Abstract

This paper studies informal default (or delinquency) in consumer credit as a process involving negotiations over unpaid debts. We consider an economy with uninsurable individual risk where, as an alternative to informal default, households can also declare formal bankruptcy. Negotiated settlements, which may be reached with some delay, involve limited recovery, are often followed by further defaults, and may end in bankruptcy as households reduce their assets in the process. When calibrated to aggregate measures of default, debt and wealth, the model yields differences in the financial positions of formal and informal defaulters which are quantitatively consistent with observed data. Informal defaulters have higher debts, assets, and net worth, but lower incomes. The opportunity to bargain in informal default - in contrast with informal default via prescriptive collection procedures - provides substantial insurance opportunities and welfare gains. It considerably mitigates the known adverse welfare consequences of formal bankruptcy. Bargaining also enhances the welfare gains following from a tighter asset exemption. Attempts at limiting debt collection outside bankruptcy lower generally, but not uniformly, welfare.

Keywords: unsecured credit, bankruptcy, bargaining, formal and informal default, debt negotiations, credit delinquency, risk sharing, consumption insurance.
1 Introduction

This paper investigates consumer defaults which do not result immediately in bankruptcy. In the U.S., default outside formal bankruptcy procedures is common. This is the theme of recent empirical and theoretical studies.\(^1\) In the Survey of Consumer Finances 2007, about 1% of the U.S. population had filed under Chapter 7, whereas over 5% held delinquent loans. This informal default might account for the bulk of loan write-offs.\(^2\) This evidence notwithstanding, formal bankruptcy filings have attracted most of the analysis and discussion in the recent macroeconomics literature.\(^3\)

In this paper, we consider informal default and can therefore address a number of novel questions. We find that informal default via bargaining improves risk sharing and welfare in the economy. It interacts meaningfully with formal bankruptcy and tempers the adverse consequences of the existence of a bankruptcy code. Informal default enhances the welfare gains from tightening the asset exemption in bankruptcy. We can also study policies that pertain primarily to delinquent rather than bankrupt households, and find that protection of consumers against creditor collection efforts has generally adverse consequences. Concerning the process of default itself, our setting encompasses common narratives depicting negotiations that can be often ineffectve in avoiding further defaults and in recovering debts.\(^4\) Also in coherence with the data, households who take different default options have specific observable characteristics, with informal defaulters being wealthier while holding more debts.

In order to address these questions and establish the results, we proceed as follows. First, we put forward an equilibrium model where informal default via bargaining is an alternative to formal bankruptcy procedures. Second, a calibrated version of the model is used to study the properties of the default process. Third, we assess the model’s implications against evidence about the characteristics of households making different default decisions. Fourth, we study the risk-sharing and welfare implications of informal default and


\(^2\)A point made in Dawsey and Ausubel (2004). Wilshusen (2011) reports that at least 10 million households participated in privately negotiated debt settlements. This figure includes debt that has been abandoned and debt that has been negotiated downward. Our model will capture both sources of write downs.

\(^3\)For example, Chatterjee, Corbae, Nakajima, and Rios-Rull (2007), Athreya, Tam, and Young (2009), Livshits, MacGee, and Tertilt (2007), Mateos-Planas (2009) or Mateos-Planas and Rios-Rull (2010). One noteworthy exception is Chatterjee and Gordon (2012) which, appeared after the present paper began to circulate, has a narrower concept of informal default than us and a more specific question. See further discussion below.

\(^4\)Wilshusen (2011) paper documents the high dropout rates from settlements negotiated.
its relevance for formal default. Finally, we analyse the effect of two policy parameters, the asset exemption in bankruptcy and the strength of creditor collection in delinquency.

The theoretical framework is based on a version of the equilibrium model of heterogeneous households with uninsurable risk typical of much bankruptcy literature – like Chatterjee, Corbae, Nakajima, and Ríos-Rull (2007) – extended to account for bargaining over debts and adjustment costs for assets. The latter feature allows us to consider in a simple way households who maintain both debt and assets, and will capture the fire-sale costs of default. It is a proxy for the existence of household illiquid assets (although we do not model durable consumption as such) and small business assets. The bargaining feature is more novel and the core contribution of the paper. Default can take the form of a formal procedure modeled on Chapter 7 of the U.S. bankruptcy code – including an asset exemption – or can lead to a process of informal default with bargaining while collection activities are also underway. The bargaining, which involves complete information, takes place through randomly alternating offers between the debtor and the creditors. The two bargain over a plan by which the debtor can avoid bankruptcy and rejoin the market. A plan consists of proposed transfers of current resources and debt swaps which are priced at market rates. It also involves a recommendation for consumer assets next period, a form of collateral requirement. Negotiations may fail to resolve quickly, an failed negotiations can continue for some time or revert to Chapter 7. In effect, proposers may participate in negotiations by making non-serious offers or refuse to accept serious ones.

We solve the equilibrium of the model for a calibrated benchmark. Parameters of the model, including the costs of defaulting, the collection technology, income persistence and adjustment costs, are chosen to broadly match appropriately calculated empirical targets from the Survey of Consumer Finances. These targets include the formal and informal default rates, the debt to income ratio, the asset to income ratio, and the proportion of population in debt.

We find the following main properties of the benchmark economy. Informal default via bargaining greatly expands the set of those who default. Informal defaulters hold more debt and more assets, and have lower income than formal defaulters. The reason is that households with significant assets but poor income use informal default first to insure against income shocks; formal default is used once consumers have run down their assets to the exemption level at which point the benefits to entering formal bankruptcy default

\footnote{It is costly for debtors to reduce their assets to avoid collection or seizure under bankruptcy. See Jackson (2001) on the central role of these costs in the context of bankruptcy.}
dominate even if income improves.

Upon the household defaulting, negotiated settlements – which can be reached with some delay – are very frequently followed by further defaults as long as income does not subsequently improve. In the process, debtors reduce their assets and may end up in formal bankruptcy. The bargaining option allows households to disavow gradually rather than hastily, thus smoothing the costs while, at the same time, weakening the bargaining position of the creditors. These individuals can eventually end up in formal Chapter 7 when the household’s assets have declined sufficiently close to the exemption level. Settlements that are durable require improvements in income. On the other hand, negotiated settlements involve limited loan recovery for the creditor. These settlements achieve substantial debt reductions.

The main results are as follows. The benchmark economy gives a good quantitative approximation to evidence on the characteristics of different types of defaulters. In SCF data informal defaulters appear different from individuals in formal bankruptcy proceedings. In particular, individuals in informal bankruptcy are wealthier and have more assets and more debt, but lower income, than individuals in formal bankruptcy. We have already noted that the model meets all these properties qualitatively. Closer inspection at the data shows that the model also performs well quantitatively in this regard, an overstatement of debt levels for informal defaulters being the only significant quantitative miss.

