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The *Western Undergraduate Economics Review* is an annual publication containing papers written by undergraduate students in Economics at Western. First published in 2002, the *Review* reflects the academic distinction and creativity of the Economics Department at Western. By showcasing some of the finest work of our students, it bestows on them a lasting honour and a sense of pride. Moreover, publication in the *Review* is highly beneficial to the students as they continue their studies or pursue other activities after graduation. For many, it is their first publication, and the experience of becoming a published author is a highlight of their undergraduate career. The *Review* is a collaborative effort of the students, faculty, and staff of the Economics Department. All papers submitted to the *Review* are essays written for courses taken in the Department. Some are by students in the early stages of their Economics studies, while others are papers written by senior students for the Department's unique thesis course, Economics 4400. Selections are made by the edition editors, in consultation with a faculty advisor, based on creativity, academic merit, and the written quality of the article.

 For submission information, please visit the *WUER* website.

http://economics.uwo.ca/undergraduate/undergraduate_economics_review.html

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Contents

Editors' Comments

The 2017 edition of the *Western Undergraduate Economics Review* (*WUER*) showcases the diversity of high quality research completed by Western's Undergraduate Students. This year, the Review includes a collection of papers that provide pertinent exploration for economic theories in financial economics and macroeconomics, as well as theories that connect important economic concepts such as production and financial investment. Together, these papers highlight Western undergraduate students' critical thinking and self-exploration for fundamental theories taught by our faculty members.
This *Review* begins with an outstanding contribution written by Tom Qiao and Parker Liu, who were the winners of the Mark K. Inman Senior Essay Prize. They evaluated long-run post seasoned equity offering stock returns in the United States by univariate analysis, which provides strong evidence for the effect of heterogeneous beliefs and short-sell constraints from Miller's theory. Tom was also awarded the Gold Medal prize in the Department of Economics.

A second senior thesis paper follows, written by Nadezhda Peretroukhina and Siddharth Untawala, on a new methodology from physics to measure the "raw energy" in a Cobb-Douglas production model. Their paper provides a physical ideology to approach total factor productivity which is difficult to measure in economics.

Lastly, Eric Huang and Matthieu Laurin wrote an excellent senior thesis on the comparison of quantitative easing (QE) programs of the US and UK. The paper provides a rigorous analysis of macroeconomics and currency market effects on UK and US QE programs after the 2008 financial crisis.

We hope that you enjoy reading the 2017 edition of the *Western Undergraduate Economics Review* and also gain a deeper appreciation for the quality of undergraduate economic research conducted at Western. Further, we would like to thank each of the authors for their contribution to this year's excellent edition of the *WUER* and we hope this edition inspires future economics students at Western to work even more diligently at their research.

Yan Wang
Kevin Madden
London, Ontario
May 2017

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We also extend our sincere thanks to the Social Science Student Donation Fund for its ongoing financial support of the *Western Undergraduate Economics Review*.

The Case for Heterogeneous Investor Beliefs: Evidence from U.S. Seasoned Equity Offerings

Parker Liu and Tom Qiao

Abstract

This paper examines the effect of heterogeneous beliefs and short-sell constraints on the long-run post seasoned equity offering stock returns in the US. We find that SEOs with high abnormal trading volume prior to the offering and high relative offering size exhibit significant and negative returns after one year. Firms in the highest quartile of market adjusted turnover and relative offering size had an average abnormal buy and hold return of -19.18% one year after the issue date. These results further support the previous theoretical works that tried to show short-sale constrained stocks with high divergence in opinion were likely to be overvalued due to short-sellers being absent in the market.

Faculty Consulted: Professor Rui Castro and Professor Lars Stentoft

I. Introduction

A. Background

Modern financial economics assumes that investors have homogenous expectations but ignores the implications of investor divergence of opinion.[1] Mayshar (1983) points out that both William Sharpe and John Lintner thought heterogeneous beliefs could be closely approximated by homogeneity if it was the average investor's opinion that determined asset prices. The capital asset pricing model (CAPM) developed independently by Sharpe and Lintner has subsequently contributed to the prevailing view today that markets are efficient. Yet Mayshar (1983) also notes that earlier works by John Maynard Keynes and John Burr Williams had argued it was the marginal investor who determined asset prices and thus divergence of opinion should be essential to any financial theory. Intuitively, investors likely have different estimates of the future cash flows of a company as some investors are no doubt more optimistic than others about a company's future prospects which are veiled by uncertainty. Miller (1977) proposed that when there are short-sale constraints[2] preventing pessimistic investors from participating in price discovery, stock prices would reflect only the beliefs of the optimistic investors.

[1] Note: "divergence of opinion" and "heterogeneous beliefs" are used interchangeably throughout this paper as referring to a state where investors have different estimates of the value of a publicly traded company.

[2] Short selling involves borrowing a stock and selling it immediately at the market price with the intention to buy back the stock at, ideally, a lower future price to make a profit. Short-sale constraints exist when it is difficult to short sell due to high shorting costs, usually the result of limited availability of stocks to borrow.

If there is divergence of opinion, relaxing short-sale constraints through additional stock issuance should decrease stock prices as the additional supply is absorbed by less optimistic investors with lower estimates of stock value. This is the focus of our paper and we outline the research question in the following section.

B. Research Question and Approach

This paper seeks evidence of heterogeneous investor beliefs by examining stock returns in the weeks following a seasoned equity offering (SEO) in the United States from Jan 2002 to Jan 2015.[1] The SEO is a unique event that allows us to test Miller (1977) as the supply of stock increases significantly on a single day and thus relaxes short-sale constraints by making more shares available for shorting. This paper draws heavily on the empirical work by Cooney, Kato, and Suzuki (2012) on Japanese SEOs and also builds on the theoretical model of Hong, Scheinkman, and Xiong (2006). We differentiate our approach by using different proxies for divergence of opinion, using U.S. SEO data, and focusing on stock returns in the weeks following the SEO while previous literature examined only the following days.

C. Hypothesis

Based on Miller's theory, we hypothesize that in the presence of short-sale constraints:

i) The additional stock float from issuing equity will be negatively related to post-SEO stock returns:

$H_0 : \beta_{\mathrm{Re}\,lativeOfferSize} = 0$ vs. alternative $H_1 : \beta_{\mathrm{Re}\,lativeOfferSize} < 0$

ii) The degree of opinion divergence will be negatively related to post-SEO stock returns:

$H_0 : \beta_{Divergence} = 0$ vs. alternative $H_1 : \beta_{Divergence} < 0$, and

iii) The interaction of opinion divergence and additional stock float will be negatively related to post-SEO stock returns; i.e. the greater the degree of opinion divergence, the greater the negative effects of issuing new equity on post-SEO stock returns:

$H_0 : \beta_{\mathrm{Re}\,lOff*Div} = 0$ vs. alternative $H_1 : \beta_{\mathrm{Re}\,lOff*Div} < 0$.

D. Roadmap of Paper and Results

Section II outlines the relevant literature including an explanation of Miller (1977) and the contribution of this paper. Section III discusses our empirical approach and section IV describes our econometric model and variable descriptions. Section V describes the data, section VI presents our findings, and section VII concludes. In summary, we find strong supporting evidence for our hypothesis using a particular proxy of opinion divergence, market-adjusted turnover, and weaker evidence using the other two proxies, relative analyst spread and 6 month put implied volatility.

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[1] A seasoned equity offering is any equity issuance following the company's initial equity offering (IPO).

II. Existing Literature

A. Divergence of Opinion

Miller's theory (1977) is one of the earliest asset pricing models to incorporate heterogeneous beliefs and short-sale constraints. Miller's paper challenged the CAPM's assumption of homogeneous expectations by arguing that uncertainty about the future naturally creates diverging forecasts of a company's future cash flows and valuation. This results in a downward sloping demand curve for the stock of a company as investors have varying expectations of future returns from holding the stock (Figure 1). Furthermore, the degree of opinion divergence is represented by the slope where steeper slopes indicate higher divergence and flatter slopes indicate lower divergence.

In Figure 1, the y-axis is each investor's estimate of stock value and the x-axis is the number of investors (Miller assumes each investor can only hold 1 unit of stock). GBH is the demand curve if investors have homogeneous expectations while ABC is the demand curve if investors have the opinion divergence. Given a limited supply of stock at N, the stock price is higher for the opinion divergence case at R compared to a stock price of G for homogenous expectations. This is because more optimistic investors will purchase the stock from less optimistic investors until the N most optimistic investors are the final owners at a price that is higher than the average expectation of value. In addition, the degree of opinion divergence is positively related to the stock price as greater divergence leads to higher prices paid by the N optimistic investors, as shown by FBJ and price of Q, while lower divergence leads to lower prices, as shown by DBE and price of M. In this framework, we can think of homogeneous expectations as a special case with zero opinion divergence and a perfectly flat slope.

The SEO is an opportunity to observe the change in stock price as the supply of stock increases past N (i.e. shifting the vertical line at N to the right). If the stock is short-sale constrained, Miller's theory predicts a negative change in stock prices because the marginal investor absorbs the additional stock float at a lower price. If investors have homogeneous expectations, then the theory predicts no change in stock prices because the marginal investor has the same estimate of value as the average investor.

B. Empirical Work

The most relevant empirical study is by Cooney, Kato, and Suzuki (2012) who examined the effects of opinion divergence and short-sale constraints on the stock prices of Japanese companies following their SEOs. The authors found that divergence of opinion was negatively related to stock returns on both the announcement date and the issue date for a sample of 830 SEOs from 1998 to 2011.[1] They also found that issue size was negatively related to stock returns on both dates and that this relationship was stronger for stocks with a higher divergence of opinion. These results are consistent with

[1] The announcement date is the day that the SEO is announced to the public. The issue date is the day on which the new shares begin trading on the stock exchange.

Figure 1 [reproduced from Miller (1977)]

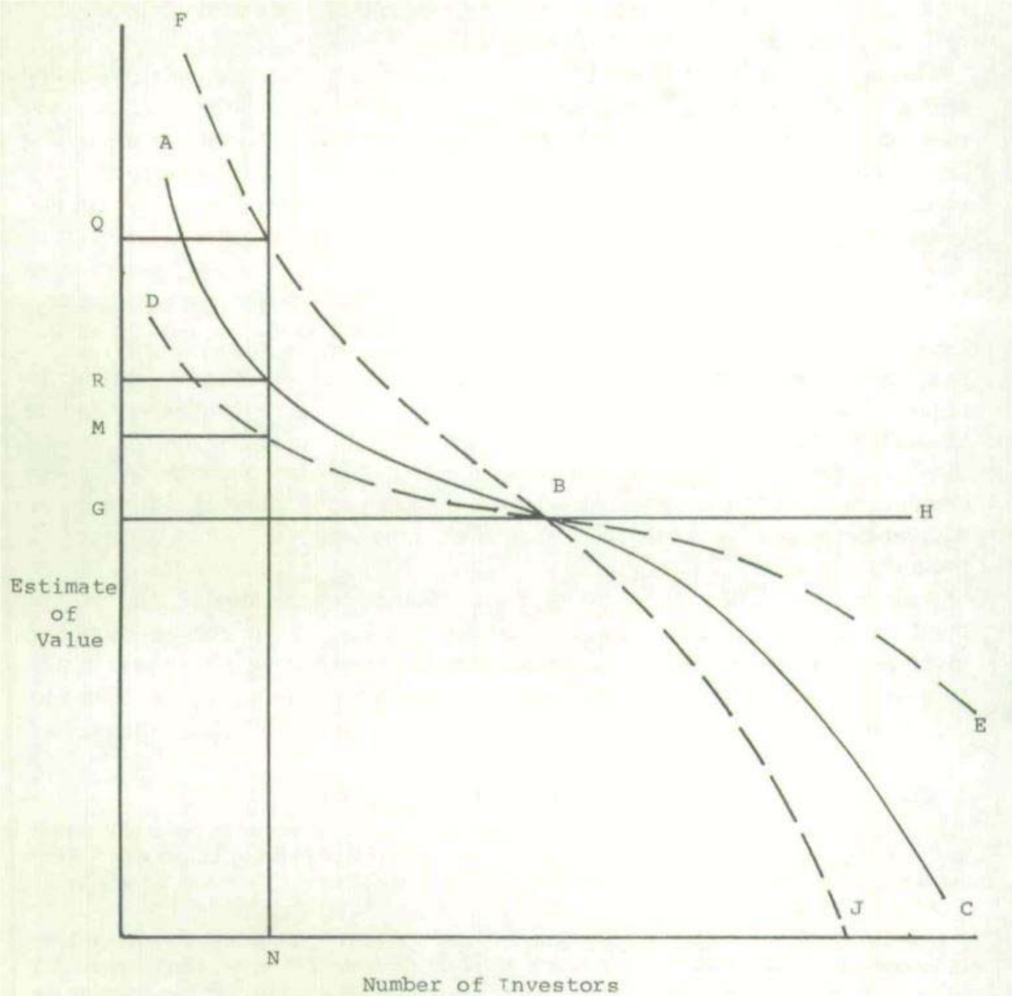

FIGURE 1.

Miller's theory (1977) and support the theory of opinion divergence. We improve upon their research using data on U.S. SEOs in the following ways.

C. Contribution

Firstly, we use several new proxies for opinion divergence. Cooney, Kato, and Suzuki (2012) used the mean square error[1] (MSE) and daily return volatility over the thirty trading days ending 10 trading days before the announcement day as proxies for divergence of opinion. While we do not completely disagree with this approach,

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[1] Computed as the deviation from the value predicted by the Fama and French three-factor model for the period from -70 days to -11 trading days before the announcement date.

Garfinkel (2009) found that the explanatory power of these proxies was inferior compared to other proxies such as change in market-adjusted turnover, deviation of analysts' forecasts divided by the stock price, and implied volatility of put options.[1] We test these three measures in our analysis to see if they can better explain post-SEO performance. We also believe qualitative factors such as industries can explain divergence of opinion. Stocks in industries where future outcomes vary drastically such as information technology or biotechnology are more likely to exhibit opinion divergence.

Secondly, Cooney, Kato, and Suzuki (2012) pointed out problems in the U.S. SEO data that we believe can be addressed in our paper. The pricing of SEOs in the U.S. is determined either on or a day before the issue date while pricing in Japan is determined at least five days before the issue date. Since the float begins trading on the same day that price is determined, the effects of opinion divergence cannot be distinguished from other factors such as information asymmetry and short-term pricing pressure. In our view, this issue is only relevant when examining returns on the issue date. In practice, the entire float does not immediately become available to short following the issue date because institutional buyers must transfer their shares to appropriate brokerages before the shares can be lent out for short sellers to borrow. This process takes several days to weeks depending on the buyers of the issue. In addition, for less followed stocks, it may take additional time for short sellers to realize that the float has increased and then decide to short the stock. Therefore, we expect a negative price change to persist for a longer time period following the issue date and this is why we examine weekly price changes whereas Cooney, Kato, and Suzuki (2012) examined only the 10 days surrounding the issue date.

III. Empirical Techniques

A. Regression

Our empirical analysis will focus on testing the theoretical model proposed by Hong, Scheinkman, and Xiong (2006). Their model estimates the change in stock price after an increase in stock float while assuming certain parameters for the degree of opinion divergence, discount rate, and risk bearing capacity of insiders.

The empirical approach we take is based on standard event study methodologies outlined in Lyon, Barber, and Tsai (2001). Our regression consists of cross-section data where the dependent variable is the cumulative buy and hold abnormal return (CBHAR) at time T after the event date and the independent variables are offering-specific characteristics before the event date. The CBHAR is the residual from a regression of stock returns for each company on factors from the Carhart four-factor model (Exhibit A).

[1] Avellaneda, Lipkin, and Trading (2009) derived an options pricing model that found hard to borrow stocks have higher prices for put options due to higher borrowing costs. Therefore, the implied volatility of put options measure both divergence in opinion and degree of short sale constraints.

B. Univariate Analysis

We also test our hypothesis by grouping offerings based on offering characteristics to see whether these groups earn significant abnormal returns. Our sample is first split into quartiles by relative offering size and we categorize the top quartile as the most short-sale constrained and the bottom quartile as the least short-sale constrained. Then we test whether the cumulative buy and hold abnormal return (CBHAR) for these groups are significantly different from zero. We follow a similar procedure for our proxies of opinion divergence. To study the interaction between short-sale constraints and divergence of opinion, we further split the most short-sale constrained quartile into two samples: one with the highest divergence of opinion and the other with the lowest divergence of opinion, again using quartiles. Our significance tests used skew-adjusted t-statistic developed by Johnson (1978) and refined by Hall (1992) to correct for the positive skew of CBHAR distribution (Exhibit B).

IV. Econometric Model and Variable Description

Our empirical model is as follows:

$$CBHAR_{i,t} = \beta_0 + \beta_1 RelOff_i + \beta_2 SI_i + \beta_3 Div_i + \beta_4 \ln(Mktval)_i + \beta_5 Prestige_i + \beta_6 (RelOff_i * Div_i) + \beta_7 SpecInd_i + \epsilon_{i,t}$$

where i is the issuing company and t is the number of weeks following the issue date. As mentioned earlier, the dependent variable, cumulative buy and hold abnormal returns (*CBHAR*), is the residual from a regression using the Carhart four-factor model (Exhibit A).[1] The relative offer size (*RelOff*) is the number of new shares issued divided by the stock float one day prior to the announcement date and serves as a measure for the change in float. The short interest (*SI*) is defined as last reported total shares shorted divided by float size and measures the degree of short-sale constraints. We test three different proxies for divergence of opinion (*Div*) which are as follows:

1) Relative analyst estimate spread (*ASpread*) is defined as the difference between high and low analyst estimates of EPS divided by the share price one day prior to the stock offering date.

2) Market-adjusted turnover (*MATO*) is defined as $\left[\left(\frac{Vol_{i,t}}{Float_{i,t}}\right)_{Firm} - \left(\frac{Vol_{i,t}}{Float_{i,t}}\right)_{Market}\right]$, where $\left(\frac{Vol_{i,t}}{Float_{i,t}}\right)_{Firm}$ is the volume divided by float at time t for the firm and $\left(\frac{Vol_{i,t}}{Float_{i,t}}\right)_{Market}$ is the volume divided by float at time t for the S&P 500 index.

3) Implied volatility on 6 month at-the-money put options (*DVOL*) is defined as the implied volatility of the at-the-money put option with 6 months left until expiry.

[1] The Carhart model is an extension of the Fama-French 3 factor model by adding a momentum factor.

In addition, the natural logarithm of market capitalization (*ln(Mktval)*) is measured on the day before the offer date and adjusted for inflation using base year of 2005. This variable is used in previous literature to capture the effects of information asymmetry. The prestige of the underwriter (*Prestige*) is a dummy variable equal to 1 if the underwriting investment bank falls within our list of the top underwriters by deal size. Industry classification (*SpecInd*) is a dummy variable equal to 1 if the company falls in our list of speculative industries according to the GICS classification system.[1]

V. Data Description

A. Data Sources

Data on seasoned equity offerings were obtained from the Securities Data Company (SDC Platinum) from January 2002 to January 2015 for stocks trading on the NYSE, AMEX and NASDAQ exchanges. The sample was restricted to companies with a minimum market capitalization of $1 million and excluded real estate investment trusts, American Depository Receipts, and investment funds, as is standard in the literature. The final sample had 2328 observations in total. Data relating to share price, volatility, and analyst estimates were obtained from Wharton Research Data Services (WRDS) and Thomson Reuters. Stock float data were obtained from Capital IQ. Data relating to the calculation of CBHAR were obtained from CSRP and the Ken French Data Library.

B. Summary Statistics

The SEOs are fairly spread out over the time period with slightly more weighted toward the last 5 years (Table 1). There is also a good diversity of industries with healthcare being the most frequently occurring followed by high technology and financials (Table 2).

Table 1: Distribution of Offerings by Year

Year	Number of offerings
2002	123
2003	166
2004	196
2005	167
2006	188
2007	161
2008	62
2009	196
2010	223
2011	214
2012	203
2013	228
2014	201
Total	**2328**

Table 2: Distribution of Offerings by Industry

Industry	Number of Offerings
Consumer Products and Services	133
Consumer Staples	50
Energy and Power	201
Financials	377
Government and Agencies	1
Healthcare	614
High Technology	365
Industrials	180
Materials	86
Media and Entertainment	75
Real Estate	64
Retail	102
Telecommunications	80
Total	**2328**

[1] Examples include: Oil and Gas exploration, Metals and mining, Pharmaceuticals, Biotechnology, Life Sciences, Information Technology

Table 3 reports the summary statistics for each regression variable and the abnormal buy and hold returns in various time windows.

Table 3 **Summary Statistics of Regression Variables**

Firm Characteristics:	Mean	Min	Max	Standard Deviation	Number of Observations	Missing Values
Market Capitalization (millions)	1116.070	1.000	138352.100	4495.700	2328	0
RefOff	1.160	0.001	291.120	8.920	1728	600
SI (%)	8.460	0.011	118.000	10.370	1397	931
Divergence of Opinion-						
Analyst Estimate Spread	0.037	0.000	4.120	0.143	999	1329
Implied Volatility (%)	55.560	12.480	169.660	27.450	555	1773
MATO	0.124	-0.528	0.320	0.107	1467	861
SpecInd	0.421	0.000	1.000	0.494	2328	0
Prestige	0.2165	0.000	1.000	0.313	2328	0
Abnormal Buy and Hold Returns (%):						
t-30 through t-1	3.38%	-68.78%	377.88%	25.11%	2318	10
t-1 through t-0	-1.43%	-51.47%	66.32%	6.40%	2318	10
t-0 through t+7	0.32%	-41.48%	263.63%	12.10%	2318	10
t-0 through t+30	0.63%	-58.08%	305.89%	18.18%	2318	10
t-0 through t+90	0.66%	-145.64%	407.68%	32.51%	2318	10
t-0 through t+180	-0.31%	-159.95%	559.44%	47.75%	2318	10
t-0 through t+360	-5.91%	-434.81%	754.27%	71.47%	2318	10

VI. Presentation and Discussion of Findings

A. Presentation

The Table 1 below shows the ordinary least square regressions of our empirical model when analyst opinion spread is used as the proxy for opinion divergence.

