Examining Competitve Balance and Assessing Strategies to Improve it in Sports Leagues: A Model with Profit-Maximizing Teams

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Abstract

This paper outlines a simple profit-maximization model for a sports league with *n* teams which explains that talented players concentrate in large market teams. This reproduces one of the worries of many sports leagues – that varying market sizes reduce competitive balance. It provides a framework for investigating the effectiveness of salary caps and shared revenue systems in sports leagues. It finds that neither strategy is effective at increasing competitive balance. It also finds that leagues with high TV revenues as a share of total revenues will have better competitive balance.

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1. Introduction

Team sports leagues are one of the few forms of legalized business cartels that we witness in a society draped in Anti-Trust regulation (El-Hodiri and Quirk 1971). Because of this protection from Anti-Trust regulation, sports leagues provide economists with a number of unique natural experiments to investigate how profit-maximizing firms interact with one another (Syzmanski 2003). Sports leagues control how many teams are allowed to operate within their respective leagues by directly controlling the number of entrants, and thus directly controlling competition within the league. For most industries, this would violate Anti-Trust regulations; however, for sports leagues this is not the case (Fort and Quirk 1995). Sports fans prefer to attend games where the outcome is uncertain. For a league to be successful, teams must be close in competition since weak teams create negative externalities for strong teams (Dietl, Grossman, and Lang 2011; Crooker and Fenn 2007).

Unlike other industries, sports leagues depend on close competition between teams to survive. Sports are entertainment – they are driven by close games, close races to make playoffs, unpredictability in the playoffs, and by opportunities for underdogs to win games (Dietl, Lang, and Rathke 2011). With perennial winners, sports leagues can lose their fan base. The dichotomy within any sports league is that while individual teams need the league to be successful through close competition, their profits are driven by winning. Because of this, sports leagues are able to bypass Anti-Trust regulation (El-Hodiri and Quirk 1971).

In most professional sports leagues persistent inequality between teams, often a result of big-market teams having higher revenues to spend on higher quality players, is a chronic issue. Sports teams aim to maximize profits, which can result in large market teams having better teams than small market teams (Fort and Quirk 1995). For this paper, the terms small- and large-market teams are used to describe variation in a team's market size. Small markets have intrinsically lower demand than large markets. League policy-makers are interested in promoting competitive balance, and a number of techniques have been introduced by authorities to create parity in leagues (Fort and Quirk 1995). The main strategies for combating competitive imbalance have been revenue sharing schemes and salary caps.

Competitive balance issues could be observed easily in baseball and hockey before 2003, where big-market teams like the New York Yankees and Toronto Maple Leafs dwarfed the average payroll in their respective leagues (Zimbalist 2002). This problem became even more apparent during the late 1990s in baseball, and in the early 2000s in hockey. In both leagues, standard deviations in team payrolls increased drastically (Wiseman and Chatterjee 2003; Zimbalist 2002). While this did not necessarily lead to anti-competitiveness in hockey (Zimbalist, 2002), it created a severe concentration of success for big-market teams in baseball (Wiseman and Chatterjee 2003).

In each of the four major North American sports leagues – National Football League (NFL), National Hockey League (NHL), National Basketball Association (NBA), and Major League Baseball (MLB) – there are approximately 30 teams. As a result of the high number of teams, there is a large variation in each team's fan base size. These variations may have a variety of sources, including size of a city, sports culture in a city, how many other professional sports teams compete for fans in a city, etc. (Dietl, Grossman, and Lang 2011). There is a plethora of exogenous variables that determine the size of a fan base; however, the important fact is simply that different cities have different sized markets. This results in individual teams having relatively different demands for their respective franchises.