We conduct some exercises to understand the significance of informal default. We find that the presence of informal default improves welfare and risk sharing through enhanced direct insurance opportunities, in spite of its adverse consequences for access to credit. So policies that facilitate negotiation have a positive impact. This is in contrast with the known result that easing formal default is detrimental to welfare. Nonetheless, the possibility of informal default substantially mitigates the loses associated with formal bankruptcy since, to a large extent, the former becomes operative because households find a good threat value in the bankruptcy option. There is thus an element of complementarity between formal and informal default. The benefits from informal default emanate mainly from the possibility of bargaining. In our analysis, informal default in the form of only collection/garnishment would have mainly opposite consequences to the ones just described.

We also analyse the effects of reducing the asset exemption level in formal default. A tighter exemption increases the levels of debt in the economy as it makes it easier to borrow, lowers the variability of consumption, and raises the rate of informal default. Tight-
ening the exemption level brings about substantial welfare gains at all debt levels. This result contrasts with the much weaker effect that obtains in the absence of informal default. In effect, the option of informal default allows households to avoid the cost of fire-selling assets associated with stricter bankruptcy. Finally we show the effects of collection actions outside of bankruptcy. We show that protecting consumers from outside collection efforts reduces access to credit, with a general but not uniform negative welfare effect.

This paper is a first attempt to modeling default both outside and inside of formal bankruptcy. It is related to the consumer bankruptcy literature. Chatterjee, Corbae, Nakajima, and Ríos-Rull (2007), Athreya, Tam, and Young (2009) and Livshits, MacGee, and Tertilt (2007) consider formal default only without renegotiation. Mateos-Planas and Ríos-Rull (2010) is a model of long-term credit contracts with switching costs which could also be interpreted as a form of renegotiation. Although Li and Sarte (2006) add Chapter 13 bankruptcy to Chapter 7, the arrangement is not subject to negotiation. In the sovereign literature, Eaton and Gersovitz (1981) discuss the behavior of short term debt and, more recently, Benjamin and Wright (2009) have first noted that including bargaining can sharply improve the ability of these models to reproduce data outcomes.⁶ At the time of completion of the present draft, Athreya, Sanchez, Tam, and Young (2012) have begun a related project addressing informal default with a different model of resolution that excludes the full negotiations we consider here.⁷ Herkenhoff and Ohanian (2012) study delinquency in the context of mortgage default but are not directly concerned with renegotiation. Most recently, Chatterjee and Gordon (2012) have considered informal default that consists uniquely of a prescribed garnishment/collection process, thus leaving aside the possibility of voluntarily negotiated outcomes considered here. As already pointed out, the implications of negotiation and collection can be very different, even qualitatively, thus warranting our broader approach. Furthermore, that paper has a single specific focus on the implications of formal bankruptcy discharge.

2 Model

Our theory concerns the borrowing, lending, and negotiations between private lenders and their creditors. The formal model has two types of agents. The first is infinitely

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⁶Other recent examples in the sovereign debt literature include Yue (2010).
⁷In that work, debtors and creditors do not bargain over immediate access to markets or consider the possibility of new short term borrowing. They do not consider the mechanism that allows debtors to reduce assets while in default, a key factor in our model for the choice over type of default.
lived risk-averse households who face idiosyncratic income risk. They can hold debt and assets to smooth consumption. They choose whether to default or not, whether to default via bankruptcy or via bargaining, and, if given the chance, which terms of settlement to propose and, otherwise, whether to accept the bank’s proposed terms. The other type of agents is risk neutral creditors who lend competitively to households. When bargaining banks choose, if given the chance, which terms of settlement to propose and, otherwise, whether to accept the household’s offered terms.

**Individual states.** At the beginning of each period, a household is described by an idiosyncratic productivity state $s$, asset holdings $a$, debt holdings $b$, and credit status $z$. There are four possible situations in terms of credit status for the household $z \in \{0, 1, 2, 3\}$. If $z = 0$, the household has a clear record and can freely borrow and lend or default; if $z = 1$, the household had filed for or been under Chapter 7 bankruptcy in the previous period and not yet cleared its bankruptcy flag; if $z = 2$ the household had bargained in the previous period and not yet reached a settlement, and the household is the current proposer; if $z = 3$ the household had bargained in the previous period and not reached a settlement, and the lender is the current proposer.

**Preferences.** Preferences are defined over sequences of consumption $c$ along the household’s infinite lifetime. At any point, it is represented as the expected sum of period utility $u(c)$ discounted at the rate $\beta$. The utility function belongs in the CRRA class and is specified as $u(c) = c^{1-\sigma}/(1-\sigma)$, where $\sigma$ is the inverse of the rate of risk aversion. Utility is also affected by a non-pecuniary cost to defaulting on debt via Chapter 7 denoted $\chi^7$, and via bargaining denoted $\chi^B$. These costs enter separably and additively in the utility of the household.

There is ex-post individual random noise in the preferences over some choices in some states in the form of additive and separable shocks to utility. The purpose is primarily technical as this achieves some needed smoothness.\(^8\) When $z = 0$, there is a random component to the utility of defaulting with distribution $F_{DvsR}$. When having to decide between bankruptcy and bargaining there is a random component to the utility of opting for Chapter 7 with distribution $F_{7vsB}$. These shocks will be Normally distributed with zero mean and standard deviations $\sigma_{DvsR}$ and $\sigma_{7vsB}$, respectively.

**Technology.** Household income $e(s)$ is given by the idiosyncratic productivity type $s$. The income type follows a Markov process with transition matrix $\Gamma_{s,s'}$ over a set $S$. Changing the level of assets from $a$ to $a'$ incurs an adjustment quadratic cost of $\phi_a \times (a-a')^2$.

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\(^8\)Care will be taken that they do not cause spurious results.
Banks operate an intermediation technology of deposits into one-period loans. They face a transaction cost at maturity which is a proportion $\lambda$ of debt outstanding.

**Markets.** Households buy perishable consumption $c$ in competitive markets at a price normalized to 1. They can hold non-contingent debt $b$ and assets $a$. Households and banks interact in two environments, in the market and, when in default, outside of the market. Consider the market here. Households buy assets and banks sell deposits at the risk-free discount price $q_0$. Households sell and banks buy debt $b'$ at a discount price that depends on all the relevant information: assets and debts carried forward, $a'$ and $b'$, and the current income state $s$. Thus the menu of debt prices is written as $q(a', b', s)$.

