Military Moral Hazard and the Fate of Empires*

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Abstract

To understand why authority was delegated to the military in some empires and centralized in others, we construct a model where the military may revolt, the civilians may shirk, and the social planner chooses how much power to delegate to the military, which enables the military to defend, but also to usurp, the empire. From the parameters for an empire a unit-free index is derived which captures the belligerence and relative wealthiness of the peripheral peoples and determines the empire's socially optimal level of military delegation. Comparative statics of the optimum with respect to the index is consistent with historical data, based on records of battles and city populations, of the Roman and Chinese empires. The institutional contrast between the two, with power delegated to the military in Rome while centralized to the emperor in China, may therefore be ascribed to their different environments, with imperial China surrounded by more indigent adversaries.

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1 Introduction

Each being a vast dominion of rich produce surrounded by so-called barbarians, both the Roman and Chinese empires relied on large armies to defend the frontiers. But their institutions differed. While Rome delegated power to the military, China centralized it to the emperor. For example, Roman provinces were mostly governed by the legates stationed locally while governance across China was carried out by a top-down bureaucracy to counterbalance the military. Whereas Roman generals were both kingmakers and contenders to the throne, a Chinese general was usually not even supposed to get involved with imperial succession and his authority was divided and checked by bureaucrats, eunuchs and other generals. Such centralization policies were instituted in China despite no lack of awareness of the necessity of delegation in ancient warfare.[1] The legacy of the institutional divide between China and the West still being felt today, an important question is what set the two worlds apart in terms of centralization.

More generally, the question is Why some states centralize power while some others delegate it to the military. From the mechanism design perspective, the issue is how much power should be delegated to the military or, in the language of the delegation literature, how large the delegation set should be given to the military.[2] This is a crucial policy issue because the military, like a double-edge sword, is capable of both protecting and usurping the state. That was especially so in ancient times. Given slow transmission of information, a high degree of delegation enabled the military to fight peripheral nations effectively. But the same delegated discretion also paved the way for the military to usurp the throne should it decide to revolt. The issue is also related to an age-old problem How to guard the guardian. Here the guardian, the military, may usurp the property that it is supposed to guard. Different

[1] The notion that a general in the battlefield should not blindly obey his monarch ("将在外君命有所不受") had been prescribed by Sun Tzu (《孙子兵法·变篇》) in China long before the empire was formed.

[2] Delegation set was also called control set by Holmström [18]. Much of the delegation literature has been reviewed by Mookherjee [21]. More recent work includes Alonso and Matouschek [2] and Acemoglu, Aghion, LeLarge, van Reenen and Fabrizio Zilibotti [1]. The delegation literature is mainly about getting the privately known information out of the agent, and the delegation set, within which the agent is permitted to act without reporting his information to the principal, is an indirect implementation of the optimal direct mechanism. In this paper, differently, the size of the delegation set affects the agent's renegotiation power, and a delegation set is assumed necessary for the agent to function for the principal.

from the well-studied holdup problem,[3] the problem here is not that contracts are incomplete but rather that it may be breached by its guardian, however complete the contract is, and that the more is the guardian enabled, the bigger the domestic threat and the more the principal has to yield during renegotiation with the guardian.

To address the above question, a simple model is presented here to theorize the allocation of political power between two factions of a society, the military and the civilians. The power allocated to the military is assumed to increase the probabilities in which the military succeeds in wars, foreign or civll. At equilibrium, the military's probability of success in civil wars determines its share of the social surplus, as any lower share would trigger the military to revolt. Anticipating that, the civilians decide whether to exert efforts in production. The focus of our analysis is how a social optimum, which maximizes the total surplus for both factions subject to equilibrium conditions, is affected by the parameters.

The theoretical finding is that a social optimum is mainly determined by an exogenous variable θ defined by the parameters, which captures both the belligerence and relative wealthiness of peripheral nations compared to the state. This θ is unit-free, and comparison of the θ values between two states does not require comparison of the absolute sizes of their wealth. The main result is that the socially optimal degree of military delegation is strictly increasing in the value of θ (Proposition 2). Thus, the institutional difference between the Roman and Chinese empires, with delegation in Rome and centralization in China, may be ascribed to their different environments, with adversaries of the Romans wealthy relative to the Romans while those of the Chinese poor compared to the Chinese.

To assess this environmental difference between the two empires, a measurement of the determinant θ is presented here based on records of battles and city populations (§5). The record of battles gives us the frequencies of warfare between an empire and the peripheral peoples, and the record of city populations provides a proxy for the wealth of peripheral peoples relative to the empire. The two proxies together yield a measurement of θ, which is higher for the Romans than for the Chinese (§5.3), consistent with our main proposition. With the wealth pillaged from Carthaginians, Greeks, Persians and the like partially making up for the reward to the legions, Rome could afford to delegate power to her military.[4]

[3] A recent summary of the holdup literature is Che and Sákovics [10].

[4] While the other nations became Romanized as time went by, Persia remained a wealthy adversary throughout the lifespan of the Roman empire, which is evidenced by a speech made by Julian during his invasion to Persia, the final moment of Roman military dominance (Gibbon [16, v2, pp485–486]).

Without wealthy adversaries, by contrast, imperial China had to rely on domestic resources to keep her military loyal. Hence delegating power to the military would tax civilians more heavily, thereby discouraging their production efforts more, in China than in Rome.[5]

Such difference in θ is even larger when we compare the environment for the Roman empire with the Han dynasty of China, the two being rough contemporaries (§5.2). Comparing the measurements between different periods within the same empire, we find that the θ for Rome halved from the early period to the later one (§5.4) and that for China was unusually high during the Sui-Tang dynasties (§5.5). Such pattern, combined with the main proposition, is consistent with the institutional changes in each empire to be mentioned next at Table 2. Finally, we look at the early Qing dynasty, just before the collision between China and the West. Ruled by Manchus the only powerful surviving barbarian tribe, the empire was threatened by barbarians no more and hence faced an extremely low θ (§5.6). Unaware of the abrupt end of such environment, the empire opted to further centralize power from the military, turning the previously formidable Manchu army into a hereditary welfare system and eradicating instead of recruiting the Chinese pirates, who had been familiar with naval and firearm warfare. Little suspense was left when the West and China met.

2 Military Delegation: Rome versus China

The Roman and Chinese empires developed in parallel courses without direct contact and were comparable in size and significance.[6] Comparing their histories, one can be easily impressed by a contrast between the twain in terms of the domestic power delegated to the military. The Roman empire was by and large run by the military. Most Roman emperors emerged from generals and remained their battlefield presence after accession. The crucial

[5] An alternative possibility is that the wealthiness of Rome's adversaries was due to her expansionistic policy that seeked out wealthy nations to invade, whereas China was mostly inward-driven. An inspection of the history, however, would reveal that imperial China during the beginning of most of her dynasties (Qin, Han, Tang, Ming and Qing) was expansionistic as even the republican period of Rome, and Rome during her imperial period was mostly defensive as the Great-Wall-building China (more in Remark 4, §3). Nevertheless, Appendix A extends our model to a case where an empire can choose an expansionistic policy.

[6] In the comparative history literature, Scheidel [28] draws an interesting parallel between the two empires up to the fifth century. Edwards [11] finds a policy commonly pursued in the Han and Tang dynasties of China and the Later Roman empire to curtail provincial commanders. More of such works are cited in http://web.stanford.edu/~scheidel/Divergence.pdf.

vote for the accession was the soldiers' proclamation (hence the appellation imperator, or emperor), which was often sufficient, with the Senate merely concurring. In return, it was a norm for a new emperor to give soldiers large donatives upon accession. Although Constantine the Great later separated the military from civil administration, generals remained to be the power behind the throne (Stilicho, Constantius III, Aëtius, Asper, Recimer, etc.).

By contrast, ever since her start, the Chinese empire was governed through a bureaucracy recruited from the intelligentsia, who shared a value system, mainly Confucianism, which ranked the integrity of the empire above all. The policy of marginalizing generals from policymaking and subordinating them to the civilian bureaucracy had been institutionalized ever since the start of the Song dynasty.[7] The administration and dispatch of the soldiers were carried out in separate branches of the bureaucracy.[8] In the Ming dynasty, the administration of soldiers was further divided into five different branches of the bureaucracy, with battlefield commanders directly appointed by the emperor, and generals were monitored by eunuchs stationed in the army.[9]

Putting the contrast to quantitative contexts, Table 1 compares three aspects of the domestic power of the military between the two empires. Here *military accession* is the ratio obtained from dividing the number of emperors who owed their accession to the military by the total number of emperors,[10] *hereditary succession* the ratio from dividing the number of emperors who owed their accession to their bloodline hereditary status by the total number of emperors, and *generals executed by emperors* the number of top generals executed by the imperial court, not in a civil-war period, divided by the total number of emperors.[11]

Table 1: **Domestic Power of the Military**

	Roman empire	Chinese empire
Military accession	61.7%	20%

Continued on next page

[7] Some original sources are documented in 顾宏义 [9] on the subordinate role of military officers relative to their civilian counterparts. Particularly telling is an episode where a most capable general Di Qing (狄青) was suppressed by civilian ministers such as Han Qi (韩琦).

[8] See 王曾瑜 [4, p5].

[9] See 杨四维 [3, p112].

[10] Appendix B explains how emperors are counted for the two empires.

[11] The first two categories overlap, e.g., Titus and Emperor the First of China (秦始皇), and do not contain all emperors, e.g., Nerva and Tacitus.

Table 1 – *Continued from previous page*

	Roman empire	Chinese empire
Hereditary succession	33.3%	91%
Generals executed by emperors	10%	21%

Source: Table 6, Appendix B.

Table 1 is compiled from the data of imperial succession in the two empires, summarized in Appendix B, from 27 BCE (establishment of the Roman empire) to 476 CE (death of the western Roman empire) for Rome and from 221 BCE (first unification of China) to 1912 CE (abdication of the last emperor) for China.[12] The republic period of Rome is excluded because the emperor as an institution had yet to be established. The Byzantium that survived the western empire, despite the transient exploits during the reigns of Justinian and Heraclius, is excluded because she did not have the aforementioned vast-dominion feature and degenerated to a spectator rather than the core of a civilization.[13] If the Roman empire is compared with only her rough contemporary, the Han dynasty of China, the contrast, presented in Table 5, would be similar to Table 1 and would fit well with our theory (§5.2). For a fairer comparison, however, the subsequent periods of imperial China are included. While the Roman empire never recovered from 476 CE, the Chinese empire continued to be a vast dominion and the core of a civilization despite disruptions of partitions, civil wars and foreign occupations. The Han dynasty is but Act One of an ongoing enterprise.

During the parallel struggles of the two peoples, each could have adopted the other's alternative, for Rome to centralize or China to delegate. In fact, Diocletian and Constantine did reform the Roman empire towards the Chinese alternative, trying to counterbalance the military with a bureaucracy, elevating the emperor away from the military by bureaucrats, eunuchs and religions, and keeping the flower of the army near the emperor.[14] Symmetrically, the Sui and Tang dynasties of China moved near to the Roman path, with emperors dethroned and erected often through mutinies, provincial military commanders[15] obtaining both military and civil authorities over the regions, and defense often delegated to Sinicized

[12] The imperial succession during the 89 years of Mongolian occupation in China is excluded, though the quantitative effect of the exclusion is negligible. See Appendix B and Footnote 43 for explanations.

[13] Gibbon [16, v4, p560; v5, pp82–84].

[14] Gibbon [16, v2, pp124–127].

[15] Called commissioners, or 节度使.

generals of barbarian extraction.[16] Such deviations from the norm are evidenced by the contrast between the corresponding columns in Table 2. The dynastic cycle in imperial China, with the thoroughly destructive wars at the end of one dynasty clearing the way for the next, afforded the new rulers tremendous power to reform the institutions, and the founding emperors of each dynasty did exactly that. The Roman counterpart of such institutional reforms also occurred, conducted by Augustus, Diocletian and Constantine, as well as by Julian and Theodosius I in less degrees. Such reforms in each empire indicate that the institutions were rather choices they adopted than something given to them, which leads to our question why each empire opted for the particular system.

Table 2: **Each empire had tried the other alternative**

	Roman empire	Later Roman empire	Chinese empire	Sui-Tang dynasties
Military accession	61.7%	50%	20%	32%
Hereditary succession	33.3%	50%	91%	84%
Generals executed by emperors	10%	22%	21%	16%

Source: Table 6, Appendix B.

3 The Soldier and the Farmer

Reduced to its bare bones, an empire can be viewed as a people whose public good is of such a high stake that specialized military defense is needed. To support a specialized military requires nontrivial transfers from the rest of the empire, or the civilians. Thus arises a fundamental interaction within an empire between the military and the civilians. To focus on this irreducible relationship our model shall have two players, a soldier representing the military and a farmer the civilians. Characters such as emperors, generals, ministers and the like are merely institutional details, who represent one faction or the other, that need to be abstracted away, or our model would at best be as tractable as the game of chess.

Thus, let us assume a model empire consisting of two players, a farmer and a soldier. Parameters are functions $f, g : [0,1] \to [0,1]$, numbers $\omega_a \in \mathbb{R}_{++}$, $\omega_b \in \mathbb{R}_+$, $\pi \in (0,1)$,

[16] Some prominent examples, among many, are Gao XianZhi (高仙芝, Korean), An LuShan (安禄山, Sogdian-Turkic) and Li KeYong (李克用, Sart).