Using a simple Cournot model, this paper investigates how these different demands create inequality between teams and reduce competition in sports leagues. This model can be of interest to competition authorities and league authorities because it provides new insights into the effect of revenue sharing and salary caps on competitive balance. In contrast to previous models, my analysis shows that revenue sharing and salary caps do not improve incentives for small market teams to invest in playing talent. It follows Atkinson, Stanley, and Tschirhart's (1988) profit-maximization model for a league with *n* teams. However, my model will implement increasing marginal costs (Syzmanski and Smith 1997) and a demand function based on the relative quality of a team, rather than number of wins. Intuitively, these changes do not stray far from Atkinson, Stanley, and Tschirhart (1988); these changes in assumptions, however, will lead to much different conclusions. Most literature focuses on leagues using only two teams to examine the effects of competition levels on franchise utility (Késenne 2000a; Késenne 2005; Dietl, Grossman, and Lang 2011; El-Hodiri and Quirk 1971; Vrooman 1995). These models tend to overestimate the positive or negative effect of strategies for improving competitive balance. Through the use of a profit-maximization model with *n* teams, my model will show that Rottenberg's (1956) invariance proposition in sports leagues is incorrect, as revenue sharing systems do change talent distribution in leagues. However, it also confirms Rottenberg's (1956) hypothesis, as my model shows that, while the distribution of talent changes under revenue sharing and salary cap systems, the overall effect on competitive balance is small.

The paper will proceed as follows: Section 2 outlines literature related to competitive balance in sports leagues and analysis of how effective policies are in improving it. Section 3 outlines the profit-maximization model and the implications for leagues with teams with different demand functions, i.e., small- and large-market teams. Section 4.1 outlines and examines how a revenue sharing system affects competitive balance. Similarly, section 4.2 outlines and examines how salary cap implementation affects competitive balance. Then I will show how leagues with large television audiences are more competitive than those without, which could lead to an organic change in competitive balance for leagues, bypassing the use of exogenous strategies for improving competitive balance.

2. Literature

2.1 *Empirical Literature*

2.1.1 Competitive Balance in Sports Leagues

There has been a wide array of work detailing competitive balance in sports leagues. Essentially, there are two broad groups of literature on the subject: investigations into the optimal level of competitive balance in leagues and examinations of how competitive balance has been improved in leagues. For example, Zimbalist (2002) investigates what factors influence competitive balance in each of the major North American professional sports – baseball, hockey, football, and basketball – and summarizes the relative competitiveness of each league. He identifies the optimal level of competitive balance as a combination of the distribution of fan preferences, fan population base, and fan income across cities. He finds that, generally, leagues with control over the number of teams maximize revenues when big-market teams win more often. He identifies several different measurements of competitive balance in sports leagues, which revolve mainly around standard deviations of winning percentages. Zimbalist (2002) concludes that in all major sports leagues there exists problems of competitive balance, and that each league has introduced policies to try to create higher parity rates.

Hamlen (2007) finds that big-market teams, on the margin, have a higher probability of making the playoffs than small-market teams. He uses an empirical approach to investigate the effect of relative wealth on winning percentage in the National Football

League. One prediction in his paper is that teams in smaller markets have a greater incentive to relocate to larger markets, which is evident in North American sports leagues.

Wiseman and Chatterjee (2003) examine the growing disparity among payrolls in Major League Baseball teams. They investigate the relationship between payroll and winning percentage over the time period of 1985 to 2002. They find that the increasing disparity in team payrolls is having an adverse effect on the competitive balance in baseball.

2.1.2 Examining the Effectiveness of Techniques in Creating Parity

In all four of the major North American sports leagues, there are league policies designed to create more equality between teams. These policies are not designed just to close the gap between team profits, but also to create better competition between teams through greater parity in the quality of franchises. The three main overarching policies that sports leagues implement are revenue sharing, luxury taxes, and salary cap systems. Zimbalist (2010) examines the effectiveness of salary caps on salary shares in the four main professional sports in North America. He concludes that salary caps may not be effective at reducing relative salaries, as the salary share of total league revenues is lower in Major League Baseball (MLB) than in the two leagues with stringent salary caps – the National Hockey League (NHL) and the National Football League (NFL). He also examines the effect of revenue sharing in the MLB, which has created a system of incentives for small-market teams to adopt a strategy of having lower payrolls, further increasing the gap between big and small market competitiveness levels.

Booth (2004) finds that both revenue sharing and salary caps in the Australian Football League, which he identifies as having win-maximizing teams, have helped to achieve better levels of competitive balance. Atkinson, Scott, and Tschirhart (1988) examine revenue sharing in the NFL. They conclude that revenue sharing in the NFL has desirable properties; however, the effect is negligible.