**Bankruptcy and bargaining.** There is imperfect enforcement of contracts. Failure to repay debts may place households and banks outside of the market, being either in Chapter 7 bankruptcy or in a process of bargaining. There affairs will be conducted under specific institutional conditions. Let us consider the timing first. If the household has a clear record $z = 0$, it is when it defaults that the exact identity of the would-be proposer, whether the household or the bank, becomes known. Then there is the choice about how to default, whether via bankruptcy or bargaining. If bankruptcy follows, then the Chapter 7 flag carries over into the next period $z' = 1$ with probability $\pi_7$, otherwise $z' = 0$. If bargaining follows, then the household will start clean next period $z' = 0$ if the parties reach a settlement, or will carry a bargaining flag $z' = 2$ or 3 if there is no settlement, depending on which party – the household or the lender – will be the next the proposer. If the household has currently a past-failed-negotiation flag $z = 2, 3$, the identity of the would-be proposer is known by definition, and events unfold just like in the previous case, starting with the choice whether to file for Chapter 7 or continue the bargaining. Finally, if the household has a Chapter 7 flag $z = 1$, it is again brought over into the next period $z' = 1$ with probability $\pi_7$, otherwise $z' = 0$.

From the above description, the initial default decision occurs when the proposer is unknown but subsequent decisions involve the knowledge of the proposer. The identity of the proposer is stochastic and may change in every period. Without default history, it is the
result of an independent draw, and $\gamma_{ho}$ denotes the ex-ante probability that the household is the party who would propose. Once carrying a bargaining flag, there may be persistence in the proposer’s identity, and $\gamma_{ho,ho}$ and $\gamma_{lo,lo}$ denote the probabilities of repeating proposer in this case.\textsuperscript{10}

We now turn to the institutional rules governing the different default situations. Consider first the case where the household is under Chapter 7 bankruptcy. The household’s assets above a certain exemption level, $a^7$, will be seized by banks as repayment, up to a value equivalent to debt outstanding. Households cannot borrow but can save. The probability of clearing the bankruptcy flag, $1 - \pi^7$, describes the punishment to the household in terms of exclusion from the credit market. Consider next the case where the household is in a process of bargaining. The designated party proposes a plan involving new debt $b'$ and current transfer of goods $\tau$ and, if the bank proposes, new assets $a'$ (a form of collateral requirement).\textsuperscript{11} If there is no settlement, the bank uses an exogenous collection process whereby it recovers a fraction given by the collection ‘tax’ on debt $\tau_b$.\textsuperscript{12} In this situation, the household is free to manage its assets. If there is settlement, the household borrows $b'$ in the credit market and arranges the transfer $\tau$ to repay the bank in accordance with the plan agreed.

\textbf{Equilibrium.} We consider competitive equilibria with free entry in the credit market, and SPE in bargaining.\textsuperscript{13} Generically, an equilibrium is a situation where households’ choices maximize the expected value of utility given prices and bank’s (bargaining) responses; banks’ choices of proposed repayment plans maximize the expected recovered value given prices and households’ responses; and prices make banks’ (expected) profits equal to zero given households’ and banks’ decision rules. To make this definition operative, we move on to state formally the decision problems and conditions implied in an equilibrium of the model.

\textsuperscript{10}The process for the choice of the future proposer is intended as a representation of bargaining power. While the given description is general, the numerical implementation will contain a simple and intuitive version where all periods are treated similarly.

\textsuperscript{11}Note that $\tau$ can be negative when debt increases after settlement in the sense that $b' > b$.

\textsuperscript{12}Rather than the garnishment of a proportion of income considered in Chatterjee and Gordon (2012). Given the different focus of the present paper, and in the interest of computational convenience, we have deliberately adopted a streamlined representation of the collection process.

\textsuperscript{13}This is not unlike the stationary Subgame Perfect Equilibrium of Merlo and Wilson (1998) and Merlo and Wilson (1995).
3 Equilibrium conditions and characterization

We now set out the decision variables and reservation values in such an equilibrium. They can be expressed recursively. We take households and firms in turns. To help notation, denote by $o \in \{ho, lo\}$ the party that gets to propose a bargaining plan, and use $ho$ to index situations when the household offers and $lo$ for situations when the lender offers. Also, use $ac$ to index situations where an offer is accepted and $noac$ for when it is not.

3.1 Households

Households take as given the market price schedule for debt $q$ and the bank’s reservation value in the failed bargaining states $W_0^{D,B}$ conditional on the proposer’s type $o \in \{lo, ho\}$.

We first consider in the next subsection the key decisions about whether and how to default under each of the states, for given values associated with these choices. Then in the following subsection we will characterize these underlying values.

3.1.1 Default choices

The default decisions yield a number of functions of the state $(a, b, s)$ for the household. Starting with a clean credit record $z = 0$, we have the ex-ante value $V$ and the corresponding default probability $d$, the value to defaulting $V^D$, the values to defaulting conditional on the proposer’s type $V_o^{D}$, and the probability of choosing Chapter-7 bankruptcy default over bargaining conditional on the proposer’s identity $p_{7,o}^{D}$.

The choice whether to default of repay yields the default probability

$$d(a, b, s) = \int I[V^D(a, b, s) - V^R(a, b, s) - x > 0] dF_{DvsR}$$

(1)

where $V^R$ is the value to repaying, $F_{DvsR}$ is the distribution of the utility noise $x$, and $I$

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$^{14}$The supindex $D, B$ indicates a state of default following a period of failed bargaining, that is one of $z = 2, 3$. With the subindex $ho$ it corresponds to $z = 2$; with the subindex $lo$ it corresponds to $z = 3$. We will often omit the arguments of this and other functions in in-line text.
denotes the indicator function. The ex-ante value

\[ V(a, b, s) = (1 - d(a, b, s))V^R(a, b, s) + \int I_{[V^D(a, b, s) - V^R(a, b, s) - x > 0]}(V^D(a, b, s) - x) dF_{D vs R}. \]  

(2)

The ex-ante value to defaulting, before the proposer’s identity becomes known, is then

\[ V^D(a, b, s) = \gamma_{ho} V^D_{ho}(a, b, s) + (1 - \gamma_{ho}) V^D_{lo}(a, b, s) \]  

(3)

Having decided to default, they choose bankruptcy or bargaining knowing the proposer’s identity \( o = lo, ho \). The probability of bankruptcy

\[ p^D_{7, o}(a, b, s) = \int I_{[(V^7(a, s) - \chi^7) - (V^B_o(a, b, s) - \chi^B) - x > 0]}(V^7(a, s) - \chi^7 - x) dF_{7vsB} \]  

(4)

so the value of defaulting, knowing the proposer,

\[ V^D_o(a, b, s) = (1 - p^D_{7, o}(a, b, s))(V^B_o(a, b, s) - \chi^B) + \int I_{[(V^7(a, s) - \chi^7) - (V^B_o(a, b, s) - \chi^B) - x > 0]}(V^7(a, s) - \chi^7 - x) dF_{7vsB} \]  

(5)

where \( V^7 \) and \( V^B_o \) denote the bankruptcy value and bargaining value, respectively, and \( F_{7vsB} \) denotes the distribution of the noise \( x \).