$p \in (0,1)$ and $c_* \in (0,\infty)$, and a cumulative distribution function $H : [0, c_*] \to [0,1]$, with continuous and positive density $h : [0, c_*] \to \mathbb{R}_{++}$. Events are unfolded in four stages:

a. An element x of $[0,1]$ is chosen as the degree of military delegation, and an allocation $(y,z) \in \mathbb{R}^2_+$, whose meaning will be clear at stage (d), is determined between the farmer and the soldier.

b. Nature randomly draws a c from the distribution H and reveals c privately to the farmer. Then the farmer chooses whether to work or shirk. If he works, the farmer bears a sunk cost equal to c and produces a harvest equal to either ω_a with probability p or zero with probability $1-p$. If the farmer shirks, the harvest is zero for sure. The farmer's action, working or shirking, is unobservable to others.

c. With probability π a war breaks out between the empire and the peripheral people, so-called barbarians. In this event, if the empire is defeated, the barbarians pillage the entire harvest, leaving zero to both the soldier and the farmer. If such foreign warfare does not occur, the wealth of the empire is intact. If the foreign war occurs and the farmer's produce is zero then the empire is defeated. If the foreign war occurs and the farmer's produce is ω_a, then the soldier, fighting the barbarians, wins with probability $f(x)$, in which case the wealth of the victorious empire becomes $\omega_a + \omega_b$, including the farmer's harvest ω_a and the amount ω_b pillaged from the barbarians.

d. If the total wealth obtained by the empire is nonzero, $\omega_a + \omega_b$ or ω_a, the soldier decides whether to revolt or not. If not, then the total wealth is divided according to the agreement at stage (a): if the total wealth is $\omega_a + \omega_b$, the soldier gets y and the farmer gets $\omega_a + \omega_b - y$; if the wealth is ω_a, the soldier gets z and the farmer gets $\omega_a - z$. If the soldier revolts, then with probability $g(x)$ the soldier usurps the empire and obtains her entire wealth, leaving zero to the farmer; with probability $1-g(x)$ the rebellion is cracked down so the soldier gets zero and the farmer gets the entire wealth.

At stage (a), the way in which the endogenous variables (x,y,z) are determined depends on the particular political mechanism of the empire. To uncover the fundamental driving force of institutional patterns, to free our analysis from the artifice of suboptimal institutions or political bias toward one party or another, and to capture the long-run trend

that interests of various parties tend to get expressed through procedures ranging from voting and bargaining to protests and bloodshed, albeit too complex to be incorporated in our model, let us assume that the endogenous variables are chosen by a neutral benevolent social planner. Hence we shall consider a principal's problem of maximizing the expected value of social surplus for the empire subject to the incentive constraints of the two players.

The above assumption, that the mechanism is chosen by a neutral social planner, can be removed and replaced by a bargaining game between the soldier and the farmer in which they alternately propose a mechanism coupled with a utility transfer between them. Provided that failure to reach an agreement would result in zero payoff for both players, one can prove that any stationary subgame perfect equilibrium of this bargaining game (à la Perry and Reny [26]) is equivalent to a social-surplus maximizing mechanism.

Remark 1: The wealth in the model should be interpreted as the surplus above the subsistence level, in the form of goods or labor susceptible to pillage.

Remark 2: While the military's moral hazard problem is the focus, the farmer is an indispensable construct in our consideration. Without the farmer's moral hazard problem, the model would offer an extreme implication that the socially first best could be achieved by military dictatorship, making the military residual claimant of the society.

Remark 3: The model assumes that the occurrence of warfare between the empire and the periphery is exogenous. Appendix A modifies the model so that warfare is chosen endogenously by the barbarians and the empire conditional on the realization of their harvests. The analytical result there bears some resemblance to our main propositions.

Remark 4: Nevertheless, the exogenous warfare assumption is consistent with the stylized pattern that in the long run the occurrence of external conflicts was by and large exogenous to the Roman and Chinese empires. Neither could choose which foreign nation to show up at the border, and the geopolitical conflicts between the neighbor and the core determined the long run likelihood of warfare. After the formation of their empire, the Romans, like the Chinese, mostly took a defensive stance instead of making new enemies through expansion.[17] Furthermore, the nations whose hostility to the empire may be ascribed to the empire's expansionary policy, the British and Dacian for Rome and the Koreans, Vietnamese and Arabs for China, were not among those who fought the empire in higher frequencies.

[17] Gibbon [16, v1, pp1–8].

4 Theoretical Analysis

We shall see that a central determinant for the socially optimal solution for the empire is an exogenous variable θ, defined by the parameters, that encapsulates both the barbarians' wealth relative to the empire and the likelihood of warfare between the two. At any social optimum, the empire with higher θ delegates more authority to her military (Proposition 2). This result suggests that the institutional difference, with more military delegation in Rome than in China, can be traced back to a difference in the primitives faced by the two empires: that the Chinese empire was given a lower θ than the Roman empire, i.e., the Chinese were surrounded by poorer barbarians, or less harassed by them, than the Romans.

4.1 Optimum

According to stage (d) of the model, the soldier weakly prefers loyalty to revolt if and only if $y \geq g(x)(\omega_a + \omega_b)$, when he prevails in the event of foreign warfare, and $z \geq g(x)\omega_a$, when no foreign warfare occurs.

Lemma 1 (revolt-proof principle) *Any equilibrium-feasible choice (x, y, z) is outcome-equivalent to some (x, y', z') that satisfies the revolt-proof conditions $y \geq g(x)(\omega_a + \omega_b)$ and $z \geq g(x)\omega_a$.*

Proof Suppose that (x, y, z) violates at least one of the revolt-proof conditions, say $y < g(x)(\omega_a + \omega_b)$. Then in the event of his victory against barbarians, the soldier revolts thereby obtaining a wealth equal to $g(x)(\omega_a + \omega_b)$ in expected value, leaving $(\omega_a + \omega_b)(1 - g(x))$ to the farmer in expected value. Thus, at any continuation equilibrium given (x, y, z), the expected payoff for each player is equivalent to replacing y by $y' := g(x)(\omega_a + \omega_b)$, which satisfies the condition $y' \geq g(x)(\omega_a + \omega_b)$. The case where $z < g(x)\omega_a$ is analogous. ■

Lemma 2 *Any equilibrium-feasible choice (x, y, z) determines a unique pair $(v_1, v_2) \in \mathbb{R}^2_+$ such that v_1 is equal to the soldier's expected payof, and v_2 the farmer's, at any continuation equilibrium given (x, y, z), and v_i $(\forall i \in \{1, 2\})$ remains constant even when the soldier varies his probability of revolts in the event that he is indifferent between revolts and otherwise.*

Proof By Lemma 1 we may assume, without loss of generality, that (x, y, z) satisfies the revolt-proof conditions. Thus, the soldier's best response at stage (d) is: If foreign warfare

occurs and the soldier wins, he does not revolt if $y > g(x)(\omega_a + \omega_b)$ and otherwise ($y = g(x)(\omega_a + \omega_b)$) mixes between revolt and not revolt; if no foreign warfare occurs and the farmer's produce is ω_a, the soldier does not revolt if $z > g(x)\omega_a$ and otherwise ($z = g(x)\omega_a$) mixes between revolt and not revolt. Thus, if the farmer's produce is ω_a and if the empire wins in foreign warfare, the soldier's expected payoff is equal to y and the farmer's equal to $\omega_a + \omega_b - y$; if no foreign warfare occurs, with produce ω_a, the soldier's expected payoff is equal to z and the farmer's $\omega_a - z$. Thus, the farmer's best response at stage (b) is to work if $p\left(\pi f(x)(\omega_a + \omega_b - y) + (1-\pi)(\omega_a - z)\right) > c$, shirk if the inequality is reversed, and mix between work and shirk if the two sides are equal. The above inequality becomes $p\left(\varphi(x;\omega,\pi) - (\pi f(x)y + (1-\pi)z)\right) \geq c$ with the notation

$$\varphi(x;\omega,\pi) := \pi f(x)(\omega_a + \omega_b) + (1-\pi)\omega_a. \tag{1}$$

We write $\varphi(x)$ for $\varphi(x;\omega,\pi)$ unless further clarity is necessary. The farmer's best response determines each player's expected payoff:

$$\begin{aligned} v_1 &= \int_0^{\min\{c_*, p(\varphi(x) - (\pi f(x)y + (1-\pi)z))\}} p\left(\pi f(x)y + (1-\pi)z\right) dH(c), \\ v_2 &= \int_0^{\min\{c_*, p(\varphi(x) - (\pi f(x)y + (1-\pi)z))\}} \left(p\left(\varphi(x) - (\pi f(x)y + (1-\pi)z)\right) - c\right) dH(c). \quad \blacksquare \end{aligned}$$

The social surplus generated by any equilibrium-feasible mechanism (x, y, z) is equal to the sum $v_1 + v_2$ calculated above, or

$$\int_0^{\min\{c_*, p(\varphi(x) - (\pi f(x)y + (1-\pi)z))\}} (p\varphi(x) - c)\, dH(c). \tag{2}$$

Hence the principal's decision at stage (a) is to choose $(x, y, z) \in [0,1] \times [0, \omega_a + \omega_b] \times [0, \omega_a]$ to maximize (2) subject to the soldier's incentive constraint, or the revolt-proof conditions

$$y \geq g(x)(\omega_a + \omega_b), \tag{3}$$

$$z \geq g(x)\omega_a. \tag{4}$$

Assumption 1 *f and g are continuous and strictly increasing on $[0,1]$; $g(0) = 0$; $g(1) = 1$.*

Lemma 3 *By Assumption 1, at any social optimum (x, y, z), $c_* \geq p\left(\varphi(x) - (\pi f(x)y + (1-\pi)z)\right)$.*

Proof First, if $x = 1$ then $g(x) = 1$ by assumption and hence the revolt-proof conditions require that $y = \omega_a + \omega_b$ and $z = \omega_a$, so the conclusion of the lemma holds trivially. Second,

let $x < 1$ and suppose $c_* < p\left(\varphi(x) - (\pi f(x)y + (1-\pi)z)\right)$. In the expression (2) for the social surplus, the upper limit is equal to the constant c_* and, by continuity of f, remains so when (x, y, z) changes within a sufficiently small neighborhood. Thus, a small increase of x, through increasing the integrand in (2) due to strict monotonicity of f, increases the social surplus (2). The revolt-proof conditions can be maintained through increasing (y, z) slightly, with f and g continuous. Thus, (x, y, z) cannot be a social optimum. ■

Lemma 4 *By Assumption 1, at any social optimum (x, y, z), the revolt-proof constraint (4) is binding, and if $f(x) > 0$ then the other revolt-proof constraint (3) is also binding.*

Proof By Lemma 3, the social surplus generated by any social optimum (x, y, z) equals

$$\int_0^{p(\varphi(x)-(\pi f(x)y+(1-\pi)z))} (p\varphi(x) - c)\, dH(c). \tag{5}$$

If constraint (4) were not binding, then z can be reduced slightly without violating the constraints. The reduction increases the upper limit $p\left(\varphi(x) - (\pi f(x)y + (1-\pi)z)\right)$ of the integral (5) since $\pi < 1$ and $p > 0$ by assumption. The mass of c added to the integration domain thereby increases the integral, because for almost every such c, $p\varphi(x) - c > 0$ as c is less than the upper limit. The argument for constraint (3) is analogous when $f(x) > 0$. ■

Proposition 1 *Suppose Assumption 1. Then social optimum exists and a degree x^* of military delegation is socially optimal if and only if x^* solves the problem*

$$\max_{x\in[0,1]} \int_0^{p(1-g(x))\varphi(x;\omega,\pi)} (p\varphi(x;\omega,\pi) - c)\, dH(c) \tag{6}$$

$$\text{s.t.} \qquad c_* \geq p(1-g(x))\varphi(x;\omega,\pi). \tag{7}$$

Proof By Lemmas 1, 3 and 4, a degree x^* of military delegation is socially optimal if and only if there exists (y^*, z^*) for which (x^*, y^*, z^*) maximizes (5) among all (x, y, z) such that

$$c_* \geq p\left(\varphi(x) - (\pi f(x)y + (1-\pi)z)\right) \tag{8}$$

and the revolt-proof constraints are binding in the sense that

$$f(x)y = f(x)g(x)(\omega_a + \omega_b) \tag{9}$$

$$z = g(x)\omega_a. \tag{10}$$

Eqs. (9)–(10), coupled with the definition of $\varphi(x)$ in Eq. (1), imply that Ineq. (8) is equivalent to the constraint (7) and that the upper limit of the integral (5) is equal to $p(1-g(x))\varphi(x)$. Thus, the objective in (6) is equal to (5), which proves Claim (ii). With f and g continuous by assumption, the maximization problem (6)–(7) admits a solution, so Claim (i) follows. ■

The next corollary, proved in Appendix C, says that when production requires little cost for the effort, the optimum is arbitrarily close to complete delegation to the military.

Corollary 1 *Suppose Assumption 1. With other parameters unchanged, when $c_* \to 0$ any socially optimal degree of military delegation converges to one.*

Remark 6: Coupled with our later analysis on the case where c_* is high, Corollary 1 offers an implication that, other things equal, the optimal degree of military delegation in a society is much higher when its production technology is primitive, in terms of its dependence on the civilians' efforts, than when the technology is relatively advanced.

In the sequel our attention is turned to established empires with sophisticated productions so that the costly effort necessary for a good harvest can be large. Hence—

Assumption 2 $c_* \geq p\left(\pi(\omega_a+\omega_b)+(1-\pi)\omega_a\right)$.

To sharpen the characterization of optima we also assume—

Assumption 3 *f and g are differentiable on $[0,1]$ and $g'(1)>0$.*

Denote the objective function in (6) by $U(x;\omega,\pi,p)$. By Assumption 2, the constraint (7) is redundant. Thus, any socially optimal degree of military delegation is a maximum of $U(x;\omega,\pi,p)$ among all $x\in[0,1]$. By Assumption 3,

$$\begin{aligned}\frac{\partial}{\partial x}U(x;\omega,\pi,p) &= p^2 g(x)\varphi(x)h[p(1-g(x))\varphi(x)]\left(\varphi'(x)(1-g(x))-g'(x)\varphi(x)\right) \\ &\quad +p\varphi'(x)H[p(1-g(x))\varphi(x)].\end{aligned} \tag{11}$$

The next Lemma 5 and Corollary 2 readily follow, with the proofs relegated to Appendix C.

