates of actual importance of factors in Question G.3

Independent Variable	mean
independent variable	(standard deviation)
a+b) Worse than expected ability/preparation	45.7%
c) Lower than expected study effort	28.2%
d) Worse than expected luck	26.1%

Students who have $\text{GPA}_{(t0:t1),i}$ -E(θ^{t0}_{i}) in bottom third, recognized on Question G.1 that grades received were lower than expected and had legitimate values of $\text{GPA}_{(t1:t2),i}$ (n=90).

Derivativ	es of dropou	t probability	vevaluated a	t the mean o	f the indepen	dent variables	
Independent Variable	estimate	estimate	estimate	estimate	estimate	estimate	estimate
independent variable	(std. error)	(std. error)	(std. error)	(std. error)	(std. error)	(std. error)	(std. error)
	n=262	n=262	n=245	n=261	n=261	n=260	n=259
	(1)	(2)	(3)	(4)	(5)	(6)	(7)
GPA _{(t0:t1),i}	092** (.035)	125** (.030)	149 (.104)		084** (.035)	089** (.035)	110** (.305)
$E(\theta^{t1}_{i})$	152** (.063)	(.050)	()		145** (.064)	127** (.066)	(.303)
$E(\theta^{t_1}*)$	()	065** (.031)				~ ,	063** (.031)**
traditional-mean _i			008 (.149)				
enjoyability ^{t1} _i				.052** (.017)	.021 (.016)	.016 (.015)	.022 (.015)
health ^{t1} _i						015 (.028)	020 (.027)
financial_returns ^{t1} _i						029	041
$parental_job_loss_{(t0:t1),i}$						(.038) .063	(.038) .048
Pseudo R ²	.175	.171	.132	.045	.188	(.066) .200	(.062) .201

DETERMINANTS OF dropo	out _i (state at end of year), Probit:
-----------------------	--------------------------------------------------

As in all of Section III, t_1 is the end of the first year.

Explanatory variables measure the person's state at the end of the first year. Column (3) has less observations due to missing values of HSGPA which are needed to compute traditional-mean_i. The smaller number of observations in Columns (4)-(6) arise because of one missing value of enjoyability^{t1} and one missing value of financial_returns^{t1}_i.

*underlying coefficient in Probit significant at .10

**underlying coefficient in Probit significant at .05

	ability evaluated at the mean of t dent variables	he	
Independent Variable	estimate	estimate	estimate
independent variable	(std. error)	(std. error)	(std. error)
	n=259	n=248	n=248
	(1)	(2)	(3)
prob_grad _i		197**	174*
proo_graa		(.098)	(.096)
$E(\theta^{t_1}) - E(\theta^{t_0})$	227**	224**	096
	(.056)	(.057)	(.074)
enjoyability ^{t1} _i -enjoyability ^{t0} _i		.024	.016
$\text{GPA}_{(t0:t1),i}$ - $\text{E}(\theta^{t0}_{i})$		(.016)	(.016) 098**
(0.1.),			(.040)
health ^{$t1$} - health ^{$t0$}		039 (.030)	040
financial_returns ^{t1} i		003 (.033)	(.030) 008
-financial_returns ^{t0}		. ,	(.032)
parental_job_loss _{(t0:t1),i}		.083	.090
		(.071)	(.071)
Pseudo R ²	.081	.131	.163

DETERMINANTS of dropout_i (learning during the year), Probit

As in all of Section III, t₁ is the beginning of the second semester *Underlying coefficient in Probit significant at .10 **Underlying coefficient in Probit significant at .05

estimate std. error n=245 Constant .217 (.289) ** MALE_i -.099 (.067) HSGPA_i .438 (.075)** ACT_i .056 (.009)** variance of permanent component, Var(θ^{t0}_i) 0.131 variance of transitory component, 2 ·Var($\epsilon^{t0}_{(t0:t1),i}$) 0.226

Random Effects Estimation of GPA

Table contains estimates from a Random Effects Estimation of grade performance in the first and second semesters. *significant at .10

**significant at .05

Appendix A: Survey Questions

Note: At the start of each question, we describe when the question was answered. **Beginning of the first year** is the time immediately before the start of classes in the first year. **Beginning of the second semester** is a time immediately before the start of classes in the second semester of the first year. **End of first year** is the end of the second semester.