On the other hand, starting from a situation of previous failed negotiation with \( z = 2 \) or \( 3 \), the household must decide whether to continue bargaining or file for Chapter 7 bankruptcy. In this case, the functions characterizing the optimal choices are the probability of choosing bankruptcy conditional on the proposer’s identity \( p^D_{7, o} \), and the associated values \( V^D_o \). The probability of bankruptcy

\[ p^D_{7, o}(a, b, s) = \int I_{[(V^7(a, b, s) - \chi^7) - V^B_o(a, b, s) - x > 0]}(V^7(a, b, s) - \chi^7 - x) dF_{7vsB} \]  

(6)

and the value

\[ V^D_o(a, b, s) = (1 - p^D_{7, o}(a, b, s))(V^B_o(a, b, s) - \chi^B) + \int I_{[(V^7(a, b, s) - \chi^7) - V^B_o(a, b, s) - x > 0]}(V^7(a, b, s) - \chi^7 - x) dF_{7vsB} \]  

(7)
There is no decision about default for the household while bearing the bankruptcy flag $z = 1$.

### 3.1.2 Value of repayment, bankruptcy, and bargaining

The objects that all the above default choices take as given are the value of repaying $V^R$, the value of filing for bankruptcy $V^7$, and the values of a failed past negotiation $V^B_0$. The value of repaying in the clean state

$$V^R(a, b, s) = \max_{b', a'} \{ u(c) + \beta E[V(a', b', s') | s] \} \tag{8}$$

subject to

$$c - q(a', b', s)b' + q^0a' \leq e(s) - b + a - \phi(a'/a)$$

The value for the household to filing for Chapter 7 when $z = 0$ or being in the bankrupt state $z = 1$ is determined as follows: (Note that when $z = 1$ the agent holds no debt so it must be that $b = 0$.)

$$V^7(a, b, s) = \max_{a'} \{ u(c) + \beta E[\pi^7V^7(a', 0, s') + (1 - \pi^7)V(a', 0, s')] \} \tag{9}$$

subject to

$$c + q^0a' = e(s) + a - \max(0, \min(a - \bar{a}^7, b)) - \phi(a'/a)$$

Consider now the values of bargaining. Bargaining is over proposed transfers of current resources $\tau$ and debt swaps $b'$. It also involves a choice for assets next period $a'$.

**Household proposes** - Suppose the proposer is the household. The value if the offer is not accepted (i.e., of delaying agreement by making an unacceptable offer) is

$$V^B_{ho,noac}(a, b, s) = \max_{a'} \{ u(c) + \delta E[\gamma_{ho,ho}V^{D,B}_{ho}(a', b, s') + (1 - \gamma_{ho,ho})V^{D,B}_{lo}(a', b(1 - \tau_b), s')] \}, \tag{10}$$

subject to the budget $c + q^0a' = e(s) + a - \phi(a'/a) - \min(\tau_b b, e(s) + a)$. This gives the decision rule for assets $a'_{ho,noac}(a, b, s)$. The value to accepting the offer is

$$V^B_{ho,ac}(a, b, s) = \max_{a', b'} \{ u(c) + \delta EV(a', b', s') \}, \tag{11}$$

subject to the budget $c + q^0a' = e(s) - T(a, b, s, a', b') + a - \phi(a'/a)$ where the transfer $T(.)$ is
such that

\[
T(a, b, s, a', b') + b'q(a', b', s) = \min \{q^0 E[\gamma_{ho, ho} W^{B}_{ho}(a'_{ho, noac}(a, b, s), b(1 - \tau_b), s') + (1 - \gamma_{ho, ho}) W^{B}_{lo}(a'_{ho, noac}(a, b, s), b(1 - \tau_b), s') | s, b]\}
\]  

Expression (12) says that the household must get the bank to accept by offering either full-repayment or the bank’s option value of postponing agreement. This yields the settlement plan consisting of \(a'_{ho, ac}(a, b, s), b'_{ho, ac}(a, b, s)\), and \(\tau_{ho, ac}(a, b, s) = T(a, b, s, a'_{ho, ac}(a, b, s), b'_{ho, ac}(a, b, s))\).

The household’s value of bargaining is thus

\[
V^B_{ho}(a, b, s) = \max \{V^B_{ho, noac}(a, b, s), V^B_{ho, ac}(a, b, s)\}
\]  

Lender proposes - Suppose now the proposer is the lender. If the offered plan is not accepted

\[
V^B_{lo, noac}(a, b, s) = \max_{a'} \{u(c) + \delta E[\gamma_{lo, lo} V^{B}_{lo}(a', b(1 - \tau_b), s') + (1 - \gamma_{lo, lo}) V^B_{ho}(a'_{lo, noac}(a, b, s), b(1 - \tau_b), s')]\}
\]  

with \(c + q^0 a' = e(s) + a - \phi(a'/a) - \min(\tau_b b, e(s) + a)\), which yields the policy rule \(a'_{lo, noac}(a, b, s)\).

Clearly, the household’s acceptance condition must hold as an equality at the plan proposed by the bank. Therefore, the household gets its reservation value \(V^B_{lo, ac}(a, b, s) = V^B_{lo, noac}(a, b, s)\). The household’s value of bargaining is thus

\[
V^B_{lo}(a, b, s) = V^B_{lo, noac}(a, b, s).
\]  

3.2 Banks

A lender can be in a state of defaulted debt from a previously failed negotiation (i.e., the borrower has \(z = 2\) or \(3\)). The value of the bank in such a state depends on the household’s decision rules about bankruptcy, \(p_{7, o}^{B}\). This value is, for \(o = lo, ho\),

\[
W^{B}_{o}(a, b, s) = (1 - p_{7, o}^{B}(a, b, s)) W^{B}_{o}(a, b, s) + p_{7, o}^{B}(a, b, s) W^7(a, s),
\]  

which weights the bank’s values to bargaining, \(W^B_{o}\) for \(o = lo, ho\), and to bankruptcy, \(W^7\). These option values will also enter the lender’s valuation of fresh loans (i.e., when the household has \(z = 0\)). They are determined as follows.
The recovered value in bankruptcy is

\[ W^7(a, b, s) = \begin{cases} 
0 & a \leq \bar{a}^7 \\
\gamma^7 \min(a - \bar{a}^7, b) & a > \bar{a}^7 
\end{cases} \quad (17) \]

The value under bargaining \( W^B_0 \) takes as given the household’s bargaining value \( V^B_{o,noac} \) (when the household proposes), her value of a clear record \( V_{B, noac} \), and the household’s decision rules about no-acceptance savings \( a'_{o, noac} \). Its determination depends on who stands to propose a settlement plan, \( o = lo \) or \( ho \).