2.2 *Theoretical Literature*

Rottenberg's (1956) seminal paper on the invariance proposition claims that revenue sharing does not affect the distribution of talent among profit-maximizing clubs. Through the law of diminishing returns on player quality and the fact that teams benefit from their opponents' quality, he argues that, in a non-collusive market, player talent distribution will not be concentrated in large-market teams. While large-market teams may perform better team than small-market teams, the difference is minimal; consequently, if a revenue sharing system is implemented, it will have minimal effect on the distribution of talent in the league. According to Rottenberg (1956), the only incentive that leagues have for tampering with free agency – such as with a salary cap – is to increase profits for owners. Subsequently, El-Hodiri and Quirk (1971) provide a proof that predicts that the economic structure of professional sports leagues produces competitive imbalance, that is, large markets will have higher quality teams than small markets. While this conclusion is different from Rottenberg's, their conclusion about revenue sharing agreements confirms the invariance proposition given by Rottenberg.

Vrooman (1995) examines Rottenberg's invariance proposition, but incorporates the effects of winning and market size on cost and revenue. He finds that the degree of competitive balance in a sports league depends on the size of these effects. In equilibrium, large-market teams will attract higher quality talent and have better winning percentages than small-market teams. He examines the effects of revenue sharing and a salary cap on the equilibrium of player distribution and competitive balance. He concludes that, while salary caps are effective at creating competitive balance, it may be through the decrease of large-market teams' quality, and not through the increase of small-market teams' quality. So, while competitive balance may be increased through a salary cap, it may be due to the overall effect of decreasing the league's talent supply. Vrooman also finds that revenue sharing does not increase competitive balance.

Késenne (2005) challenges the invariance proposition; if the incentives of revenue sharing parameters are changed so that teams become win-maximizers rather than profit-maximizers, then revenue sharing improves competitive balance. Using a mixed-talent model, he concludes that a pool-revenue sharing arrangement concentrates talent in a league. However, he also finds that in some leagues poorer teams are profit-maximizers and richer teams are win-maximizers. This results in improving competitive balance with revenue sharing. Késenne (2000b) finds that if teams are profit-maximizing firms, then revenue sharing will not improve competitive balance. However, if a team is utility-maximizing, that is, it prefers winning and profits, then revenue sharing can improve competitive balance. In his examination of salary caps, Késenne (2000a) uses a two-team model to examine a sports league. His model indicates that salary caps can improve competitive balance in a league while only marginally disrupting total league revenues and team profits.

Dietl, Grossman, and Lang (2011) provide a convincing argument for utility-maximizing teams with small-market teams' utility based on profit-maximization and large-market teams' utility based on profits and wins. They find that revenue sharing does not necessarily reduce incentives for teams to invest in playing talent. They emphasize the importance a mixed-utility function based on wins and profits for teams, and they point out how their approach differs from previous literature in this regard. However, their approach uses a contest model with only two teams. This does not capture the free-riding effect of having revenues shared between $n > 2$ teams. Similarly, Syzmanski (2004) introduces a Cournot game between two teams. His findings indicate that revenue sharing decreases competitive balance. Through the introduction of a league with $n > 2$ teams, my model will show that revenue sharing will not increase incentives to invest in playing talent in contrast to Dietl, Grossman, and Lang (2011), and will not necessarily improve competitive balance (Szymanski, 2004).

Atkinson, Scott, and Tschirhart (1988) employ a profit-maximization model for a league with n teams. They assume that team revenues are positively correlated with winning and that marginal costs are constant. Under this model, they find that if owners behave as profit-maximizers, then equal revenue sharing maximizes league revenues by optimally distributing talent among teams. My model will augment their model to show that these conclusions are false: revenue sharing and salary caps have little effect on competition levels, and potentially can have negative effects on competitive balance.

3. A Model of Pricing and Franchise Quality in a Sports League

3.1 *A League under no Regulations*

This model reproduces a sports league that has two types of franchises: small and large market. The small-market franchise is specified as follows:

The team spends money on inputs (stadium, players, coaches, etc.), which results in having a team with quality q. Assume that the cost of attaining quality level q is $c(q) = \gamma q^2$, which means the cost of quality is increasing at a rate $2\gamma q$. Szymanski and Smith (1997) indicate that quality costs are highly correlated with player talent, which implies that as teams spend more on players, the talent of the team increases and the team is relatively better. As Lewis, Sexton, and Lock (2007) demonstrate through empirical analysis, increasing player salaries leads to increased ability or quality. $\frac{\partial c(q)}{\partial q} = 2\gamma q > 0$ indicates that diminishing returns on quality leads to increasing marginal costs for teams. Revenue initially comes only from ticket sales, and is given by $p_s d_s$, where d_s is the demand for tickets to a small-market team's games, and p_s is the price of a ticket.