Lemma 5 *By Assumptions 1–3, given any $(\omega_a,\omega_b,\pi,p)$, any socially optimal degree of military delegation is interior to $[0,1]$.*

Corollary 2 *By Assumptions 1–3, at any social optimum given parameters* $(\omega_a, \omega_b, \pi, p)$*, (i) the degree* x *of military delegation satisfies the first-order condition*

$$g(x)\frac{\partial}{\partial x}\left(\ln H\left[p(1-g(x))\varphi(x;\omega,\pi)\right]\right)+\frac{\partial}{\partial x}\ln\varphi(x;\omega,\pi)=0, \tag{12}$$

(ii) $\frac{\partial}{\partial x}\left((1-g(x))\varphi(x;\omega,\pi)\right)<0$ *and (iii) both revolt-proof constraints are binding.*

Claim (i) of Corollary 2 says that at any social optimum two growth rates, each generated by an infinitesimal increase of the degree of military delegation, cancel out each other. The first one, $\frac{\partial}{\partial x}\left(\ln H\left[p(1-g(x))\varphi(x;\omega,\pi)\right]\right)$, is the growth rate of the mass of farmer-types that exert efforts, as $p(1-g(x))\varphi(x)$ is the supremum of such farmer types. The second, $\frac{\partial}{\partial x}\ln\varphi(x;\omega,\pi)$, is the growth rate of the expected wealth for the empire provided that the farmer exerts efforts. Claim (ii) says that, at optimum, delegating more power to the military discourages the farmer's effort. Claim (iii), that at any social optimum the soldier is indifferent about revolts, is consistent with historical occurrence of military revolts. That is because our design of the optimal mechanism remains valid for all randomizations between revolt and loyalty when the soldier is indifferent (the last statement of Lemma 1).

4.2 Comparative Statics

Delegating more power to the military, or increasing x, has two opposite effects. On one hand, with greater delegated power the military becomes more effective in foreign warfare thereby enlarging the empire's expected wealth in the event that the farmer exerts efforts. I.e., the upside of increasing x is an increase of the integrand in (6). On the other hand, more military delegation also causes a higher probability $g(x)$ with which the military can usurp the empire; with such a bigger threat, the military gets a larger share of the empire's wealth. Anticipating that, a farmer whose effort cost is near the high end of those that exert efforts would rather shirk, thereby reducing the expected wealth for the empire. I.e., the downside of increasing x is a decrease of the upper limit of the integral (6), or the maximum effort-exerting farmer-type (Claim (ii) of Corollary 2).

Comparative statics of the social optimum boils down to an analysis on how the parameters affect the above opposite effects differently. One readily sees, with small increase of x, that the upside effect is approximately $p\pi(\omega_a+\omega_b)f'(x)$ while the downside approximately $pg'(x)\varphi(x)-p\pi(\omega_a+\omega_b)f'(x)(1-g(x))$, where

$$pg'(x)\varphi(x)=p\pi(\omega_a+\omega_b)g'(x)\left[f(x)+\left((1-\pi)/\pi\right)\left(\omega_a/(\omega_a+\omega_b)\right)\right].$$

Thus, the smaller is the exogenous variable

$$\theta := \frac{\pi}{1-\pi}\left(1+\frac{\omega_b}{\omega_a}\right), \tag{13}$$

the heavier is the downside of increasing military delegation, and the stronger is the push for curtailing power from the military. Intuitively, the military can enlarge the wealth for the empire, from ω_a to $\omega_a+\omega_b$, only in foreign warfare, which occurs with probability π. Hence the upside of military delegation is scaled up by only $\pi(\omega_a+\omega_b)$. By contrast, the military poses an internal threat, thereby discouraging the farmer's efforts, whether foreign war breaks out or not. Hence part of the downside of military delegation is magnified by both $\pi(\omega_a+\omega_b)$ and $(1-\pi)\omega_a$. Therefore, an increase in θ, by its definition, means that the downside is magnified more than the upside is, lending more weight to the argument for curtailing power from the military. In particular, when the empire's own wealth ω_a enlarges with other parameters unchanged, the empire should curtail power from her military.

In general, the above effect of the parameters may be dampened or magnified by the factor $p\pi(\omega_a+\omega_b)$ common in both the upside and downside. But if the distribution H of the farmer-type is nearly uniform, the common factor affects each side nearly linearly and hence its effect on both is a wash. The sequel is therefore specialized to the case where H is a uniform distribution. In addition to easing the exposition, the specialization makes θ sufficient statistic for all the parameters that may affect the social optimum. And θ has the advantage of being unit-free, obtained from comparing the wealth of the empire with that of her contemporary periphery. Hence a comparison between the Roman and Chinese empires in terms of their θ does not require comparing the absolute sizes of the twain.

Lemma 6 *If Assumption 2 holds and H is the uniform distribution on $[0,c_*]$, then the social-surplus maximization problem (6)–(7), given parameters $(\omega_a,\omega_b,\pi,p)$, is equivalent to $\max_{x\in[0,1]} V(x,\theta)$, where*

$$V(x,\theta) \;:=\; 2\ln\left(f(x)+1/\theta\right)+\ln\left(1-g(x)^2\right). \tag{14}$$

Proof As shown previously, maximizing social surplus is the same as maximizing $U(x;\omega,\pi,p)$ among all $x\in[0,1]$. With H uniform on $[0,c_*]$, $U(x;\omega,\pi,p)$ is equal to

$$\begin{aligned}\frac{1}{c_*}\int_0^{p(1-g(x))\varphi(x)}\left(p\varphi(x)-c\right)dc \;&=\; \frac{1}{c_*}\left(p\varphi(x)p(1-g(x))\varphi(x)-\frac{1}{2}p^2(1-g(x))^2\varphi(x)^2\right)\\ &\overset{(1),(13)}{=}\; \frac{p^2}{2c_*}\pi(\omega_a+\omega_b)\left(f(x)+1/\theta\right)^2\left(1-g(x)^2\right),\end{aligned}$$

which, transformed by ln, becomes $V(x,\theta)$ plus a term constant to the choice variable x. ■

Lemma 7 *By Assumption 1, $V(x,\theta)$ exhibits strictly increasing difference in (x,θ), i.e., if $\theta' < \theta''$ and $0 \leq \underline{x} < \overline{x} \leq 1$ then $V(\overline{x},\theta') - V(\underline{x},\theta') < V(\overline{x},\theta'') - V(\underline{x},\theta'')$.*

Proof Given any $x \in [0,1]$, we have $\frac{\partial}{\partial\theta}V(x,\theta) = -2/\left(\theta^2(f(x)+1/\theta)\right)$, which is strictly increasing in x since f is strictly increasing in x (Assumption 1). Thus, for any $\theta'' > \theta'$, $V(\underline{x},\theta'') - V(\underline{x},\theta') < V(\overline{x},\theta'') - V(\overline{x},\theta')$. ■

Lemma 8 *Suppose that H is the uniform distribution on $[0,c_*]$ and Assumptions 1–3. If x' is socially optimal given θ', and x'' socially optimal given θ'', then $\theta' \neq \theta'' \Rightarrow x' \neq x''$.*

Proof Suppose not, i.e., $x' = x'' =: x_*$ is optimum for both θ' and θ''. With x_* interior to $[0,1]$ (Lemma 5), x_* satisfies the first-order condition $\frac{\partial}{\partial x}V(x,\theta)\big|_{x=x_*} = 0$ whether $\theta = \theta'$ or $\theta = \theta''$. But that violates the fact $\frac{\partial^2}{\partial\theta\partial x}V(x_*,\theta) > 0$, which follows directly from Eq. (14). ■

Proposition 2 *Suppose Assumptions 1–3 and that H is the uniform distribution on $[0,c_*]$. If x' is a socially optimal (or SPBE) degree of military delegation given θ', and x'' socially optimal (or SPBE) given θ'', with θ' and θ'' defined by the parameters $(\omega_a',\omega_b',\pi')$ and $(\omega_a'',\omega_b'',\pi'')$ through Eq. (13), then $\theta' < \theta'' \Rightarrow x' < x''$.*

Proof Let $\theta' < \theta''$ and we shall prove $x' < x''$. As shown previously, at any social optimum, the degree of military delegation is a maximum of $U(x;\omega,\pi,p)$ among all $x \in [0,1]$, which, with H uniform and by Lemma 6, is equivalent to maximizing $V(x,\theta)$ among all $x \in [0,1]$. By Lemma 8, $x' \neq x''$. Hence it suffices to show that $x'' \geq x'$. Suppose, to the contrary, that $x'' < x'$, then Lemma 7 implies the desired contradiction:

$$0 \leq V(x',\theta') - V(x'',\theta') < V(x',\theta'') - V(x'',\theta'') \leq 0,$$

with the first and last inequalities due to optimality of x'' given θ'' and that of x' given θ'. ■

4.3 Multiple-Tribe Extension

For simplicity, only one peripheral people has been assumed present in our model so far. To confront the data let us relax the assumption. The game defined in §3 is modified as follows: At stage (c), either no war between the the empire and the periphery occurs or the empire is

engaged in warfare with exactly one barbarian tribe, labeled as an element of a finite set B; for each $b \in B$, a war between the empire and tribe b occurs with probability π_b such that $0 < \sum_{b\in B} \pi_b < 1$. In the event that the empire defeats tribe b in the war, the total wealth of the empire becomes $\omega_a + \omega_b$. At stage (a), the allocation $y \in \mathbb{R}_+$ is replaced by vector $(y_b)_{b\in B} \in \prod_{b\in B}[0, \omega_a + \omega_b]$, so that at stage (d) if the soldier does not revolt after defeating barbarian tribe b then he gets y_b and the farmer gets $\omega_a + \omega_b - y_b$. Assumption 2 is replaced by $c_* \geq p\left(\sum_{b\in B} \pi_b(\omega_a + \omega_b) + \left(1 - \sum_{b\in B} \pi_b\right)\omega_a\right)$, with other assumptions unchanged.

Corollary 3 *In the multiple-tribe extension, all previous results remain true with the expression* $\pi(\omega_a + \omega_b)$ *replaced by* $\sum_{b\in B} \pi_b(\omega_a + \omega_b)$ *and Eq. (13) replaced by*

$$\theta := \sum_{b\in B} \frac{\pi_b}{1 - \sum_{b'\in B} \pi_{b'}} \left(1 + \frac{\omega_b}{\omega_a}\right). \tag{15}$$

Proof The reasoning parallels previous steps. The revolt-proof conditions become $y_b \geq g(x)(\omega_a + \omega_b)$ for each $b \in B$ and $z \geq g(z)\omega_a$; the farmer's expected wealth from working is equal to $p\left(f(x)\sum_{b\in B} \pi_b(\omega_a + \omega_b - y_b) + (\omega_a - z)\left(1 - \sum_{b\in B} \pi_b\right)\right)$ at any revolt-proof social contract $(x, (y_b)_{b\in B}, z)$ according to the extended model; and the binding constraints at Lemma 4 become $f(x)\sum_b \pi_b y_b = f(x)g(x)\sum_b \pi_b(\omega_a + \omega_b)$ and $z = g(x)\omega_a$. ■

5 Historical Evidence

According to Proposition 2, the institutional divergence between the two empires, with military power delegated in Rome and centralized in China, can be understood as a consequence of a single difference in their environments, that the exogenous variable θ had a higher value for Rome than for China, i.e., the wealth of the peripheral peoples relative to that of the empire, weighed by the likelihood of wars between them, was larger for Rome than for China. Following are some quantitative measures of such environmental difference.

5.1 Measuring θ through City Sizes and Battle Records

Given the fact that each empire is surrounded by multiple tribes of peripheral peoples, we shall measure θ according to the multiple-tribe formula, Eq. (15).[18] For a proxy of the

[18] The formula assumes away the possibility that the empire is engaged in the same battle with multiple tribes of barbarians, a rare occurrence for both Rome and China.

probability π_b of wars between an empire and a peripheral people say b, I use the frequency of battles $\hat{\pi}_b$ between this people and the empire, which is calculated by dividing the number of such battles by the number of years in the period under consideration. Battles for each empire are listed in Wikipedia.[19] To measure the wealth of a historical people, I use Ian Morris's [22, pp148–152] idea that a people's social development is indicated by its level of urbanization, or the population of its large cities. Records of historical populations in large cities are listed in Wikipedia.[20] For each nation b that had warfare with the empire, we can identify its cities that show up in the list and calculate the total population across these cities as a proxy $\hat{\omega}_b$ of the nation's wealth ω_b.[21] Divide this "city population" of the nation by that of the empire to obtain a proxy $\hat{\omega}_b/\hat{\omega}_a$ for the nation's relative wealth with respect to the empire's. Plug the proxies $\hat{\omega}_b/\hat{\omega}_a$ and $\hat{\pi}_b$ into Eq. (15) and we obtain a proxy $\hat{\theta}$ for θ.