**Household proposes** - Consider now the bargaining option when the household proposes. The bank’s value to accepting is

\[ W^B_{ho, ac}(a, b, s) = \tau_{ho, ac}(a, b, s) + b'q(a'_{ho, ac}(a, b, s), b'_{ho, ac}(a, b, s), s), \quad (18) \]

and the value to rejecting is

\[ W^B_{ho, noac}(a, b, s) = q^0 E[\gamma_{ho, ho} W^{D,B}_{ho}(a'_{ho, noac}(a, b, s), b(1 - \tau_b), s') + (1 - \gamma_{ho, ho}) W^{D,B}_{lo}(a'_{ho, noac}(a, b, s), b(1 - \tau_b), s')|s] + \min(\tau_b b, e(s) + a) \quad (19) \]

The bank would get its reservation value unless it exceeds full repayment so that \( W^B_{ho, ac}(a, b, s) = \min[W^B_{ho, noac}(a, b, s), b]. Therefore the value of bargaining to the bank is

\[ W^B_{ho}(a, b, s) = \min[W^B_{ho, noac}(a, b, s), b] \quad (20) \]

**Lender proposes** - Consider finally bargaining when the bank proposes. The values to the bank to having the household accepting is

\[ W^B_{lo, ac}(a, b, s) = \max_{a', b'} T(a, b, s, a', b') + b'q(a', b', s), \quad (21) \]

subject to full repayment being the best the bank can propose

\[ W^B_{lo, ac}(a, b, s) \leq b \quad (22) \]
with the transfer satisfying
\[ T(a, b, s, a', b') = -q^0 a' + e(s) + a - \phi(a'/a) \]
\[ - ((1 - \sigma)(V_{lo,lo,ac}^B(a, b, s) - \beta EV(a', b', s')))^{(1-\sigma)} \] (23)

This last condition (23) says that the bank will make an offer that leaves the household indifferent between accepting and delaying the outcome.\(^{15}\) This yields the settlement decision rules \(a'_{lo,ac}(a, b, s), b'_{lo,ac}(a, b, s)\) and \(\tau_{lo,ac}(a, b, s)\). The value if the offer is not accepted
\[ W_{lo,lo,ac}^B(a, b, s) = q^0 E[\gamma_{lo,lo} W_{lo,lo,ac}^D(a', b', s, b(1-\tau_b), s') + (1 - \gamma_{lo,lo}) W_{ho,ho}^D(a', b', s, b(1-\tau_b), s')|s] + \min(\tau_b, e(s) + a) \] (24)

The bank’s value of bargaining here is therefore
\[ W_{lo}^B(a, b, s) = \max\{W_{lo,lo,ac}^B(a, b, s), W_{lo,ac}^B(a, b, s)\} \] (25)

3.3 Zero profits

With fee-entry competition in the credit market, the prices of debt imply zero ex-ante profits for banks extending loans. The evaluation of cash flows in the coming periods depends on the households default rule \(d\) and the banks ex-ante value in the event of default \(W^D\). Given these, the zero-profit condition for the lender yields the price schedule
\[ q(a', b', s) = (1 - \pi(a', b', s) - \lambda) q^0 + q^0 \sum_{s'} \Gamma_{s,s'} d(a', b', s') W^D(a', b', s')/b' \] (26)

where the probability of default is
\[ \pi(a', b', s) = \sum_{s'} \Gamma_{s,s'} d(a', b', s'). \]

It remains to determine the ex-ante value of the bank under default with a clear record \(W^D\). It depends on the clean household’s decision rules about bankruptcy \(p_{lo,o}^D\) and the bank’s option values of bargaining \(W^B_o\), for \(o = lo, ho\), and of bankruptcy \(W^7\). The ex-ante

\(^{15}\) Furthermore, positive consumption requires the constraint that \(V_{lo,lo,ac}^B(a, b, s) - \beta EV(a', b', s') < 0\) be satisfied.
recovered value when there is default, conditional on the proposer \( o = lo, ho \), is

\[
W_o^D(a,b,s) = p^D_{r,o}(a,b,s)W^T(a,b,s) + (1 - p^D_{r,o}(a,b,s))W_o^B(a,b,s)
\]  

(27)

The ex-ante value is therefore

\[
W^D(a,b,s) = \gamma_{ho}W^D_{ho}(a,b,s) + (1 - \gamma_{ho})W^D_{lo}(a,b,s)
\]  

(28)

### 3.4 Formal definition and computation

The definition of equilibrium requires that the above conditions be satisfied. Therefore finding an equilibrium involves a number of nested fixed-point problems in the above functional equations. More specifically, it is instructive to break the equilibrium down into two operators, a no-bargaining equilibrium and a bargaining equilibrium. A no-bargaining equilibrium takes as given the bargaining values for the households and the banks, and determines the price schedule, and the values and decision rules for repayment, default and Chapter-7 bankruptcy. This is akin to the standard equilibrium in recent consumer bankruptcy models (that is, if with arbitrarily bad outcomes from bargaining). On the other hand, a bargaining equilibrium takes as given the values of a clean credit record, the values under Chapter 7, and prices, and determines the bargaining policy rules and reservation values. An equilibrium is a combination of no-bargaining and bargaining equilibria that are consistent with each other. We leave to Appendix A the more formal statement of the definition.

We establish existence by computing a fixed point of the two operators. Given such an equilibrium, we find the stationary distribution for household over debts, assets and income, as well as on default status, as defined by the states \((a,b,s,z)\). This is done, as standard, through iterations on a measure using the transition implied by the policy functions and the process for the exogenous shocks.