BELIEFS ABOUT GRADES AND STUDY EFFORT: (Beginning of first year)

Question A.1. During your first year of college, how many hours do you expect to spend in the following activities on an average weekday (Monday-Friday).

<u>Activity</u>

Avg Weekday hours

Question A.2. We realize that you do not know exactly how well you will do in classes. However, we would like to have you describe your beliefs about the grade point average that you expect to receive in the first semester.

Given the amount of study-time you indicated in question A.1, please tell us the percent chance that your grade point average will be in each of the following intervals. That is, for each interval, write the number of chances out of 100 that your final grade point average will be in that interval.

Note: The numbers on the six lines must add up to 100.

[3.5, 4.00]	
[3.0, 3.49]	
[2.5, 2.99]	
[2.0, 2.49]	
[1.0, 1.99]	
[0.0, .99]	

Note: A=4.0, B=3.0, C=2.0, D=1.0, F=0.0

BELIEFS ABOUT GRADES AND STUDY EFFORT: (Beginning of second semester)

Question A.3 Identical to Question A.1 except that questions asks about hours "during the second semester."

Question A.4 Identical to Question A.2 except that questions asks about grades "in the second semester."

BELIEFS ABOUT GRADES AND STUDY EFFORT: (End of first year)

Question A.5 Identical to Question A.1 except that questions asks about hours "during the second year."

Question A.6 Identical to Question A.2 except that questions asks about grades "in the second year" assuming "that the courses that you take next year are of equal difficulty to those you took this semester."

BELIEFS ABOUT THE IMPORTANCE OF STUDY EFFORT: (Beginning of first year)

Question B.1 For each of the following possible amounts that you might study this semester, write down the percent chance that you will study that amount and the grade point average you expect to receive if you study that amount.

Number of Study Hours a Day	Percent Chance	Expected Grade Point Average
0 hours a day		
1 hour a day		
2 hours a day		
3 hours a day		
4 hours a day		
5 hours a day		
6 or more hours a day		

BELIEFS ABOUT THE IMPORTANCE OF STUDY EFFORT: (Beginning of second semester and end of first year)

Question B.2 For each of the following possible amounts that you might study this semester, write down the grade point average you expect to receive if you study that amount.

Number of Study Hours a Day	Expected Grade Point Average
0 hours a day	
1 hour a day	
2 hours a day	
3 hours a day	
4 hours a day	
5 hours a day	
6 or more hours a day	

BELIEFS ABOUT MAJOR: (Beginning of first year and beginning of second semester)

Question C. We realize that you may not be sure exactly what area of study you will choose at Berea College. In the first column below are listed possible areas of study. Please write down the percent chance that you will end up with each of these areas of study.

Humanities include Art, English, Foreign Languages, History, Music, Philosophy, Religion and Theatre.
Natural Science and Math includes Biology, Chemistry, Computer Science, Physics and Mathematics.
Professional Programs include Industrial Arts, Industrial Technology, Child Development, Dietetics, Home Economics, Nutrition, and Nursing.

Social Sciences include Economics, Political Science, Psychology and Sociology.

Area of study	Percent chance (Number of chances out of 100)
1.Agriculture and Physical Education	
2.Business	
3.Elementary Education	
4.Humanities	
5.Natural Sciences and Math	
6. Professional Programs	
7. Social Sciences	

Question D Circle the one answer that describes your beliefs at this time: (Beginning of first year, beginning of second semester, end of first year)

- 1. I believe that being in college at Berea will be much more enjoyable than not being in college.
- 2. I believe that being in college at Berea will be somewhat more enjoyable than not being in college.
- 3. I believe that I will enjoy being in college at Berea about the same amount as I would enjoy not being in college.
- 4. I believe that being in college at Berea will be somewhat less enjoyable than not being in college.
- 5. I believe that being in college at Berea will be much less enjoyable than not being in college.

Question E. How would you rate your current health? Poor Fair Good Excellent (Beginning of first year, beginning of second semester, end of first year)

Question F.What is the percent chance that you will eventually graduate from Berea College?_____. (Beginning of first year, end of first year)

Question G.1. (Beginning of second semester)

Circle the one that is true

a). I received grades in the Fall term that were higher than I had expected to get when I came to Berea.

b). I received grades in the Fall term that were lower than I had expected to get when I came to Berea.