The demand function for a small-market team's tickets is derived as follows: there is a unit measure of potential game attendees (fans) in the team's area whose willingness to pay for tickets is given as v_i. Assume that the payoff to fan *i* of attending a game is:

$$u_i = \left(\frac{q_s}{Q}\right) v_i - p_s$$

where q_s is the quality for the small-market team, and Q is the average quality of teams in the league. The idea here is that if the quality of the team in a city is below average, the payoff to attending its games will diminish, whereas, if the quality is above average, the fan's payoff is increased. This fan utility function follows Atkinson, Scott, and Tschirhart (1988) and Szymanski (2004), who underline the fact that fans prefer winning teams to losing teams. What changes in this utility function is that perceived team success is based on relative talent level. While real 'fanatics' do exist, this payoff function reproduces the notion that a team's relative quality has an impact on attendance, at the margin. Also, note that

$$\frac{\partial\left(\frac{q_s}{Q}\right)}{\partial q_s} = \frac{Q - q_s\left(\frac{\partial Q}{\partial q_s}\right)}{Q^2} = \frac{Q - \frac{q_s}{n}}{Q^2} > 0$$

because $Q = \frac{1}{n}\sum q_j$ means that $\frac{\partial Q}{\partial q_s} = \frac{1}{n}$. This means that improving a team's quality always increases the utility from attending games. Also, assume that the v_i of potential fans are distributed uniformly over the interval [0,1], so the set of fans who buy tickets are those i for whom $u_i > 0$, or $v_i > \frac{Qp_s}{q_s}$. This does mean that if other teams get better and yours does not, attendance will be hurt unless you lower the ticket price.

So, the demand for tickets for a small-market team is given by:

$$d_s(p_s,\ q_s, Q) = \begin{cases} 1 - \dfrac{Qp_s}{q_s} & \text{if } q_s > Qp_s \\ 0 \text{ otherwise} \end{cases}$$

The profits of a small-market team are then given by:

$$\pi_s(p_s,\ q_s, Q) = p_s\left[1 - \frac{Qp_s}{q_s}\right] - \ \gamma q_s^2$$

This model assumes that there are no costs associated with d_s, that is, it is not more costly to have more people come to games. While this assumption is initially false, higher attendance generates extra revenue from beer, food, and merchandise, so we can assume that p_s is the net addition to revenue the team gets from each costumer who buys a ticket. The team can alter p_s by altering the prices of tickets, food, or beer, and it is this composite price that the fans use to decide whether or not to attend.

A big-market team differs from the above in only one way: the demand for its tickets by any one fan is the same as for a small-market team, but there are λ times as many fans in the large market, where $\lambda > 1$, so that demand for a big-market team is:

$$d_l(p_l,\ q_l, Q) = \lambda\left(1 - \frac{Qp_l}{q_l}\right)$$

by the same reasoning. Thus, the profits of a big-market team are:

$$\pi_l(p_l,\ q_l, Q) = p_l\lambda\left[1 - \frac{Qp_l}{q_l}\right] - \ \gamma q_l^2$$

This model assumes that the costs of quality are the same in both markets, which seems reasonable since the biggest cost in producing high quality is player salaries. Késenne (2004) indicates that under a perfectly competitive labour market, teams are wage takers, so that quality costs are the same across the league. Either type of team then chooses its *p* and q to maximize its profits. The two first-order conditions for the small-market team's profit maximization problem [$\max_{p_s q_s} \pi_s(p_s,\ q_s)$] are as follows:

$$\frac{\partial \pi_s}{\partial p_s} = 1 - \frac{2Qp_s}{q_s} = 0$$

which clearly implies that $q = 2Qp$ is the profit-maximizing relationship between q and p. The first-order condition for q is slightly more complicated, since Q is a function of each team's q. Q' will be substituted for $\partial Q/\partial q_s$ – the derivative of average team quality with respect to this particular team's quality. This results in the following first-order condition for *q*:

$$\frac{\partial \pi_s}{\partial q_s} = -p_s^2 \left[\frac{Q' q_s - Q}{q_s^2}\right] - 2\gamma q_s = 0$$

which can be simplified to

$$-\left(\frac{p_s}{q_s}\right)^2 [Q' q_s - Q] = 2\gamma q_s$$

$$\frac{\partial^2 \pi_s}{\partial {q_s}^2} > 0$$

For every team, $Q' = 1/n$, so the expression in brackets is just

$$= \frac{q_s}{n} - Q$$

$$= -\left[Q - \frac{q_s}{n}\right]$$

$$= -\frac{1}{n}\sum_{j \neq s} q_j \equiv -Q_{-s}$$

That is, Q_{-s} is just the average quality of the league if the team in question (s, in this case) had a quality of 0. So, this first-order-condition can be written as:

$$\left(\frac{p_s}{q_s}\right)^2 Q_{-s} = 2\gamma q_s$$

If we use the first first-order condition to substitute in $1/2Q$ for p/q, we get

$$\frac{Q_{-s}}{4Q^2} = 2\gamma q_s$$

which implies that

$$q_s = \left(\frac{Q_{-s}}{8\gamma Q^2}\right)$$

so that

$$p_s = \frac{q_s}{2Q} = \frac{Q_{-s}}{16\gamma Q^3}$$

These are not ‘closed-form’ expressions, since q_s appears in Q. The Nash equilibrium values of *all* teams’ q and p depend on Q, which in turn depends on the q’s of other teams. But, we can still use the relationships above; in equilibrium, ticket sales are $d_s(p_s,\ q_s, Q) = \frac{1}{2}$ for the small-market team, because all $v_i > \frac{Qp_s}{q_s} = \frac{Q}{2Q} = \frac{1}{2}$ buy tickets.

The same exercise for a large-market team results in one first-order condition, which implies$q_l = 2Qp_l$, whereas the other first-order condition now implies

$$\lambda \left(\frac{p_l}{q_l}\right)^2 Q_{-l} = 2\gamma q_l$$

so that we get

$$q_l = \frac{\lambda Q_{-l}}{8\gamma Q^2}, p_l = \frac{\lambda Q_{-l}}{16\gamma Q^2}, \text{and } d_l = \frac{\lambda}{2} > d_s$$

As noted, these are not ‘closed-form’ expressions for the equilibrium values of *p* and *q*, stated entirely in terms of exogenous parameters; however, they allow us to answer several questions regarding this model’s predictions about prices, quality, profits, and attendance.

3.2 *Is it true that the l team charges higher prices and has a higher quality team?*

Suppose the answer to the *q* part of the question is no, so that $q_l \leq q_s$. This would imply that

$$\frac{\lambda Q_{-l}}{8\gamma Q^2} \leq \frac{Q_{-s}}{8\gamma Q^2}, \text{so that}$$

$$\lambda Q_{-l} \leq Q_{-s}, \text{and since } \lambda > 1, \text{this implies}$$

$$Q_{-l} < Q_{-s}$$

but the definitions of the Q_{-l} mean that this can only be true if $q_s > q_l$, which is a contradiction of the hypothesis that the opposite is true, so the hypothesis must be false. Thus, the model predicts $q_l > q_s$, as we would expect. This in turn means that

$$p_l = \frac{q_l}{2Q} > \frac{q_s}{2Q} = p_s$$

and the large-market team also charges higher prices, and, since only $v_i > \frac{Qp_l}{q_l} = 1/2$ buy tickets in the large market, the model predicts $d_l = \lambda/2 > 1/2$. Even though ticket prices are higher, the large market team draws more fans to its higher quality franchise.