The periods based on which the θ is measured for the two empires are chosen to reflect, not the imperial institutions per se, but the environments in response to which the institutions were established. Hence the data count for Rome starts from 200 BCE because by then the Roman dominance had started to be felt around the Mediterranean. By the same token, the periods of separations, civil wars and foreign occupation within the lifespan of imperial China are included as part of her environment. Similarly, the period before 221 BCE the first unification of China is excluded from the θ calculation for China because Qin, the kingdom that founded the empire, was not that dominant back then, with major defeats of its army observed up to a decade before 221 BCE.[22] In addition, the Qin dynasty lasted merely 15 years and the next, long-lasting Han dynasty started with quite a different system;

[19] Roman battles: http://en.wikipedia.org/wiki/List_of_Roman_battles, Sep. 8, 2014; Chinese battles: http://zh.wikipedia.org/wiki/中国战争列表, Sep. 12, 2014, supplemented with the English version, http://en.wikipedia.org/wiki/List_of_Chinese_wars_and_battles, Sep. 8, 2014.

[20] It is http://en.wikipedia.org/wiki/Historical_urban_community_sizes, dated September 6, 2014. Two Chinese cities, Datong aka Pingcheng and Jiankang aka Nanjing, are each listed by this webpage in multiple entries under their different names. For each city I have combined the corresponding entries.

[21] If the population record of a city is absent while a contemporary record of another city is present, I count the population of the former city as zero at that instant, as record preservation should also indicate the level of social development. Then I take the average of these population counts within the period under consideration for the "city population" over that period.

[22] For example, the Battle of E'yu (阏与之战) in 269 BCE, the Battle of Handan (邯郸之战) around the year 258 BCE, the Battle of Hewai (河外之战) in 247 BCE, the Battle of Fei (肥之战) in 233 BCE and the Battle of PoWu (番吾之战) in 232 BCE.

hence whatever worldview the Qin emperor might have obtained from the earlier period did not predetermine the system adopted by Han. I also exclude the ending period of imperial China, 1801–1912, because the environment then, marked by the collision between the West and the oblivious empire, was utterly unexpected and world-shattering to the Chinese.

5.2 Contemporary Contrast

Table 3 demonstrates at its last row the measurement of θ as 0.2241 for the Roman empire, while Table 4 shows the much lower θ as 0.0751 for China during the Qin-Han dynasties, the rough contemporary of the Roman empire.

Table 3: **The θ for the Romans, 200 BCE–476 CE**

	battles	$100\hat{\pi}_b$	city population	$\hat{\omega}_b/\hat{\omega}_a$	$100\hat{\pi}_b\,(\hat{\omega}_b/\hat{\omega}_a)$
Core (Romans)			852500		
Alans/Huns	3	0.4438	0	0	0
Armenians	4	0.5917	0	0	0
British	6	0.8876	0	0	0
Carthaginian	4	0.5917	18750	0.0220	0.0130
Celtic (Spain)	3	0.4438	0	0	0
Dacians	6	0.8876	0	0	0
Egyptians	2	0.296	106250	0.1246	0.0369
Gauls	13	1.9231	0	0	0
Germanic	34	5.0296	0	0	0
Greeks (Asia)	16	2.3669	233750	0.2742	0.6490
Greeks (Europe)	10	1.4793	13750	0.0161	0.0239
Jewish	4	0.5917	106250	0.1246	0.0737
Numidians	2	0.2959	0	0	0
Persians	12	1.7751	31250	0.0367	0.0651
Total	119	17.6038			0.8616
$\hat{\theta}$	**0.2241**				

Sources: (i) Battles: http://en.wikipedia.org/wiki/List_of_Roman_battles, Sep. 8, 2014; (ii) city populations: http://en.wikipedia.org/wiki/Historical_urban_community_sizes, Sep. 6, 2014; (iii) Ferrero [13, v4, pp21–23], based on which I added to the list Antony's battle of Phraaspa; (iv) first-century population in Jerusalem: http://en.wikipedia.org/wiki/Demographic_history_of_Jerusalem, Sep. 13, 2014.

Table 4: **The θ in the Qin-Han dynasties, 221 BCE–280 CE**

	battles	$100\hat{\pi}_b$	city population	$\hat{\omega}_b/\hat{\omega}_a$	$100\hat{\pi}_b\,(\hat{\omega}_b/\hat{\omega}_a)$
Core (Chinese)			632600		
Central Asian (西域)	3	0.5988	0	0	0
Hiong-Nou (匈奴)	16	3.1936	0	0	0
Korean	3	0.5988	0	0	0
Qiang (羌)	5	0,9980	0	0	0
Sienpi (鮮卑)	2	0.3992	0	0	0
Viet (越)	4	0.7980	0	0	0
Yunnan/Guizhou	2	0.3992	0	0	0
total	35	6.9858			0
$\hat{\theta}$	**0.0751**				

Sources: (i) Battles: http://zh.wikipedia.org/wiki/中国战争列表, Sep. 12, 2014, supplemented with the English version, http://en.wikipedia.org/wiki/List_of_Chinese_wars_and_battles, Sep. 8, 2014; (ii) city populations: http://en.wikipedia.org/wiki/Historical_urban_community_sizes, Sep. 6, 2014 (Footnote 20).

The Romans had relatively wealthy adversaries such as the Carthaginians, the Greeks spreading across Europe and Asia, the Jewish and the Persians, whose wealth was indicated by their city populations in Table 3. Even the Gauls and British were of some considerable wealth by anecdotal evidence despite their lack of city population records.[23] While most of these wealthy nations were later more or less Romanized, wars between Rome and the wealthy Syria occurred up to the Crisis of the Third Century, and the Persians remained a wealthy adversary throughout the life of the Roman empire. As Julian blatantly articulated in his speech (Footnote 4), the wealth of the Persians remained in Romans' calculations up to the very end of the era in which the Roman army could dominate its adversaries.

The Chinese during the roughly contemporary period, by contrast, fought much fewer foreign wars and had only indigent adversaries. Table 4 shows that none of those peoples had any urban wealth for Chinese to pillage. Such a stark contrast in the environments of the two empires, coupled with our theoretical finding, Proposition 2, implies that at the social optimum the military enjoyed much more delegated power in Rome than in China.

[23] It was the prospect of Gaul's becoming as wealthy as Egypt that determined Augustus's mind to firm up his military grip on Gaul (Ferrero [13, v5, pp111–116]). Britain back then was thought to be endowed with pearl fishery for a while (Gibbon [16, v1, p5]).

Such implication is consistent with the contrast between the domestic power of the Roman military versus that of the Chinese military in the Qin-Han dynasties:

Table 5: **Domestic Power of the Military: Rome v. Qin-Han**

	Roman empire	Qin-Han dynasties
Military accession	61.7%	19.4%
Hereditary succession	33.3%	87.1%
Generals executed by emperors	10%	16.1%

Source: Table 6, Appendix B.

5.3 Aggregate Contrast

Table 9 in Appendix E presents the measurement of θ as 0.2031 for imperial China as a whole. It remains to be lower than the Roman θ (Table 3), though the contrast is lessened quantitatively. Three factors dampened the contrast. First, the overall frequency of foreign warfare over the two millennia of imperial China was much higher than that in the Qin-Han dynasties and became similar to that for the Romans (16.7 battles per century for China versus 17.6 per century for Rome, Tables 3 and 9). Second, some peripheral peoples who fought the Chinese in high frequencies had under their control some cities, mostly captured from the Chinese.[24] Whereas, most of the wealthy adversaries of the Romans were gradually replaced by indigent tribes, and the record of city populations for the Persians, the only remaining wealthy adversary, was sparse. Third, a single, protracted war affected the data significantly. During the 46 years of Mongolian invasion to southern China, 1234–1279, the Mongols waged against the Chinese 33 campaigns, which accounted for 1/10 of the total number of foreigns battles that the Chinese had fought from 221 BCE to 1800 CE. Such a high frequency of warfare with the Mongols, coupled with the city population in northern China captured by them during the invasion, increased the θ for imperial China by 12%.

[24] In the period 316–581, the northern Chinese cities Chang'an (长安), Datong/Pingcheng (大同／平城), Luoyang (洛阳) and Ye (邺城) were captured by barbarians and eventually became political centers of a Sienpi state (detailed in §5.5). Hence I count them toward the wealth of the Sienpi. Similarly, I count Youzhou (幽州), Jinzhou (锦州) and Shangjing (上京, present-day 赤峰) as Khitan cities in the period 1000–1100, after which the present-day Beijing as a Jürchen city until 1215 and then a Mongol city until 1279, when the Mongols completed their conquest of China to become the ruler of the core.

5.4 Periodwise Contrast and the Diocletian-Constantine Reform

We can dissect each empire into two periods and compare the values of θ for the same empire between periods. For Rome I separate the two periods by the year 192, the end of the Antonine dynasty, which ended the first declining stage of the empire.[25] For China the two periods are divided by the year 1279, which marked the completion of the Mongolian conquest of China and, according to many,[26] the beginning of the slow decline of China.

Table 10 in Appendix E shows that the θ for the Roman empire halved from 0.2992 in the early period to 0.1475 in the later period. Consequently, according to Proposition 2, the Diocletion-Constantine reform in the later period, which deviated significantly from the previous Roman institution (Table 2), can be understood as the empire's response to the changed environment. Table 11 in Appendix E shows that the θ for imperial China decreased only slightly from 0.2034 earlier to 0.1936 later, indicating an environment more stable than that for the Romans.

As in the contemporary contrast (§5.2), the early period θ for China was significantly lower than the early period θ for Rome, consistent with the observation that the two empires started by heading to opposite directions, delegation in Rome and centralization in China. In the later period, however, the θ for Rome fell below the Chinese counterpart while the degree of centralization in the later Roman empire did not exceed that in China, though qualitatively the later Roman institution did evolve towards the Chinese direction. Such discrepancy may be ascribed to the institutional legacy of the early Roman empire, which is outside our model but will be discussed further in the Conclusion.

5.5 Military Empowerment in the Sui-Tang Dynasties

The Sui-Tang dynasties, whose early stage shined with such extraordinary military vigor unsurpassed in Chinese history, were built upon a Sinicized barbarian state in the north. After the Han dynasty, the Chinese empire was disrupted by over three centuries of partition. Most of the nobility took refuge in the south, leaving the north to be rampaged by various

[25] Gibbon [16, v6, p647].

[26] For example, Chen YinKe (陈寅恪) [7]. The Mongolian invasion made such a deep cut to the empire, with persistence and totality on both sides and the Masada-like extinction of the Chinese army and royal house in the end, that a late Ming poet lamented that after the final battle at YaShan (崖山) between the two sides the land was China no more (钱谦益《后秋兴之十三》: "海角崖山一线斜，从今也不属中华").

barbarian tribes. A Sienpi state founded in 386 CE eventually united the north and adopted the Chinese language and meritocratic bureaucracy. It was named North Wei initially and renamed successively, by one usurping general after another, up to Sui, which reunified the empire in 589 CE and soon became the Tang dynasty, established by a noble house of Sui.

To the founding emperors of the Sui-Tang dynasties, brought up in the north and partially of barbarian extraction,[27] the wealthy south would have been part of the periphery. Table 12 in Appendix E shows that the θ for the Sienpi state, which these emperors came from, was 0.2841, almost as high as the early Roman empire (Table 10, Appendix E). Thus, the relatively high degree of military delegation in this period demonstrated in Table 2 can be understood as the institutional response to the environment familiar to these northern emperors in accord with Proposition 2. Frontier governance was entrusted to generals, whose armies prevailed in Vietnam, Korea, Mongolia and for quite a while Central Asia. But eventually the wealthy south became internalized, and the north and west periphery too indigent or tough to provide loots for the troops.[28] That suppressed the value of θ back down, rendering the high degree of military delegation socially suboptimal. Lasting for about 170 years, the glorious period was followed by 150 years of numerous mutinies and devastating rebellions, which (albeit interrupted by a reunification during the 15-year reign of emperor XianZong) eventually disintegrated the empire into warring states.

5.6 Military Disability in the Qing Dynasty

Up to the the transition from the Ming dynasty to Qing, the last dynasty of the empire, the Chinese military as a whole could still resist invasions from the West and Japan. The Ming army dispelled the Portuguese from the Cantonese coast around 1520 and repelled the Japanese from Korea in 1590s. The Ming navy, reinforced by the Fukien pirates, defeated the

[27] The founder of the Sui dynasty had adopted a Sienpi surname and become a powerful general in the Sienpi state North Zhou before he usurped its throne. The first emperor of the Tang dynasty after the short-lived Sui was born to a house whose members had been generals of Sienpi states for generations. His maternal grandfather was a Sienpi general. The second emperor of Tang, the celebrated warrior-emperor Tang TaiZong (唐太宗), was son of a granddaughter of the founder of the Sienpi state North Zhou.

[28] To reward soldiers, the Tang empire's reliance on loots from the enemies (因粮于敌) gradually gave way to domestic funding, as noticed by 贾志刚 [5, pp48–50]. The empire's westward expansion peaked at the Battle of Talas (怛罗斯战役) in 751 CE, where a Tang army was vanquished by the Abbasid Caliphate (黑衣大食), though before long a Tang general 封常清 scored a couple of victories in the region.

Dutch at sea in 1633. A naval spinoff from the collapsed Ming, again reinforced by pirates, drove the Dutch out of Taiwan in 1661. And finally the Qing army kept the Russians out of southeast Siberia in the border conflicts during 1652–1689. But in 150 years the Chinese military became defunct. The Qing army was decisively defeated in all but three of the twenty-three battles against the West and Japan from 1839 till the empire's end. And the humiliation culminated in a war between Russia and Japan fought in Manchuria, the home base of the Qing royal house, with the Qing army not even a participant.