### 4 Calibration

We have to assign values to the model’s parameters. In order to facilitate this task, we will first specify a subset of parameters that are either largely responsible for one observation or part of a broad consensus in the literature. These are shown in table 1. A model’s period corresponds to one year. Risk aversion \( \sigma \) is set to a standard value. The price of assets \( q_0 \)
is set to accord with a 3 per cent risk free interest rate. The intermediation cost $\lambda$ matches existing estimates of 4 per cent. The exemption level for assets under bankruptcy varies widely across U.S. states. The choice for $\bar{a}^7$ is equivalent to exempting assets below 70 per cent of average income.\footnote{Consistent with the average level across states in the U.S. See Hintermaier and Koeniger (2009).} We pick the persistence of the formal default state $\pi_7$ to imply a one year and a half average exclusion from credit after a bout in Chapter 7, which is frequently cited in the literature as under 2 years. Finally we begin with a benchmark calibration which has offers solely suggested by creditors, thus reflecting the widely held view that bargaining power is not evenly divided. This also simplifies considerably the task of reporting outcomes. We have however also studied several cases with alternating proposer and results appear to be largely unaffected.\footnote{Appendix B reports outcomes from a case with even probabilities $\gamma_{ho} = 0.50$.}

\begin{table}
\centering
\begin{tabular}{lll}
\hline
\textbf{Description} & \textbf{Parameter} & \textbf{value} \\
\hline
Risk aversion & $\sigma$ & 2.00 \\
Bank’s cost & $\lambda$ & 0.04 \\
Asset exemption & $\bar{a}^7$ & 0.50 \\
Risk-free interest & $1/q_0 - 1$ & 0.03 \\
Persistence formal default & $\pi_7$ & 0.68 \\
Proposer & $\gamma_{ho}$ & 0.00 \\
\hline
\end{tabular}
\caption{Parameters set directly}
\end{table}

\begin{table}
\centering
\begin{tabular}{lll}
\hline
\textbf{Description} & \textbf{Parameter} & \textbf{value} \\
\hline
Discount & $\beta$ & 0.92 \\
Collection tax & $\tau_b$ & 0.08 \\
Bargaining costs & $\chi_B$ & 0.20 \\
Bankruptcy costs & $\chi_7$ & 0.20 \\
Income realizations & $(e_1, e_2)$ & (0.12, 0.90) \\
Persistence income & $(\Gamma_{1,1}, \Gamma_{2,2})$ & (0.40, 0.80) \\
Adjustment cost & $\phi$ & 0.50 \\
\hline
\end{tabular}
\caption{Endogenous parameters}
\end{table}

The remaining few parameters will be chosen in order to produce outcomes consistent with a limited number of reasonable targets related to observed measures of default, debt, assets and income. The parameters are the collection tax $\tau_b$, the asset adjustment cost $\phi$, the discount rate $\beta$, the utility costs to defaulting via bankruptcy $\chi_7$ and via bargaining $\chi_B$, and the income process. The income process is described by two possible states that
follow a Markov chain, so one needs to specify two values for income realizations and their degree of persistence. This simple process is adopted for ease of exposition as well as to keep computational costs. The parameters will be aiming to produce outcomes consistent with the 2007 wave of the Survey of Consumer Finances, excluding the top 20 per cent net worth quantile.\textsuperscript{18}

In order to provide empirical counterparts of debt and assets, we measure unsecured debt and net worth in the data as follows. We follow the methodology used in the 2007 Survey of Consumer Finance reports which we interpret as providing market values for assets and face values for debt. What is unique to our analysis is the split of net worth into debt and asset levels to conform the division in our model.\textsuperscript{19} As such we consider debt to come from three sources: credit card and store debt, negativity home equity, and the value of autoloans over the value of automobiles attached to those loans.\textsuperscript{20} We consider the remainder of debt to be negative net assets. For consistency, our measure of assets includes positive equity in housing and automobiles.

A remark is also in order regarding the measurement of default in the data and the model. In the data we are able to measure households by their entry into bankruptcy, but not their chapter choice or necessarily their exit from bankruptcy. This corresponds to the flow into bankruptcy in the model which can come from one of two sources: clean households or households in informal default. We can also measure the number of individuals who were more than two months behind on payments scheduled within the last year but did not

<table>
<thead>
<tr>
<th>Description</th>
<th>Model</th>
<th>Data</th>
</tr>
</thead>
<tbody>
<tr>
<td>Wealth/inc</td>
<td>2.01</td>
<td>2.10</td>
</tr>
<tr>
<td>Debt/inc</td>
<td>0.137</td>
<td>0.125</td>
</tr>
<tr>
<td>proportion in debt</td>
<td>0.349</td>
<td>0.32</td>
</tr>
<tr>
<td>Formal default</td>
<td>0.0085</td>
<td>0.011</td>
</tr>
<tr>
<td>Informal default</td>
<td>0.0570</td>
<td>0.055</td>
</tr>
<tr>
<td>Var log inc</td>
<td>0.80</td>
<td>0.80</td>
</tr>
</tbody>
</table>

\textsuperscript{18}Note also that the variance of the noises, $\sigma_{V_B}$ and $\sigma_{D_{B}}$, will need to be set to perform their designed technical function; at 0.05, they do not affect results in any way.

\textsuperscript{19}We discuss the implications of the assumptions behind our construction below and report statistics for key sub groups in Appendix C.

\textsuperscript{20}Our guidance for debt is that it should be unsecured, forgiveable in bankruptcy, and less costly to adjust than our measure for assets. Since our model lacks expense shocks, we also add the additional requirement that the debt not be attached to a medical bill or a business venture. For availability reasons we consider the mortgages on only primary housing and automobiles.
enter bankruptcy. We interpret this question as capturing primarily the flow into informal default.\footnote{One issue with our data is splitting the stock in informal default from the flow into. The data observation we have measures the number of individuals who were more than two months behind on a payment made in the previous year or skipped such a payment altogether. Note that this data does not capture individuals who are more than a year behind on their payments, so it is somewhere between a stock and flow. We report the data as a flow, though we note few of our substantial results would change by reporting it as a stock, because most individuals in informal default are new to informal default in the model. We have a secondary data issue in that we do not know how many of our individuals who report a bankruptcy in a year have had some debt forgiven. We look at this in Appendix C.}

We target the following observations. The unsecured debt-to-income ratio of about 13 per cent, the proportion of household in debt is close to 33 per cent, the asset to income ratio is about 2.1. As for debt defaults, the number of households not current on their debt repayments stands at above 5 per cent, and the portion of defaults that take place under the formal bankruptcy procedures (i.e., Chapter 7) is close to 1 per cent. Finally, the cross sectional variance of the log of income is 0.80 approximately. Table 2 displays the chosen parameters and Table 3 the implied values for the targeted variables.

The choices made yield implications fairly close to the empirical targets. Furthermore, the average interest rate implied by the model is 18 per cent, only a little larger than the measured 14 per cent, but in line with other similar studies. The assumed 8% of debt lost to collection is roughly consistent with the data on collection from the NYFRB.\footnote{The Quarterly Report - Household Debt and Credit, August 2011, reports the evolution of average collection amount per person with collection since 1999 in p15. Since 2007, the figures stand at about 1,400 USD dollars, or 7.5 per cent of average debt. The 8 per cent collection rate over outstanding debt held by informal defaulters assumed in the calibration is close to this number.} The income transition probabilities speak of bad realizations implying large but relatively transitory drops in income, such as those associated with unemployment, which are frequently associated with default.