Similar to Cooney, Kato, and Suzuki (2012), we find issue date returns (t-1 to t-0) are negatively impacted by relative offering size and positively impacted by market size. Post-event returns, however, are not significantly affected by divergence of opinion, relative offering, and prestige of the underwriter. For returns 180 days after the issue date, we find both the interaction term and the industry classification dummy are statistically significant and negative.

Table 2 below shows the ordinary least square regressions of our empirical model when market-adjusted turnover is used as the proxy for divergence in opinion.

Table 1: Analyst Spread

	(1) t-1 to t-0	(2) t-0 to t+7	(3) t-0 to t+30	(4) t-0 to t+90	(5) t-0 to t+180	(6) t-0 to t+360
Short Interest	0.000131	-0.0000507	0.000346	-0.000371	0.00114	0.00348
	(0.69)	(-0.20)	(0.82)	(-0.49)	(0.90)	(1.26)
Aspread	-0.0292	0.0624	-0.0434	0.0597	0.327	0.0210
	(-0.49)	(0.65)	(-0.32)	(0.18)	(0.82)	(0.05)
RelOff	-0.000749***	0.00486	0.00623	0.00701	0.00972*	0.00263
	(-3.27)	(0.97)	(0.74)	(1.09)	(1.52)	(0.53)
Aspread X RelOff	-0.00907	-0.0591	-0.0874	-0.105	-0.216**	-0.134*
	(-1.36)	(-1.05)	(-0.86)	(-1.29)	(-2.55)	(-1.45)
lnmktval	0.00295**	-0.00246	-0.00623*	-0.00903	-0.00448	0.0179
	(1.84)	(-1.08)	(-1.49)	(-1.22)	(-0.43)	(1.11)
Prestige	-0.0140***	-0.00936	-0.00855	0.0233	0.00353	0.0601
	(-3.42)	(-1.37)	(-0.71)	(1.25)	(0.11)	(1.33)
SpecInd	0.00339	0.00306	0.00319	-0.0425**	-0.0470*	-0.0682*
	(0.82)	(0.51)	(0.30)	(-2.01)	(-1.52)	(-1.49)
Constant	-0.0289***	0.0217	0.0487**	0.0770*	0.0222	-0.176*
	(-2.66)	(1.32)	(1.65)	(1.48)	(0.31)	(-1.59)
Observations	843	843	843	843	843	843

t statistics in parentheses

t statistics calculated using White's Heteroskedastic adjusted standard errors

* $p < 0.15$, ** $p < 0.10$, *** $p < 0.01$

In this regression, divergence in opinion, relative offering size, and industry classification are negative and statistically significant for returns after 180 days, consistent with our hypothesis. Although the sign of the interaction is negative, it fails to reach significance at the 15% confidence level.

Table 3 below shows ordinary least square regressions of our empirical model when implied volatility of put options is used as the proxy for divergence in opinion.

Except issue day returns and one week returns, most coefficients are statistically insignificant. The sign of the interaction term is negative for all time periods after the issue date which is consistent with our hypothesis.

B. Discussion

From the three different proxies used above, the market adjusted turnover regression produced the most significant coefficients. The other two proxies failed to produce significant results because they may be poor indicators of divergence in investors' opinions.

Table 2: Market Adjusted Turnover

	(1)	(2)	(3)	(4)	(5)	(6)
	t-1 to t-0	t-0 to t+7	t-0 to t+30	t-0 to t+90	t-0 to t+180	t-0 to t+360
Short Interest	0.000201	0.000330	0.000565	0.000585	0.00252**	0.00774***
	(1.03)	(0.84)	(1.33)	(0.67)	(1.85)	(3.03)
Market Adjusted Turnover	0.0272	-0.0213	-0.0853**	-0.372***	-0.321**	-0.380**
	(1.22)	(-0.58)	(-1.67)	(-2.65)	(-2.14)	(-1.98)
RelOff	0.00186	-0.00596	0.00656	-0.0126	-0.0240*	-0.0328*
	(0.72)	(-0.91)	(0.69)	(-1.14)	(-1.45)	(-1.59)
MATO X RelOff	0.00648	-0.0251	0.00981	-0.0419	-0.0744	-0.0803
	(0.87)	(-0.93)	(0.25)	(-1.08)	(-1.24)	(-1.34)
lnmktval	0.00227*	-0.00851	-0.00167	-0.00706	-0.0237**	-0.00456
	(1.47)	(-1.22)	(-0.30)	(-0.84)	(-1.83)	(-0.28)
Prestige	-0.0123***	0.00350	-0.00378	0.0119	-0.00582	0.0257
	(-2.80)	(0.35)	(-0.27)	(0.55)	(-0.17)	(0.49)
SpecInd	0.000217	0.00608	-0.00963	-0.0333*	-0.0830**	-0.102**
	(0.05)	(0.76)	(-0.90)	(-1.45)	(-2.43)	(-2.06)
Constant	-0.0237**	0.0436	-0.00740	-0.0226	0.0911	-0.126
	(-1.99)	(1.20)	(-0.21)	(-0.43)	(1.07)	(-1.15)
Observations	1049	1049	1049	1049	1049	1049

t statistics in parentheses

t statistics calculated using White's Heteroskedastic adjusted standard errors

* $p < 0.15$, ** $p < 0.10$, *** $p < 0.01$

Analyst estimates are typical opinions of an investment bank that issues research papers. Their estimates are likely to be biased upwards in order to generate underwriting fees for the investment bank. Indeed, Hong and Kubik (2003) found that analysts with more optimistic forecasts had better career outcomes. This bias will be even more prevalent when a company is planning to undergo a seasoned equity offering because multiple investment banks will be competing with each other to underwrite the deal. Therefore, all of the analyst estimates are likely to be overly optimistic, misrepresenting their actual opinions in the stock. In this case, the spread of their forecasts will bear less relationship with the spread in investors' opinions. In addition, the larger the offering, the larger the incentive to generate underwriting fees, which may introduce multicollinearity between analyst spread and relative offering.

Although implied volatility on put options explicitly measures uncertainty, the majority of companies in our sample did not have options' contracts available. This is likely due to a larger proportion of our sample being relatively small firms. Thus the implied volatility regression suffers from both a smaller sample size and a biased sample towards larger firms. This could have reduced the significance levels of our coefficients in the regression because larger firms are less likely to be short-sale constrained due to larger public stock float and lower insider ownership.

Table 3: 6 Month Put Implied Volatility

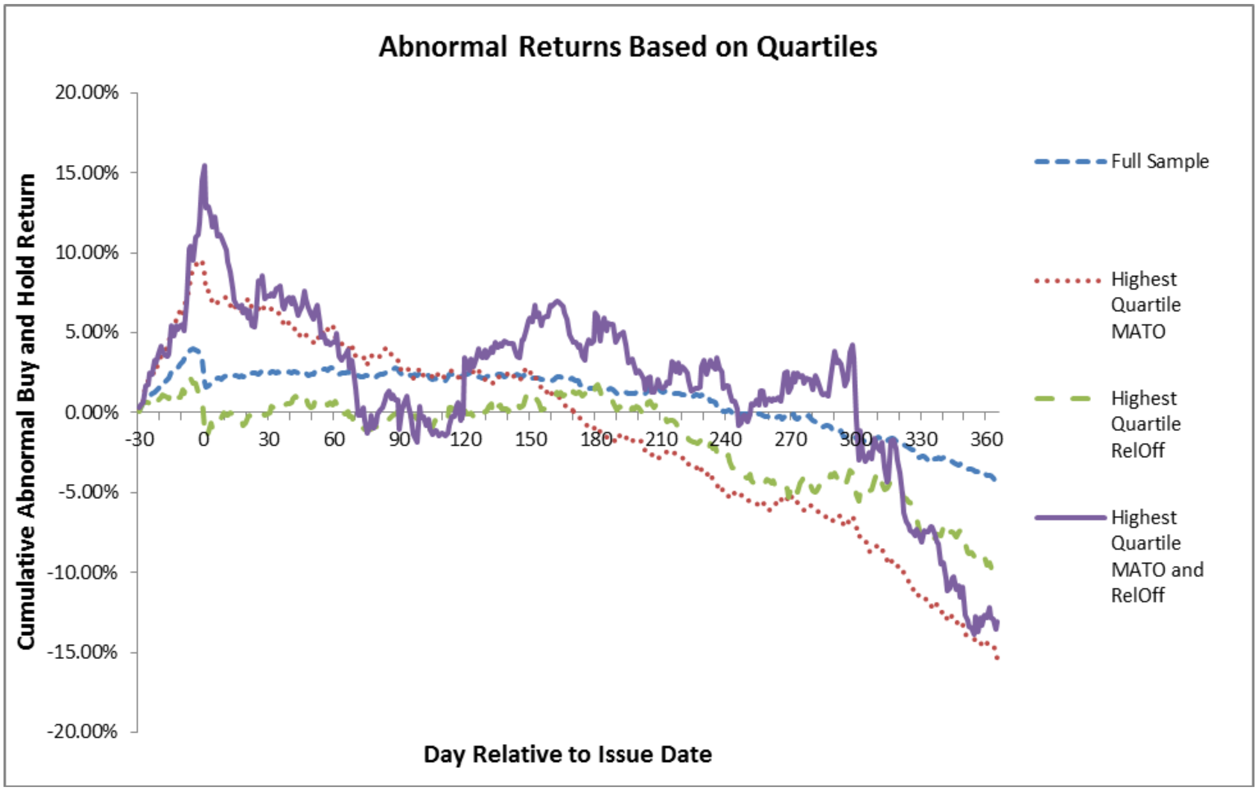
	(1)	(2)	(3)	(4)	(5)	(6)
	t-1 to t-0	t-0 to t+7	t-0 to t+30	t-0 to t+90	t-0 to t+180	t-0 to t+360
Short Interest	0.000184	-0.000486*	-0.000208	-0.000890	0.000629	0.00170
	(0.61)	(-1.58)	(-0.44)	(-0.79)	(0.28)	(0.53)
6 Month Implied Put Volatility	-0.0000224	0.0000724	-0.000482*	-0.000116	0.0000354	-0.00179
	(-0.10)	(0.36)	(-1.59)	(-0.11)	(0.02)	(-0.91)
RelOff	0.0658*	0.0878**	0.0545	0.185	0.335	0.299
	(1.51)	(2.26)	(0.81)	(1.02)	(1.15)	(0.84)
Implied Vol X RelOff	-0.000771	-0.00101**	-0.000546	-0.00201	-0.00407	-0.00519
	(-1.30)	(-2.18)	(-0.70)	(-0.96)	(-1.15)	(-1.30)
lnmktval	0.00519*	-0.00214	-0.00987**	-0.0154	-0.00318	0.000785
	(1.57)	(-0.64)	(-1.97)	(-1.40)	(-0.17)	(0.03)
Prestige	-0.0200***	-0.00163	0.00240	0.0237	-0.0112	-0.00429
	(-3.37)	(-0.27)	(0.21)	(1.06)	(-0.27)	(-0.07)
SpecInd	0.0156**	0.00217	0.00934	-0.00609	0.00660	-0.0165
	(2.09)	(0.29)	(0.74)	(-0.28)	(0.16)	(-0.24)
Constant	-0.0540**	0.0102	0.0839**	0.104	-0.00655	0.0925
	(-1.74)	(0.33)	(1.79)	(0.87)	(-0.03)	(0.34)
Observations	441	441	441	441	441	441

t statistics in parentheses
t statistics calculated using White's Heteroskedastic adjusted standard errors
* $p < 0.15$, ** $p < 0.10$, *** $p < 0.01$

The market adjusted turnover regression was the most consistent with our hypotheses (*i, ii, iii*). Relative offering size, divergence of opinion, and the interaction term were all significant and negative for returns after one week. This might indicate that abnormal trading volume is a good proxy for divergence opinion. Indeed, frequent trading implies frequent changes of shares between different investors, indicating changes in the expectations of individual investors. The industry classification dummy was also both statistically and economically significant. The coefficient estimate infers that a company operating in a speculative industry such as biotechnology and information technology experience on average 10.2% lower annual returns than a company that did not operate in such industries after a SEO. The above impacts could be due to that fact that speculative firms are more likely to be short-sale constrained due to their binary nature - the company either discovers a drug or technological breakthrough that generates a large amount of profits in the future, or fails and earns zero profits. The possibility of the company being worth zero creates a high demand for short-sales.

C. Univariate Analysis

The following chart shows the cumulative abnormal returns of different portfolios split by different quartiles of divergence of opinion and relative offer size.



The following table shows abnormal returns relative to the issue date and their t-statistic based on a hypothesis test where the null is that the abnormal return is equal to zero (Exhibit B).[1]

	Highest Quartile MATO		
Period	CBHAR	Cross T-stat	Skew T-stat
t-30 to t-1	9.50%	4.63	6.88
t-1 to t-0	-1.34%	-3.55	-3.32
t-0 to t+7	-0.54%	-1.15	-1.13
t-0 to t+30	-1.27%	-1.52	-1.49
t-0 to t+90	-3.54%	-2.38	-2.37
t-0 to t+180	-7.00%	-3.18	-3.01
t-0 to t+360	-19.11%	-6.02	-5.38

[1] Four, three, two, and one dot represent significance levels at , 0.01%, 0.025%, 0.05%, and 0.1% respectively.

	Highest Quartile RelOff		
Period	CBHAR	Cross T-stat	Skew T-stat
t-30 to t-1	0.92%	0.59	0.66
t-1 to t-0	-2.20%	-5.26	-5.20
t-0 to t+7	1.09%	1.06	1.24
t-0 to t+30	1.44%	1.19	1.30
t-0 to t+90	1.47%	0.69	0.74
t-0 to t+180	3.34%	1.04	1.11
t-0 to t+360	-7.99%	-1.73	-1.59

	Highest Quartile MATO and RelOff		
Period	CBHAR	Cross T-stat	Skew T-stat
t-30 to t-1	14.54%	2.03	2.81
t-1 to t-0	-1.00%	-0.76	-0.72
t-0 to t+7	-1.66%	-1.46	-1.60
t-0 to t+30	-3.66%	-1.51	-1.55
t-0 to t+90	-6.89%	-1.74	-1.70
t-0 to t+180	-3.45%	-0.49	-0.43
t-0 to t+360	-19.18%	-1.97	-1.59

	Full Sample		
Period	CBHAR	Cross T-stat	Skew T-stat
t-30 to t-1	3.38%	6.35	8.05
t-1 to t-0	-1.43%	-10.56	-10.21
t-0 to t+7	0.32%	1.24	1.36
t-0 to t+30	0.63%	1.63	1.73
t-0 to t+90	0.66%	0.96	0.99
t-0 to t+180	-0.31%	-0.31	-0.30
t-0 to t+360	-5.91%	-3.90	-3.62

Similar to our regression analysis, higher MATO and relative offering size leads to statistically significant negative abnormal returns that are greater than the full sample average. The interaction of the two variables, however, do not lead to a significant difference when compared with the highest MATO quartile group alone – both groups experience approximately -19% abnormal returns after one year. Therefore, the univariate analysis is also consistent with our hypotheses (*i*, *ii*).

VII. Conclusion

This paper finds supporting evidence for heterogeneous beliefs in the presence of short-sale constraints using the market-adjusted turnover as the proxy for divergence of opinion. The remaining proxies: relative analyst estimate spread and 6 month put implied volatility, are not found to be significant. In addition, the interaction of relative offer size and opinion divergence is significant and negative, as shown in both the regression and univariate results.

The implications of Miller (1977)'s asset pricing theory are enormous for academics and financial market participants. It provides a simple explanation of how divergence of opinion can produce persistently overvalued stock prices and provides insight into asset bubbles. It implies that investors can improve portfolio returns by avoiding stocks with high divergence of opinion following a secondary equity offering, which suggests a continuing role for active management.

We hope our empirical analysis of seasoned equity offerings in the U.S. from 2002 to 2015 has shed more light on the effects of heterogeneous beliefs and short-sale constraints on stock prices.

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Appendix

Exhibit A: Cumulative Buy and Hold Abnormal Return Calculation

Our dependent variable, Cumulative Buy and Hold Abnormal Returns (*CBHAR*), is the residual from a regression using the Carhart four-factor model. Abnormal return is defined as:

$$R_{it} = \hat{\alpha}_i - \hat{\beta}_i RM_t - \hat{s}_i SMB_t - \hat{h}_i HML_t - \hat{w}_i WML_t$$
$$CBHAR_{it} = \prod_{t=T_1}^{T_2}(1 + R_{i,t}) - \prod_{t=T_1}^{T_2}(1 + E[R_{i,t}]),$$

where R_{it} is the return of the stock on day *t* for company *i*, *RM* is the value-weighted return of all listed firms or a large aggregate index such as S&P 500, *SMB* (small minus big) is the difference between the returns on diversified portfolios of small stocks and big stocks, *HML* is the difference between the returns on diversified portfolios of high book-to-market (value) stocks and low book-to-market (growth) stocks, and *WML* is the difference between the returns on diversified portfolios of the winners and losers of the past year.[1] The coefficients $\hat{\alpha}_i, \hat{\beta}_i, \hat{s}_i, \hat{h}_i, \hat{w}_i$ are estimated using an OLS regression of stock returns for company *i* on *RM, SMB, HML*, and *WML* for an out-of-sample period between -130 and -30 days before the issue date. $CBHAR_{it}$ is the cumulative average buy and hold abnormal return assuming the investor holds the stock from T_1 to T_2.

Exhibit B: Calculation of t-statistics

Specifically, the cross sectional t-statistic is defined as:[2]

$$t_{ACBHAR} = \sqrt{N}\frac{ACBHAR}{S_{ACBHAR}} \quad where \quad S^2_{ABHAR} = \frac{1}{N-1}\sum_{i=1}^{N}(CBHAR - ACBHAR)^2$$

where ACBHAR is the average cumulative buy and hold abnormal return,

and the skew adjusted t-statistic is defined as:

$$t_{skew} = \sqrt{N}\left(S + \frac{1}{3}\gamma S^2 + \frac{1}{27}\gamma S^3 + \frac{1}{6N}\gamma\right)$$
$$where \quad S = \frac{ACBHAR}{S_{ACBHAR}} \quad and \quad \gamma = \frac{N}{(N-1)(N-2)}\sum_{i=1}^{N}(CBHAR - ACBHAR)^3 S^{-3}_{ABHAR}$$

For the majority of our ACBHARs, the skew adjustment does not change our results. The skew adjustment generally increases the t-statistic so our negative ACBHARs become less statistically significant and our positive ACBHARs become more statistically significant.

[1] For further information, see Carhart (1997).
[2] For further information, see Lyons, Barber, and Tsai (1997).

Modeling Efficiency Units of Electricity in Production

Nadezhda Peretroukhina and Siddharth Untawala

Abstract

"Raw energy" in traditional Cobb-Douglas production models is assumed to be homogeneous in both value and productive capacity among producers. In this paper, we describe a new method to model heterogeneous and parsimonious preferences, as well as the constraints of various industries. Simple and versatile, "efficiency units of electricity" is able to significantly model cross-industry variation in energy productivity using principles of statistical physics to mitigate the introduction of several parameters. Our findings demonstrate that the introduction of efficiency units of electricity in production improves the statistical efficiency of estimators for labour and capital. We recommend that supplementary literature should explore the economic significance of the Boltzmann weighted parameter (φ) using alternative proxies and datasets for efficient labour using industry level considerations.

Keywords: Efficiency Units of Electricity, Cobb-Douglas Production Model, Heterogeneous Preferences, Total Factor Productivity, Cross-Industry Variation, Boltzmann distribution

Faculty Consulted: Dr. Charles Saunders and Dr. Rui Castro

1. Introduction

The production function is a key economic idea that expresses the relationship between physical inputs and the output produced. Convention dictates that the factors of production feature labour (L) and capital (K), exclusively. However, after extensive research and interest in the field of Econophysics, the goal of this paper is to explore how efficiency units of electricity can account for differences in the use of raw energy in production among industries. The economic question that we are exploring examines how the heterogeneous preferences and constraints, faced by various industries for raw 'energy', can be modeled in production functions.

Our interest in the role of energy stems from the integration of key principles in both economics and physics; whereby the behaviour of matter and properties of energy in physics can describe economic preferences and constraints. Specifically, the idea of energy conservation can mirror the behaviour of industries in cost-minimization problems associated with production. In addition, the variances in productivity among industries will be represented by industry-specific labour force controls that mirror the characteristics of particles in thermodynamics. Howlett, Netherton & Ramesh suggest that the fundamental differences in industry production can be useful to policy makers to incorporate the role of energy when regulating the production of Canadian industries (Howlett, Netherton, and Ramesh 1999, 5-15).

By identifying efficiency units of energy as a significant factor of production, we will estimate a production function that incorporates a productivity parameter using the Boltzmann distribution. To do so, we will draw ideas from our literature survey, as well as the consultation of Professor Saunders and Rui Castro. We will then define our production model and variables using data drawn exclusively from CANSIM. In the last section we intend to outline other considerations we could have made to this model and how we would intend to proceed with the objectives of the paper.

2. Background and Literature Review

2.1 Motivation and Economic Origins

Traditional economic models of production emphasize raw factor data and flows, such as labour, capital, land and technology, in order to describe changes in the output produced by firms and industries. Specifically, in the Cobb-Douglas model, fixed proportions of labour (L) and capital (K) explain how much output (Y) is produced (Williamson 2012). Any unobserved variation is aggregated in Total Factor Productivity (Z). This model is presented below:

$$\mathbf{Y = Z\{f(K, L)\} = Z\{K^{\alpha}L^{\beta}\}}$$

such that α = proportion/income share of capital, β = proportion/income share of labour.