What happened in those 150 years, with the military capable before and defunct after, was that the empire was spoiled by unprecedented border tranquility. Among the two main adversaries of the Ming empire, the Mongols and the Manchus (called Jürchens in earlier periods), the former had degenerated into small, divided tribes after two centuries of wars against the Ming empire, and the latter, the survivor, had now usurped the empire and become her residual claimant. Hence the θ in the early Qing period in Table 13, Appendix E, became 0.1148, much lower than the overall θ in the second millennium of imperial China.[29]

Thus, by Proposition 2, power was further curtailed from the military. Military officers were given tiny divided authorities that overlapped with those of the others, and the Manchu army degenerated into a hereditary welfare system for Manchu descendants to live on subsidies.[30] During the transition from Ming to Qing, the seas in east Asia were mainly controlled by the pirates, mostly Chinese from coastal regions, who had fought for the empire and defeated Europeans at sea[31] despite the latter's initial firearms advantage due to the military revolution in the West.[32] Instead of recruiting them and delegating to them naval defense, which the Roman emperors (e.g., Theodosius I) would have done, the Qing empire treated the pirates as enemies and, in order to eradicate their home base, banned coastal

[29] The Qing dynasty still had a higher θ than the early stage of the Chinese empire, the Qin-Han dynasties (Table 4), whose degree of centralization was arguably much lower than that in Qing. Such a difference, unexplained in our model, might be ascribed to the historical inertia that the Qin-Han empire builders had to confront. To the pre-imperial Chinese who had flourished for at least eight centuries of original thinkers and diverse heroes under a decentralized, feudal system, centralized governance over a vast landmass might have been as radical an idea as replacing the European Union with a centralized regime.

[30] 金普森 and 姚杏民 [6, pp91–92].

[31] During the collapse of the Ming dynasty, a pirate leader Zheng ZhiLong (郑芝龙) defeated the Dutch for the empire and his son Zheng ChengGong (郑成功, known as Koxinga in the West) led an obstinate resistance against the Manchus and reclaimed Taiwan from the Dutch.

[32] Parker [25].

Chinese from even going to the sea.[33]

Upon the onslaught of foreign invasions starting from 1839, the empire did try to learn the modern military technology, sending students overseas and building up a navy largest in east Asia. In history, it was not uncommon for a technologically disadvantaged nation to quickly master a new technology thereby leveling the playing field. The Mongols, adopting Arabian cannons and building up naval units, eventually conquered the Song empire that was advanced in firearms and water battles back then. The Manchu ancestors of the Qing royal house, learning firearms from defecting Ming soldiers, turned the tables on the firearm-equipped Ming empire in less than two decades. In three decades the Japanese transformed their military from a disadvantaged one like the Qing army to a modern one that vanquished Qing's larger navy and defeated the Russians. Yet the Qing empire failed to join the list of quick learners.[34] Time was not enough to accomplish a radical turnaround of a military disempowerment system that was optimal for most of the past two millennia.

6 Conclusion

Presented above are a tractable model to explain the allocation of political power between the military and civilians, an index of the parameters that determines the socially optimal degree of military delegation, and historical data of the Roman and Chinese empires, consistent with the comparative statics of the optimum with respect to the index.

With military defense an essential function of an empire, our results might help to reconcile the debate over the "great divergence" between China and the West. Claims of the relatively advanced technology and economy in ancient China[35] do not necessarily contradict her recent backwardness,[36] because instead of being caused by inferior technology or economy the recent backwardness could simply be a consequence of her persistent centralization institution to counterbalance the potential threat of her own military; such an institution,

[33] The most reconciliatory policy undertaken by the empire occasionally was to offer pirates amnesty (招安), whereby the pirate leader retired in peace with nominal titles and his followers were disbanded and supposed to retire as peasants, though they often resumed their pirate careers (陈钰祥 [8, pp122–123]).

[34] See Sng and Moriguchi [29] for an explanation, based on geographical sizes and the cost of tax collection, for the difference between the state capacities of China and Japan during the Ming and early Qing dynasties.

[35] For example see volumes after volumes due to Needham [23], as well as Pomeranz [27] and Edwards [12].

[36] China's backwardness has been ascribed to institutional or endogenous aspects by Parente and Prescott [24], Mokyr [20], and Li and van Zanden [19].

from the perspective of Proposition 2, can be seen as a best response to her environment, relatively stable in the past two millennia, where most peripheral adversaries were indigent.[37]

Our notion of empires, abstracting away specifics such as cultures and political regimes, may be applicable to other forms of states.[38] The model also has the potential of being developed into a framework relevant to present-day international politics. It is conceivable to combine two copies of the model into a game of interacting empires, in each of which the military and civilians negotiate a social contract to best reply the other empire's choice.

While this paper concerns mainly the question how an empire is governed given the premise that she continues to be, the model has the potential to be extended, with dynamic elements incorporated, to explain the evolution and decline of an empire. An overlapping-generation framework where each generation confronts the mechanism-design problem modeled in this paper, with an additional constraint due to the institutional legacy from previous generations, could capture the path-dependent aspect of history. For example, given their different environments, the Roman and Chinese empires started out with different institutions, with the former governed mainly through the military and the latter through an intelligentsia bureaucracy. With the legions occupied by military affairs, local governance and cultures in the Roman provinces were left mostly intact. By contrast, the Chinese bureaucracy, with post-rotation across the empire and a uniform Confucian ideology, which the imperial court deliberately promoted to elevate the authority of the center, had mostly evened out various cultures by the end of her early stage, the Han dynasty.[39] The east-west divide within the Roman empire, left unchecked during the earlier period, became a most severe constraint in the later period for reformers such as Diocletian and Constantine, who attempted centralization in response to the inroads of indigent nations.[40] To govern the

[37] See Hoffman [17] for an alternative explanation, based on opportunities of learning-by-doing, for the recent military backwardness of imperial China.

[38] For example, one could imagine an argument to ascribe the institutional difference between France and Britain around 1800s, with the former more centralized than the latter, to their environmental difference: while the army of the land-based France needed only to march inward to usurp the state, the oceanward British navy would be hard-pressed to sail ashore and conquer its homeland.

[39] Finer [15, v1, p532] has a concise contrast between the Roman and Han empires on local governance.

[40] Christianity back then did not help to mend the divide. Whereas Confucianism owed its dominance to the imperial court, Christianity was independent of the empire because it survived despite imperial persecution. Such institutional legacy affected the different roles played by Confucianism and Christianity in their empires. In addition, when needed as a unifying force during the Constantinian dynasty, Christianity

diverging empire, the Roman emperor had to entrust half of it to some of his colleagues. Consequently, civil wars between the two halves interrupted the centralizing effort, which failed to keep pace with the widening divide amidst the deluge of indigent barbarians.

A A Model of Endogenous Warfare

Let us modify our basic model so that the occurrence of foreign warfare is endogenous: The wealth of the barbarian tribe, instead of a constant ω_b in the basic model, is either zero with probability π or ω_b with probability $1-\pi$. After the wealth of the empire and that of the barbarians have been realized, say $\boldsymbol{\omega}_a$ (zero or ω_a) for the empire and $\boldsymbol{\omega}_b$ (zero or ω_b) for the barbarians, the barbarian tribe decides, as an individual player, whether to invade the empire. The empire also chooses whether to invade the barbarians when her realized wealth is ω_a, but she cannot do so when her wealth is zero; furthermore, if the empire chooses to invade the barbarians, her wealth is reduced to $\delta\omega_a$, with $\delta \in (0,1)$ a parameter. Implicitly assumed in this setup is an asymmetry between the empire and the barbarians. While indigent barbarians are able to pillage the empire (e.g., the Goths displaced and impoverished by the Huns were able to ravage the Roman empire), the empire needs to spend a fraction of her wealth to mobilize an invasion to barbarian territories (e.g., the Ming dynasty amidst a decade-long famine was incapable of cracking down the Manchus).

Clearly the barbarians' strategy is to invade the empire if $f(x) < \boldsymbol{\omega}_a/(\boldsymbol{\omega}_a + \boldsymbol{\omega}_b)$ and not so if the inequality is reversed. Hence the empire invades the barbarians if $f(x) > \boldsymbol{\omega}_a/(\delta\boldsymbol{\omega}_a + \boldsymbol{\omega}_b)$ and not so if the inequality is reversed. Denote

$$\beta := \sup\left\{x \in [0,1] : f(x) \leq \frac{\omega_a}{\omega_a + \omega_b}\right\}, \tag{16}$$

$$\gamma := \sup\left\{x \in [0,1] : f(x) \leq \frac{\omega_a}{\delta\omega_a + \omega_b}\right\}. \tag{17}$$

A war between the empire and the barbarians occurs if and only if either (i) $\boldsymbol{\omega}_a = \omega_a$ and $\boldsymbol{\omega}_b = 0$, or (ii) $\boldsymbol{\omega}_a = \omega_a$ and $\boldsymbol{\omega}_b = \omega_b$ and $x \leq \beta$, or (iii) $\boldsymbol{\omega}_a = \omega_a$ and $\boldsymbol{\omega}_b = \omega_b$ and $x > \gamma$.[41] Thus, if the farmer of type c exerts efforts then the social surplus for the empire is equal to

was itself caught in bloody internal rivalries between the Trinitarians and Arians.

[41] When $f(x) = \boldsymbol{\omega}_a/(\boldsymbol{\omega}_a + \boldsymbol{\omega}_b)$ the expected wealth for the empire is the same whether barbarian invasion occurs or not. Likewise, when $f(x) = \boldsymbol{\omega}_a/(\delta\boldsymbol{\omega}_a + \boldsymbol{\omega}_b)$, the empire is indifferent between whether to invade barbarians or not. Hence it makes no difference to treat the borderline cases as war or peace.

$-c + p\tilde{\varphi}(x;\omega,\pi,\delta)$, where

$$\tilde{\varphi}(x;\omega,\pi,\delta) := \pi f(x)\omega_a + (1-\pi)\left(\mathbf{1}_{x\leq\beta} f(x)(\omega_a+\omega_b) + \mathbf{1}_{x>\gamma} f(x)(\delta\omega_a+\omega_b) + \mathbf{1}_{\beta<x\leq\gamma}\omega_a\right). \tag{18}$$

In the end, there are four possible levels of the total wealth for the empire, zero (poor harvest or defeat in a war), ω_a (good harvest and no war), $\omega_a+\omega_b$ (good harvest and victory against invading barbarians) and $\delta\omega_a+\omega_b$ (good harvest and victory upon invading barbarians). Correspondingly, an allocation between the farmer and the soldier specifies the soldier's contingent payoffs $z \in [0,\omega_a]$, $y_1 \in [0,\omega_a+\omega_b]$ and $y_2 \in [0,\delta\omega_a+\omega_b]$. As before, the revolt-proof principle holds and, through calculating the supremum farmer-type willing to exert efforts, one can show that there is no loss of generality to assume binding revolt-proof constraints, i.e., $f(x)y_1 = f(x)g(x)(\omega_a+\omega_b)$, $f(x)y_2 = f(x)g(x)(\delta\omega_a+\omega_b)$ and $z = g(x)\omega_a$. Thus, this supremum farmer-type is equal to $p(1-g(x))\tilde{\varphi}(x;\omega,\pi,\delta)$. The empire's expected total surplus is therefore equal to

$$\int_0^{p(1-g(x))\tilde{\varphi}(x;\omega,\pi,\delta)} \left(-c + p\tilde{\varphi}(x;\omega,\pi,\delta)\right) dH(c). \tag{19}$$

As before, existence and characterization of the social optimum are at hand. The comparative statics retains part of the previous feature, as part (a) of the next proposition presents a case where the optimal degree of military delegation is strictly increasing in π, the probability of poor harvest for the barbarians. Part (b) of the proposition, interestingly, says that if the barbarians are sufficiently wealthy and invading them is sufficiently costly for the empire, then at optimum the probability for the empire to defeat the barbarians is proportional to her wealth relative to the total wealth of the two. Thus arising endogenously is the effect of wealth on the outcome of a war.

Proposition 3 *Assume all the hypotheses of Proposition 2, with Assumption 2 replaced by* $c_* \geq p\left(\omega_a + (1-\pi)\omega_b\right)$, *and that* $f'' < 0 < g''$. *Then the optimal degree of military delegation, denoted* $\tilde{x}$, *has the following properties:*

a. For all sufficiently large π,

i. if δ *is sufficiently small,* $\tilde{x}$ *is strictly increasing in* π;

ii. if δ *is sufficiently large,* $\tilde{x}$ *jumps to a level constant to parameters* $(\omega_a,\omega_b,\pi,\delta,p)$.

b. If π *and* δ *are sufficiently small and* ω_b/ω_a *sufficiently large,* $f(\tilde{x}) = \omega_a/(\omega_a+\omega_b)$.

Proof With H uniform on $[0, c_*]$, the empire's social surplus is equal to $\frac{p^2}{2c_*}\tilde{\varphi}(x;\omega,\pi,\delta)^2\left(1-g(x)^2\right)$ according to (19). Thus, as in Lemma 6, an optimum is equivalent to a maximum of

$$\tilde{V}(x;\omega,\pi,\delta) := 2\ln\tilde{\varphi}(x;\omega,\pi,\delta) + \ln\left(1-g(x)^2\right)$$

over all $x \in [0,1]$. Note the continuity of $\tilde{V}(\cdot;\omega,\pi,\delta)$ at the cutoffs β and γ. By Eq. (18),

$$\frac{\partial}{\partial x}\tilde{V}(x;\omega,\pi,\delta) = \begin{cases} \frac{2f'(x)}{f(x)} - \frac{2g(x)g'(x)}{1-g(x)^2} & \text{if } x<\beta \text{ or } x>\gamma \\ \frac{2f'(x)}{f(x)+(1-\pi)/\pi} - \frac{2g(x)g'(x)}{1-g(x)^2} & \text{if } \beta<x<\gamma. \end{cases} \tag{20}$$

As in Lemma 5, there exist $x^*, x_* \in (0,1)$ such that

$$\frac{f'(x^*)}{f(x^*)} - \frac{g(x^*)g'(x^*)}{1-g(x^*)^2} = 0 = \frac{f'(x_*)}{f(x_*)+(1-\pi)/\pi} - \frac{g(x_*)g'(x_*)}{1-g(x_*)^2}.$$

With $f'' < 0 < g''$, the two branches on the right-hand side of Eq. (20) are each strictly decreasing in x, hence x^* and x_* are each uniquely defined by the above equation, which also implies that $x_* < x^*$, x^* constant, and x_* strictly increases in π.