5 Behaviour and outcomes

We turn to a discussion of the outcomes of the model. In this economy both formal and informal default occur. A first point is that including informal default expands the default set. We show that the model produces differences in the households who choose each type of default: informal defaulters have lower incomes, but higher assets, debts and net worth. We also show that in situations of informal default, settlements may happen quickly but also often after some delay. These negotiated settlements are frequently followed by fur-
ther defaults which, in the end, may only resolve as a bankruptcy filing when individual assets have been drawn down sufficiently. A lasting resolution of default requires individual earnings to improve after settlement. Not only are then settlements ineffective at preventing default but they are also only partially effective in facilitating recovery since they result in heavy haircuts. Negotiations however typically achieve substantial reduction of household liabilities and they are only partly based on new debt. To make these points, we show policy functions for who defaults and how, present summary statistics from the stationary distribution, and report simulated examples.

5.1 Policy functions

Consider first total default, including both the formal and informal modalities. Figure 1 displays the policy rule for default, \( d \), or default set, over asset and debt levels, for the two income levels. As expected, default is monotonically increasing in debt and decreasing in assets. It is also decreasing in income. More interestingly, it can be seen that default can occur for individuals with positive net worth and assets well above the exemption level. This is a marked change from the model without bargaining where default sets have contours narrowly close to the exemption level.

Looking now at the default mix between formal Chapter 7 bankruptcy and informal bargaining, we see the wider default set is due to the possibility of bargaining. Figure 2 shows the fraction of defaulting households that choose filing for bankruptcy instead of bargaining. The preference for formal default shows a sharp drop near the asset exemption level. With assets that can be seized under formal Chapter 7 procedures to pay off a considerable debt outstanding, the option to bargain is more attractive when assets exceed the exemption level.\(^{23}\)

5.2 Description of the stationary distribution

Given these policy functions, we turn to the outcomes produced by the model. Table 4 displays figures regarding the mass and characteristics of households in different states as well as their decisions. Table 5 describes the outcomes of negotiated settlements.

We begin by examining the flows into default. Starting with a clean credit record, the

\(^{23}\)The mapping is however U-shaped in the level of assets, most visibly for high income households. As the U-shape indicates, at a sufficiently large level of assets, the negotiation position of the household weakens and bankruptcy may become a better option. These situations however are not relevant in this economy.
Table 4: **Summary distribution and choices** (non-normalised)

<table>
<thead>
<tr>
<th></th>
<th>mass</th>
<th>debt</th>
<th>assets</th>
<th>borrowing</th>
<th>saving</th>
<th>inc</th>
<th>recov</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Clean credit:</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>All</td>
<td>0.9635</td>
<td>0.097</td>
<td>1.424</td>
<td>0.091</td>
<td>1.477</td>
<td>0.708</td>
<td></td>
</tr>
<tr>
<td>Formal defaulters</td>
<td>0.0085</td>
<td>0.107</td>
<td>0.386</td>
<td>0.000</td>
<td>0.302</td>
<td>0.362</td>
<td></td>
</tr>
<tr>
<td>Informal defaulters</td>
<td>0.0570</td>
<td>0.441</td>
<td>1.059</td>
<td>0.335</td>
<td>0.767</td>
<td>0.182</td>
<td></td>
</tr>
<tr>
<td>Inf. def. who settle</td>
<td>0.0394</td>
<td>0.388</td>
<td>0.919</td>
<td>0.235</td>
<td>0.702</td>
<td>0.210</td>
<td>0.162</td>
</tr>
<tr>
<td>Inf. def. who delay</td>
<td>0.0177</td>
<td>0.558</td>
<td>1.369</td>
<td>0.558</td>
<td>0.911</td>
<td>0.120</td>
<td></td>
</tr>
<tr>
<td>Repeat defaulters</td>
<td>0.0165</td>
<td>0.386</td>
<td>0.664</td>
<td></td>
<td></td>
<td></td>
<td>0.242</td>
</tr>
<tr>
<td><strong>Failed to agree:</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>All</td>
<td>0.0183</td>
<td>0.527</td>
<td>0.911</td>
<td></td>
<td></td>
<td>0.588</td>
<td></td>
</tr>
<tr>
<td>Formal defaulters</td>
<td>0.0000</td>
<td>0.472</td>
<td>0.781</td>
<td>0.000</td>
<td>0.780</td>
<td>0.900</td>
<td></td>
</tr>
<tr>
<td>Informal defaulters</td>
<td>0.0183</td>
<td>0.527</td>
<td>0.911</td>
<td>0.168</td>
<td>0.805</td>
<td>0.5880</td>
<td></td>
</tr>
<tr>
<td>Inf. def. who settle</td>
<td>0.0176</td>
<td>0.522</td>
<td>0.895</td>
<td>0.128</td>
<td>0.802</td>
<td>0.604</td>
<td>0.286</td>
</tr>
<tr>
<td>Inf. def. who delay</td>
<td>0.0006</td>
<td>0.670</td>
<td>1.360</td>
<td>0.670</td>
<td>0.889</td>
<td>0.120</td>
<td></td>
</tr>
<tr>
<td>Repeat defaulters</td>
<td>0.0034</td>
<td>0.270</td>
<td>0.708</td>
<td></td>
<td></td>
<td></td>
<td>0.234</td>
</tr>
</tbody>
</table>

Note: The first column divides households by their default choices. The top panel refers to all individuals who begin the period outside default; the bottom panel looks at all individuals who begin in informal default. Repeat defaulters are new defaulters who have had a settlement in the previous period. The second column contains the proportion of each type in the population. The rest of the columns contain summary statistics about stocks and flows of debts and assets, income, and recovery for those who negotiate and reach a settlement.