The motivation of this paper is to examine the TFP using methods aimed to quantify unobserved factors that generate large variances in output at the industry level. In the traditional Cobb-Douglas model, the TFP aggregates a variety of influences on the growth of output including technology, political, cultural and unobservable economic factors that may be random or unobservable (Jorgenson and Griliches 1967, 276-279). The potential to identify an omitted variable, hidden within the TFP, may cause the estimates of labour and capital to be under or overstated in the existing model. This paper will explore the impact of introducing an additional factor input to the classic Cobb-Douglas model, with the intention of arguing that there is potential for improving the validity and efficiency of the labour and capital estimators.

The classic Cobb-Douglas model emphasizes the role of income shares of capital and labour as α and β respectively. These shares represent ratios of each fixed level of input for labour and capital needed to produce a unit of output, such that:

$$\mathbf{Total\ Labour\ Income\ Share = \frac{w\{\sum_{i=1}^{n} l_i\}}{Y} = \beta}$$

$$\text{Total Capital Income Share} = \frac{\{\sum_{i=1}^{n} \pi_i\}}{Y} = \alpha$$

The derivations of these factor input shares are provided in Figure 1 in the Appendix (Williamson 2012). At the industry level, these shares present the aggregate proportions of labour and capital inputs of all firms. This paper will focus on the national level to interpret these elasticities as aggregate proportions of capital and labour of all industries in a given year to produce GDP. Historically, the shares of capital and labour at the national level were 0.3 and 0.7 respectively according to the Cobb-Douglas Model.

2.2 Preliminary Tests and Parameter Analysis

Preliminary tests on our sample of Canadian industry data were used to test the assumptions of the Cobb-Douglas Model in a few different approaches.

1) An initial regression was conducted as represented below:

$$\ln Y_{it} = \ln Z + \alpha \ln K_{it} + \beta \ln L_{it} + \varepsilon_{it}$$

such that i = industry, t = time period. As shown in Table 1 of the appendix, the results were statistically significant at the 99% confidence level and provided values of 0.253 and 0.232 for α and β respectively. This result provided us with further justification to analyze the inputs of production.

2) Subsequent tests were done to assess the relationship of Total Factor Productivity in relation to the discrepancies in income shares of labour and capital. The Solow Residual was used to measure the TFP indirectly by examining whether the low elasticity values of labour and capital could be attributed to discrepancies in data or whether elasticity estimates could potentially exhibit considerable bias. The Solow Residual is determined as follows for each given year (Williamson 2012):

$$\text{Solow Residual} = TFP_t = Z_t = \frac{Y_t}{K_t^{\alpha} L_t^{\beta}}$$

Table 2 in the appendix compares and contrasts the value of Solow residuals from our approximated values of α and β and that of traditional assumptions, more accurately 0.3 for α and 0.7 for β. The indirect calculations show that the TFP values calculated using the elasticities generated from the first regression of 0.253 and 0.232 for α and β respectively were significantly larger than both the TFP values calculated by CANSIM and the TFP values using the traditional Solow assumptions of 0.7 and 0.3 for α and β respectively. Specifically, the TFP values using our regression-specific elasticities were 4x greater than the TFP values of the sample, while the traditional Solow elasticities were 79x smaller.

This test shows that the *true* elasticities of α and β respectively in our sample are significantly overestimated by the classic assumptions and slightly underestimated by our calculated Solow residuals. The important implication of this preliminary test is to show that the proportions of the TFP from 2002 to 2014 stay fairly consistent, implying that the proportion of output unexplained by labour and capital remains consistent through time.

3) Final initial tests were conducted to test whether labour and income shares are also constant through different time periods. Growth accounting was used to examine the growth in output (Y_t) specific to the data sample. The full derivations of the growth factors of L_t, K_t, TFP_t and Y_t are presented in Figure 2 of the appendix (Williamson 2012). Table 3, exhibits the results from conducting regressions of the Cobb-Douglas production function over the short time periods. These results suggest that the income shares of labour and capital are not constant through the time periods, while the shares could not also account fully for the growth rate of GDP per capital from 2002 to 2011. The findings from the regressions in Table 3 of the appendix suggest that the estimates of labour and capital are not consistent due to the large fluctuations in the standard errors. Large variations in standard errors may be caused by endogeneity of the model, which will be tested in subsequent sections.

To better account for discrepancies between the theoretical Cobb-Douglas Model and the empirical data drawn from Statistics Canada, we are proposing a modification to the Cobb-Douglas model, such that a new factor of input is introduced to the model. This factor of input, breaks down the TFP into a quantifiable omitted variable and a random component, with the intention of reducing the bias of existing estimators.

Subsequent sections will describe and analyze the significance of proposing a new input factor that accounts for the role of raw energy in production. What sets our intentions apart from other literature or models that incorporate raw energy data - as energy measured by oil demand or electricity usage among industries - is that we are looking to model energy in accordance with the perceived differences in ability of various industries to use energy as an input. These differences refer to infrastructural and operational differences, as well as parsimonious preferences in energy use.

2.3 The Role of Energy and Econophysics

In economics, the raw energy is traditionally assumed to be homogeneous, whereby the marginal benefit from each additional unit of raw energy is constant (Kümmel, Ayres, and Lindenberger 2010, 147-52). The productivity of each quantity of electricity, for example, is thus considered to be the same. A kilowatt or terajoule of electricity in the agricultural industry has the same productivity capacity as a kilowatt or terajoule of electricity in the manufacturing sector.

However, while raw electricity can be considered homogeneous, the assumption that Kümmel, Ayres, and Lindenberger make that the productive capacity of each unit of raw electricity is also homogeneous is very weak (Kümmel, Ayres, and Lindenberger 2010, 145). In each industry, a certain amount of electricity is needed to keep buildings and equipment running in order to generate heat and light as conditions for labour, etc. These conditions vary according to the specific industry, and thus should not be considered to have the same productive capacity when generating output. Rather, the productivity capacity or “efficiency units” of electricity are heterogeneous.

In order to model the argued heterogeneous behaviour of efficiency units of electricity, this paper will draw on principles from Econophysics. Specifically, thermodynamics can represent economic models of various industry preferences and constraints to derive implicitly the objective functions (Landau 1958, 12). These objective functions consider units of electricity with different efficiency values at the margin. Specifically, industry preferences can be modeled by the non-uniform behaviour of energetic particles in natural equilibrium, while minimized-costing constraint is reflected by the conservation of energy principle.

The primary source for the methodology used in this paper draws from the work of Park, Kim, and Isard (2012). In their paper, the allocation of emission permits is modelled in various countries based on a function of national pollution preferences over time. Rather than allowing for free-trade or the use of social planner, the proportion of permits allocated to each country was argued to be most efficient when distributed according to the Boltzmann distribution. This thermodynamic principle considers the historical emission levels of each country in previous periods against their relative sizes (Park, Kim, and Isard 2012, 4885-890).

The efficiency of a dynamic distribution is upheld by Barbanel and Brams in a purely conceptual cake-cutting problem. In order to allocate the optimal amount of cake to each family member, such that the distribution is Pareto-optimal, envy-free and equitable, the consumption preferences of each family member must be weighted against the caloric intake that is suggested for the relative weight of each individual. That is to say that heavier individuals will require more cake than a thinner individual in order to satisfy each daily caloric requirement. The challenge that Barbanel and Brams found is that as the number of players in the cake-cutting problem increases, the fair distribution of this “cake” becomes far more complicated (Barbanel and Brams 2004, 251-3). Figure 3 below is a pictogram which represents a breakdown of the variables and concepts from statistical physics and how Park, Kim, and Isard used those features as a proxy for their economic model (Park, Kim, and Isard 2012, 4889).

In our paper, the use of the Boltzmann distribution – from physical sciences – will be extended to describe how efficiency units of electricity are assumed to be heterogeneous, modeled on page 26 of Section 3.2, in Figure 6.

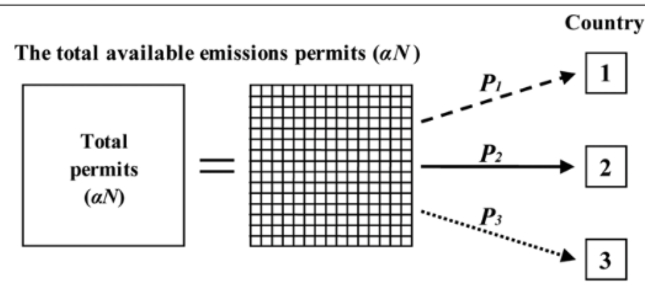
2.4 Incorporation of a Boltzmann Weight

Landau and Lifshitz defined the Boltzmann probability, mentioned in the Emissions Trading paper, as the distribution of energy levels among all particles in a physical system. The distribution is a function of the available energy, relative preferences of energy and the number of particles in the system (Landau and Lifshitz

1958, 11-14). The common model for the Boltzmann distribution is exhibited below in Figure 4.

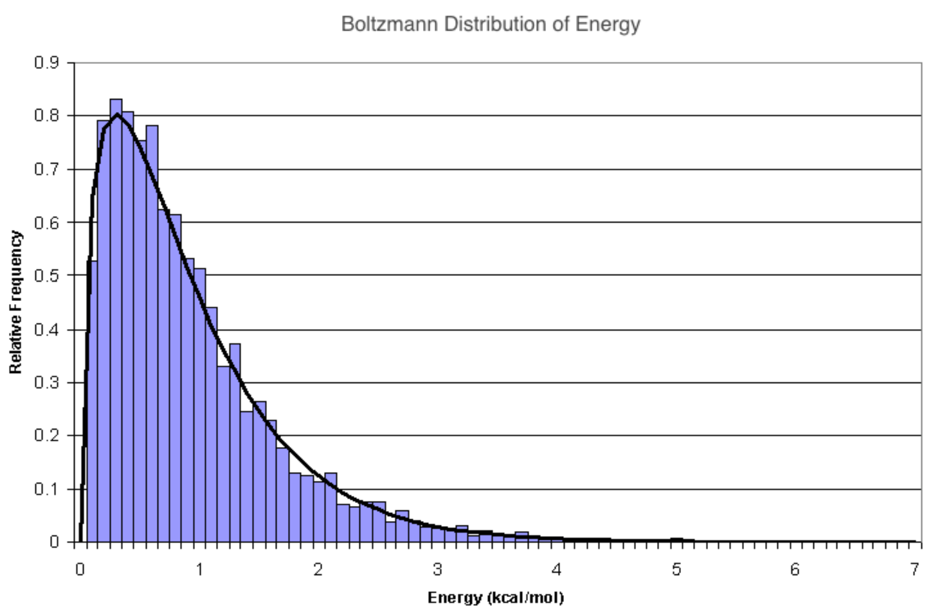
Figure 3: Breakdown of The Park, Kim, and Isard Economic Model

The Boltzmann distribution for permit allocation.

Boltzmann distribution	Description
In physical sciences	$P_i \propto e^{-\beta E_i}$ Where, P_i = probability that a particle stays in substate i e = constant of the exponential function ≈2.71828 $\beta = 1/kT$ (k = Boltzmann constant, T = absolute temperature) E_i = energy of substate i
Potential application in permit allocation	$P_i \propto C_i e^{-\beta E_i}$ Where, P_i = probability that emissions permits are allocated to a country i e = constant of the exponential function ≈2.71828 β = constant (≥0) E_i = allocation potential energy per capita of a country i C_i = total population of a country i
Permit allocation using the Boltzmann distribution*	

* The total available emissions permits (αN) are split into N pieces of unit carbon credit α, and then the emissions permits are allocated to country i (i = 1, 2, and 3) based on the probability distribution (P_i) from the Boltzmann distribution. Note that the number (N) of unit emissions permits can always be made large enough for the Boltzmann statistics by making the unit emissions permit (α) smaller.

Figure 4: Boltzmann Distribution of Energy



More generally, this distribution can describe any set of entities have varying preferences and constraints for energy as mentioned by Banerjee and Yakovenko. Preferences are also constrained to the temperature, or environment conditions, of the system, where the more "energetic" an entity is, the more energy input it requires. The distribution will naturally follow the non-uniform distribution exhibited in Figure 4 (Banerjee and Yakovenko 2010, 755-64). The distribution implies that density of observations is higher at low energy levels, meaning that in any population, the frequency of high-efficiency entities will significantly outweigh observations of "energetic" or low-efficiency entities.

The application of this Boltzmann distribution can show that entities tend to exist in low energy states, since this distribution is more sustainable in the natural equilibrium. When modeling efficiency units of electricity, the Boltzmann weight is the most effective method of modeling heterogeneity in the energy input efficiency among industries for the following reasons:

1) The Boltzmann weight is simple and versatile. This single variable is used to describe industry energy input preferences by weighting relative characteristics and constraints according to their effect on efficiency levels of using electricity to generate output. Thus, the weight is easily calculated based on the unique characteristics of the industry that can be observed, without requiring a specific weight or parameter for each observation by province, industry and year. The weight significantly reduces the amount of terms regressed on output, by requiring no additional parameters on efficiency units of electricity.

2) Energy is inherently a non-linear dynamic flow, according to the assumptions upheld by thermodynamics and argued by other literature analyzed in Section 2.2. Electricity, as a form of energy, varies in volume according to province, industry and year, due to localized and industrial factors associated with energy needs and efficiencies production. The heterogeneous assumption of behaviour in the Boltzmann distribution upholds the argument that efficiency units of electricity follow a similar heterogeneous assumption. It is thus a way to model energy in a parsimonious way, such that there is a relative scarcity of high-energy and low efficient industries compared to high-efficient industries.

3) Lastly, the distribution allows us to interpret the role of energy in production in a meaningful way. Efficiency units of electricity, quantified by electricity input in terajoules required per worker, describe the unique interaction of electricity preferences of an industry with the relative size of efficient workers in the industry.

The specifications for the model for the "Boltzmann weight" will be outlined in Section 3. It is important to note, however, that the first and second reasons listed above outline the importance of including observations at the provincial level, in addition to year and industry, i.e. the subscript of pit. The use of provincial level data can net out the fixed effects of energy regulation at the provincial and federal levels, in order to prevent covariance between the residual of the TFP and the efficiency units of electricity. At the federal level, the National Energy Board oversees the inter-provincial as well as import and export transfer of electricity through power lines, specifically in the energy section (Natural Resources Canada 2016). At the provincial level, utility boards regulate the electricity competition. The provincial government in Alberta has fully privatized the retail competition of electricity, while Ontario is in the process of doing so. All other provinces adhere to public generation and distribution of electricity. The discrepancies in energy regulation among provinces facilitate the need to run fixed-effect using a provincial dummy, or as we did, provincial level data by industry.

Table 4 in the appendix compares the results from conducting a fixed effects model with a random effects models for specific years for each province among specific industries. The results are statistically significant for both models and the Hausman test statistic value of 14.94 in Figure 5 suggests that since unobserved regulation and political decisions at the provincial level are not significant to the findings. However, we will include the subscript of *pit* in our regressions in order to mitigate any fixed effects associated with provincial-level data.

2.5 Hypothesis and Assumptions

The goal of the paper is to estimate a production function that incorporates an efficiency units of electricity parameter. The use of raw energy in production analysis of prior literature does not account for cross-industry variation in productivity.

We are thus using the Boltzmann distribution specifically, because it is a way to model this variation using observables, while reducing the amount of parameters that would necessarily constrain each industry in each province and each year. We are able to reduce the number of parameters to zero parameters, by modeling φ by means of the Boltzmann weight, based on a constant parameter lambda and observables, such that:

$$\ln(\text{Efficiency Units Of Electricity})_{pit} = \ln\{\varphi_{pit}(\text{Raw Energy in TeraJoules})_{pit}\}$$

Very simply stated, our hypothesis tests whether the coefficient for the efficiency units of electricity can significantly add to the share of output not explained in the initial regressions in Section 2.1. The hypothesis aims to account for the discrepancies between the theoretical assumptions with the Cobb-Douglas Model and the empirical model.

$$H_0 : \beta_{\ln(\text{Efficiency Units of Electricity})} = 0$$
$$\text{against } H_1 : \beta_{\ln(\text{Efficiency Units of Electricity})} \neq 0$$

Our hypothesis is contingent on the following assumptions:

1) We believe energy is used with varying degrees of efficiency across industries.

2) Efficiency of energy use is based on our selected characteristics and proxies.

3) The preferences and constraints of the representative firms within each industry are homogenous, while cross-industry variations in preferences and constraints are heterogeneous.

The intention of the hypothesis test is to compare the standard errors of the original Cobb-Douglas parameters against the new parameters set by our model. The comparison of standard errors allowed us to analyze whether our model could more efficiently explain the growth in industry output through time and account for the discrepancies in the initial tests conducted in Section 2.1.

3. Model

Our model incorporates the role of efficiency units of electricity (E) as an additional factor of production. We will firstly account for raw energy (Raw E), in addition to labour (L) and capital (K) as the factors of production. The model draws on empirical data in Section 4, using Canadian industry data over multiple time periods and cross-sectional variables.

3.1 Model of Production

In contrast to the conventional Cobb-Douglas production function, we have proposed an alternative model that incorporates energy as a factor of production in aggregate industry output:

$$\ln Y_{pit} = \ln Z + \alpha \ln K_{pit} + \beta \ln L_{pit} + \gamma \ln RawE_{pit} + \varepsilon_{it}$$

such that p = province, i = industry, t = time period

In this model: $\ln Y_{pit}$ represents the aggregate output in industry and year; $\ln Z$ represents the total factor productivity, $\ln K_{pit}$ and $\ln L_{pit}$ are the natural logarithms of labour and capital inputs respectively broken down by province, industry and year, while α and β are their income shares. Ln $RawE_{pit}$ represents the natural logarithm of raw electricity input, while γ measures its respective income share or elasticity.

3.2 Boltzmann Considerations in the Model of Production

Ln E_{pit} represents the natural log of efficiency units of electricity, is also broken down by industry, province and year. This transformation is outlined as follows:

$$lnE_{pit} = ln\{\varphi_{pit}(RawE_{pit})\}$$

where

$$\varphi_{pit} = Boltweight_{pit} = \frac{1}{Employ_{pit}}\{PT_{pit}(e^{-\lambda\{AgeCohort_{pit}\}}\}$$

$$RawE_{pit} = Raw\ \ Electricity\ \ In\ \ Terajoules$$

$$such\ \ that\ \ p\ =province,\ \ i\ =industry,\ \ t\ =time\ \ period$$

The Boltzmann weight is a relative weight of electricity units required per worker. It is directly proportional to the fraction of part time employed and inversely proportional to the exponential function of the number of workers within the age range of 25-54 in the economy. In this subsection we would like to explain the rationale behind the suitability of using these economic variables as counterparts for the statistical physics variables.

We modeled the heterogeneous nature of efficiency units of electricity by applying the aforementioned Boltzmann weight. These efficiency units are proportional to the fraction of part-time employed as, in a given province, industry and year, part-time workers, which will be used as a proxy for unskilled workers, would have a higher marginal propensity to consume a unit of electricity input due to their burdens of more energy-intensive tasks. While unskilled labour may be measured with other proxies such as education attainment and work experience, in the context of energy, we are incorporating part-time employment to account for the fraction of employed that requires more energy-intensive tasks with greater allocation of energy to certain tasks in order to meet deadlines and output quotas (Hirsch 2005, 547-51). An industry with abundant part-time workers can thus be considered to be an energy-intensive industry, since output requires more productivity per labour hour to obtain hourly wages, compared to the productivity levels per labour hour of salaried employees.

Another important parameter of the Boltzmann weight is the age cohort which is similarly broken down by province, industry and year. The age cohort proxies the "velocity" of a particle – which is a characteristic of its energy – in a physical system to our economic system through the concept of efficiency. Based on economic research and intuition, age has an effect on productivity levels, as in the proportion of workers within 25-54 years have the most mobility between roles and positions within the industry allowing them to be more efficient in converting factors of inputs to outputs (Skirbekk 2003, 2004-6). This allows us to convert efficiency units of electricity from an exogenous to an endogenous variable that is a function of the non-homogenous nature of the efficiencies of various industries (particularly due to the age cohort). This framework forms a key role in understanding and modeling the heterogeneous nature of the efficiency units of electricity.

An additional feature of this Boltzmann weight is the Greek constant lambda ‘λ’, whose variation has a consequence on the actual value of the weight and thus the efficiency units of electricity. In a thermodynamic system, λ is a constant that is inversely related to temperature of the system, a higher temperature (lower λ) relates to a higher internal energy for a particle and therefore the overall system. There isn’t a well-defined or developed economics equivalent concept for the idea of a temperature or λ but references have been made in the work of Landau and Lifshitz where they describes that at temperatures close to zero in a negative temperature state, the economy “corresponds to an allocation of all workers to a state of the highest productivity” (Landau and Lifshitz 1958, 45-6).

We would like to apply a similar ideology, which assumes that a lower temperature system (thus a higher ‘λ’) signals an economy that requires more efficiency units of electricity input thus a relative scarcity of highly-energy intensive or low-efficient industries. Varying the value of λ between 0 and 1 affords us the ability to examine the effects of these efficiency units of electricity on the elasticity of labour and capital. A ‘λ’ value of close to 0 suggests that there is a larger fraction of highly efficient industries, whereas a value of 1 suggests otherwise. In line with the suggestions of Park, Kim, and Isard, the calculation of the optimum value of λ is not the focus of the paper and recommends that the value at which the least square has a minimum can be used as a reference point (Park, Kim, and Isard 2012, 4890).