To prove Claim (a), suppose π is large enough for

$$\frac{f'(\beta)}{f(\beta)+(1-\pi)/\pi} - \frac{g(\beta)g'(\beta)}{1-g(\beta)^2} > 0. \tag{21}$$

Then $\beta < x_*$ by the definition of x_*. From Eqs. (18) and (20) we see the shape of $\tilde{V}(\cdot;\omega,\pi,\delta)$: As x increases from zero, $\tilde{V}(x;\omega,\pi,\delta)$ increases until x reaches a cutoff $\min\{\gamma, x_*\}$ (which is above β). Now either (I) $\min\{\gamma,x_*\} = x_* < \gamma$ or (II) $\min\{\gamma,x_*\} = \gamma$. To prove Claim (a.i), suppose δ is small enough for $(1-\delta)\omega_a \geq \omega_b$; then $\gamma = 1$ by Eq. (17). Hence we are in Case (I) and the lower branch of Eq. (20) applies throughout $(x_*, 1)$, so $\tilde{V}(\cdot;\omega,\pi,\delta)$ is strictly decreasing on $[x_*, 1]$. Thus, $\tilde{x} = x_*$ and Claim (a.i) is proved since x_* is strictly increasing in π. To prove Claim (a.ii), suppose δ is so large that $\gamma \leq x_*$ (which is not vacuous, because $\beta < x_*$ and, by Eqs. (16)–(17), $\gamma \to \beta$ as $\delta \to 1$). Then we are in Case (II) and the upper branch of Eq. (20) applies over $(x_*, 1)$. Thus, after x has reached x_*, $\tilde{V}(x;\omega,\pi,\delta)$ still keeps increasing (since $\gamma < x_* < x^*$) until x reaches x^*, after which $\tilde{V}(x;\omega,\pi,\delta)$ keeps decreasing. Hence $\tilde{x}$ is equal to x^*, a constant above x_*, so Claim (a.ii) is proved.

To prove Claim (b), suppose π is small enough for Ineq. (21) to be reversed, δ small enough for $(1-\delta)\omega_a \geq \omega_b$, and ω_b/ω_a large enough for $\frac{f'(\beta)}{f(\beta)} - \frac{g(\beta)g'(\beta)}{1-g(\beta)^2} > 0$ (which holds for all small β because $f'/f - gg'/(1-g^2)$ is strictly decreasing by the assumption $f'' < 0 < g''$, and β is small for all large ω_b/ω_a by Eq. (16)). Then $x_* < \beta < x^* < 1 = \gamma$. Thus, by (20), $\tilde{V}(\cdot;\omega,\pi,\delta)$ is strictly increasing on $[0,\beta]$ (since $\beta < x^*$) and strictly decreasing on $[\beta, 1]$ (since $x_* < \beta$ and $\gamma = 1$). Hence $\tilde{x}$ equals β, which by Eq. (16) is $f^{-1}(\omega_a/(\omega_a+\omega_b))$. ■

B Criteria for Tallying the Imperial Succession

Table 6 tallies the emperors and those among them who owed their accession to the military and those to their bloodline hereditary status, as well as the top generals executed by the emperors not during civil-war periods. To count them for each empire I choose the period to reflect the imperial institution while the empire represented the core of a civilization. Hence the Roman empire is counted from her foundation in year 27 BCE to the death of the western empire in 476 CE, and the Chinese empire from the foundation year 221 BCE to the abdication of the last emperor in 1912 CE. To count Roman emperors I take the Senate's endorsement as the criterion.[42] To count Chinese emperors, the criterion I take is the status of reigning over a dominion such that (i) the majority in the dominion was Chinese, (ii) there was no major Chinese state outside the dominion, and (iii) the institution within the dominion was stable, with Confucianism the dominant ideology.

Table 6: **Counting the Emperors**

	Roman empire	Later Roman empire	Chinese empire	Qin-Han dynasties	Sui-Tang dynasties
Number of emperors	60	18	100	31	25
Military accession	37	9	20	6	8
Hereditary succession	20	9	91	27	21
Top Generals executed by emperors	6	4	21	5	4

Source: Table 7 in Appendix D, with "Later Roman empire" from the row "Diocletian" to the end of the table; Table 8 in Appendix D, with "Qin-Han dynasties" from rows "Qin ShiHuang (秦始皇)" to "Han XianDi (汉献帝)", and "Sui-Tang dynasties" from rows "Sui WenDi (隋文帝)" to "Tang AiDi (唐哀帝)".

For most of the lifespan of imperial China, the emperor can be identified easily as the supreme ruler of the unified entity. For the other periods the three conditions listed above help to reduce the degree of arbitrariness. Condition (ii) rules out the self-proclaimed emperors during civil-war periods such as the Three-Kingdom (三国) period after Han and the Five-Dynasty Ten-State (五代十国) period after Tang. In the period of north-south partition (南北朝) from East Jin (东晋) dynasty until the reunification of Sui (隋), the

[42] When the Roman empire was divided in the Theodosian dynasty, both the eastern and western emperors are counted because they were colleagues rather than rivals and the two halves still constituted the core.

self-proclaimed emperors in the barbarian-overrun north are screened out by conditions (i) and (ii), and those in the south, which witnessed quick turnover of mini-dynasties and was reshaped by the not yet Sinicized Buddhism, are screened out by condition (iii). Condition (i) also rules out the khans during the Mongolian occupation.[43] By contrast, the Southern Song emperors are counted, as all three conditions are met even though during their reign a big part of the traditionally Chinese territory was occupied by peripheral peoples.

C Proofs of Corollaries 1 and 2 and Lemma 5

Corollary 1 By Proposition 1, any optimum x satisfies (7), hence

$$g(x) \geq 1 - \frac{c_*}{p\varphi(x;\omega,\pi)} \geq 1 - \frac{c_*}{p(1-\pi)\omega_a} \longrightarrow 1 \quad \text{as} \quad c_* \to 0.$$

Thus, with g strictly increasing by Assumption 1, $x \to 1$ as $c_* \to 0$. ■

Lemma 5 Let $x^* \in [0,1]$ be a socially optimal degree of military delegation. By Eq. (11),

$$\frac{\partial}{\partial x}U(x)\bigg|_{x=0} = p\varphi'(0)H[p\varphi(0)] = p\pi(\omega_a+\omega_b)f'(0)H[p(\pi(\omega_a+\omega_b)f(0)+(1-\pi)\omega_a)] > 0$$

since $g(0) = 0$ and $g(1) = 1$ (Assumption 1), hence $x^* \neq 0$. Analogously, $\frac{\partial}{\partial x}U(x)\big|_{x=1} = -p^2\varphi(1)h[0]g'(1)\varphi(1) < 0$ since $g(1) = 1$ and $g'(1) > 0$ (Assumption 3), hence $x^* \neq 1$. ■

Corollary 2 By Lemma 5 and the comment preceding it, for any optimal degree x, we have $x > 0$, hence $f(x) > 0$ and $\frac{\partial}{\partial x}U(x;\omega,\pi,p) = 0$. Factoring out from this equation the strictly positive term $p\varphi(x)H\left(p(1-g(x))\varphi(x)\right)$, we obtain Eq. (12), hence Claim (i). To prove Claim (ii), expand the left-hand side of Eq. (12) and note $\frac{\partial}{\partial x}\ln\varphi(x;\omega,\pi) > 0$, $g(x) > 0$ and $h > 0$. Claim (iii) follows from Lemma 4 and the fact $f(x) > 0$. ■

[43] During the 89 years of Mongolian occupation, China was only part of a much larger dominion of the Mongols, which after its disintegration never got back to one entity, and the Chinese were treated as the bottom-class subjects, with the Mongols on the top and central/western Asians in the middle. Had this period been counted as a dynasty of China, the statistics would not have changed much from Table 1: the percentage of military accession among Chinese emperors would be 21.6%, that of hereditary succession 85.6%, and the ratio of executed generals over emperors would still be at least 18.9%. There were eleven khans during the occupation. Four of them ascended the throne through military conquests, coups or assassinations: Kublai (忽必烈), Külüg (武宗), Yesün Temür (泰定帝) and Jayaatu (文宗). While all became final contenders due to hereditary status, only four of them, Ayurbarwada Buyantu (仁宗), Gegeen (英宗), Rinchinbal (宁宗) and Toghon Temür (顺帝), ascended the throne without serious contention within the royal family.

D Online Appendix: The Imperial Succession

Table 7: **Succession of Roman Emperors**

Emperor	dynasty/period	military acces'n	hereditary succession	generals executed	kins slayed	killed/ suicide
Augustus	Julio-Claudian	1	0	0	0	0
Tiberius	Julio-Claudian	1[44]	0	0	0	0
Caligula	Julio-Claudian	0	1	0	0	1
Claudius	Julio-Claudian	1[45]	1	0	0	0
Nero	Julio-Claudian	0	1	1[46]	1	1
Galba	Yr of 4 Emp'rs	1	0	0	0	1
Otho	Yr of 4 Emp'rs	1	0	0	0	1
Vitellius	Yr of 4 Emp'rs	1	0	0	0	1
Vespasian	Flavian	1	0	0	0	0
Titus	Flavian	1[47]	1	0	0	0
Domitian	Flavian	1[48]	1	0	0	1
Nerva	Nerva-Antonine	0	0	0	0	0
Trajan	Nerva-Antonine	1	0	0	0	0
Hadrian	Nerva-Antonine	0	0	1[49]	0	0
Antoninus Pius	Nerva-Antonine	0	0	0	0	0
Marcus Aurelius	Nerva-Antonine	0	0	0	0	0
Commodus	Nerva-Antonine	0	1	0	0	1
Pertinex	Yr of 5 Emp'rs	1	0	0	0	1
Didius Julianus	Yr of 5 Emp'rs[50]	1	0	0	0	1
Septimius Severus	Severan	1	0	0	0	0

Continued on next page

[44] Albeit a Claudian, Tiberius was selected to be the successor because he was the only capable general surviving during the old age of Augustus (Ferrero [13, v5, pp222-226]).

[45] Claudius was selected by the Praetorian Guard soldiers while the Senate was deliberating over the successors, with some hope, indicated by their honoring the assassin of Caligula, of restoring the old republic (Ferrero and Barbagallo [14, pp161–162]).

[46] General Gnaeus Domitius Corbulo.

[47] Upon his victory in Judea, Titus was hailed imperator by the soldiers and feared to rebel. His father, Vespasian, accommodated him with a triumph and power sharing.

[48] Upon the death of Titus, Domitian went to the Praetorian Guard and was declared emperor there.

[49] Shortly after Hadrian's accession was the execution of Lusius Quietus, an accomplished general under Trajan, and other three former consuls, on a vague charge of conspiracy by a secret court.

[50] Immediately after Didius Julianus bought the throne from the Praetorian Guard soldiers (via an ascending-bid open-outcry auction), Pescennius Niger was proclaimed emperor by the legions in Syria,

Table 7 – *Continued from previous page*

Emperor	dynasty/period	military acces'n	hereditary succession	generals executed	kins slayed	killed/ suicide
Caracalla	Severan	0	1	0	1	1
Macrinus	Severan	0	0	0	0	1
Elagabalus	Severan	1	0	0	0	1
Alexander Severus	Severan	1[51]	1	0	0	1
Maximinus Thrax	Crisis3rdCent'ry	1	0	0	0	1
Pupienus	Crisis3rdCent'ry	0	0	0	0	1
Balbinus	Crisis3rdCent'ry	0	0	0	0	1
Gordian III	Crisis3rdCent'ry	1[52]	0	0	0	1
Philip the Arab	Crisis3rdCent'ry	1	0	0	0	1
Decius	Crisis3rdCent'ry	1	0	0	0	1
Trebonianus Gallus	Crisis3rdCent'ry	1	0	0	0	1
Aemilianus[53]	Crisis3rdCent'ry	1	0	0	0	1
Valerian	Crisis3rdCent'ry	1	0	0	0	1
Gallienus	Crisis3rdCent'ry	1	1	0	0	1
Claudius Gothicus	Crisis3rdCent'ry	1	0	0	0	0
Quintillus	Crisis3rdCent'ry	1[54]	0	0	0	0
Aurelian	Crisis3rdCent'ry	1	0	0	0	1
Marcus Tacitus	Crisis3rdCent'ry	0	0	0	0	0
Marcus Probus	Crisis3rdCent'ry	1	0	0	0	1
Carus	Crisis3rdCent'ry	1	0	0	0	0

Continued on next page

Clodius Albinus by the armies in Britain and Gaul, and Septimius Severus by the troops in Illyricum and Pannonia. The former two were sequentially defeated and killed in civil wars with Severus.

[51] With soldiers upset by emperor Elagabalus's religious and sexual eccentricities, the emperor was pressured to name his cousin Alexander Severus as heir and share power with him. When Elagabalus recanted, stripped the title from Alexander and spread rumors of his death, both were summoned by the Praetorian Guard to their camp. The soldiers hailed Alexander emperor and killed Elagabalus and his mother.

[52] After killing Emperors Pupienus and Balbinus, the Praetorian Guard hailed Gordian III as the emperor.