3.3 *Is it true that l teams earn higher profits than s teams?*

Suppose the l firm chose exactly the same p and q as the s firm. Then its costs would be the same as the s firm's, but its revenues would be higher, since it would be $p_s\,\lambda/2$. This means that even this naïve choice of p_l and q_l would give it higher profits than the small-market firm, so it would only choose the higher p and q that has been shown, which shows that l firms are more profitable than s firms. This can also be shown directly through calculation:

$$\begin{aligned}
\pi_s^* &= \frac{p_s}{2} - \gamma q_s^2 \\
&= \frac{q_s}{4Q} - \gamma q_s^2 \\
&= q_s\left[\frac{1}{4Q} - \frac{\gamma Q_{-s}}{q_s \gamma Q^2}\right] \\
&= \frac{q_s}{Q}\left[\frac{1}{4} - \frac{Q_{-s}}{q_s Q}\right]
\end{aligned}$$

which has to be positive since $Q_{-s} < Q$ and

$$\begin{aligned}
\pi_l^* &= \frac{\lambda p_l}{2} - \gamma q_l^2 \\
&= \frac{\lambda q_l}{4Q} - \gamma q_l^2 \\
&= q_l\left[\frac{\lambda}{4Q} - \frac{\gamma \lambda Q_{-l}}{q_l \gamma Q^2}\right] \\
&= \frac{\lambda q_l}{Q}\left[\frac{1}{4} - \frac{Q_{-l}}{q_l Q}\right]
\end{aligned}$$

Since $\lambda q_l > q_s$ and $Q_{-l} < Q_{-s}$, it follows that $\pi_l^* > \pi_s^*$.

Thus, the model reproduces a worry of any sports league: large-market teams have higher profits and better teams than small-market teams. This finding is consistent with empirical analysis of sports leagues (Hamlen 2007; Wiseman and Chatterjee 2003) and is supported by Vrooman's (1995) theoretical model of a sports league. This model fails to capture the increased utility fans associate with close competition, as outlined by Szymanski (2004) and Crooker and Fenn (2007). For future research, a change in the demand structure may be required to take this properly into account.

4. Strategies for Improving Competitive Balance

4.1 *League under Revenue Sharing*

To show the effects of a revenue sharing system on a sports league, I will introduce a tax defined as *t*, which is applied equally to each team in the league. These tax revenues are then pooled and distributed equally among all *n* teams in the league. The

revenues for small-market teams in the original model are defined as

$$R_s = \frac{p_s}{2} = \frac{q_s}{4Q}$$

and similarly, the revenues for a large-market team are defined as,

$$R_l = \frac{p_l}{2} = \frac{q_l}{4Q}$$

then the average team revenue must be

$$\bar{R} = \frac{\bar{p}}{2Q} = \frac{\bar{q}}{4Q} = \frac{Q}{4Q} = \frac{1}{4}$$

The new profit function for a small-market team in a league with revenue sharing will be:

$$\pi_s = (1-t)p_s\left[1 - \frac{Qp_s}{q_s}\right] - \gamma q_s^2 + t\bar{R}$$

$$\pi_s = (1-t)p_s\left[1 - \frac{Qp_s}{q_s}\right] - \gamma q_s^2 + \frac{t}{4}$$

$$\frac{\partial \pi_s}{\partial p_s} = (1-t)\left[1 - \frac{2Qp_s}{q_s}\right] = 0$$

which still simplifies to $q = 2Qp$, as in the original model. Similarly,

$$\frac{\partial \pi_s}{\partial q_s} = -(1-t)p_s^2\left[\frac{Q' q_s - Q}{q_s^2}\right] - 2\gamma q_s = 0$$

which results in

$$q_s^* = (1-t)\left(\frac{Q_{-s}}{8\gamma Q^2}\right) = (1-t)q_s$$

where $q_s = \left(\frac{Q_{-s}}{8\gamma Q^2}\right)$. This result clearly shows that small-market teams decrease their quality under a revenue sharing system.

This results in small market-teams' profits being

$$\pi_s = (1-t)\frac{q_s^*}{4Q} - \gamma q_s^{*2} + \frac{t}{4}$$

substituting q_s^* into π_s gives

$$\pi_s = (1-t)\frac{(1-t)q_s}{4Q} - \gamma(1-t)^2 q_s^2 + \frac{t}{4}$$

$$\pi_s = (1-t)^2\left[\frac{q_s}{4Q} - \gamma q_s^2\right] + \frac{t}{4}$$

$$\pi_s = (1-t)^2\pi_s^* + \frac{t}{4}$$

where $\pi_s^* = \frac{q_s}{4Q} - \gamma q_s^2$, which is the profit under the original model for small-market teams. To show the effects of *t* on the profitability of a small market,