Figure 6: Boltzmann Weight for Efficiency Units of Electricity

The Boltzmann Weight for Efficiency Units of Electricity

Boltzmann Weight	Description	
Potential application for heterogeneous production efficiencies among industries	$\mathbf{Boltweight_{pit}} \propto \mathbf{(1/employ_{pit})(PT_{pit})^{*}e^{-\lambda(AgeCohort_{pit})}}$	
	Where,	
	$Boltweight_{pit}$	= relative weight of electricity required per worker; terajoules of electricity per worker per province, industry and year
	$Employ_{pit}$	= Number of workers employed in the industry per province, industry and year
	e	= constant of exponential function ≈ 2.71828
	Λ	= constant ⩾ 0
	$AgeCohort_{pit}$	= number of workers within 25-54 years per province, industry and year; proxy for max velocity/ mobility to engage in new tasks
	PT_{pit}	= total amount of part-time employees in a given province, industry and year; proxy for less skilled or ‘energy-intensive’ labour;

The economic significance of using the Boltzmann-weight, as opposed to any other weight or the lack thereof, is the idea that the labour force follows similar discrepancies in behavior as energy, which is inherently non-uniform. Since both the characteristics concerning full-time and age statistics are measured in the number of workers, we are able to compare the energy input with capital and labour inputs by analyzing electricity input as the amount of additional energy contributed by industry-specific labour force characteristics.

Lastly, before testing of efficiency units of electricity, it is important to understand the conceptual implications of its elasticity γ. According to the derivations in Figure 7 in the appendix, γ represents the proportion of efficiency units of time to industry output, such that:

$$\boldsymbol{\gamma} = \frac{\boldsymbol{\Phi(\mathrm{RawE}_{pit})}}{\mathbf{Y}_{pit}} = \frac{\mathbf{E}_{pit}}{\mathbf{Y}_{pit}}$$

This elasticity represents the industry level output response to a change in efficiency units of electricity. It is interesting to note that the derivative for β does not change and still remains a proportion of labour wages in output. α however, has a noticeable decrease in its elasticity with the introduction of γ. These conceptual changes will be addressed when analyzing the data and results in Section 4.

4. Data, Results and Analysis

As mentioned earlier in the paper, a majority of our data was collected from statistics provided on CANSIM. We proceeded to collect data on provincial GDP, industry GDP, labour force characteristics, productivity, and energy measured in Tera Joules. We used these measures to create variables to fit our model, such as the Boltzmann weights and the factor intensity of industries. Further details of these variables are provided in Figure 8 and Table 5 of the Appendix.

We began by conducting a regression of the natural log of industry GDP on the natural log of labour and capital. The results are represented in Figure 1 of the Appendix and suggest that the total contribution of these factors to output does not equal one, suggesting there might be some other variables which could significantly contribute to remaining share of output. Based on our results, a one percent increase in the flow of capital and one percent increase in the flow of labour leads to 23.2% and 25.3% increases in industry GDP. These coefficients are also statistically significant at a 99.9% confidence level with low standard errors.

Encouraged by these results, we compared results from conducting regressions of the Cobb-Douglas production function, incorporating raw electricity input as energy flow (measured in terajoules), to understand whether energy could be the factor which could help explain the remaining share of output.

The results shown in Table 6 below express that raw energy input (in the form of electricity flow in Terajoules) – as well as labour input and capital input - is economically and statistically significant at 99.9%, with a one percent increase in the flow of energy leading to an approximately 3.45% increase in industry GDP, without any significant economic changes to the other factors of input.

Table 6: Comparison of Regression Results from Cobb-Douglas & Cobb-Douglas (With Raw Electricity Input)

ln IndustryGDP		
ln Labour	0.253***	0.251***
	(3.67)	(3.64)
ln Capital	0.232***	0.239***
	(5.80)	(6.03)
ln Raw E		0.0345***
		(3.47)
_cons	6.270***	5.803***
	(7.75)	(6.65)
N	1210	1210

t statistics in parentheses
* p<0.05, ** p<0.01, *** p<0.001

Although, the overall contribution of factors of inputs still does not entirely represent the growth in GDP we were successful in rejecting the null hypothesis (based on our sample) that energy might be insignificant. The results from our data confirm, to some extent, the findings of Kümmel, Ayres, and Lindenberger (2010) where they suggested that neo classical economic models regard the returns from energy flow as an input to be insignificant.

We then proceeded to test the hypothesis mentioned in section 2.4, with the intention of examining the influence of introducing a new parameter on the standard errors of the model. Using the Boltzmann weights – which were functions of certain labour force characteristics – we suggested a combination of raw electricity flow and labour flow (as inputs) could help generate statistically and economically efficient estimates of labour and capital.

The results of our regressions using the efficiency units of electricity inputs represented in Table 7 below show that most of our variables are statistically significant at the 99.9% level and all are definitely significant at the 95% level, thus allowing us to reject the null hypothesis of statistical insignificance of the parameters in our modified version of the Cobb-Douglas production function.

Table. 7: Comparison of Regression Results from the Boltzmann Weighted Energy Input (0.1<λ<1.0)

ln Industry GDP	(λ = 0.10)	(λ = 0.25)	(λ = 0.50)	(λ = 0.75)	(λ = 1.00)
ln Labour	0.277**	0.265**	0.258**	0.251**	0.246*
	(2.90)	(2.87)	(2.82)	(2.62)	(2.54)
ln Capital	0.222***	0.211***	0.203***	0.210***	0.209***
	(4.37)	(4.19)	(3.98)	(3.79)	(3.59)
ln E	-0.0279***				
	(-5.35)				
ln E1		-0.0197***			
		(-6.95)			
ln E2			-0.0124***		
			(-8.06)		
ln E3				-0.00914***	
				(-3.89)	
ln E4					-0.00862**
					(-2.85)
_cons	6.302***	6.357***	6.445***	6.406***	6.458***
	(9.90)	(10.33)	(10.66)	(10.20)	(10.24)
N	992	960	890	801	751

t statistics in parentheses
* p<0.05, ** p<0.01, *** p<0.001

As the lambda value increases from 0.1 to 1 the economic significance of the returns from efficiency units of electricity input decreases as exhibited by a decrease in industry GDP by 2.4% and 0.83% when lambda was 0.1 and 1 respectively. These efficiency units are a function of the labour force characteristics and indicate that a more efficient industry would require lower levels of energy inputs to produce a change in the overall output. This is evident in the fact that as the lambda value increases from 0.1 to 1 – acting as a proxy for the overall efficiency level of industries from higher to lower levels – the returns from labour and capital input due to these efficiency units of electricity increases when compared to the Cobb-Douglas production function. There is a maximum return from labour input of approximately 27% and return from capital input of 22% at lower levels of lambda.

It can be observed from the derivations of elasticities in Figure 6 in the Appendix, with the introduction of efficiency units of electricity we expected the returns from labour to stay constant and the returns from capital to decrease. While the returns from capital certainly have decreased in every variation of the efficiency units we have applied, there is also an increase of the returns from labour. This result although contrary to our expected derivations – possibly due to discrepancies in the data we have collected – might have grounds in an economic intuition. When observed independently, the premium of the efficiency units of electricity does not signify much economically or statistically (as they are negative). As the functional form of these units relies on the characteristics of the labour force, they would optimize, the share of labour and capital in national accounts according to the proportion of efficient firms within the given industry, in the province, in that period of time.

However, returning to our principal reason for the use of heterogeneous efficiency units as outlined in the introduction results exhibited in Tables 6 and 7 above shows that the standard errors of returns from labour and capital inputs from regressions using the efficiency units of electricity as a control variable appear to be more statistically efficient. The standard errors are reduced significantly when compared with the results from the standard Cobb-Douglas regression. These results help us reject the null hypothesis that the returns from the efficiency units of electricity inputs are insignificant and also comment on the variation in the standard errors of returns from labour and capital inputs. Conceptually, these results are similar to the findings of Kümmel, Ayres, and Lindenberger, where they experienced an increase in returns from inputs of labour and capital by incorporating a cost share theorem of energy input (Kümmel, Ayres, and Lindenberger 2010, 178-9).

It is quite evident that modeling a heterogeneous nature of efficiency units of electricity inputs on labour force characteristics would provide us with larger returns from labour input than capital, and our results in Tables 6 and 7 confirm as such.

5. Future Work

This section provides possible comments for future work on the economic significance of the Boltzmann weighted parameter (φ) and the resulting labour decisions. Also mentioned are a few general remarks on possible future research to contribute towards the field of Econophysics.

1) A more comprehensive formulation of the elasticity of the efficiency units of electricity might better represent the empirical results. Modeling the cost of the efficiency units as a function of the wages – due to the nature of inherent labour characteristics – in the maximization problem shown in the appendix might mitigate theoretical and empirical discrepancies.

2) Trying to implement further aspects of Kümmel, Ayres, and Lindenberger (2010) cost share theorem into our model of the Boltzmann weighted energy (Kümmel, Ayres, and Lindenberger 2010, 146). Along with a more nuanced application of Landau's concepts of 'negative temperature' might provide more statistically efficient and economically significant explanations for the efficiency units (Landau and Liftshitz 1958, 4-6).

3) Continue on the path illuminated by Park, Kim, and Isard by applying these weights to the problem of efficient resource allocation (Park, Kim, and Isard 2012, 4889). Without invoking the role of energy into the production function, one could allocate resources based on the Boltzmann weights described in the model. The objective would be to maximize entropy by minimizing the sum of errors squared when conducting tests on the factors of inputs with the Boltzmann weights. The null hypothesis we would then like to reject, suggests that the coefficient of a weighted production function would be equal to the coefficients produced without weights. In addition there would be an overall reduction in the sum of residuals squared. There would be changes to the parameters of the model, with the number of people employed replacing the total amount of full time employees and the aggregate industry proportion of GDP replacing the age cohort variable.

4) Our model can be subject to refinement, both, in terms of the amount of sample data and the actual specification of the weights. It is important to mention that there is no other research that has been conducted in this field that incorporates a three dimensional panel data, especially conducted over 13 years, 10 provinces and 15 major industries. Even Park, Kim, and Isard conducted their research over a conservative data set of eight countries and two time periods (Park, Kim, and Isard 2012, 4885). The other aspect of the refinement would involve a readjusted definition of the Boltzmann weights as there may potentially exist other viable variables which would act as a better proxy to their counterparts in statistical physics as well as the application of better quality human capital variables, which act as efficient proxies for labour flow as input.

6. Conclusion

In conclusion, we attempted to enhance our understanding of the neo-classical Cobb-Douglas production function by augmenting it with efficiency units of electricity as a factor of production. We assumed the efficiency units of electricity input to be heterogeneous in nature due to the heterogeneous nature of efficiency within Canadian industries. Heavily inspired by literature by Kümmel, Ayres, and Lindenberg, as well as Park, Kim, and Isard, we decided to model the nature of these efficiency units using the statistical physics concept of Boltzmann distribution.

We were able to successfully test – based on our data – the initial null hypothesis in demonstrating statistically and economically significant results. The results suggested that raw electricity flow has a role in the Cobb Douglas function as a factor of input. The results weren't highly economically significant and agreed with neo-classical economics that energy – as a factor of production – has a relatively small contribution to overall output (about 5%).

The Boltzmann weights and efficiency units of electricity were a function of the labour force characteristics of Canadian industries, chiefly the proportion of part-time workers, the optimum age cohort and a constant signaling of the overall productivity level of the industry. We then conducted a Cobb-Douglas regression, but applying these weighted efficiency units of the electricity as a factor of input. The results were statistically significant at the 95% confidence level, although we weren't able to conclusively prove an economic significance, namely weighted efficiency units of electricity contribute to more than 5% of total industry GDP.

What we were able to observe from the results was that the contribution of the other factors of inputs increased with the addition of these new variables, more specifically the return from labour input seemed to be higher than previously achieved in a normal Cobb-Douglas regression. This could – to some extent – verify our assumptions about modeling the heterogeneity of efficiency units of electricity through a Boltzmann Distribution.

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Appendix

Figure 1: Derivative of Standard Labour and Capital Income Shares

Within each industry, a representative firm i faces the profit function:

$$\pi\ (k,l) = z(k^{\alpha} l^{\beta}) - wl \qquad given\ w$$

Firms maximize profit with respect to labour supplied (l) by each firm

$$\underset{l}{Max}\ \pi \quad \{z(k^{\alpha} l^{\beta})\ - wl\}$$

$$(\beta) k^{\alpha} l^{\beta-1} = w = 0 \quad \Rightarrow l^{*} = \left\{ \frac{(\beta z k^{\alpha})}{w} \right\}^{\frac{1}{\beta-1}} \quad \Rightarrow \textit{optimal level of labour supplied represents the total hours worked}$$

$$w = (\beta) z k^{\alpha} l^{\beta-1} \qquad \Rightarrow wl = (\beta) z k^{\alpha} l^{\beta} = (\beta) Y$$

$$\textit{Total Profits } (\Pi) = \sum_{i=1}^{n} \pi_i\ ,$$

$$\textit{Total Industry Labour Supplied } (L) = \sum_{i=1}^{n} l_i\ ,$$

$$\textit{Total Industry Capital Supplied } (K) = \sum_{i=1}^{n} k_i$$

$$\Rightarrow \Pi = Y - wL = Y - (\beta) Y = \alpha Y$$

$$\textit{Total Labour Income Share} = \frac{wL}{Y} = \beta = 1-\alpha$$

$$\textit{Total Capital Income Share} = \frac{\Pi}{Y} = \alpha$$

Table. 1: Initial Cobb Douglas Production Function Regression

	ln IndustryGDP
ln Labour	0.253***
	(3.67)
ln Capital	0.232***
	(5.80)
_cons	6.270***
	(7.75)
N	1210

t statistics in parentheses
* p<0.05, ** p<0.01, *** p<0.001

Table. 2: Comparison of TFP Based on Regression and Historical Estimates

Year	2002	2003	2004	2005	2006
Solow Residual	52766.67	53526.46	52925.34	51695.59	52331.91
TFP	14000.00	13980.60	13918.20	14057.30	14038.20
Historical Solow	178.57	177.11	167.18	153.32	151.15

Year	2007	2008	2009	2010
Solow Residual	56641.11	58126.79	56026.34	57621.27
TFP	13994.50	14100.60	13865.60	13995.60
Historical Solow	151.15	170.89	167.04	168.59

Figure 2: Derivation of the Full Growth of GDP

Given:

$$Labour\ Share = \frac{wL}{Y} = \beta\ ,\ Capital\ Share = \frac{\Pi}{Y} = \alpha$$

Apply Growth Accounting to $t\ \varepsilon\ [2002,2011]$:

$$Y_t = Z_t K_t^{\alpha} L_t^{\beta} \Rightarrow \frac{Yt}{Population_t} \Rightarrow y_t = z_t k_t^{\alpha} l_t^{\beta}$$

Decompose growth in y_t *over time into growth rates of* $z_t,\ k_t^{\alpha},\ l_t^{\beta}$

$$\frac{y_{t+1}}{y_t} \Leftrightarrow \left(\frac{z_{t+1}}{z_t}\right)\left(\frac{k_{t+1}}{k_t}\right)^{\alpha}\left(\frac{l_{t+1}}{l_t}\right)^{\beta}$$

$$ln\left(\frac{y_{t+1}}{y_t}\right) = ln\left(\frac{z_{t+1}}{z_t}\right) + \alpha ln\left(\frac{k_{t+1}}{k_t}\right) + \beta ln\left(\frac{l_{t+1}}{l_t}\right)$$

$\rightarrow$ *Let* g_{yt} = *growth rate of* y *from* t *to* $t+1$, $t\ \varepsilon\ [2002,2011]$

then $\frac{y_{t+1}}{y_t} = 1+g_{yt}$

$\rightarrow$ *Let* g_{zt} = *growth rate of* z *from* t *to* $t+1$, $t\ \varepsilon\ [2002,2011]$

then $\frac{z_{t+1}}{z_t} = 1+g_{zt}$

$\rightarrow$ *Let* g_{kt} = *growth rate of* k *from* t *to* $t+1$, $t\ \varepsilon\ [2002,2011]$

then $\frac{k_{t+1}}{k_t} = 1+g_{kt}$

$\rightarrow$ *Let* g_{zt} = *growth rate of* l *from* t *to* $t+1$, $t\ \varepsilon\ [2002,2011]$

then $\frac{l_{t+1}}{l_t} = 1+g_{lt}$

$$\therefore\ ln(1+g_{yt}) = ln(1+g_{zt}) + \alpha ln(1+g_{kt}) + \beta ln(1+g_{lt})$$

$\rightarrow$ *For small values of* g $ln(1+g) \approx g$

$$\therefore\ g_{yt} \approx g_{zt} + \alpha g_{kt} + \beta g_{lt}$$

$\Rightarrow$ *The growth rate of GDP per capita* (g_{yt}) *over one time period (ie 1 year), should be appoximately equal to the sum of the three components.*

Table. 3: Results from Cobb-Douglas Production Function Over Short Time Periods

	(2002-2003)	(2003-2004)	(2004-2005)	(2005-2006)
	ln Industry GDP			
ln Labour	0.0803	0.103	0.188***	0.153***
	(0.97)	(1.27)	(4.75)	(3.92)
ln Capital	0.331***	0.121	0.251***	0.195***
	(4.00)	(1.85)	(4.06)	(4.82)
_cons	6.863***	8.676***	6.673***	7.566***
	(9.35)	(10.04)	(9.71)	(16.31)
N	260	260	260	260

t statistics in parentheses
* p<0.05, ** p<0.01, *** p<0.001

	(2006-2007)	(2007-2008)	(2008-2009)	(2009-2010)
	ln Industry GDP			
ln Labour	0.140*	0.0300	0.298	0.263*
	(2.10)	(0.27)	(1.75)	(2.51)
ln Capital	0.220***	0.103	0.458**	0.110
	(3.47)	(0.92)	(2.62)	(1.27)
_cons	10.21***	10.32***	12.66***	7.385***
	(18.89)	(8.64)	(7.33)	(9.16)
N	270	280	280	280

t statistics in parentheses
* p<0.05, ** p<0.01, *** p<0.001

Table. 4: Comparison of Regression from a Fixed Effects and a Random Effects Model

ln IndustryGDP	(Fixed Effects)	(Random Effects)
ln Labour	0.253***	0.251***
	(8.77)	(8.71)
ln Capital	0.236***	0.239***
	(11.16)	(11.26)
ln RawE	0.0178	0.0345**
	(1.35)	(2.74)
_cons	5.998***	5.803***
	(22.51)	(21.81)
N	1210	1210

t statistics in parentheses
* p<0.05, ** p<0.01, *** p<0.001

Figure. 5: Results from Hausman Test

b = consistent under Ho and Ha; obtained from xtreg

B = inconsistent under Ha, efficient under Ho; obtained from xtreg

Test: Ho: difference in coefficients not systematic

chi2(3) = (b-B)'[(V_b-V_B) ^ (-1)] (b-B)

= 14.94

Prob>chi2 = 0.0019

(V_b-V_B is not positive definite)

Figure 6: Derivations of Elasticities with Efficiency Units of Electricity

Industry Homogeneous Maximization Problem

$$Max_{L,E} \quad \sum_{i=1}^{n} \Pi(L,K,E) = n\{Y - wL - \varphi E\}$$

Derive β using the industry maximization problem for representative firm i, given w, φ

$$\Rightarrow Max_{L} \{ zK^{\alpha}L^{\beta}(\varphi E)^{\gamma} - wL - \varphi E \}$$

$$\beta z[K^{\alpha}L^{\beta-1}(\varphi E)^{\gamma}] - w = 0$$

$$\beta z[K^{\alpha}L^{\beta-1}(\varphi E)^{\gamma}] = w$$

$$\beta Lz[K^{\alpha}L^{\beta-1}(\varphi E)^{\gamma}] = wL \Longrightarrow \beta z[K^{\alpha}L^{\beta}(\varphi E)^{\gamma}] = wL$$

$$\beta Y = wL \Longleftrightarrow \beta = \frac{wL}{Y} \Longleftrightarrow \beta = \beta$$

Derive γ using the industry maximization problem for representative firm i, given w, φ

$$\Rightarrow Max_{E} \{ zK^{\alpha}L^{\beta}(\varphi E)^{\gamma} - wL - \varphi E \}$$

$$\gamma z[K^{\alpha}L^{\beta}(\varphi E)^{\gamma-1}] - \varphi = 0$$

$$\gamma z[K^{\alpha}L^{\beta}(\varphi E)^{\gamma-1}] = \varphi$$

$$\gamma \varphi Ez[K^{\alpha}L^{\beta}(\varphi E)^{\gamma-1}] = \varphi^2 E \Longrightarrow \gamma \varphi z[K^{\alpha}L^{\beta}(\varphi E^{\gamma})] = \varphi^2 E$$

$$\gamma \varphi Y = \varphi^2 E \quad \Longrightarrow \gamma = \frac{\varphi E}{Y}$$

Derive α given β, γ, w, φ

$$\Pi = Y - wL - \varphi E \Rightarrow \frac{\Pi}{Y} = 1 - \frac{wL}{Y} - \frac{\varphi E}{Y}$$

$$\frac{\Pi}{Y} = 1 - \beta - \gamma \Rightarrow \frac{\Pi}{Y} = \alpha$$

$$\alpha = 1 - \beta - \gamma \Longleftrightarrow 1 = \alpha + \beta + \gamma$$

Figure 7: Variable Summary

Variable	Obs	Mean	Std. Dev.	Min	Max
Year	3250	2008	3.742233	2002	2014
Industry	3250	10.56	8.622603	0	25
Province	3250	5.5	2.872723	1	10
LF	2340	206.0422	515.5626	.2	5722.8
Employ	2340	158.0796	386.9576	.2	4241.4
PT	2340	38.17363	111.7924	0	1254.5
AgeCohort	2337	66.75627	106.1986	.2	916.1
Y_Annual	3250	1584161	234788.4	1189452	1973043
Y	3250	11721.87	34416.01	0	554833.9
L	1260	15316.08	18189.18	5841.837	481357.1
K	1260	16644.94	7498.856	8098.594	136252.7
TFP	1260	99.96079	15.63845	10.2	216.1
lnY	2850	11.26034	.9569581	9.340505	14.12529
lnL	1260	9.563359	.2609403	8.672801	13.08436
lnK	1260	9.668686	.2886822	8.999446	11.82227
lnRawE	1756	2.998837	11.22657	-80.23475	12.05883
Phi	2077	.0315416	.0508655	0	.3479315
Phi1	2077	.0132406	.0333946	0	.2503878
Phi2	2077	.0061116	.021295	0	.2155107
Phi3	2077	.0035844	.0149871	0	.1854918
Phi4	2077	.0023119	.0111183	0	.1596543
lnE	1753	2.935786	10.72305	-71.79883	11.85816
lnE1	1679	-3.449839	17.18033	-78.62382	11.58816
lnE2	1538	-8.782712	21.10222	-76.23698	11.4705
lnE3	1377	-9.661809	20.18285	-78.80045	11.3955
lnE	1753	2.935786	10.72305	-71.79883	11.85816