If you circled a), GO TO Question G.2 below. If you circled b), GO TO Question G.3 below.

Question G.2. (Answer this question if you circled that grades better than expected in Question G.1.)

Please circle those reasons why you think you received grades in Fall term that were higher that you had expected.

		Percent
A)	My ability is better than I thought it was when I came to Berea.	
B)	I am better prepared for Berea College than I thought I was when I came to Berea.	
C)	I studied harder than I had expected I would when I came to Berea .	
D)	I had better luck than I expected when I came to Berea in that those things that	
	influence grades but were out of my control turned out to be very much in my favor.	

Now consider the difference between the grades you received in Fall term and the grades you had expected. On the **lines to the right of the reasons**, write the percentage of this difference that you would attribute to each of the reasons you circled. (The items you did not circle should have zero percentage or be left blank.) **Note: The numbers on the lines should add to 100.**

Question G.3. (Answer this question if you circled that grades are worse than expected in Question G.1)

Circle those reasons why you think you received grades in Fall term that were lower than what that you had expected.

		Percent
A)	My ability is not as good as I thought it was when I came to Berea.	
B)	I am not as well prepared for Berea College as I thought I was when I came to Berea	l
C)	I did not study as hard as I thought I would when I came to Berea.	
D)	I had worse luck than I expected when I came to Berea in that those things that	
	influence grades but were out of my control turned out to be hurting my grades.	

Now consider the difference between the grades you received in Fall term and the grades you had expected. On the **lines to the right of the reasons**, write the percentage of this difference that you would attribute to each of the reasons you circled. (The items you did not circle should have zero percentage or be left blank.) **Note: The numbers on the lines should add to 100.**

Question H. (Beginning of year)

Your grades are influenced by your academic ability/preparation and how much you decide to study. However, your grades may also be influenced to some extent by good or bad luck which may vary from term to term and may be out of your control. Examples of "luck" may include 1) The quality of the teachers you happen to get and how hard or easy they grade; 2) Whether you happened to get sick (or didn't get sick) before important exams; 3) Whether a noisy dorm kept you from sleeping before an important exam; 4) Whether you happened to study the wrong material for exams; 5) Whether unexpected personal problems or problems with your friends and family made it hard to concentrate on classes.

We would like to know how important you think "luck" is in determining your grades in a particular semester. We'll have you make comparisons relative to a semester in which you have "average" luck. Average luck means that a usual number of things go right and wrong during the semester. Assume you took classes at Berea for many semesters.

GOOD LUCK IN A TERM MEANS THAT YOU HAVE BETTER THAN AVERAGE LUCK IN THAT TERM

Assume for this section that you are in a semester in which you have good luck

H.1 In what percentage of semesters that you have good luck would good luck raise your grade point average (GPA) by between 0.00 points and 0.25 points compared to a semester in which you received "average" luck.

Note. (If you are taking four courses, good luck would raise your GPA by 0.25 points if good luck led to a full letter grade increase in one of your courses).

H.2 In what percentage of semesters that you have good luck would good luck raise your grade point average (GPA) by between 0.26 points and 0.50 points compared to a semester in which you received "average" luck.

Note: (If you are taking four courses, good luck would raise your GPA by .50 points if good luck led to a full letter grade increase in two of your courses or a two letter grade increase in one of your courses.)

H.3 In what percentage of semesters that you have good luck would good luck raise your grade point average (GPA) by 0.51 or more points compared to a semester in which you received "average" luck.

Note: (For a student taking four courses, this would mean that good luck would lead to a full letter grade increase in three or more courses.)

The numbers in the three spaces above in the good luck section should add up to 100 (because if you are in a semester where you have good luck, good luck must increase your grades by between 0 and .25 points, or by between .25 and .5 points, or by more than .5 points).