[53] While his reign was short and his name absent in Gibbon's [16, Appendix II, v6, p658] list of emperors, the rise and fall of Aemilianus were typical in that era. A commander at the Danube frontier, Aemilianus was proclaimed emperor by his soldiers upon defeating a Gothic invasion. The Senate concurred after the incumbent emperor Gallus was murdered by his own soldiers, attracted by Aemilianus's offer of pay increases. When a larger force led by Valerian closed in, however, Aemilianus was in turn killed by his own soldiers (Gibbon [16, v1, pp280–281]).

[54] Immediately after Claudius Gothicus's death, Quintillus assumed emperorship while commanding a considerable force, and the Senate concurred.

Table 7 – *Continued from previous page*

Emperor	dynasty/period	military acces'n	hereditary succession	generals executed	kins slayed	killed/ suicide
Carinus	Crisis3rdCent'ry	0	1	0	0	1
Numerian	Crisis3rdCent'ry	0	1	0	0	1
Diocletian	Tetrarchy	1	0	0	0	1
Constantius Chlorus	Constantinian	1	0	0	0	0
Constantine I	Constantinian	1	0	1[55]	2	0
Constantine II	Constantinian	0	1	0	0	1
Constans	Constantinian	0	1	0	1	1
Constantius II	Constantinian	0	1	0	9	0
Julian	Constantinian	1	0	0	0	1
Jovian	Constantinian	1[56]	0	0	0	0
Valentinian I	Valentinian	1	0	0	0	0
Valens I	Valentinian	0	1	0	0	1
Gratian	Valentinian	0	1	1[57]	0	1
Valentinian II	Valentinian	1[58]	0	0	0	1
Theodosius I	Theodosian	1	0	0	0	0
Arcadius	Theodosian	0	1	0	0	0
Honorius	Theodosian	0	1	1[59]	0	0
Theodosius II	Theodosian	0	1	0	0	0
Valentinian III	Theodosian	1[60]	0	1[61]	0	1[62]

Continued on next page

[55] It was Constantine's oldest son Crispus, well-recognized as a successful general in foreign and civil wars.

[56] When Emperor Julian died, Jovian was the commander of the imperial bodyguard and, possibly misidentified with more illustrious figures, was elected by the army to be the next emperor.

[57] Theodosius the Elder, the general who restored Britain and Africa for the empire.

[58] When Valentinian I died in the camp, his generals, despite the fact that the older son Gratian had been entitled Augustus, acclaimed as Augustus the four-year old, hence manipulable, Valentinian II.

[59] Master general Stilicho.

[60] Upon the death of Honorius, the eastern emperor Theodosius II installed Valentinian III as the western emperor by naming him Caesar of the west and defeating the usurper Joannes.

[61] After the death of Attila and consequently the end of the threat from the Huns, Valentinian III killed Aëtius in the court and was soon assassinated by two Hunnish followers of Aëtius.

[62] After Valentinian III died, the throne of the western empire was briefly assumed by a senator Petronius Maximus before he was stoned to death by a mob when the Vandal king Genseric was about to capture Rome. Genseric sacked Rome. Then the Visigoth king Theodoric II and Gallic chiefs proclaimed Avitus as emperor, who was dethroned by rebelling generals Ricimer and Majorian. With Ricimer of Gothic origin and intending to be the power behind the throne, Majorian became the next emperor and carried out a series of reforms to restore the western empire. His effort offended the aristocrats and got him killed by Ricimer,

Table 7 – *Continued from previous page*

Emperor	dynasty/period	military acces'n	hereditary succession	generals executed	kins slayed	killed/ suicide
Pulcheria/Marcian	Theodosian	0	1[63]	0	0	0
Total: 60 emperors		37	20	6	14	35

Table 8: **Succession of Chinese Emperors**

Emperor	dynasty	military acces'n	hereditary succession	generals executed	kins slayed	killed/ suicide
Qin ShiHuang (秦始皇)	Qin (秦)	1[64]	1	0	0	0
Qin ErShi (秦二世)	Qin (秦)	0	1	1[65]	1	1
ZiYing (秦王子婴)	Qin (秦)	0	1	0	0	0
Han GaoZu (汉高祖)	Han (汉)	1	0	4[66]	0	0
HuiDi (汉惠帝)[67]	Han (汉)	0	1	0	0	0
ShaoDi (少帝刘弘)	Han (汉)	0	1	0	0	1
WenDi (汉文帝)	Han (汉)	1[68]	1	0	0	0

Continued on next page

who then installed Libius Severus as a puppet emperor until Severus died. With the consent of Ricimer, the eastern emperor Leo I sent to the throne his officer Anthemius, who was killed by Ricimer in a power contest. Then Olybrius, backed by the Vandal king Genseric and consented by Ricimer, assumed the throne briefly but soon died of illness. As Ricimer also died, his nephew Gundobad, the new master general, put Glycerius to the throne. Regarding Glycerious as a usurper, the eastern imperial court named Julius Nepos, ruler of Dalmatia, as the western emperor, who invaded Italy and dethroned Glycerius. But soon he was forced to flee back to Dalmatia by his own master general Orestes. Not a Roman citizen himself, Orestes appointed his 12-year old son Romulus Augustulus as the western emperor. Within a year, a foreign mercenary revolt led by Odoacer killed Orestes and forced Romulus, the last so-called Western Emperor, to abdicate.

[63] Pulcheria was an elder sister of the eastern emperor Theodosius II, who was under her regency during his minority, when she was declared as Augusta. After Theodosius II died, Pulcheria came to power and married Marcian, a general and senator, thereby making him emperor of the eastern empire.

[64] Qin ShiHuang inherited the throne of the Qin kingdom and only after his military conquest of the other kingdoms did he proclaim himself Emperor the First ("ShiHuang").

[65] General Meng Tian (蒙恬).

[66] After reuniting the empire, Han GaoZu executed, through arrests or wars, most of the generals to whom he had awarded kingdoms, including Han Xin (韩信), Peng Yue (彭越), Ying Bu (英布) and Chen Xi.

[67] Puppet of Dowager Lü Hou (吕后), so was the next emperor, ShaoDi.

[68] After Dowager Lü Hou died, a coalition of ministers, generals and members of the royal family launched a coup, cleansed the offsprings of her paternal family, dethroned and executed Emperor ShaoDi, and then

Table 8 – *Continued from previous page*

Emperor	dynasty	military acces'n	hereditary succession	generals executed	kins slayed	killed/ suicide
JingDi (汉景帝)	Han (汉)	0	1	0	0	0
Han WuDi (汉武帝)	Han (汉)	0	1	0	2	0
ZhaoDi (汉昭帝)	Han (汉)	0	1	0	0	0
Liu He (刘贺)	Han (汉)	0	1	0	0	0
XuanDi (汉宣帝)	Han (汉)	0	1	0	0	0
YuanDi (汉元帝)	Han (汉)	0	1	0	0	0
ChengDi (汉成帝)	Han (汉)	0	1	0	0	0
AiDi (汉哀帝)	Han (汉)	0	1	0	0	0
PingDi (汉平帝)	Han (汉)	0	1	0	0	1
RuZi Ying (孺子婴)	Han (汉)	0	1	0	0	1
Wang Mang (王莽)	Xin (新)	1[69]	0	0	0	1
GuangWu Di (光武帝)	Han (汉)	1	0	0	0	0
MingDi (汉明帝)	Han (汉)	0	1	0	0	0
ZhangDi (汉章帝)	Han (汉)	0	1	0	0	0
HeDi (汉和帝)	Han (汉)	0	1	0	0	0
ShangDi (汉殇帝)	Han (汉)	0	1	0	0	0
AnDi (汉安帝)	Han (汉)	0	1	0	0	0
ShunDi (汉顺帝)	Han (汉)	0	1	0	0	0
ChongDi (汉冲帝)	Han (汉)	0	1	0	0	0
ZhiDi (汉质帝)	Han (汉)	0	1	0	0	1
HuanDi (汉桓帝)	Han (汉)	0	1	0	0	0
LingDi (汉灵帝)	Han (汉)	0	1	0	0	0
XianDi (汉献帝)	Han (汉)	1[70]	1	0	0	0
Jin WuDi (晋武帝)	Jin (晋)	1[71]	0	0	0	0
Jin HuiDi (晋惠帝)	Jin (晋)	0	1	0	0	1

Continued on next page

selected the ministers selected the oldest surviving son of Han GaoZu as the successor, known as WenDi.

[69] Member of a powerful family and nephew of the dowager, Wang Mang became the master general of the empire and took the throne from the puppet emperor RuZi Ying.

[70] Upon the death of Emperor LingDi, the ministers cleansed the eunuchs through inviting the help from a general Dong Zhuo (董卓). Quickly establishing his despotism in the capital, Dong Zhuo elevated XianDi as the puppet emperor instead of the designated heir. Although the ministers managed to assassinate Dong Zhuo, he was replaced by a capable and intelligent general, Cao Cao (曹操), who eventually controlled the north of the disintegrated empire. After his death, Cao's son usurped the throne from Emperor XianDi.

[71] Born to a house that controlled the military of the kingdom established by general Cao's offsrpings, Jin WuDi usurped the throne and reunited the empire briefly, soon overrun by barbarians.

Table 8 – *Continued from previous page*

Emperor	dynasty	military acces'n	hereditary succession	generals executed	kins slayed	killed/ suicide
Sui WenDi (隋文帝)	Sui (隋)	1	0	0	0	0
Sui YangDi (隋炀帝)	Sui (隋)	0	1	2[72]	1	1
Tang GaoZu (唐高祖)	Tang (唐)	1	0	0	0	0
Tang TaiZong (唐太宗)	Tang (唐)	1[73]	1	0	2	0
GaoZong (唐高宗)	Tang (唐)	0	1	0	0	0
Wu ZeTian (武则天)	Tang (唐)	1[74]	0	0	0	0
ZhongZong (唐中宗)	Tang (唐)	1[75]	1	0	0	1
Tang ShangDi (唐殇帝)	Tang (唐)	0	1	0	0	0
RuiZong (唐睿宗)	Tang (唐)	1[76]	1	0	0	0
Tang XuanZong (唐玄宗)	Tang (唐)	0	1	0	5	0
SuZong 唐肃宗)	Tang (唐)	0	1	0	0	0
DaiZong (唐代宗)	Tang (唐)	0	1	0	0	0
DeZong (唐德宗)	Tang (唐)	0	1	0	0	0
ShunZong (唐顺宗)	Tang (唐)	0	1	0	0	0
XianZong (唐宪宗)	Tang (唐)	1[77]	1	2	0	1
MuZong (唐穆宗)	Tang (唐)	0	1	0	0	0
JingZong (唐敬宗)	Tang (唐)	0	1	0	0	1
WenZong (唐文宗)	Tang (唐)	0	1	0	0	0
WuZong (唐武宗)	Tang (唐)	0	1	0	0	0
XuanZong (唐宣宗)	Tang (唐)	0	1	0	0	0
YiZong (唐懿宗)	Tang (唐)	0	1	0	0	0
XiZong (唐僖宗)	Tang (唐)	0	1	0	0	0

Continued on next page

[72] They were 贺若弼 and Gao Jiong.

[73] A valorous soldier and brilliant general, Tang TaiZong was a driving force of his father's revolt and military accession. In a coup, he killed two of his brothers, including the heir apparent. His father was forced to acquiesce and abdicate the throne to him. Tang TaiZong completed the reunification of the empire.

[74] Wu ZeTian became the de facto ruler when her husband, Emperor GaoZong, was disabled by illness. After his death, she ruled through puppet emperors (her sons), cracked down several revolts led by members of the royal house, and proclaimed herself Empress.

[75] When Empress Wu ZeTian was sick, several ministers and royal guard officers, in fear of her passing the throne to offsprings of her paternal family, launched a coup and forced her to abdicate to the designated heir, ZhongZong.

[76] RuiZong's son Li LongJi (李隆基) launched a coup that elevated RuiZong to the throne, who in a year abdicated to Li LongJi, known as Tang XuanZong.

[77] XianZong ascended the throne since the eunuchs and commissioners pressured his father to abdicate.

Table 8 – *Continued from previous page*

Emperor	dynasty	military acces'n	hereditary succession	generals executed	kins slayed	killed/ suicide
ZhaoZong (唐昭宗)	Tang (唐)	0	1	0	0	1
Tang AiDi (唐哀帝)	Tang (唐)	1[78]	1	0	0	0
Song TaiZu (宋太祖)	Song (宋)	1	0	0	0	1[79]
TaiZong (宋太宗)	Song (宋)	0	1	0	4	0
ZhenZong (宋真宗)	Song (宋)	0	1	0	0	0
Song RenZong (宋仁宗)	Song (宋)	0	1	0	0	0
YingZong (宋英宗)	Song (宋)	0	1	0	0	0
ShenZong (宋神宗)	Song (宋)	0	1	0	0	0
ZheZong (宋哲宗)	Song (宋)	0	1	0	0	0
Song HuiZong (宋徽宗)	Song (宋)	0	1	0	0	1
Song QinZong (宋钦宗)	Song (宋)	0	1	0	0	1
Song GaoZong (宋高宗)	Song (宋)	0	1	1[80]	0	0
XiaoZong (宋孝宗)	Song (宋)	0	1	0	0	0
GuangZong (宋光宗)	Song (宋)	0	1	0	0	0
NingZong (宋宁宗)	Song (宋)	0	1	0	0	0
LiZong (宋理宗)	Song (宋)	0	1	0	0	0
DuZong (宋度宗)	Song (宋)	0	1	0	0	0
GongDi (宋恭帝)	Song (宋)	0	1	0	0	1
DuanZong (宋端宗)	Song (宋)	0	1	0	0	1
Song ShaoDi (宋少帝)	Song (宋)	0	1	0	0	1
HongWu (明太祖)	Ming (明)	1	0	10[81]	0	0

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[78] AiDi was erected by his father's killer, general Zhu Wen (朱温), who later took the throne from him.