Table 5: Variable Descriptions

Variable Definition				
Variable	**Type**	**Unit**	**Description**	**CANSIM Table**
Year	**Numeric**	**Year**	**Panel Data for the years [2002-2014]**	**All specified below**
Industry	**String; converted into numeric Industry1**	**N/A; Identifier;**	**20 Industries Identified: Agriculture, forestry, fishing and hunting; Mining, quarrying, and oil and gas extraction; Utilities; Construction; Manufacturing; Wholesale trade; Retail trade; Transportation and warehousing; Information and cultural industries Finance and insurance; Real estate and rental and leasing; Professional, scientific and technical services; Management of companies and enterprises; Administrative and support, waste management and remediation services; Educational services; Health care and social assistance; Arts, entertainment and recreation; Accommodation and food services; Other services (except public administration); Public administration**	**All specified below**
Province	**String; converted into numeric Province1**	**N/A; Identifier (Province1 given values of 1-10)**	**10 Provinces Specified; Territories not included to provide a more balanced panel;**	**All Specified Below**
Employ	**Numeric**	**Persons x 1000**	**Number of Persons Employed; Number of persons who, during the reference week, worked for pay or profit, or performed unpaid family work or had a job but were not at work due to own illness or disability, personal or family responsibilities, labour dispute, vacation, or other reason. Those persons on layoff and persons without work but who had a job to start at a definite date in the future are not considered employed. Estimates in thousands, rounded to the nearest hundred.**	**Table 282-0008**
FT	**Numeric**	**Persons in thousands**	**Number of Persons Employed who work 30 hours or more per week at their main or only job. Estimates in thousands, rounded to the nearest hundred.**	**Table 282-0008**
PT	**Numeric**	**Persons in thousands**	**Number Of Part Time Employed = Number Of Persons Employed - Full Time Workers. Estimates in thousands, rounded to the nearest hundred.**	
AgeCohort	**Numeric**	**Persons x 1000**	**Total persons, between 25-54 years of age in the labour force, broken down by province and industry**	**Table 282-0008**
Y_Annual	**Numeric**	**GDP in Current Dollar**	**Total aggregate GDP annually**	**Table 384-0038**

Y	**Numeric**	**GDP in current Dollars**	**The product of Provincial GDP(in Dollars) * [Industry Share of GDP(Provincial)/100] to calculate the contribution that each industry within each province has to GDP**	**Calculated**
L	**Numeric**	**Hours Worked**	**= (1/Labour Productivity)*GDP in current prices**	**Calculated**
K	**Numeric**	**Dollars**	**= (1/Capital Productivity)*GDP in current prices**	**Calculated**
TFP	**Numeric**	**GDP /(Capital + Labour Inputs)**	**Multifactor productivity, as known as total factor productivity, measures the efficiency with which all inputs are used in production. It is the ratio of real gross domestic product (GDP) to combined labour and capital inputs.**	**Table 383-0026**
RawE	**Numeric**	**Terajoules**	**Measured the physical flow of energy use annually; aggregated by industry, consistent among provinces; consolidated data using a terminated data set and current dataset; all industry classifications are the same between the two sets except for manufacturing, transportation, education and other services.**	**Table 153-0032(terminated); Table 153-0013**
lnY	**Numeric**	**Log Dollars**	**Natural log of the Industry GDP = ln(Industry GDP)**	**Calculated**
lnL	**Numeric**	**Log Hours Worked**	**Natural log of Labour input = ln(L)**	**Calculated**
lnK	**Numeric**	**Log Dollars**	**Natural log of Capital input = ln(K)**	**Calculated**
lnRawE	**Numeric**	**Log TeraJoules**	**Natural log of Raw Electricity Flow as a factor of input = ln(RawE)**	**Calculated**
Phi	**Numeric**	**TeraJoules per worker**	**Efficiency Units Of Electricity = Fraction Of Part Time Workers in total Employed*(2.71828)^(-λ*AgeCohort); where values of the constant λ are tested at [0.1,0.25,0.5,0.75,1.0]. Lambda = 0.1; Lambda1 = 0.25 etc.**	**Calculated**
lnE	**Numeric**	**Log Of Weighted Energy Input**	**Natural Log of the Efficiency units of electricity weighted energy input = phi*EnergyInput; ln(E) is calculated at phi = 0.1, ln(E1) is calculated at phi = 0.25 etc.**	**Calculated**

Evaluation and Comparison of the Quantitative Easing Programs of the US and UK: Macroeconomic and Currency Market Effects

Eric Huang and Matthieu Laurin

Abstract

This paper analyses the unconventional monetary policy response to the 2008 financial crisis by the Federal Reserve and Bank of England. Specifically, this paper discusses the design, implementation, goals, along with the macroeconomic and currency market effects of Quantitative Easing (QE) employed by these central banks from 2008 to 2013. Using established results of QE on financial variables, namely the compression of the long-term bond yield spread, we employ a Vector Autoregression, and conduct a counterfactual estimation to quantify the macroeconomic and currency market impact of QE insofar as it has been transmitted via this specific channel. The results suggest that for the macroeconomic impact, the US program found more success in the long run, while the UK program experienced slightly more desirable results in the short run. For the currency market impact, our results suggest that the relationship between the exchange rates and the bond spread strengthened during the financial crisis, and that QE appreciated the dollar index in the US and depreciated the UK Sterling index. Finally, the effects in the US were much less clear cut compared to the UK, as the US financial system is more complex and susceptible to speculation.

Acknowledgements

We would like to express our sincere gratitude for the faculty members who dedicated their valuable time to discuss and help us with our thesis paper. We would also like to express our gratitude to the Department of Economics of the University of Western Ontario for the opportunity to write this thesis, as well as the dedication from all the professors who trained and prepared us for this challenge over the years.

Faculty Members Consulted: Rui Castro, Jim MacGee, Igor Livshits, Alan Bester, Simona Cociuba

1. Introduction

Quantitative Easing (QE) is an Unconventional Monetary Policy (UMP) that was first widely used by central banks around the world during the 2008 financial crisis. Through different transmission mechanisms, such as decreased term premiums and creating liquidity in the financial market, it raises aggregate demand and stimulates the economy back to the desired state. As the effects of the financial crisis come to end, there has been little research done analyzing the effectiveness of this program on the economy and its full effects using more recent data. In this research paper, we analyze the effectiveness of such a policy in the United States (US) and United Kingdom (UK) by studying its effects on various macroeconomic variables and the currency markets, utilizing theory and empirical analysis.

1.1 The Great Recession

There are many hypotheses for what caused the global economic deterioration of 2007-2008. Many critics blamed central banks in advanced economies for keeping interest rates too low for too long, while others blamed the large quantity of foreign reserve holdings in emerging markets. Regardless of the origin, the financial markets saw excess liquidity, the creation of complex financial instruments such as collateralized debt obligations of sub-prime mortgages, and bubbling asset prices, which ultimately led to the crash of the financial markets (Baily and Taylor 2014).

The first signs surfaced in mid-2007, as banks around the world began to show losses from the subprime real estate market in the US. Financial markets dried up and became illiquid in the following months. In December 2007, the Federal Reserve (FED), Bank of England (BOE), European Central Bank (ECB) and Bank of Canada (BOC) announced a coordinated effort to bring about new liquidity-enhancing measures. With a few major commercial banks already on default, central banks cut interest rates and introduced a number of new liquidity facilities. By September 2008, the FED had to inject billions of dollars into the economy to prevent a systematic breakdown, while several European banks collapsed and had to be bailed out or nationalized (Annunziata 2011). This is often regarded the beginning of the recession.

1.2 Monetary Policy and Central Banks

Prior to this, conventional monetary policy was secure in its application and logic. The goal of this type of monetary policy was to target an inflation rate of 2 percent annually,[1] with the FED having a dual mandate which also included keeping employment at full capacity. These goals were met by either buying or selling securities to affect the overnight rate for banks, which in turn affected short-term interest rates. After the recession, the FED and BOE were forced to apply UMPs, as interest rates reached their effective lower bounds. QE involves a central bank creating new reserves (currency) in order to purchase financial assets, such as mortgage-backed securities (MBS), government bonds and so forth. The ultimate goal of QE was to increase spending and meet the central bank’s target inflation rate. By 2015, the US and UK had wrapped up their respective QE programs.

There have been numerous economic papers that have discussed the effects of QE, but these focus largely on the effects on financial variables. There has been little research done into the effects of QE on currency markets in these countries.

[1] The FOMC uses PCE inflation measure whereas Bank of England focuses on CPI inflation.

1.3 Summary of Research

The goal of this paper is to find the effect of QE in the US and UK on macroeconomic variables and currency markets, namely their respective currency indices. This is done by estimating a Vector Autoregression (VAR), and running a counterfactual simulation to evaluate what would have happened had the central banks not proceeded with these UMPs. The theory suggests that as QE decreases the term premium, the currency index in the country will decrease, and that the relationship between bond spread and currency indices becomes stronger following the implementation of QE.

Our empirical results suggest that the use of QE strengthens the relationship between the bond spread and currency indices, while appreciating the US currency and depreciating currency in the UK. For the macroeconomic effects, the main way in which QE impacted the economy was through increasing the prices of risky assets, which made individuals wealthier. Therefore, an increase in GDP and a decrease in inflation would be expected after QE is applied. Since QE has only recently been applied in practice, there is little historical evidence on which to compare these programs. We found that for macroeconomic variables in the long run, the US program outperformed that of the UK, while in the short run, the UK program had more desirable effects on inflation. This paper hopes to provide information of the wider currency market effects these UMPs had, which has not been covered in any previous literature to the extent of our knowledge, and determine if this type of monetary policy is an effective substitute to previous conventional monetary policies.

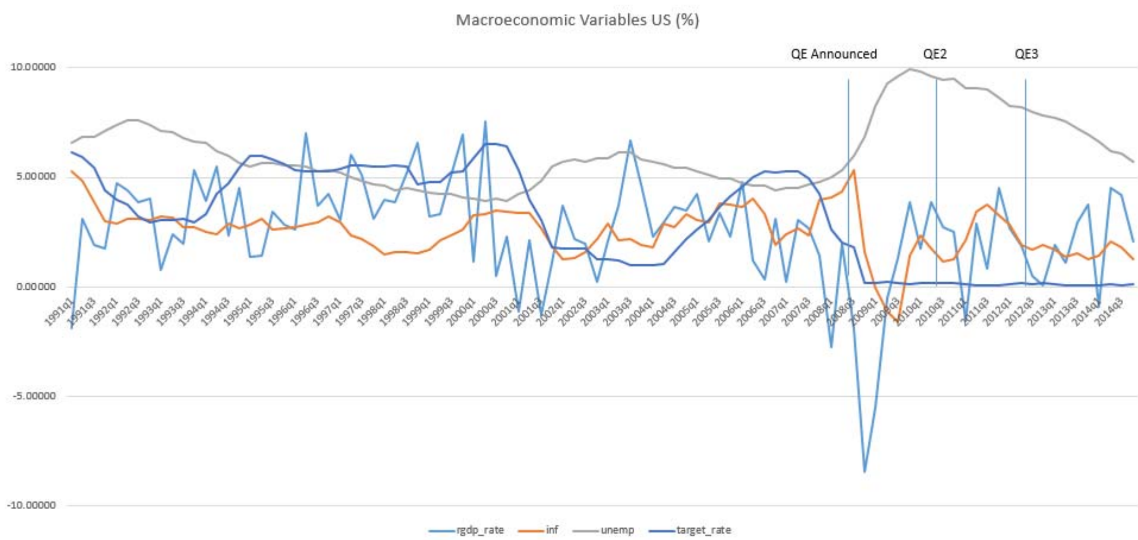
2. Policy and Theory of QE

2.1 Details of QE Policy

US Financial Markets and Federal Reserve's QE Policies

Shortly after the failure of Lehman Brothers on September 15th, 2008, the FED began to initiate the large-scale asset purchase (LSAP) program of QE (Federal Reserve 2012). It was around this time the Federal Open Market Committee (FOMC) lowered the overnight rate to between 0 and 0.25 percent, effectively its lower bound. There was a need for further monetary policy intervention in order to meet their dual mandate. In order to do so, the FOMC announced they would be purchasing large amounts of housing agency debt and mortgage-backed securities (MBS). According to the FED, the ultimate goal of their QE program was to "reduce the cost and increase the availability of credit for the purchase of houses, which in turn should support housing markets and foster improved conditions in financial markets more generally" (Federal Reserve 2008). The first round of purchases, QE1, occurred in November 2008. In QE1, the FED purchased \$100 billion in government-sponsored enterprise (GSE) debt, as well as \$500 billion in MBS. In March 2009, QE1 was extended to purchase an additional \$750 billion in MBS and GSE, as well as \$300 billion in Treasury securities. In November 2010, the second phase, QE2, began, in which the FED purchased another \$600 billion in longer-dated treasuries, at a rate of $75 billion per month. QE2 ended in June 2011. The final phase, QE3, began in September 2012, and involved the purchase of $40 billion in MBS per month, which was later increased to $85 billion. In 2013, the FED announced that they would begin tapering off their LSAP program, and it finally came to an end in October 2014 (Applebaum 2014). Figure 2.1.1 represents a timeline of macroeconomic variables in the US before and during the crisis.

Figure 2.1.1: US Macroeconomic Variables and QE Timeline[1]



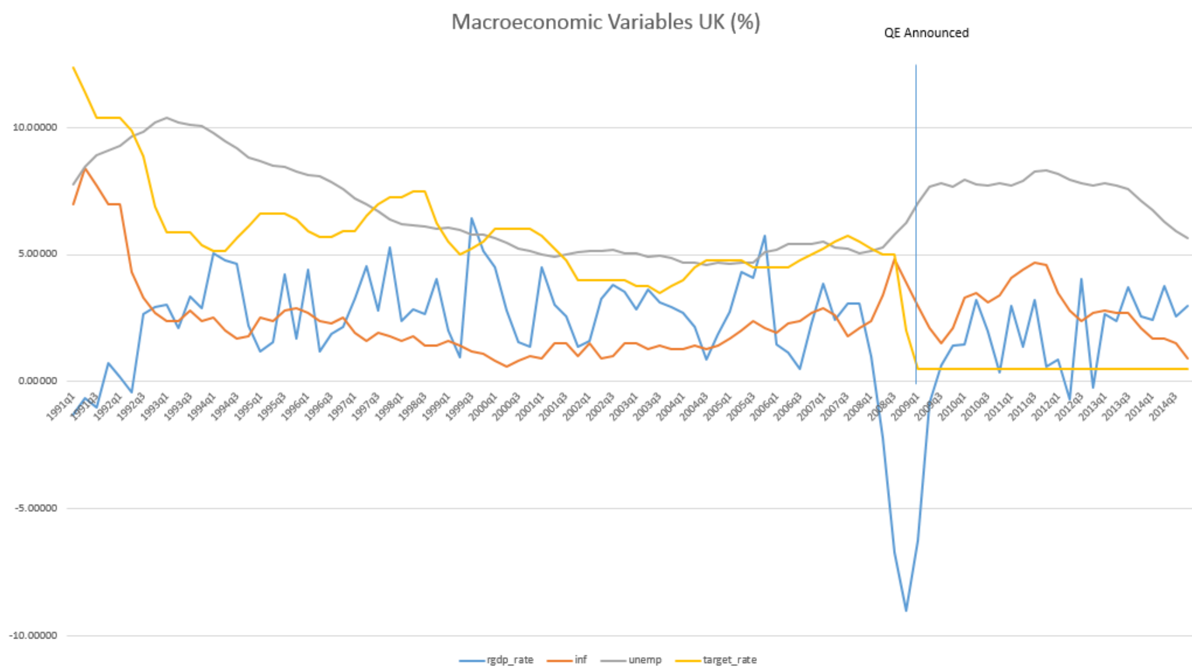
UK Financial Markets and Bank of England's QE Policy

The UK's Monetary Policy Committee (MPC) announced they would begin their LSAP program of UK government gilts (equivalent of US Treasuries) in March 2009. At this point, the MPC had reduced the Bank Rate to its effective lower bound, 0.5 percent. However, the MPC needed additional procedures to meet their inflation goal of 2 percent. To perform QE, the MPC set a target for the stock of asset purchases financed by the creation of reserves, achieved by buying and selling assets through the Asset Purchase Facility (APF).[2] According to the BOE, the goal of QE was to influence inflation enough to reach their 2 percent goal, and by performing QE they hoped to decrease government bond interest rates as well as short-term rates, making it cheaper for businesses to raise capital (Bank of England 2015). The initial purchases totalled £75 billion, but the program was increased to £200 billion by November 2009 (Kapetanios et al. 2012). The final round of purchases brought the total to £375 billion in July 2012. In 2012, the MPC announced that they had decided to keep the bank rate at 0.5 percent and total QE purchases at £375 billion, effectively bringing an end to their QE program (Aldrick,

[1] Data source in Appendix 1.2.
[2] Giudice et al. (2012).

2012). Figure 2.1.2 represents a timeline of macroeconomic variables in the UK before and during the crisis.

Figure 2.1.2: UK Macroeconomic Variables and QE Timeline[1]



2.2 Transmission Channel of QE

Channels Through Which QE Operates

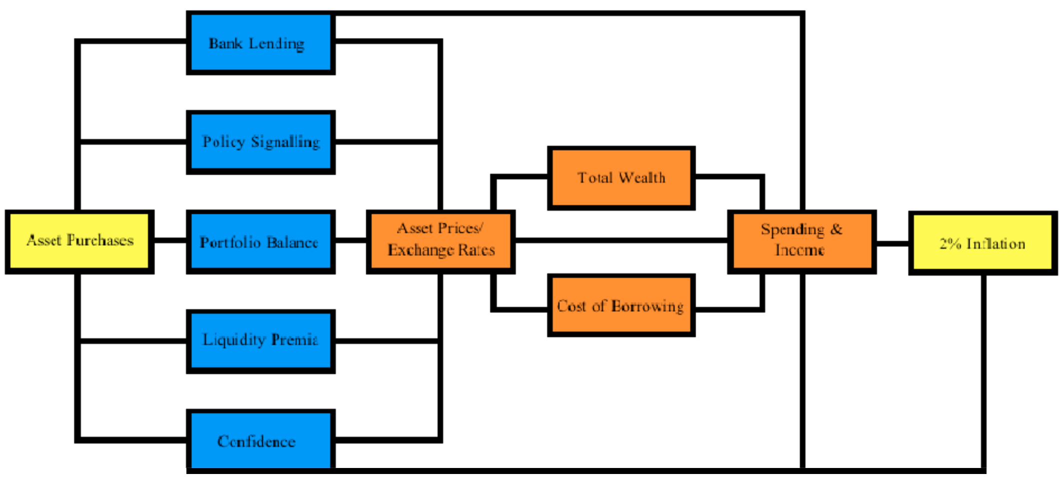
Considering the goal of QE was to stimulate nominal spending in order for inflation to meet the 2 percent target, there are a variety of channels through which QE could work. The first is the policy signalling effect, which includes any information banks or economic agents learn about the central bank plans for monetary policy, including asset purchases and plans to keep target interest low. The second channel is the liquidity premia effect. In this channel, the central bank can improve the functionality of markets by increasing the liquidity of the counterpart of the asset purchases done through QE. However, this channel may only apply while the central bank is performing these purchases. Third, performing QE may increase consumer confidence, as it could be seen by the public that there is an improved future economic outlook. This would also increase willingness to spend now, in turn stimulating the economy and helping the government reach their inflation goal. The fourth channel is the bank lending effect. Since the asset purchases came mainly from banks and other lending institutions, these institutions end up with more reserves on hand. Due to the higher level of liquid assets, banks are

[1] Data source in Appendix 1.2.

encouraged to hand out more loans, which again stimulates the economy (Joyce, Tong, and Woods 2011).

The final channel is the portfolio balance channel. This channel works through the fact that central banks mostly purchase short-term assets held by lending institutions. This pushed up the prices of these assets, and lowered their yields. Since short-term yields decreased, investors were encouraged to look for higher returns elsewhere. They did so by purchasing riskier assets, whose prices had been depressed by the recession. The new demand on these riskier assets drive up their prices. As asset prices increased, individuals become wealthier, increasing aggregate demand (Joyce et al. 2012). Additionally, through this channel, QE reduced the short- and long-term interest rates, lowering the borrowing costs for corporations and making businesses more willing to spend on investments and wages. Again, this worked to increase spending and the income of individuals (Joyce, Tong, and Woods 2011). Most research, including our own, focuses on this channel, as it is considered the most important channel for QE (Kapetanios et al. 2012). It is the most important because QE directly affects asset prices, which is the mechanism for this channel. Figure 2.2.1 is a representation of the transmission mechanisms of QE, and how they affected inflation.