Question I. (Four times during the year)

We are interested in how much you enjoy or do not enjoy studying. For this question, we want you to **ignore the fact that one of the reasons you may study is to increase your grades.** Suppose for this question that studying the extra hour below **WOULD HAVE NO EFFECT ON YOUR GRADES.**

Think about your schedule today. If someone was willing to pay you exactly 1.00 to study one extra hour today and have one less hour of leisure today (and studying this hour would have no effect on your grades) would you be willing to do this? YES NO

Think about your schedule today. If someone was willing to pay you exactly \$3.00 to study one extra hour today and have one less hour of leisure today (and studying this hour would have no effect on your grades) would you be willing to do this? YES NO

Think about your schedule today. If someone was willing to pay you exactly \$5.00 to study one extra hour today and have one less hour of leisure today (and studying this hour would have no effect on your grades) would you be willing to do this? YES NO

Think about your schedule today. If someone was willing to pay you exactly \$7.00 to study one extra hour today and have one less hour of leisure today (and studying this hour would have no effect on your grades) would you be willing to do this? YES NO

Think about your schedule today. If someone was willing to pay you exactly \$10.00 to study one extra hour today and have one less hour of leisure today (and studying this hour would have no effect on your grades) would you be willing to do this? YES NO

Think about your schedule today. If someone was willing to pay you exactly 15.00 to study one extra hour today and have one less hour of leisure today (and studying this hour would have no effect on your grades) would you be willing to do this? YES NO

Think about your schedule today. If someone was willing to pay you exactly 20.00 to study one extra hour today and have one less hour of leisure today (and studying this hour would have no effect on your grades) would you be willing to do this? YES NO

Question J. (Beginning of second semester)

Why didn't you study more in the Fall term?

Circle all the items that are reasons why you did not study more in the Fall term. Please read carefully. Note: You will use the lines when completing the question that follows below.

- A. I find studying to be unenjoyable.
- B. I found available leisure/recreational activities to be particularly enjoyable.

C.	Given how much I am studying, additional studying would have necessarily cut
	directly into essential activities such as sleep.

D.	Additional	studying woul	d not raise my	grades or only	v raise mv	grades a little.

- E. Additional studying would have raised my grades significantly, but I did not believe that this would affect the type of job I would get in the future.
- F. Additional studying would have raised my grades significantly, but I was interested only in getting the grades necessary to graduate.
- G. Learning is important for me for reasons other than grades and I did not think I could learn substantially more by studying more.
- H. Much of my potential study time was spent dealing with problems such as personal problems, health problems, or family problems.

Each item you circled in Question J is part of the total reason why you did not study more in the Fall term. Put on the line to the right of each part you circled, the percentage of the total reason that you would attribute to the circled item. The numbers you write will represent the relative importance of the various reasons in your decision not to study more. The items you did not circle should have a zero percentage or be left blank.

Appendix B. Course Difficulty

Major^j_{(t0:t1),i}, j=1,...,7 is equal to one if at t_0 person i indicates on Question C that he is most likely to end up with a major in major group j (Question C, Appendix A). Major^j_{(t1:t2),i}, j=1,...,7 is equal to one if at t_1 person i indicates on Question C that he is most likely to end up with a major in major group j (Question C, Appendix A). The proportion of students who have each major in the first semester and the proportion of students who have each major in the second semester is shown in Column 1 of Table B.

To estimate the $\tau_{(10:11)}^{i}$'s (the effects of different majors on GPA in the first semester) and the $\tau_{(11:12)}^{i}$'s (the effects of different majors on GPA in the second semester), we pool GPA observations from the first two semesters and estimate a regression of GPA in a semester on i's major in that semester, the amount i studied in that semester, HSGPA, ACT, and MALE. Major⁴_{(10:11),i1} (Humanities in the first semester) is chosen as the omitted category. Then, the estimates of $\tau_{(10:11)}^{i}$, j=1,2,3,5,6,7 represent the grade difficulty of the other majors in the first semester relative to the grade difficulty of Humanities in the first semester and the estimates of $\tau_{(11:12)}^{i}$, j=1,...,7 represent the grade difficulty of the majors in the second semester relative to the grade difficulty of the majors in the second semester relative to the grade difficulty of Humanities in the first semester.

The results are shown in Column 2 of Table B. Consistent with the fact that students have limited flexibility over courses in the first year due to the fact that roughly two-thirds of first-year courses are required under the school's General Studies Curriculum, we find that only two of the thirteen estimates are statistically significant.³³ Further, D*, the sample average of $\sum_{j=1}^{7} \tau_{(t0:t1)}^{j}$ Major $_{(t0:t1,i)}^{j}$, is -.074 and the average difficulty of courses in the second semester, as given by the sample average of $\sum_{j=1}^{7} \tau_{(t1:t2)}^{j}$ Major $_{(t1:t2,i)}^{j}$, is -.041. Thus, on average, grade difficulty of courses is almost identical in the two semesters.