[79] Song TaiZu died suddenly on a snowy night after a mysterious conversation with his brother TaiZong, who immediately ascended the throne. After his accession, TaiZong persecuted his emperor brother's two sons (赵德昭 and 赵德芳) and his other brother (赵廷美), who all died of the persecution. Evidence of the suspected assassination, however, is circumstantial at best.

[80] When the Jürchens sacked the capital Kaifeng and captured his brother and parents, Song GaoZong escaped, assumed the vacant throne, and eventually resettled the capital in the south. The Jürchens' further inroads were defeated by local armies led by generals such as Yue Fei (岳飞). Instead of attempting to recover the lost territory and revenge his family, the emperor opted for removing the military powers from those generals and signing a peace treaty with the Jürchens, whereby the Jürchens kept the north and the empire, or the remains thereof, secured herself in the south. Immediately before signing the peace treaty, the emperor executed Yue Fei, who was most vocally pro-war and had requested the emperor to designate an heir despite the norm that generals were not supposed to get involved in imperial succession.

[81] They were 廖永忠, 陆仲亨, 唐胜宗, 朱亮祖, 李文忠, 蓝玉, 周德兴, 傅友德, 王弼 and 冯胜.

Table 8 – *Continued from previous page*

Emperor	dynasty	military acces'n	hereditary succession	generals executed	kins slayed	killed/ suicide
JianWen (建文帝)	Ming (明)	0	1	0	0	0
YongLe (明成祖朱棣)	Ming (明)	1	1	0	0	0
HongXi (洪熙帝)	Ming (明)	0	1	0	0	0
XuanDe (宣德帝)	Ming (明)	0	1	0	1	0
ZhengTong (正统帝)	Ming (明)	0	1	0	0	0
JingTai (景泰帝)	Ming (明)	1[82]	1	0	0	0
ChengHua (成化帝)	Ming (明)	0	1	0	0	0
HongZhi (弘治皇帝)	Ming (明)	0	1	0	0	0
ZhengDe (正德帝)	Ming (明)	0	1	0	0	0
JiaJing (嘉靖帝)	Ming (明)	0	1	0	0	0
LongQing (隆庆帝)	Ming (明)	0	1	0	0	0
WanLi (万历帝)	Ming (明)	0	1	0	0	0
TaiChang (泰昌帝)	Ming (明)	0	1	0	0	0
TianQi (天启帝)	Ming (明)	0	1	0	0	0
ChongZhen (崇祯帝)	Ming (明)	0	1	1[83]	0	1
ShunZhi (顺治帝福临)	Qing (清)	1[84]	1	0	0	0
KangXi (康熙帝玄烨)	Qing (清)	0	1	0	0	0
YongZheng (雍正)	Qing (清)	0	1	0	0	0
QianLong (乾隆)	Qing (清)	0	1	0	0	0
JiaQing (嘉庆)	Qing (清)	0	1	0	0	0
DaoGuang (道光)	Qing (清)	0	1	0	0	0
XianFeng (咸丰)	Qing (清)	0	1	0	0	0
TongZhi (同治)[85]	Qing (清)	0	1	0	0	0
GuangXü (光绪)	Qing (清)	0	1	0	0	0

Continued on next page

[82] JingTai was elevated to the throne by the ministers after his emperor brother, ZhengTong, was captured by the Oirats (瓦剌). Pushed back by the Ming army, the Oirats later released ZhengTong, who was kept under house arrest by his brother, the current emperor. Seven years later, ZhengTong managed to launch a coup and resume the throne when JingTai was seriously ill, who died a month later.

[83] General Yuan ChongHuan (袁崇焕).

[84] The Manchus were organized by their chieftain Nurhaci (努尔哈赤) in eight tribes, each represented by a banner (hence the appellation eight banners, or 八旗). He was succeeded by Hong TaiJi (皇太极), selected among Nurhaci's sons by the heads of the eight banners. After Hong's death, his brother Dorgon (多尔衮) was the de facto ruler during the minority of the nominal successor Fulin, Hong TaiJi's son. Led by Dorgan and let in by a Ming general, the Manchus captured the empire, and Fulin became Emperor ShunZhi.

[85] Puppet of Dowager CiXi (慈禧太后), so was the next emperor, GuangXü.

Table 8 – *Continued from previous page*

Emperor	dynasty	military acces'n	hereditary succession	generals executed	kins slayed	killed/ suicide
XuanTong (宣统溥仪)	Qing (清)	0	1	0	0	0
Total: 100 emperors		20	91	21	16	19

E Online Appendix: Other Measures of θ

Table 9: **The θ for the Chinese, 221 BCE–1800 CE**

	battles	$100\hat{\pi}_b$	city population	$\hat{\omega}_b/\hat{\omega}_a$	$100\hat{\pi}_b\,(\hat{\omega}_b/\hat{\omega}_a)$
Core (Chinese)			1118520		
Central Asian (西域)	15	0.7422	0	0	0
Di (氐)	4	0.1979	0	0	0
Dutch	2	0.099	13090	0.0117	0.0012
Hiong-Nou (匈奴)	18	0.8906	0	0	0
Japanese	6	0.297	214680	0.1919	0.0570
Java (爪哇)	1	0.0495	0	0	0
Jie (羯)	4	0.1979	0	0	0
Jürchen (女真)	47	2.3256	4060	0.0036	0.0084
Khitan (契丹)	22	1.0886	13280	0.0119	0.0129
Korean	15	0.742	19380	0.0173	0.0129
Liuqiu (流求)	1	0.0495	0	0	0
Mongol	58	2.8699	4380	0.0039	0.0112
Myanmar	4	0.1979	36780	0.0329	0.0065
Nepalese	2	0.099	0	0	0
Portuguese	2	0.099	0	0	0
Qiang (羌)	7	0.3464	0	0	0
Russian	1	0.0495	16880	0.0151	0.0007
Sienpi (鲜卑)	30	1.4844	53070	0.0474	0.0704
Sri Lankan	1	0.0495	2340	0.0021	0.0001
Tangut (党项)	7	0.3464	0	0	0
Tibetan (吐蕃)	30	1.4844	3130	0.0028	0.0042
Turkic (突厥)	23	1.381	0	0	0
Xinjiang (新疆)	6	0.2969	0	0	0

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Table 9 – *Continued from previous page*

	battles	$100\hat{\pi}_b$	city population	$\hat{\omega}_b/\hat{\omega}_a$	$100\hat{\pi}_b\,(\hat{\omega}_b/\hat{\omega}_a)$
Viet (越)	16	0.7917	0	0	0
Yunnan/Guizhou	11	0.5443	9060	0.0081	0.0044
total	333	16.7202			0.1900
$\hat{\theta}$	**0.2031**				

Sources: Same as Table 4.

Table 10: **The θ for Romans halved**

	battle frequency (%)		$\frac{\text{cityPop}}{\text{CoreCityPop}}$		battleFreq $\times$ $\frac{\text{cityPop}}{\text{CoreCityPop}}$	
period	before 192	after 192	before 192	after 192	before 192	after 192
Alans/Huns	0	1.0563	0	0	0	0
Armenians	1.0204	0	0	0	0	0
British	1.5036	0	0	0	0	0
Carthaginian	1.0204	0	0.0493	0	0.0503	0
Celtic (Spain)	0.7653	0	0	0	0	0
Dacians	1.5306	0	0	0	0	0
Egyptians	0.5102	0	0.2791	NA	0.1424	0
Gauls	3.0612	0.3521	0	0	0	0
Germanic	3.0612	7.7465	0	0	0	0
Greeks (Asia)	3.3163	1.0563	0.5649	0.0397	1.8732	0.0420
Greeks (Europe)	2.5510	0	0.0361	NA	0.0922	0
Jewish	1.0204	0	0.2791	NA	0.2848	0
Numidian	0.5102	0	0	0	0	0
Persians	1.2755	2.4648	0	0.0662	0	0.1632
Total	21.1463	12.676			2.4429	0.2052
$\hat{\theta}$					**0.2992**	**0.1475**

Sources: Same as Table 3.

Table 11: **The θ for Chinese was stable**

	battle frequency (%)		$\frac{\text{cityPop}}{\text{CoreCityPop}}$		battleFreq $\times$ $\frac{\text{cityPop}}{\text{CoreCityPop}}$	
period	before 1279	after 1279	before 1279	after 1279	before 1279	after 1279
Central Asian (西域)	1	0	0	0	0	0
Di (氐)	0.2667	0	0	0	0	0

Continued on next page

Table 11 – *Continued from previous page*

	battle frequency (%)		$\frac{\text{cityPop}}{\text{CoreCityPop}}$		battleFreq × $\frac{\text{cityPop}}{\text{CoreCityPop}}$	
period	before 1279	after 1279	before 1279	after 1279	before 1279	after 1279
Dutch	0	0.3839	0	0.0227	0	0.0087
Hiong-Nou (匈奴)	1.2	0	0	0	0	0
Japanese	0.0667	0.9597	0.0511	0.3244	0.0034	0.3113
Java (爪哇)	0	0.1919	0	0	0	0
Jie (羯)	0.2667	0	0	0	0	0
Jürchen (女真)	1.9333	3.4549	0.0075	0	0.0145	0
Khitan (契丹)	1.4667	0	0.0245	0	0.0359	0
Korean	0.9333	0.1919	0	0.0336	0	0.0065
Liuqiu (流求)	0.0667	0	0	0	0	0
Mongol	2.2	4.7985	0.0081	0	0.0177	0
Myanmar	0	0.7678	0.0491	0.0176	0	0.0135
Nepal	0	0.3839	0	0	0	0
Portuguese	0	0.3839	0	0	0	0
Qiang (羌)	0.4667	0	0	0	0	0
Russian	0	0.1919	0	0.0293	0	0.0056
Sienpi (鮮卑)	2	0	0.0979	0	0.1957	0
Sri Lankan	0	0.1919	0.0043	0	0	0
Tangut (党项)	0.4667	0	0	0	0	0
Tibetan (吐蕃)	1.6667	0.9597	0.0058	0	0.0096	0
Turkic (突厥)	1.5333	0	0	0	0	0
Xinjiang (新疆)	0	1.1516	0	0	0	0
Viet (越)	0.6667	1.1516	0	0	0	0
Yunnan/Guizhou	0.4667	0.7678	0.0167	0	0.0078	0
Total	16.6667	15.9309			0.2847	0.3457
$\hat{\theta}$					**0.2034**	**0.1936**

Sources: Same as Table 9.

Table 12: **The θ for the Northern State in China, 400–600**

	battles	$100\hat{\pi}_b$	city population	$\hat{\omega}_b/\hat{\omega}_a$	$100\hat{\pi}_b\,(\hat{\omega}_b/\hat{\omega}_a)$
Core (Northern state)			832770		
Southern State	20	10	334170	0.4013	4.0128
Korean	1	0.5	0	0	0
Northern Liang (北凉)	1	0.5	0	0	0

Continued on next page

Table 12 – *Continued from previous page*

	battles	$100\hat{\pi}_b$	city population	$\hat{\omega}_b/\hat{\omega}_a$	$100\hat{\pi}_b\,(\hat{\omega}_b/\hat{\omega}_a)$
Rouran (柔然)	1	0.5	0	0	0
Turkic	8	4	0	0	0
Tuyuhun (吐谷浑)	2	1	0	0	0
Yan (燕)	1	0.5	0	0	0
Yunnan/Guizhou	1	0.5	0	0	0
Xia (夏)	3	1.5	0	0	0
Total	38	19			4.0128
$\hat{\theta}$					**0.2841**

Sources: Same as Table 4; the second source there lists the city populations, in the period 500–600, of Chang'an (长安), Datong/Pingcheng (大同／平城), Luoyang (洛阳) and Ye (邺城) in the north, and of Jiankang/Nanjing (建康／南京), Suzhou (苏州) and Wuchang (武昌) in the south.

Table 13: **The low θ in early Qing, 1644–1800**

	battles	$100\hat{\pi}_b$	city population	$\hat{\omega}_b/\hat{\omega}_a$	$100\hat{\pi}_b\,(\hat{\omega}_b/\hat{\omega}_a)$
Core (Chinese)			1784500		
Central Asian (西域)	0	0	0	0	0
Di (氐)	0	0	0	0	0
Dutch	0	0	89750	0.0503	0
Hiong-Nou (匈奴)	0	0	0	0	0
Japanese	0	0	1215600	0.6812	0
Java (爪哇)	0	0	0	0	0
Jie (羯)	0	0	0	0	0
Jürchen (女真)	0	0	0	0	0
Khitan (契丹)	0	0	0	0	0
Korean	0	0	123750	0.0693	0
Liuqiu (流求)	0	0	0	0	0
Mongol	1	0.6410	0	0	0
Myanmar	1	0.6410	0	0	0
Nepalese	2	1.2820	0	0	0
Portuguese	0	0	0	0	0
Qiang (羌)	0	0	0	0	0
Russian	1	0.6410	135060	0.0757	0.0485
Sienpi (鲜卑)	0	0	0	0	0
Sri Lankan	0	0	0	0	0

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Table 13 – *Continued from previous page*

	battles	$100\hat{\pi}_b$	city population	$\hat{\omega}_b/\hat{\omega}_a$	$100\hat{\pi}_b\,(\hat{\omega}_b/\hat{\omega}_a)$
Tangut (党项)	0	0	0	0	0
Tibetan (吐蕃)	4	2.5641	0	0	0
Turkic (突厥)	0	0	0	0	0
Xinjiang (新疆)	6	3.8462	0	0	0
Viet (越)	1	0.6410	0	0	0
Yunnan/Guizhou	0	0	0	0	0
total	16	10.2563			0.0485
$\hat{\theta}$					**0.1148**

Sources: Same as Table 9.

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