Figure 2.2.1: Transmission Mechanisms of QE, Bank of England (2011)



Hypothesis 1: US and UK

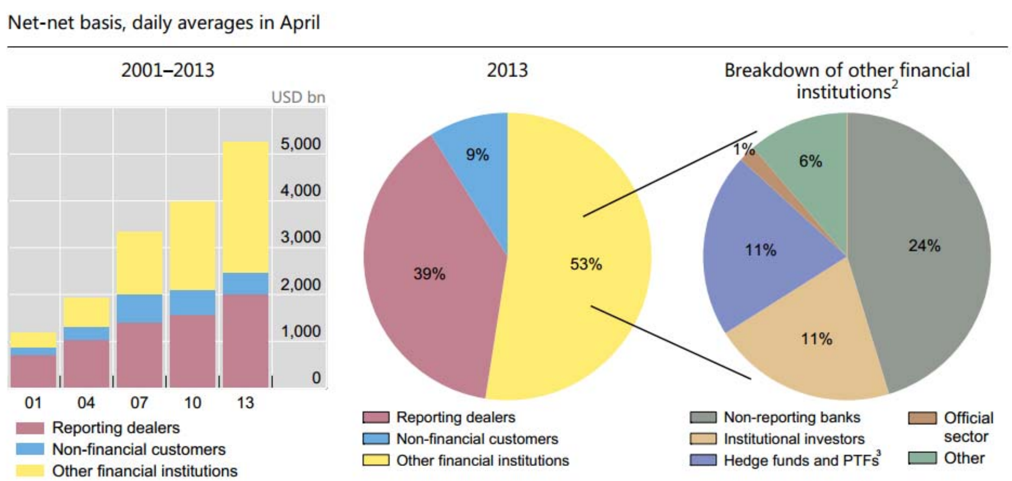
As mentioned above, the goal of these QE programs was to meet the central bank targets for inflation, as well as improve unemployment in the case of the US. In terms of the macroeconomic effects of QE, in the US, we would expect improvements in inflation and increases in real GDP growth, due primarily to the portfolio balance channel mentioned above. Similarly, in the UK we would expect inflation to ameliorate with QE over the no-policy scenario. Additionally, a turnaround in the growth of real GDP, which experienced a large downturn at the beginning of the financial crisis, would be anticipated.

2.3 QE and Exchange Rate Movements

Exchange Rate Movement in US and UK

Exchange rates are important variables to examine during the 2008 financial crisis. They can have significant effects on macro variables in large open economies such as the US and UK. Also, they can be seen as an indicator of the future economic outlook and financial well-being of a nation's economy. Most movement in exchange rates in today's foreign exchange markets is driven by speculation, although the fundamentals of exchange rates are still highly relevant. The Bank of International Settlements (2013) report shows that the foreign exchange market is driven by financial institutions, and shows that turnover is more often driven by speculation rather than trade. The foreign exchange markets are grounded in the theory of interest rate parity. Figure 2.3.1 represents a breakdown of the turnover in foreign exchange markets.

Figure 2.3.1: Foreign Exchange Market by Counterparty (Bank of International Settlements 2013)

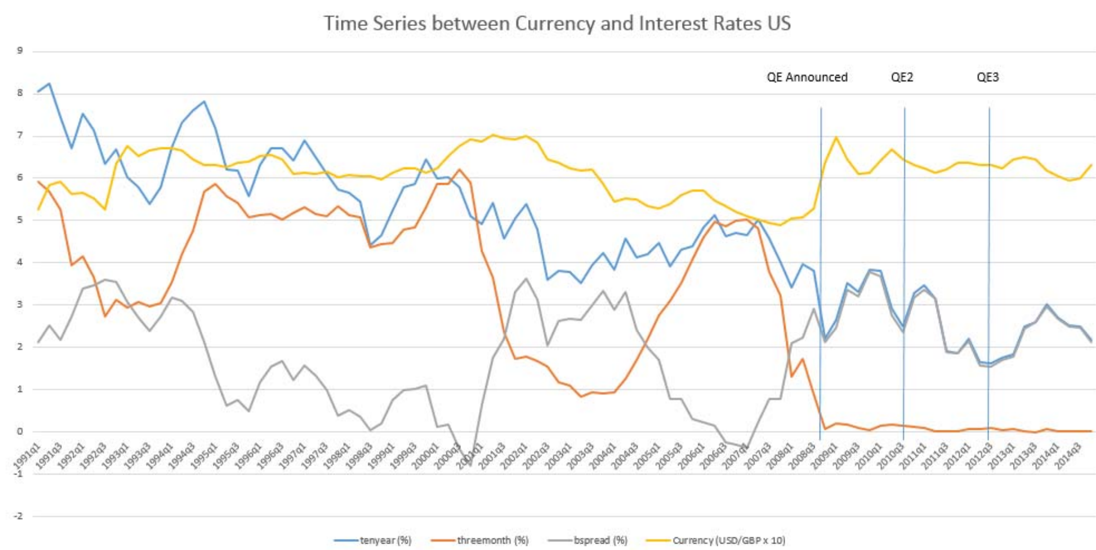


Exchange rate levels moved significantly during the financial crisis. In the US, the DXY began depreciating in 2001,[1] largely as a response to the low interest rate environment of the Treasury and Bond markets. As the financial market crisis unfolded, the index rebounded to a peak in early 2009, but soon began to drop again, following the announcement of QE by the FED in November 2008. The exchange rate index can be seen in Figure 2.3.2.

The ERI (exchange rate index) published by the BOE showed a dramatic depreciation of the sterling. Joyce et al. (2011) found that when QE was first announced, the sterling ERI experienced a 4 percent decrease, but afterwards, from March 4th 2009 to May 31st

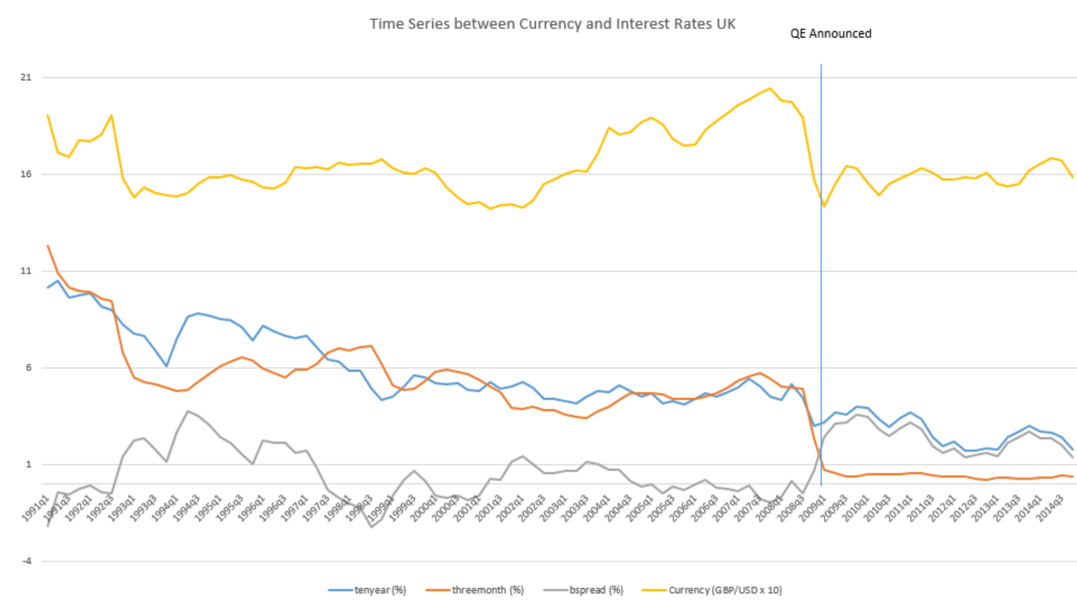
[1] The DXY is a USD dollar index, weighted by a basket of currencies. (Federal Reserve Bank of St. Louis 2016).

Figure 2.3.2: Time Series of Exchange Rate and Interest Rates in US[1]



2010, it saw an increase of 1 percent. Figure 2.3.3 shows a time series of exchange rates and interest rates in the UK.

Figure 2.3.3: Time Series of Exchange Rate and Interest Rates in UK[2]



[1] Data source in Appendix 1.2.
[2] Data source in Appendix 1.2.

2.4 Drivers of Exchange Rate

The Fundamental Theories of Exchange Rates

Understanding what drives exchange rates is a complex task. There are several theories available on the topic. In the traditional open economy model, demand and supply for domestic and foreign currencies are driven by activities such as trade and movement of capital. The law of one price and purchasing power parity (PPP) dictates equilibrium prices, and quantities are driven by prices in domestic and foreign economies. Empirical studies claim that PPP is not a long-term relationship, because real exchange rates tend to resemble a random walk in several studies (Grilli and Kaminsky 1990).

The foreign exchange market and theories on exchange rates have become more complex over the years beyond the traditional open economy model. There are more factors at play, more complex financial instruments and so forth, moving between national borders. Dornbusch (1976) postulates that because of arbitrage opportunities, domestic and foreign securities can be perfect substitutes, assuming there are no frictional costs. Adding in exchange rate expectations, without friction, the authors hypothesize that the relationship investors care about is expected net return on alternative assets, which is interest rates minus anticipated change in interest rate:

$$i=i^*+(e'/e-1),$$

where e' is expected future spot exchange rate, e is the spot exchange rate, or the "permanent rate", i is the domestic interest rate, and i* is the foreign interest rate. Using the Mundell-Fleming model, the authors assert that a monetary expansion gives rise to a depreciation in the exchange rates because of inelastic expectations, with the interest rate and exchange rate expectations playing a critical role in the adjustment process.

Unconventional Policy Effect on Exchange Rates

In their paper, Coenen and Wieland (2003) studied the relationship between interest rate and exchange rate, and they found that a drastic expansion of the monetary base leads to a depreciation of currency during a zero bound interest rate.

The portfolio balancing effects of QE would be expected to put downward pressure on exchange rates. With bond yields low domestically, investors substitute their investments in the country for higher yielding assets elsewhere, thus decreasing demand and increasing supply of domestic currency. There are three main ways through which the lower interest rates caused by QE affect the exchange rate. The first is through what is called the 'money demand effect'. Lower interest rates decrease the demand for assets denominated in domestic currency, which depreciates the currency. The second, the 'output effect', is caused by lower interest rates causing an expansion in domestic output, which appreciates currency. The final channel is the 'fiscal effect'. This channel occurs as the decrease in the interest rate decreases the debt service of the government. This decreases the inflationary expectations and appreciates the currency (Hnatkovska, Lahiri, and Vegh 2012).

Based on these theories, we hypothesize the relationship between exchange rates and term premiums strengthens during QE. One reason is that with the short-term rate at the lower-bound, speculators would look at the next best indicator for predicting asset yields in each country, the long-term rate and term spread. The second reason is that as people look for investments, they look to the yield curve for future expectations on interest rates. The second hypothesis is that QE depreciates the exchange rate, as it increases supply of domestic currency, and also depreciates bond prices. Through the various channels, QE should depreciate domestic currency.

2.5 Summary of Hypotheses

Hypothesis 1: Macroeconomic variables improved due to QE
Hypothesis 2: Relationship between exchange rate and term premium strengthened during the QE period.
Hypothesis 3: QE depreciates exchange rate

3. Data and Model

3.1 Data

The data used in our model are quarterly macroeconomic data from several national databases, such as the Federal Reserve Economic Database (FRED), OECD, BOE, and the Bureau of Economic Analysis (BEA), with financial data retrieved from Bloomberg. The full list of variables, as well as their description, source and transformation, are available in Appendix 1.1 and 1.2. Summary statistics for all variables are available in Appendix 2. The variable selection for the base VAR models will be explained in Section 3.2.

3.2 Base Empirical Mode

Estimating the effects of QE on macroeconomic variables is a complicated task, as there are many moving pieces. This paper will follow what the majority of research has done in terms of empirical analysis,[1] which utilizes Vector Autoregression (VAR) models in order to analyze the effects of QE on variables of interest. The VAR model used in this paper is a replication of that used by Lenza et al. (2010), who performed their empirical analysis on the macroeconomic effects and rate changes during the financial crisis in the Eurozone area. A similar method was used in Giannone et al. (2012) to analyze monetary policy to economic and loan activities. The base VAR model equation is of the following form:

$$Y_t = \beta_0 + \beta_1 Y_{t-1} + \beta_2 Y_{t-2} + \beta_3 X_t + \epsilon_t \quad \ldots\ldots\ldots\ldots (VAR\ 1)$$

[1] See Baumeister and Benati (2010) and Lenza et al. (2010).

Here, Y_t is a vector of endogenous variables consisting of the real GDP growth rate (*rgdp_rate*), inflation (*inf*), the overnight rate (*target_rate*) and the term premium (*bspread*). It is also possible to have the variable *bspread* as an exogenous variable, but we have decided to treat *bspread* as an endogenous, as have many other papers (Lenza et al. 2010) and (Baumesiter and Benati 2010). In Section 5.3, we look at the effects of using *bspread* as an endogenous variable and explain our results. Additionally, Y_{t-1} and Y_{t-2} are the first and second lags of Y_t, respectively, X_t is a vector of exogenous variables, listed and described in Appendix 1.1, ϵ_t is the stochastic error term, and the *t* subscript represents the quarterly time period of the observation (*t*=1994Q1,...,2014Q4). Moreover, β_0 is a vector of constants, β_1 and β_2 are matrices describing the relationship between Y_t and its first and second lags, respectively, and β_3 is a matrix describing the relationship between the exogenous variables and the vector of endogenous variables. The variables included in the endogenous vector Y_t were chosen for their macroeconomic importance, as well as their effects on exchange rates.

In order to produce the counterfactual simulation for our paper, we followed several key steps. First, we estimated our base model above (*VAR 1*) for the period of 1994Q1 to 2008Q4. We chose to end our estimation in 2008Q4 because it was after this time that the US and UK programs were put into effect. Doing this allows us to observe the relationship between the variables using about 15 years' worth of data. Once we estimated this VAR, we then conducted our counterfactual simulation. We first take our estimates from the previous *VAR 1* estimation, and forecast out for 8 periods (2 years). We chose 8 periods because we found that for any length of time longer than that, the effect of QE becomes too weak to properly analyze. This estimation was used as our *without-QE* policy scenario. Next, we shocked the variable *bspread* down by 60 basis points. We are working off the results of Gagnon et al. (2010) for the US and Meier (2009) for the UK, which found that QE depressed the long-term maturity premiums on bonds by between 60 and 100 basis points. Section 5.3 describes a sensitivity analysis done to see the effects of changing bond spread by more or less than 60 basis points. We chose to work off the lower-bound estimates in the base model, and this depression in *bspread* was used as a proxy for the effect of QE. With the now-depressed *bspread*, we again forecast out 8 periods from the end of our initial estimation (2008Q4). This estimation was then used as our *with-QE* policy scenario. Similar to Kapetanios et al. (2012), we used this estimation, rather than the actual path of variables, as our estimates for the *with-QE* scenario because the actual path of our endogenous variables is affected not only by QE, but also by several other factors. By doing it this way, the model isolates the direct effects of QE on our variables of interest. Finally, in order to evaluate the magnitude of QE's effects on our variables, we simply compare the results for the *with-* and *without-QE* scenarios by looking at the difference in change in the forecasted inflation and real GDP growth. This should give an indication of the effectiveness of the programs in relation to the situation where QE was not enacted.

In order to test the validity of our model, we followed the above steps for our US data. We then compared our results to those of Baumeister and Benati (2010), who performed a similar empirical analysis of QE's effects on US macroeconomic variables using a more complex VAR model. Our results from this test are seen in Section 4.1.

3.3 Econometric Considerations

(1) Time Varying Parameters

In order for the counterfactual forecast to provide a good estimation for the effects of QE, the relationship between the variables must remain the same before and after QE. While there is reason to believe that there could be changes in the relationship theoretically (Baumeister and Benati 2010), it is important to look at the size of the change and whether it would affect the counterfactual analysis. To look at whether the coefficients have changed before and after QE, the VAR model was used on two different time periods for the US and UK.

First, a comparison between 1991-2008 and 2009-2012 US data was made, and a similar comparison was made for the UK data. The focus of this test is to see if the forecast of the counterfactual on the endogenous variables will hold in the 2009-2012 period. The endogenous variables of interest in this VAR are the real GDP rate and inflation. The results for the US show that the coefficient on the VAR model does indeed change between the pre-crisis and crisis periods. The UK variables also had different coefficients on the VAR. However, the differences are smaller than those from the US. It is important to note however that the R^2 and p-value of the VAR model between 1991-2008 is not very high for the real GDP growth rate. This suggests that the specification of our VAR model can be improved by adding more variables that affect real GDP rate and inflation. Baumeister and Benati (2010) suggest that recent effects of spread compression on macroeconomic variables are strong compared to those from the past two decades, suggesting that using more recent time periods would improve the accuracy of the forecast.

(2) Stationarity Constraints

In order for the VAR results to be correct, the variables need to be stationary. Augmented Dicky Fuller tests were performed on all variables, and a number of variables failed the test and showed a unit root. Most of these variables were translated into log level or log difference to accommodate for the unit root, as seen Appendix 1.1. Both bond spread and target rate were left without manipulation as per industry norm.

(3) Number of Lags for Endogenous Variables

In a VAR model, the number of lags can affect the results. The majority of literature on QE and macroeconomic variables uses two lags in their empirical analysis, as this is seen as the time necessary for the interest rate transmission mechanisms to affect macroeconomic variables. A lag selection function was also used to empirically determine the number of lags most appropriate for the analysis. After observing the results in Figures 3.3.1 and 3.3.2, the conclusion that two lags would be the most effective was reached, for the US and UK.

Figure 3.3.1: US Lag Statistics[1]

```
Selection-order criteria (lutstats)
Sample:  1994q1 - 2008q4                    Number of obs      =        60
```

lag	LL	LR	df	p	FPE	AIC	HQIC	SBIC
0	-214.817				.088065	-4.19093	-4.19093	-4.19093
1	-145.509	138.62	16	0.000	.01544	-5.96789	-5.74943	-5.40939*
2	-121.035	48.948*	16	0.000	.012358*	-6.25035*	-5.81344*	-5.13337

```
Endogenous:  rgdp_rate inf bspread target_rate
 Exogenous:  unemp gfcf ca stocks manu savings bus_bankruptcy ind_insolv
             gov_exp consum housing twusdi  _cons
```

Figure 3.3.2: UK Lag Statistics[2]

```
Selection-order criteria (lutstats)
Sample:  1991q3 - 2008q4                    Number of obs      =        70
```

lag	LL	LR	df	p	FPE	AIC	HQIC	SBIC
0	-363.282				1.70084	-.972033	-.972033	-.972033
1	-183.464	359.64	16	0.000	.016128	-5.65254	-5.4484	-5.1386*
2	-156.803	53.322*	16	0.000	.012336*	-5.95714*	-5.54886*	-4.92926

```
Endogenous:  rgdp_rate inf bspread target_rate
 Exogenous:  D.l_unemp gfcf l_ca D.l_stocks manu D.l_savings D.liquidation
             D.ind_insolv D.l_gov_exp l_consum D.l_housing d_l_seri  _cons
```

4. Empirical Analysis

4.1 Testing Hypothesis 1: QE positively affected Macroeconomic Variables in the US and UK

Previous studies suggest QE would improve GDP and inflation in the US and UK, working through the channels shown in Figure 2.2.1. To test whether QE had a positive impact on these variables, compared to a scenario where it was not enacted, *VAR 1* was estimated to when QE was introduced (2008Q4). Using these results, a counterfactual forecast was conducted. The estimates from this counterfactual were then used to forecast the paths of *rgdp_rate* and *inf* from 2009Q1 onwards. These initial estimates were used as our no-policy estimates (prefix wo_). The *bspread* variable was then decreased by 60 basis points, and the counterfactual was run with these new values. The forecast with the adjusted *bspread* was used as our policy scenario estimates (prefix w_). The difference between the policy and no-policy results were then analyzed to evaluate the impact of QE on *rgdp_rate* and *inf*.

[1] Data source in Appendix 1.2.
[2] Data source in Appendix 1.2.

Results for US: For the US, *VAR 1* was run for the period 1994Q1-2008Q4. The full results of this VAR are in Appendix 3. We see that lag 1 of *bspread* is positive and insignificant, while lag 2 is negative and statistically significant for *rgdp_rate*. For *inf*, lag 1 is negative, lag 2 is positive, but neither are statistically significant. The R^2 for *inf* (R^2 = 0.6196) is much lower than that for *rgdp_rate* (R^2 = 0.8272). Figure 4.1.1 below shows the path of the counterfactual results, along with the actual path of *rgdp_rate* and *inf* (dashed lines).