³³Some classes are actually mandatory. For other classes students have a small amount of choice, within a set of classes that are likely to be very similar in nature and difficulty.

Table	B
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	Col	lege major		
Variable [Coe	fficient name]	Descriptive Statistics	OLS Dependent variable GPA	
		sample	n=601	
		proportion	estimate (std. error)	
		(1)	(2)	
Constant			232 (.293)	
Major ¹ (t0:t1),i	agriculture/physical ed	0.095	.052 (.179)	
$Major^{2}_{(t0:t1),i}$	business	0.184	.019 (.147)	
Major _{(t0:t1),i}	elementary education	0.080	.209 (.189)	
Major ⁴ (t0:t1) i1	humanities	0.221	Omitted Category	
$\begin{array}{c} \text{Major}^{4}_{(t0:t1),i1}\\ \text{Major}^{5}_{(t0:t1),i} \end{array}$	science	0.212	259 (.139)*	
$Major_{(t0:t1),i}^{6}$	professional	0.126	291 (.161)*	
Major ⁷ (t0:t1),i	social sciences	0.080	105 (.103)	
Major ¹ (t1:t2),i	agriculture/physical ed	0.082	108 (.200)	
Major ² (t1:t2),i	business	0.200	173 (.144)	
Major _{(t1:t2),i}	elementary education	0.083	020 (.183)	
Major ⁴ $(t_1:t_2)_i$	humanities	0.218	.026 (.139)	
Major ⁵ ((1)(2)) i	science	0.172	021 (.148)	
$Major_{(t1:t2),i}^{6}$	professional	0.138	114 (.159)	
Major ⁷ (t1:t2),i	social sciences	0.107	.158 (.179)	
HSGPA			.135 (.044)**	
ACT			.066 (.009)**	
MALE			020 (.070)	
Ŝ			0.36	

Appendix C: Adding study effort to the textbook model Defining N(i) to be the number of time diaries (out of four) that person i completed in the first semester and letting Study_{ji}, j=1,..., N(i) represent the N(i) observed daily study amounts for person i, a noisy proxy $\hat{S}_{(t0:t1),i}$ of $S_{(t0:t1),i}$ can be constructed as: (C.1) $\hat{S}_{(t0:t1),i} = \frac{1}{N(i)} \sum_{j=1}^{N(i)} Study_{ji}$.

The MLE approach deals with the measurement error issue under the assumption that $Study_{ji}$ is given by the permanent/transitory process

(C.2) Study_{ji} =
$$\mu_i + \nu_{ji}$$
.

The permanent component μ_i represents the average amount that person i studies per day and the transitory component ν_{ji} represents a daily deviation from this average amount. We assume that in the population $\mu_i \sim N(C, \sigma^2_{\mu})$. We assume that ν_{ji} is independent across both j and i and that $\nu_{ji} \sim N(0, \sigma^2_{\nu})$.

Intuitively speaking, if we knew the value of μ_i for each person and the distribution of v_{ji} , we could integrate out the effect of the missing information in any outcome equation of interest. Our MLE takes into account that, while we do not know the value of μ_i for each person i, the observed values of Study_{ji}, when viewed through equation (C.2), provide evidence about the likelihood of different values of μ_i . More specifically, analogous to the MLE's derived in the missing data literature, the likelihood contribution for person i, L_i , is the joint probability of $E(\theta^{t1})$ and each of the observed study amounts. Under our permanent/transitory assumption in equation (C.2), each of the daily study amounts and $E(\theta^{t1})$ are independent conditional on μ_i ,

 $L_{i} = \int g_{1}(Study_{1i}|\mu_{i})\cdots g_{1}(Study_{N(i)i}|\mu_{i}) g_{2}(E(\theta^{t^{\prime}}) |\mu_{i}) h(\mu_{i}) d\mu_{i}$

where the g's and h are density functions. Assuming normality for all densities of relevance allows L_i to be computed.