$$\frac{\partial \pi_s}{\partial t} = -2(1-t)\pi_s^* + \frac{1}{4}$$

$$= 2t\pi_s^* - 2\pi_s^* + \frac{1}{4}$$

$$= 2\pi_s^*(t-1) + \frac{1}{4} = 0$$

where $\pi_s^* > 0$ and $0 < t < 1$, which implies that $(t-1) < 0$, resulting in $2\pi_s^*(t-1) < 0$. Therefore $\frac{\partial \pi_s}{\partial t} < 0$ if $|2\pi_s^*(t-1)| > \frac{1}{4}$ and $\frac{\partial \pi_s}{\partial t} > 0$ if $|2\pi_s^*(t-1)| < \frac{1}{4}$. While it is clear what the effect of *t* is on q_s, the effect on profits is not as clear. However, it is clear that as the original model profits π_s^* increase (decrease), then the likelihood of the effect of t on profits is negative (positive). Intuitively, this result makes sense: small-market teams with smaller profits benefit more from a revenue sharing system, or at least are not as negatively impacted, while larger market teams with high profits are negatively impacted, or at least not as positively impacted, from a revenue sharing system.

The results from this model show how revenue sharing systems can impact competitive balance negatively for sports leagues: small-market teams are induced to spend less on players, making themselves less competitive. While small-market teams may be more profitable through revenue sharing systems, this is not immediately clear from the model. If league authorities are concerned with competitive balance, then revenue sharing systems do not induce small-market teams to spend more on players, and is therefore an ineffective mechanism for making a league more competitively balanced. In contrast to Atkinson, Scott, and Tschirhart (1988), the implementation of increasing marginal costs and fan preferences based on relative quality rather than winning leads to revenue sharing's being ineffective at increasing competitive balance. This follows Rottenberg's (1956) invariance proposition that revenue sharing will not change talent distribution in a sports league.

4.2 *League under a Salary Cap*

To show the effect of a salary cap on sports leagues, I will introduce a ceiling on salary expenditures (which amounts to a ceiling on q). This will be defined as $\bar{q}$, where $\bar{q} < q_l$. The model will stay the same, but with l teams only being allowed to spend up to $\bar{q}$. Because $\bar{q} < q_l$, l teams will spend as much as they can to maximize profits. Thus, $q_l = \bar{q}$, and following the same steps as before shows that $p_l = \frac{\bar{q}}{2Q} < \frac{q_l}{2Q}$.

For a small-market team, the effect of a salary cap implementation would be as follows:

The effect of q_l on $Q_{-s} = \frac{1}{n}\sum_{j \neq s} q_j$ is central to this argument, and is $\frac{\partial Q_{-s}}{\partial q_l} = \frac{1}{n}$, which is the same as $\frac{\partial Q}{\partial q_l} = \frac{1}{n}$. Calculating the effect of a change in q_l on q_s is

$$\frac{\partial q_s}{\partial q_l} = \frac{8\gamma Q^2\left(\frac{\partial Q_{-s}}{\partial q_l}\right) - Q_{-s}\left(\frac{\partial Q}{\partial q_l}\right)}{(8\gamma Q^2)^2} = \frac{8\gamma Q^2\left(\frac{1}{n}\right) - Q_{-s}\left(\frac{1}{n}\right)}{64\gamma^2 Q^4} = \frac{8\gamma Q^2 - Q_{-s}}{64n\gamma^2 Q^4} > 0$$

which implies that an increase in q_l will have a (slightly) positive impact on q_s. Under a salary cap system, $q_l = \bar{q}$ will have the effect of reducing q_s.

The model shows that the relative decrease in q_l is much larger than the decrease in q_s. Thus, we should observe closer competition in a league with a salary cap system in place than in a league without one. However, this model points out that a salary cap may not be an effective way to create higher parity in leagues. Ideally, a salary cap should have decreased q_l, which it did, and also increased q_s, which it failed to achieve under this model. This may explain why salary cap systems have not been as effective at creating parity as league policy makers might have originally anticipated (Zimbalist 2010).