Figure 4.1.1: Counterfactual Paths (Policy and No-policy) Real GDP Growth and Inflation in US[1]

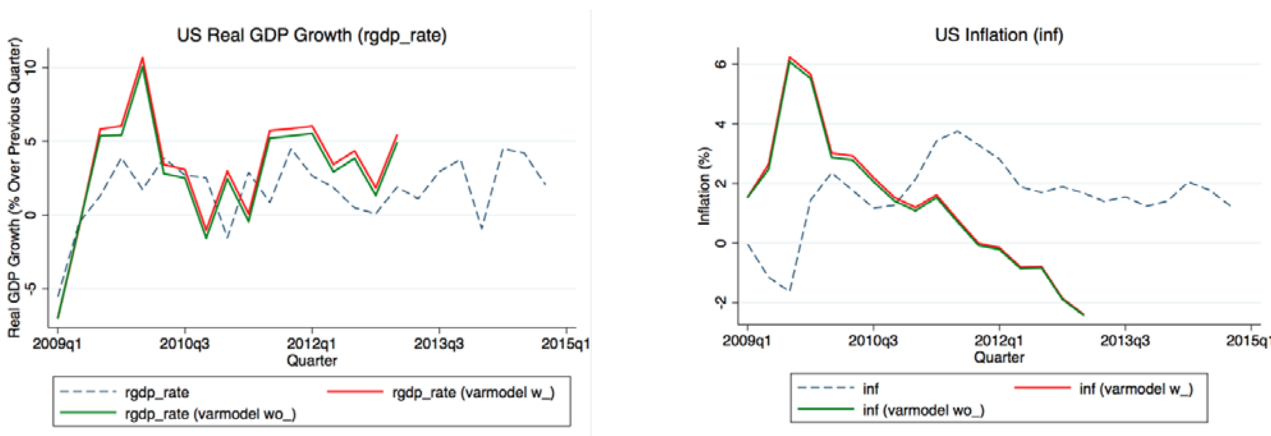
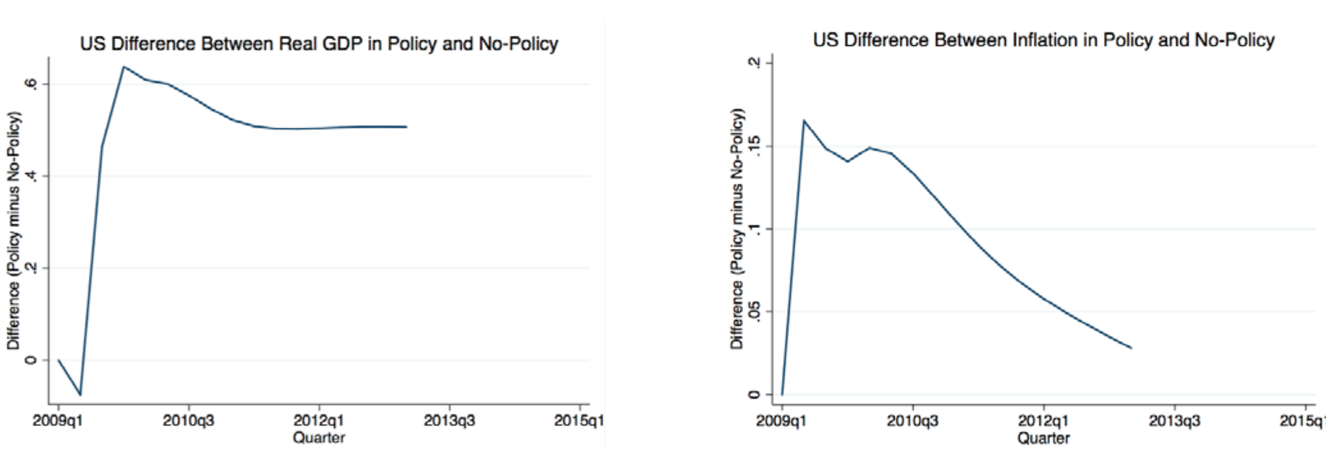


Figure 4.1.2 shows the difference between the policy and no-policy paths of the *rgdp_rate* and *inf*.

Figure 4.1.2: Difference Between Policy and No-policy Counterfactual Paths for Real GDP Growth and Inflation in US[2]



[1] Data source in Appendix 1.2.
[2] Data source in Appendix 1.2.

The difference in the level of real GDP growth (*rgdp_rate*) between the policy and no-policy scenarios decreased slightly in the first period following QE, and increases to a maximum of a little over 0.6 percent, before settling around 0.5 percent in the long run. For *inf*, the difference in levels are positive following the start of QE, reaching a peak of 0.15 percent, before diminishing.

Results for UK: Similar to the US, we first estimate *VAR 1* for the period of 1991Q3 – 2008Q4. The results of this VAR are also in Appendix 3. We see that the coefficient for lag 1 of *bspread* on *rgdp_rate* is negative, lag 2 is positive, but neither are statistically significant. The coefficient of *bspread* on *inf* for lag 1 is positive, lag 2 is negative, but neither are statistically significant. Looking at the fit of the VAR, the R^2 for *rgdp_rate* is low ($R^2 = 0.6623$) compared to that for *inf* ($R^2 = 0.9292$). Figure 4.1.3 shows the path of the counterfactual results, along with the actual paths of *rgdp_rate* and *inf*.

Figure 4.1.3: Counterfactual Paths (Policy and No-policy) of Real GDP Growth and Inflation in UK[1]

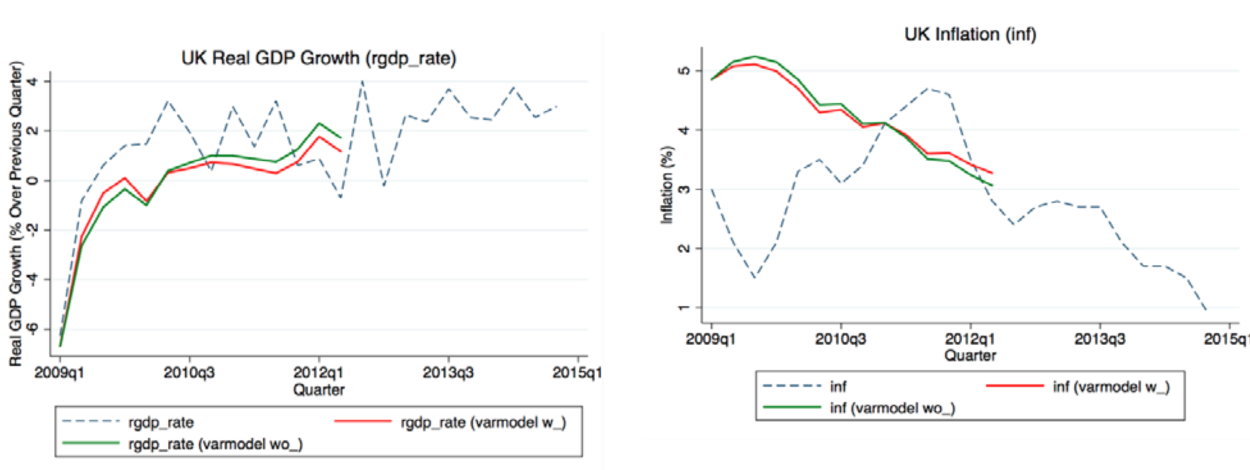


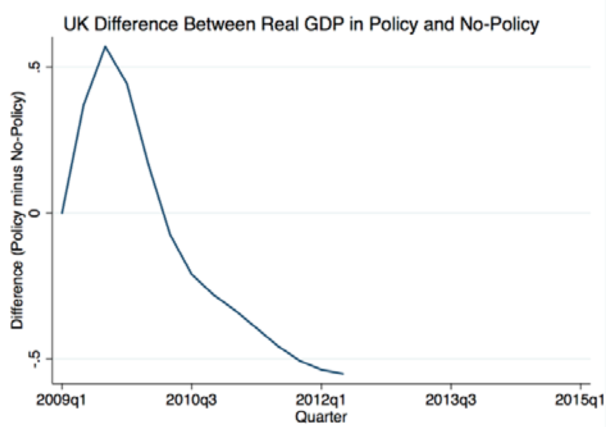
Figure 4.1.4 shows the difference between the policy and no-policy paths of the *rgdp_rate* and *inf* . The difference between the policy and no-policy *rgdp_rate* peaks at slightly over 0.5 percent 3 periods after implementation, before falling off rapidly. By these results, it seems the UK QE had a similar effect to the US program in the short run. For *inf*, the difference in levels reaches a low point of about 0.15 percent three periods after implementation, before steadily increasing.

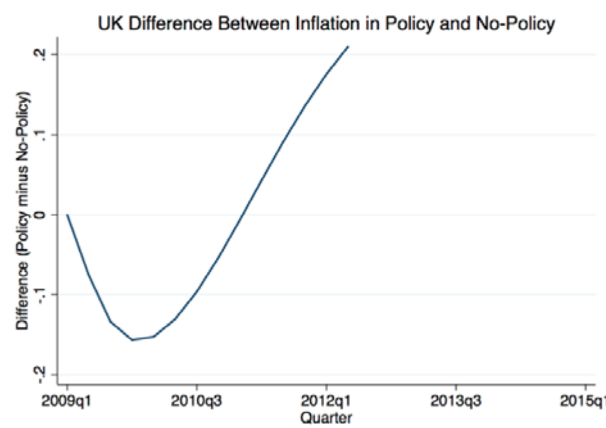
Shortcomings: Compared to the results of BB, our results for the US *rgdp_rate* are similar in direction, but smaller in magnitude (peak of 0.6 percent compared to a peak of 1.9 percent in BB). For *inf*, the results from BB (peak of 1.1 percent after 3 periods), are again larger in magnitude compared to our results (peak of just over 0.15 percent). While the direction of our results appear to match those of other studies, the differences in magnitude and duration of the impacts leave something to be desired. According to BB, the "results for other countries are quantitatively slightly different [from the US] but

[1] Data source in Appendix 1.2.

exhibit, overall, the same order of magnitude" (Baumeister and Benati 2010). Our results for the UK again match the direction of the BB results (both positive) but are smaller in magnitude (peak of 0.5 percent compared to peak of 1.9 percent). For *inf*, the results move in the opposite direction as those in BB. The results for *rgdp_rate* are similar to the BB results in direction, but fail when it comes to the magnitude of the effects, while the results for our *inf* counterfactual do not line up with their results in terms of magnitude or direction.

Figure 4.1.4: Difference Between Policy and No-policy Counterfactual Paths for Real GDP Growth and Inflation in UK[1]





4.2 Testing Hypothesis 2: Relationship Between Exchange Rate and Term Premium Strengthens During QE

Theory suggests the relationship between short-term interest rates and exchange rates diminished as short-term interest rates reach their lower bound. As speculators look for different ways to analyze trading opportunities, they look at longer term interest rates for insight. In order to test the hypothesis that the relationship between exchange rates and term premium (*bspread*) strengthened during the QE period, *VAR 1* was used with the exchange rate as an endogenous variable, instead of exogenous. Two separate VAR models were run for each country. The first VAR (VAR_{pre}) is run for the period before the financial crisis, before 2006Q2, when symptoms of the financial crisis first began to surface, and the FED began to lower the federal funds rate. This was done because the recession and QE may have affected the relationship between our variables of interest. The second VAR (VAR_{post}) was run for the period during financial crisis, that is 2006Q3 and 2012Q4.

Results for the US: To study the strength of the relationship between interest rates and term premiums, the coefficients and p-values between the two were analyzed. VAR_{post}

[1] Data source in Appendix 1.2.

had a higher R^2 and better Final Prediction Error (FPE) than VAR_{pre}. Looking at the coefficient in VAR_{pre} to determine the direction of the effect on exchange rates, we see that coefficients for both lags 1 and 2 have a negative sign, with lag 1 being significant at the 10 percent level and lag 2 not significant. Neither target rate lags had a significant effect on the dollar index. In VAR_{post}, there was an increase in the significance level across most variables. As well, in $VAR_{post,}$ the coefficients' directions stayed the same and the level of the effects became greater, suggesting the relationship strengthened during the period of QE. This is in line with Hypothesis 2.

Results for the UK: In the UK, VAR_{post} had a much higher R^2 and FPE than VAR_{pre}. In VAR_{pre}, the coefficient for the effect of *bspread* on the dollar index is positive for lag 1 and negative for lag 2, with a p-value showing the coefficients are not significantly different from zero. The sign on these coefficients remained the same for VAR_{post} as they had for VAR_{pre}. However, the coefficients' significance were greater during the QE period than before it.

Shortcomings: It was surprising that the VAR model for the pre-QE period showed insignificant coefficients between the exchange rate index and key macroeconomic variables, such as current account and target rate, in both the US and UK. This finding, combined with the R^2 on dollar exchange being low (0.45 for UK and 0.55 for US) suggests that the VAR model specification of variables was insufficient in explaining exchange rate movements. This is contrary to theory, which suggests that the fundamental drivers of exchange rates are short-term interest rates and the movement of goods between countries, as explained in Section 2.2.

Furthermore, the significance of variables between VAR_{pre} and VAR_{post} showed that the hypothesis may be incorrect. In the US VAR_{pre}, only three of the 22 coefficients were significant in determining the dollar index, but in the VAR_{post} seven were significant. The results were similar for the UK. One possible explanation could be that during the financial crisis, variables moved together because the general fear in the public shifted all behaviour of agents in the economy negatively. The negative behaviour means more cautious spending and investing behaviour, which is not in line with the fundamental drivers of exchange rates.

4.3 Testing Hypothesis 3: QE Depreciates Domestic Currency

Theory suggests QE depreciates domestic currency, as it depresses the domestic interest rate and increases money supply. To test this hypothesis, *VAR 1* was modified so that the exchange rate variable was changed from exogenous to endogenous.[1] The VAR was then estimated up to when QE was introduced, 2008Q4. A counterfactual study was then conducted. The VAR estimates were used to forecast from 2009Q1 onwards as the *without-QE* scenario (prefix wo_). Another forecast was run as the *with-QE* scenario, where QE was proxied as a 60 basis points decrease in *bspread* (prefix w_). The exchange rate variable was analyzed in both forecasts to understand the effects of QE on currency.

[1] The variable is an effective dollar index from US (twusdi) and UK (seri).

Results in the US: The VAR model with *d_l_twusdi* as endogenous changed the significance and fit of the model. By R^2 and Final Prediction Error (FPE) measures, the results were a better fit than the original counterfactual model.

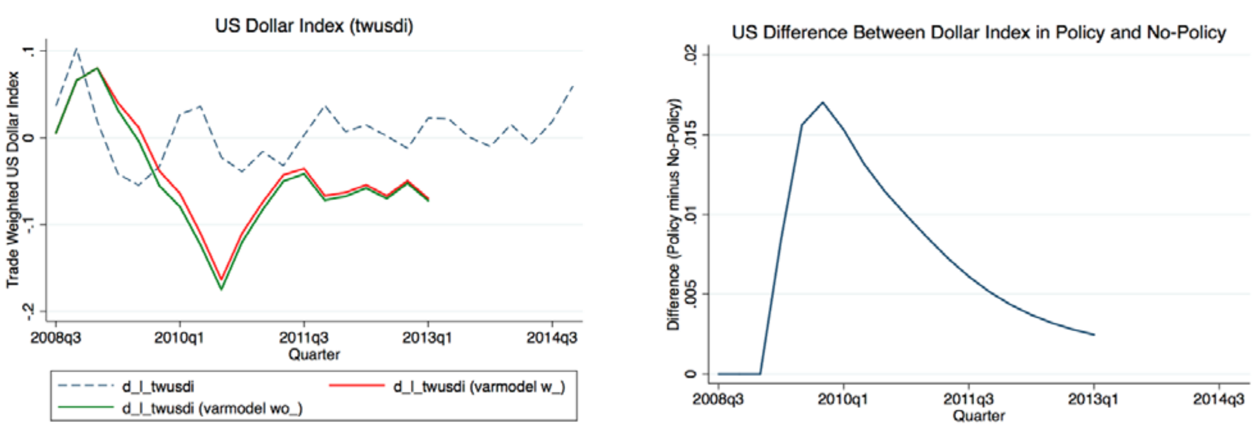
```
Vector autoregression

Sample:  1994q1 - 2008q4                        No. of obs      =        60
Log likelihood =  16.05334                      AIC             =  3.131555
FPE            =  .0000189                      HQIC            =  4.633446
Det(Sigma_ml)  =  4.03e-07                      SBIC            =  6.971187

Equation           Parms      RMSE     R-sq      chi2     P>chi2
----------------------------------------------------------------
rgdp_rate            22     1.31045   0.8360    305.87    0.0000
inf                  22     .625393   0.6366  105.1036    0.0000
target_rate          22     .344253   0.9767  2520.511    0.0000
d_l_twusdi           22      .02536   0.5480  72.73234    0.0000
bspread              22     .393579   0.9268  759.8873    0.0000
----------------------------------------------------------------
```

The results above for US were different from the hypothesis. Figure 4.3.1 below shows that the VAR forecast of dollar index with QE is actually higher than the VAR forecast without QE.

Figure 4.3.1: Counterfactual Path (Policy and No-policy) of US Dollar Index (Left) and Difference Between Policy and No-policy Counterfactual Paths for the US Dollar Index (Right)[1]

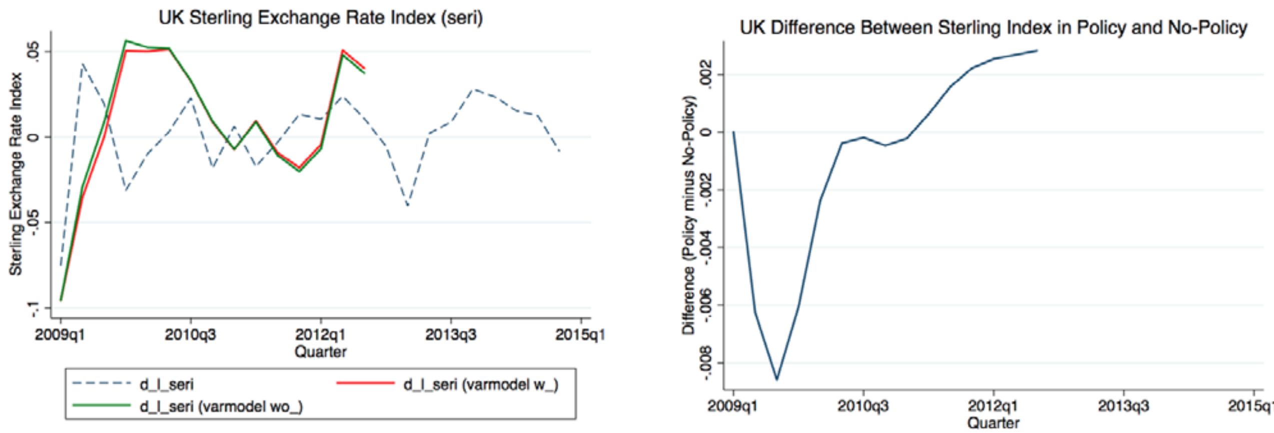


Results in the UK: The UK VAR model with *d_l_seri* as endogenous rather than exogenous increases the R^2 and decreases FPE of the model. Figure 4.3.2 below shows that the results of the counterfactual study are in line with hypothesis 3. QE decreased the Pound index.

[1] Data source in Appendix 1.2.

Figure 4.3.2 below shows that the results of the counterfactual study are in line with hypothesis 3. QE decreased the Pound index.

Figure 4.3.2: Counterfactual Path (Policy and No-policy) of UK Sterling Index (Left) and Difference Between Policy and No-policy Counterfactual Paths for the UK Sterling Index (Right)[1]



Findings: The results from the empirical analysis for Hypothesis 3 suggest differences between the US and UK QE policy, as well as in the market structure. While the introduction of QE depreciated the currency value in the UK, it appreciated the currency in the US. However, a low R^2 in the initial model could mean that there are many other factors that our VAR did not account for. Further analysis will need to be performed. Exchange rate indicators using different weighting methods, as well as using actual QE data, which measures the effects of increasing money supply rather than what the interest rate effect has on exchange rates, could prove useful.

5. Discussion of Results, Policy, Theory and Implications

5.1 Macroeconomic Comparison

The main goal of QE was to help improve economic performance and encourage inflation due to the recession. In order to do so, QE worked through 5 main channels (Figure 2.2.1), the most important of which was the portfolio balance channel.

In Section 4.1, Figures 4.1.2 and 4.1.3 show that the US program had a longer lasting impact on real GDP growth than that of the UK. For inflation, the results from the UK seem to suggest that its program was more effective in encouraging inflation in the short run, compared to the US. Nonetheless, it appears that in the long run, the US program was more capable in boosting inflation than in the UK. While neither program is perfect, the US program seems to have had a more positive effect on real GDP growth and

[1] Data source in Appendix 1.2.

inflation in the long run, while the UK program had a more desirable effect in terms of short-run inflation. The basic results for the estimation described in Section 4.1, along with those of our alternative methods (Section 5.3), are summarized in Appendix 4, for both the US and UK.

Depending on the government's main concern, whether it is improving the economy as quickly as possible or a long-term recovery, the different QE programs studied in this paper may be of interest. Certain aspects could be changed or modified, and findings such as these could serve to better understand how QE functions, as well as its overall effectiveness, for countries in similar situations to the US and UK.

5.2 Exchange Rate and QE

Exchange rates in US and UK are driven primarily by market forces. The FED and BOE did very little intervention in the currency exchange market over the past 20 years. Theories in exchange rates suggest QE would depreciate the exchange rate. As well, at the zero lower-bound, speculators would look to other indicators, especially long-term bonds, in making their buy or sell decision.

The results from our empirical analysis are surprising and give further insights into what moves the currency rate during the financial crisis. In Section 4.2, the possibility arises that even though the hypothesis was correct, it may not be because the relationship between the exchange rate and term premium strengthened, but rather that during QE, traders were in a general panic, and all assets moved in sync with economic indicators. In Section 4.3, we looked at a counterfactual analysis of QE on exchange rates and saw it had different effects in the US and UK, suggesting it was not through the mechanisms of decreasing bond spread. One explanation might be that during a global economic crisis such as the one in 2008, the US dollar is seen as more of a "safe haven" currency.

Exchange rates are important to many sectors of the economy in both the US and UK. Understanding how the implementation of QE can affect exchange rates will ultimately affect the decisions and design of the policy in QE. Both the FED and BOE needed to boost aggregate demand in order to stimulate the economy and get it back on track. Depreciating the currency could boost the exports of the economy. However, with the interconnectedness of global markets, it becomes much more difficult to account for exchange rate fluctuations. With other countries in distress at the same time, analyzing currency must be done in conjunction with the rest of the world, which was not done in this paper.

5.3 Alternative Methods and Findings

There were a few alternative methods that were attempted in order to evaluate the robustness of our model regarding the impact of QE. First, we ran a VAR for all time periods (1991Q1-2014Q4) for the US and UK, rather than our sample to the end of 2008. We also tried using *bspread* as an exogenous, rather than endogenous, variable in our *VAR 1* model. We found that in the US, *bspread* was positively correlated with GDP, so when QE was implemented through downward *bspread* shock in the counterfactual analysis, there was a decrease in GDP, although this relationship was not significant. However, in the UK, the relationship was negative, which is what theory has expected.