4.3 *League with High Shared Television Revenues*

In 2008, the NFL made an estimated $7.6 billion in total revenues (Fisher 2010). In 2011, the NFL made $4 billion in national television contracts, constituting approximately half of the total league revenues (Bloomberg 2011). Zimbalist (2002) found the NFL to be the most competitive of the four major North American sports leagues. Of the four major North American sports leagues, the NFL has the highest percentage of total revenues from television contracts (Forbes 2011). This TV revenue is divided equally among franchises, and represents a shift in importance away from gate revenues to TV revenues in the incentive structure for sports leagues and their various franchises. The shift from gate revenues to TV revenues in the NFL may explain why it has such high levels of competitive balance.

For this analysis, I will augment the original model slightly: there will be *n* teams, but *only* two different types of teams – small-market teams and large-market teams. There will be *m* number of small-market teams and *n-m* number of large-market teams in the league. The new profit function for a small-market team will be denoted as

$$\pi_s(p_s,\ q_s, Q) = p_s\left[1 - \frac{Qp_s}{q_s}\right] - \gamma q_s^2 + \frac{1}{n}TV$$

$$\pi_s(p_s,\ q_s, Q) = p_s\left[1 - \frac{Qp_s}{q_s}\right] - \gamma q_s^2 + \frac{1}{n}[\delta[Var(Q)] + c]$$

where $\delta < 0$ and $c > 0$. Also, $c > \delta[Var(q_s)]$ so that $\delta[Var(q_s)] + c > 0$. This model has shared television revenue that all teams benefit from; as the variance of the quality of teams increases, television revenue decreases. The variance of the quality of teams in the league is calculated as

$$E(Q) = \frac{1}{n}\left[\sum_{s=1}^{m} q_s + \sum_{l=m+1}^{n} q_l\right]$$

$$Var(Q) = E(Q^2) - E(Q)^2$$

$$Var(Q) = \frac{1}{n}\left[\sum_{s=1}^{m} q_s^2 + \sum_{l=m+1}^{n} q_l^2\right] - E(Q)^2$$

$$\frac{\partial}{\partial q_i}Var(Q) = \frac{2q_i}{n} - \frac{2}{n}E(Q) = \frac{2}{n}[q_i - E(Q)]$$

If i is large, then $\frac{\partial}{\partial q_i}Var(Q) > 0$ and if i is small, then $\frac{\partial}{\partial q_i}Var(Q) < 0$. If large-market teams increase their quality, the variance of the league quality increases. Conversely, if a small-market team increases its quality, the variance of the league decreases. This will induce small-market teams to spend more on quality players, and large-market teams to spend less on quality players, thus narrowing the quality gap between small-l and large-market teams, and achieving a better competitive balance. This model shows that for leagues like the NFL, where a major portion of revenue is from national television contracts, greater parity among teams may occur, resulting in better competitive balance.

5. Conclusion

This paper addresses a number of issues facing professional sports leagues using a simple profit-maximization model based on fan utility increasing with a relative increase in team quality. This model incorporates the effect of market size in determining the quality of different teams, and the distribution of talent across a league. Recreating a sports league where teams are profit-maximizers, it has illustrated that large-market teams have higher levels of talent and are more profitable than small-market teams – recreating one of the concerns of sports league policymakers: competitive imbalance. The model allows for insight into the effect of strategies used by sports leagues to increase competitive balance: revenue sharing and salary caps. It also provides insight into why leagues with high TV revenues may have better competitive balance than those with low TV revenues.

There have been many investigations into the effectiveness of revenue sharing and salary caps in increasing competitive balance. Some conclude that these strategies do not change competitive balance, e.g., Rottenberg (1956), El-Hodiri and Quirk (1971); some find that they improve competitive balance, e.g., Vrooman (1995), Késenne (2005), Atkinson, Scott, and Tschirhart 1(988); some find that they reduce competitive balance, e.g., Dietl, Grossman, and Lang (2011). The model used in this paper shows that revenue sharing is ineffective at increasing competitive balance and may, in fact, reduce incentives to invest in talent. It also finds that salary caps may impact competitive balance positively, but may not have a significant overall effect. In investigating the impact of high TV revenues on a league, this model finds that leagues with high TV revenues may have better competitive balance than leagues that rely primarily on gate revenue.

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