We also tried to use an alternate measure for QE. Instead of using *bspread*, we tried to input a proxy for the amount of QE, central bank assets, directly into *VAR 1*. We found the results for this VAR were meaningless and insignificant.

In general, while there were some minor similarities, we found for these alternative iterations, the effects of *bspread* on macroeconomic variables were very different for the US and UK. The effects of *bspread* on macroeconomic variables in the US were found to be more erratic, while in the UK, the effects were more along the lines of what we anticipated. This could be a result of the differences in the design of the programs. For example, in the US, QE1 was effective in achieving its goal, but QE2 and QE3 are generally believed to have been less successful in raising GDP and lowering inflation, as there was a diminishing effectiveness of the QE program. Meanwhile, in the UK, they only ran "one" program that was continually renewed, rather than what the US did in having multiple programs with different designs. This contrast could be an explanation for the differences in the effect of *bspread* on our variables of interest.

Additionally, forward guidance, expectations that result from the announcement of monetary policy, was more effective in the UK in that when they announced QE, the effects were seen more immediately. This is seen by the positive effect on real GDP growth in the period following the QE announcement in the UK (Figure 4.1.3), while in the US, there was an initially negative response, before becoming positive (Figure 4.1.2). The reason we believe forward guidance is less effective in the US is because its financial system is more complex and speculative by nature, as it is a larger and more open economy than the UK. Also, in the US, the Federal Reserve System, with the way it is organized with 12 districts and 7 board members that must pass monetary policy by a majority vote, is much less clear and straightforward with monetary policy for businesses and investors to act on compared to the more simple system in place in the UK.

The results for the alternative methods described above can be seen in Appendix 4. It details the results for all the variations of our VAR model that were estimated, describing the basic results for the first and second lags of *rgdp_rate* and *inf*, their respective p-values and the R^2 in both the US and UK.

Finally, a sensitivity analysis was done to vary the *bspread* variable to see whether a change in the magnitude away from the original 60 basis point assumption would cause different inferencing in the analysis of QE using the counterfactual. The general direction of the effect of QE on macroeconomic variables did not change. Varying *bspread* simply affected the magnitude of the change in *rgdp_rate* and *inf* in the counterfactual estimation.

5.4 Time-Varying Parameters

Based on the models presented in this paper, we have found that when analyzing QE on macroeconomic variables, time-varying parameters should have been used. This is in contrast to Lenza et al. (2010), who did not use time-varying parameters in their analysis, and claimed that this omission would not significantly affect the results. Through the course of our research, we found that they should be included, because the relationships between important macroeconomic variables change during the financial crisis compared to their relationship beforehand, and in many cases, this change is significant.

6. Conclusion

Using empirical analysis, we decided to focus on the macroeconomic and currency market effects of QE. This paper focuses on comparison of the macroeconomic results between the US and UK, and looks into the effects of QE on exchange rates and currency markets. A literature survey was conducted to look at the current research done on analysis of QE. After the analysis of the theory, we came to 3 hypotheses:

1: Macroeconomic variables improved due to QE
2: Relationship between exchange rates and term premium strengthened during QE
3: QE depreciated exchange rates

We estimated the coefficients of a VAR, forecasting out the macroeconomic variables without QE. We forecasted out the macro variables with QE by artificially depressing the bond spread by 60 basis points. The analysis was done by comparing the sets of forecasted variables. We found that both programs had nearly the same directional effect on macroeconomic variables in the short run, with the US outperforming the UK in the long run. For inflation, the UK program appears to have had the effect of decreasing inflation, compared to the no-policy scenario, in the short run. Meanwhile, in the US, inflation increased relative to the no-policy scenario in the short run, before steadily decreasing in subsequent periods.

The findings on exchange rates differ from the original hypotheses. The relationship between exchange rates and the bond spread strengthened during QE. The results were similar between the US and UK. The effects of QE on exchange rates were different for the two countries. Our empirical study suggests that QE appreciated the dollar index in US, while depreciating the UK Sterling index.

Comparing the two countries, the UK seems to have had the more desirable effects with their program than the US, as it was able to decrease inflation and exchange rates. On the other hand, the US' QE policy was more successful in improving the GDP growth rate. It ultimately depends on which variable was of greater importance to the country. In the US, the FED had the mandate to improve the unemployment rate as well as decreasing the amount of toxic assets in the financial system. On the other hand, the UK was solely focused on controlling inflation. This explains some of the differences between the implementation and ultimate results of the two countries' programs.

Our paper found that depending on how the model was specified, the effects and results of QE could tell different stories. A change in the time period or changing the transmission mechanism can greatly vary the estimated results.

Further research should be done on how QE affects even more specific variables such as income levels, bankruptcy rates, unemployment, and so forth. Moreover, most of the research done so far has used a decrease in *bspread* as a proxy for QE. Other transmission mechanisms of QE such as liquidity premium or consumer confidence should be studied as well because the effects can be complex in large open economies like the US and UK. As well, we find that there also needs to be further research done to study the recovery of financial crisis after the implementation of QE. Data showed a much longer recovery period in this recession compared to past recessions. All in all, QE is a tool that is effective under specific situations and has very specific goals. The design needs to carefully thought out and modelled to increase the success of its implementation.

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Appendix

Appendix 1.1: Variable List

Variable	Description	Transformation
rgdp_rate*	Real GDP growth in percentage	Levels
inf*	Inflation (CPI) in percentage	Levels
gfcf	Gross Fixed Capital Formation, proxy for investment	Levels
unemp	Unemployment rate	Log-levels
stocks	Stock index value, S&P 500 or FTSE	Log-levels
tenyear	10-year maturity government bond yield	Levels
threemonth	3-month maturity government bond yield	Levels
bspread*	Difference between 10 year and 3 months	Levels
target_rate*	Central bank target overnight rate	Levels
savings	Net savings, not chained, ratio of income	Log-levels
bus_bankruptcy	Total number of business liquidations	Levels
ind_insol	Sum of bankruptcies, DROs an IVAs	Levels
ca	Export-Import, Total Current Account Balance	Log-levels
gov_exp	Government Final Consumption Expenditure	Log-levels
consum	Private Final Consumption Expenditure	Log-levels
manu	Manufacturing Production	Levels
housing	Real Residential Housing Prices	Log-levels
gbpusd*	British Pound per US Dollar	Levels
usdgpb*	US Dollar per British Pound	Levels
seri*	Sterling Exchange Rate Index	Log-levels

twusdi*	Trade Weighted US Dollar Index	Log-levels

* indicates endogenous variable

Appendix 1.2: Variable Source List

Variable	US Source	UK Source
rgdp_rate*	OECD Quartly GDP	OECD Quartly GDP
inf*	OECD CPI Inflation	OECD CPI Inflation
gfcf	FRED of St. Louis FED	FRED of St. Louis FED
unemp	OECD stats	OECD STAT stats
stocks	Investing.com	Investing.com
tenyear	Bloomberg Terminal	Bloomberg Terminal
threemonth	Bloomberg Terminal	OECD
bspread*	Calculated	Calculated
target_rate*	FRED of St. Louis FED	Bank of England
savings	Bureau of Eocnomics Table 5.1	Office for National Statistics, UK
bus_bankruptcy	American Bankruptcy Institute	Gov.uk Statistics
ind_insol	American Bankruptcy Institute	Gov.uk Statistics
ca	OECD Current Account Balance	FRED of St. Louis FED
gov_exp	Bureau of Eocnomics Table 3.9.3	FRED of St. Louis FED
consum	Bureau of Eocnomics Table 2.3.3	FRED of St. Louis FED
manu	FRED of St. Louis FED	FRED of St. Louis FED
housing	FRED of St. Louis FED	FRED of St. Louis FED
gbpusd*	Bloomberg Terminal	Bloomberg Terminal
usdgpb*	FRED of St. Louis FED	FRED of St. Louis FED
seri*		Bank of England
twusdi*	FRED of St. Louis FED	

Appendix 2: Data Summary

US Summary Statistics[1]

US	Count	Mean	Std. Dev	Min	Max
Country	96	2.000	0.000	2.000	2.000
rgdp_rate	96	2.495	2.478	-8.450	7.554
inf	96	2.514	1.101	-1.623	5.303
gfcf	94	0.044	0.066	-0.266	0.145
unemp	96	6.148	1.608	3.900	9.933
stocks	101	1113.822	453.152	371.160	2067.890
tenyear	96	4.739	1.681	1.634	8.227
threemonth	96	2.857	2.163	0.005	6.210
bspread	96	1.883	1.171	-0.783	3.789
target_rate	99	2.960	2.275	0.070	6.530
savings	99	304.436	244.134	-364.500	675.400
bus_bankru~y	94	11363.620	3443.232	4086.000	19566.000
ind_insolv	82	312720.200	87348.990	112685.000	654633.000
ca	92	-93.591	59.046	-214.501	9.957
gov_exp	97	444.837	144.455	249.100	654.050
consum	89	7715.291	1432.623	5284.400	9726.200
manu	97	0.584	1.507	-6.592	2.738
housing	95	102.426	21.155	79.000	152.300
usdgbp	100	0.613	0.052	0.489	0.704
twusdi	100	86.812	10.325	69.528	111.575
qe	52	2154.876	1361.760	725.019	4497.660
quarter	96	171.500	27.857	124.000	219.000
l_unemp	96	1.784	0.251	1.361	2.296
min_ca	101	-214.501	0.000	-214.501	-214.501
l_ca	92	4.594	0.832	0.000	5.418
l_stocks	101	6.917	0.473	5.917	7.634
min_savings	101	-364.500	0.000	-364.500	-364.500
l_savings	99	6.342	0.872	0.000	6.948
l_gov_exp	97	6.043	0.337	5.518	6.483
l_consum	89	8.933	0.195	8.573	9.183
l_housing	95	4.609	0.197	4.369	5.026
l_twusdi	100	4.457	0.118	4.242	4.715
d_l_twusdi	99	0.001	0.030	-0.059	0.103

[1] l_ indicates log, d_ indicates first difference.

UK Summary Statistics[1]

UK	Count	Mean	Std. Dev	Min	Max
Country	96	1.000	0.000	1.000	1.000
rgdp_rate	96	2.090	2.356	-9.012	6.426
inf	96	2.471	1.493	0.600	8.400
gfcf	94	0.032	0.115	-0.236	0.297
unemp	96	6.841	1.688	4.600	10.400
stocks	101	4985.432	1322.243	2313.000	6984.400
tenyear	96	5.279	2.199	1.727	10.500
threemonth	96	4.356	2.784	0.236	12.290
bspread	96	0.923	1.417	-2.229	3.778
target_rate	99	4.383	2.877	0.500	12.375
ind_insolv	96	15827.450	10213.590	5436.000	35682.000
liquidation	92	3923.554	826.037	2900.000	6473.000
ca	93	-5.686	5.032	-23.919	1.154
savings	96	10.244	3.369	4.500	17.000
gov_exp	96	57.938	20.885	29.311	89.195
consum	88	192.836	35.003	134.242	238.143
manu	96	0.056	1.191	-5.666	2.232
housing	97	76.005	25.637	41.500	114.600
gbpusd	100	1.644	0.150	1.421	2.044
seri	100	91.936	8.345	77.899	104.725
qe	33	248708.200	120877.800	78509.000	413029.000
quarter	96	171.500	27.857	124.000	219.000
l_unemp	96	1.893	0.245	1.526	2.342
min_ca	101	-23.919	0.000	-23.919	-23.919
l_ca	93	2.886	0.489	0.000	3.261
l_stocks	101	8.473	0.301	7.746	8.851
l_savings	96	2.271	0.340	1.504	2.833
l_gov_exp	96	3.991	0.376	3.378	4.491
l_consum	88	5.244	0.191	4.900	5.473
l_housing	97	4.267	0.369	3.726	4.741
l_seri	100	4.517	0.092	4.355	4.651
d_l_seri	99	-0.001	0.026	-0.131	0.064

Appendix 3: Output Summary of Base VAR Model

US VAR Model

[1] l_ indicates log, d_ indicates first difference.

```
Vector autoregression

Sample:  1994q1 - 2008q4                        No. of obs      =        60
Log likelihood = -137.6174                      AIC             =  7.387246
FPE            =  .0214783                      HQIC            =  8.534144
Det(Sigma_ml)  =  .0011543                      SBIC            =  10.31933

Equation           Parms      RMSE     R-sq      chi2     P>chi2
----------------------------------------------------------------
rgdp_rate            21     1.32777   0.8272   287.2481   0.0000
inf                  21     .631557   0.6196   97.74566   0.0000
target_rate          21     .328492   0.9783   2701.407   0.0000
bspread              21     .391284   0.9258   748.2636   0.0000
----------------------------------------------------------------
```

	(1)	(2)	(3)	(4)
VARIABLES	rgdp_rate	inf	target_rate	bspread
L.rgdp_rate	-0.438***	0.0911*	-0.00960	0.0443
	(0.111)	(0.0529)	(0.0275)	(0.0328)
L2.rgdp_rate	0.0237	0.0466	-0.00768	0.00509
	(0.105)	(0.0500)	(0.0260)	(0.0310)
L.inf	-0.384	0.688***	0.00820	0.324**
	(0.436)	(0.207)	(0.108)	(0.128)
L2.inf	0.903**	0.141	0.0725	-0.0739
	(0.400)	(0.190)	(0.0989)	(0.118)
L.target_rate	-0.367	-0.295	1.154***	-0.838***
	(0.542)	(0.258)	(0.134)	(0.160)
L2.target_rate	-0.378	0.410	-0.277**	0.252
	(0.560)	(0.266)	(0.138)	(0.165)
L.bspread	0.125	-0.275	0.0775	0.333**
	(0.460)	(0.219)	(0.114)	(0.136)
L2.bspread	-0.964**	0.320	0.0697	-0.0493
	(0.442)	(0.210)	(0.109)	(0.130)
D.l_unemp	-4.975	0.777	-2.957*	5.628***

	(6.513)	(3.098)	(1.611)	(1.919)
gfcf	16.83***	1.233	2.802**	-3.381**
	(5.475)	(2.604)	(1.355)	(1.613)
l_ca	1.194***	-0.107	-0.0542	0.279**
	(0.383)	(0.182)	(0.0948)	(0.113)
D.l_stocks	4.420**	-0.262	-0.578	0.896
	(2.117)	(1.007)	(0.524)	(0.624)
manu	1.270***	0.0912	0.0587	0.101
	(0.252)	(0.120)	(0.0623)	(0.0742)
l_savings	0.190	-0.805	0.612*	0.483
	(1.386)	(0.659)	(0.343)	(0.408)
bus_bankruptcy	-0.000182*	-7.95e-05	-2.68e-05	-1.51e-06
	(0.000105)	(4.98e-05)	(2.59e-05)	(3.09e-05)
ind_insolv	2.03e-06	2.06e-06*	-7.52e-08	-9.46e-07
	(2.57e-06)	(1.22e-06)	(6.35e-07)	(7.56e-07)
D.l_gov_exp	39.32*	32.39***	8.281	25.39***
	(23.82)	(11.33)	(5.892)	(7.019)
l_consum	-2.008	-1.269	-1.171	-2.731***
	(3.335)	(1.586)	(0.825)	(0.983)

D.l_housing	-8.816	-5.016	-9.715**	-9.360*
	(17.35)	(8.250)	(4.291)	(5.111)
d_l_twusdi	1.845	-3.858	3.597**	0.725
	(6.689)	(3.182)	(1.655)	(1.971)
Constant	17.31	16.24	6.828	22.69**
	(33.99)	(16.17)	(8.410)	(10.02)
Observations	60	60	60	60

Standard errors in parentheses

*** p<0.01, ** p<0.05, *p<0.1

UK VAR Model

```
Vector autoregression

Sample:  1991q3 - 2008q4                          No. of obs      =        70
Log likelihood = -156.8028                        AIC             =  6.880079
FPE            =  .0123355                        HQIC            =  7.951835
Det(Sigma_ml)  =   .001037                        SBIC            =  9.578274

Equation           Parms      RMSE     R-sq      chi2     P>chi2
----------------------------------------------------------------
rgdp_rate            21     1.63319   0.6623   137.3076   0.0000
inf                  21      .42458   0.9292   918.7356   0.0000
bspread              21     .397999   0.9267   885.4714   0.0000
target_rate          21     .310707   0.9731   2530.357   0.0000
----------------------------------------------------------------
```

	(1)	(2)	(3)	(4)
VARIABLES	rgdp_rate	inf	bspread	target_rate
L.rgdp_rate	0.499***	-0.0377	-0.00371	0.0108
	(0.136)	(0.0353)	(0.0331)	(0.0259)
L2.rgdp_rate	-0.147	-0.0122	0.0106	0.0746***
	(0.139)	(0.0360)	(0.0338)	(0.0264)

L.inf	-0.728*	0.862***	0.153	-0.0512
	(0.422)	(0.110)	(0.103)	(0.0803)
L2.inf	0.464	-0.131	0.119	0.0175
	(0.410)	(0.107)	(0.0999)	(0.0780)
L.bspread	-0.617	0.126	0.480***	0.492***
	(0.473)	(0.123)	(0.115)	(0.0900)
L2.bspread	0.606	-0.0446	0.0944	-0.260***
	(0.499)	(0.130)	(0.122)	(0.0949)
L.target_rate	-0.496	-0.105	-1.149***	1.680***
	(0.643)	(0.167)	(0.157)	(0.122)
L2.target_rate	0.540	0.170	0.597***	-0.469***
	(0.762)	(0.198)	(0.186)	(0.145)
D.l_unemp	-8.419	-1.441	-6.262***	-2.685
	(9.958)	(2.589)	(2.427)	(1.894)
gfcf	-0.366	0.932**	0.0145	0.241
	(1.633)	(0.425)	(0.398)	(0.311)
l_ca	-0.761	0.155	-0.356	-0.196
	(1.597)	(0.415)	(0.389)	(0.304)
D.l_stocks	0.710	1.193*	0.514	0.244
	(2.599)	(0.676)	(0.633)	(0.494)
manu	0.626***	-0.0654	-0.0192	0.100**
	(0.218)	(0.0566)	(0.0531)	(0.0414)
D.l_savings	-0.664	-0.264	0.284	-0.341
	(1.319)	(0.343)	(0.322)	(0.251)
D.liquidation	-0.000734	0.000135	-0.000180	0.000302***
	(0.000541)	(0.000141)	(0.000132)	(0.000103)

D.ind_insolv	9.70e-06	8.59e-05	4.50e-05	1.02e-05
	(0.000204)	(5.29e-05)	(4.96e-05)	(3.87e-05)
D.l_gov_exp	12.64	-6.686**	-2.913	0.346
	(12.35)	(3.212)	(3.010)	(2.350)
l_consum	-1.164	0.921	-3.326***	1.951***
	(3.569)	(0.928)	(0.870)	(0.679)
D.l_housing	10.65	-8.430***	-0.494	0.458
	(8.463)	(2.200)	(2.062)	(1.610)
d_l_seri	-0.132	2.761	-10.76***	7.879***
	(7.925)	(2.060)	(1.931)	(1.508)
Constant	9.775	-4.918	21.12***	-11.17***
	(22.07)	(5.737)	(5.378)	(4.199)
Observations	70	70	70	70

Standard errors in parentheses

*** p<0.01, ** p<0.05, *p<0.1

Appendix 4: Alternative Model Result Summary

US Model Output

	Models	1	2	3	4	5
rgdp_rate	**Coef-1 lag**	**0.1246**	**2.5111**	**0.1498**	**-0.1304**	**-0.0018**
	p-value	0.7870	0.0000	0.7250	0.6950	0.0830
	Coef - 2 lag	**0.9636**	**1.0043**	**-0.2672**		
	p-value	0.0290	0.0250	0.4670		
inf	**Coef-1 lag**	**-0.2754**	**1.4369**	**-0.0425**	**0.4870**	**-0.0008**
	p-value	0.2080	0.0000	0.8260	0.7260	0.1400
	Coef - 2 lag	**0.3205**	**1.2094**	**0.2660**		
	p-value	0.1280	0.0000	0.1110		
R^2	**rgdp_rate**	**0.8272**	**0.9671**	**0.7661**	**0.7651**	**0.8579**
	inf	**0.6196**	**0.9425**	**0.7245**	**0.7144**	**0.8164**

Models

1) Base model: 1991 to 2008, bspread as endogenous
2) 2007-2013: bspread as endogenous
3) All data: 1991 to 2014, bspread as endogenous
4) All data: 1991 to 2014, bspread as exogenous
5) All data: 1991 to 2014, qe as exogenous

UK Model Results

	Model	1	2	3	4	5
rgdp_rate	**Coef-1 lag**	**-0.6169**	**-0.6392**	**-0.6470**	**-0.4601**	**0.0000**
	p-value	0.1920	0.4510	0.1110	0.0560	0.0470
	Coef - 2 lag	**0.6065**	**3.3021**	**0.6131**		
	p-value	0.2240	0.0010	0.1520		
Inf	**Coef-1 lag**	**0.1263**	**-0.3947**	**0.1580**	**-0.0284**	**0.0000**
	p-value	0.3050	0.0000	0.1520	0.6680	0.0320
	Coef - 2 lag	**-0.0446**	**-0.1523**	**-0.0994**		
	p-value	0.7310	0.1670	0.3930		
R^2	**rgdp_rate**	**0.6623**	**0.9090**	**0.6612**	**0.6649**	**0.8860**
	inf	**0.9292**	**0.9851**	**0.9170**	**0.9151**	**0.9753**

Models

1) Base model: 1991 to 2008, bspread as endogenous
2) 2006-2013: bspread as endogenous
3) All data: 1991 to 2012, bspread as endogenous
4) All data: 1991 to 2012, bspread as exogenous
5) All data: 1991 to 2012, qe as exogenous