Table 5: **Settlements**

<table>
<thead>
<tr>
<th></th>
<th>debt</th>
<th>transfer</th>
<th>new debt</th>
<th>saving</th>
<th>recovered</th>
</tr>
</thead>
<tbody>
<tr>
<td>clean credit history</td>
<td>0.388</td>
<td>0.0138</td>
<td>0.235</td>
<td>0.702</td>
<td>0.162</td>
</tr>
<tr>
<td>failed to agree in past</td>
<td>0.522</td>
<td>0.1973</td>
<td>0.128</td>
<td>0.802</td>
<td>0.286</td>
</tr>
</tbody>
</table>

Note: Statistics about settlement terms for debtors in the top and bottom panel of Table 4, respectively.
households who default predictably have larger debts, lower assets and lower incomes than those who do not default. More interestingly, comparison between different types of defaulters shows that informal defaulters have markedly more debt and assets, higher net worth, and less income, than formal defaulters. For a finer level of detail, the upper section of Figure 3 shows the distribution of the mass of households who actually default, and the bottom part shows the distribution of bargainers only. Note that the asset position of defaulters is increasing in the level of debt outstanding—an indication of the role of assets as (informal) collateral—and hence households failing to repay debt are characterized by either low debts and low assets or higher debts and high assets. It is clear that bargainers make up for the majority of the defaulters with high asset positions, but they are only a small proportion of the defaulters with low asset positions. The low-asset defaulters, particularly those below the bankruptcy exemption level, choose predominantly bankruptcy, whereas the high-asset defaulters opt for bargaining.

There are also differences within the group who choose informal default. In Table 4, about

---

24 The rest of the population's distribution occupies much lower debt positions along the support of asset positions.
two thirds of the households who default informally do accept the lender’s first proposed plan and settle immediately; the rest do not settle and carry on bargaining. Those who settle quickly have lower debts and, especially, fewer assets than those who delay settlement; they also have a higher average income.

About 42 per cent of those who settle today will repeat defaulting next period if income does not improve. They will try to negotiate again or slip into formal bankruptcy if, in the process, they have run their assets sufficiently low.\textsuperscript{25} So settlements are not very effective at preventing default; we will show shortly that a lasting resolution requires an improvement in income.

Looking at the borrowing and saving decisions in Table 4, it is clear that as individuals move through informal default their assets fall, an indication that the process of bargaining is being used to run assets down.\textsuperscript{26} Table 5 shows that settlements involve a sharp

\textsuperscript{25}Repeat defaulters are a strict subset of all defaulters further up in Table 4.

\textsuperscript{26}In effect, if an exogenous condition banning the sale of assets during bargaining were imposed, then bargaining would not occur in equilibrium.
reduction in debt, and a heavy average write off (or haircut) close to 50 per cent. It also shows that 90 per cent of the recovered debts consist of new debt and the rest are transfers.

Consider now the group of households shown in the bottom section of Table 4 who had defaulted informally in the past and have not yet settled. Similar patterns show. Nearly all the households with such a story will seek to negotiate again and settle in the current period. They settle as assets held have been reduced after the failed negotiation. These delayed settlements involve a recovery of about half of outstanding debt, and consist of new debt in only about 30 per cent of their value. However, note again that a considerable fraction, about 20 per cent, of these settlements will be followed by new defaults.²⁷

5.3 Time paths

We now turn to how actually events unfold for typical households. We illustrate here various possible events based on simulations of individual behaviour drawn from the sta-

²⁷Only a tiny fraction of remaining non-settled bargainers file for formal Chapter 7 bankruptcy.
tionary distribution. They are a demonstration of what the typical complex defaults look like in the model. Figure 4 shows various typical situations where in period 0 the household receives a low earnings shock which leads into an informal default decision in that period. It displays corresponding paths for asset holdings, nominal debt, and consumption, to demonstrate how individuals who have accumulated assets may begin the gradual path into Chapter 7 or through to settlement. These defaults demonstrate that successful lasting settlements require income to improve subsequently; otherwise negotiations are used to wait until assets fall close to the exemption and formal default becomes the optimal option.

Panel (a) of Figure 4 illustrates the case of an immediate settlement within the default period 0. The settlement involves some reduction of debt as well as of assets. There is no subsequent default event as earnings improve in the next period. This reflects the general fact that improved earnings is necessary for no further default to happen. Panel (b) is a case with somehow lower assets and debts (but still in the bargaining region in Figure 2 earlier) when defaulting at time 0 and where, unlike the previous case in (a), low earnings persist into the next period and leads to formal default via bankruptcy. Note that this default option is associated with a relatively low level of assets below the exemption level. But chapter 7 is not the only option if bad luck continues following the initial default. Panel (e) shows a situation where, holding a relatively large stock of assets after default at time 0, the household decides to repeat defaulting informally and reaching a settlement in the two subsequent periods while slowly disaving, only to leave default situations when finally earnings improve. This is a case where settlements, even though fragile, delay bankruptcy long enough to avoid it. Note that the settlement lowers debt which delays bankruptcy. However this strategy does not always work. Panel (f) shows that a similar process can end up in formal bankruptcy after two periods of informally defaulting. In this case, the individual has already drawn assets down sufficiently while still receiving the bad earnings shock.

In the examples discussed so far, there is no delay to settlement in informal default. Panels (c) and (d) show instead situations where negotiations at time 0 fail to reach a settlement. Formally, at time zero these households are in the no-settlement region of the state space, holding a large level of assets compared to panel (a). The household reduces their assets

28 As we have seen, however, default can also happen with high earnings. As of time 0, no default event has happened in the previous 5 periods.
29 Income is not displayed, but the direction of changes in consumption tracks the direction of changes in income.
30 This illustrates the point shown in Table 4 that delayers hold more assets and debt.
into the next period, while outstanding debt only falls slightly as some collection is under way. This is followed by a negotiated settlement in the next period. Panel (c) shows the case when low earnings persists and consumption falls slightly, and panel (d) shows the case when earnings improve in the settlement period and consumption increases.

5.4 Special features

Some of the model’s central properties are worth discussing further: Why do Chapter 7 filings happen? Why are there delayed settlements?
Bankruptcy  The first one is the existence in equilibrium of formal Chapter 7 defaults. Because of the exogenous exclusion and the fire-sale costs, formal Chapter 7 is more costly than informal default, yet both occur in equilibrium. The reason for formal default occurring is a failure of commitment. The timing in negotiations implies that the household cannot reconsider their decision within the current period not to file for bankruptcy when the bank proposes a plan to bargain over. Waiting, perhaps to declare bankruptcy tomorrow rather than today, is what determines the threat value of the household. With bargaining power on the side of the creditor, debtors cannot be guaranteed their current Chapter 7 value if they choose to negotiate.

That said, bankruptcy plays a positive role in the benchmark version of the model. While bankruptcy and bargaining are seen as alternative choices for the household, the occurrence of bargaining depends on the existence of the option to declare bankruptcy. Without bankruptcy, creditors could get full repayment in negotiations. Bankruptcy thus prevents the bargaining process from being overly biased in favour of creditors. In effect, if we were to ban bankruptcy, then bargaining would cease to happen in equilibrium since the household would be lacking a credible outside option.31

Delay  We have seen the model produces protracted negotiations. While some protracted negotiations involve a sequence of defaults with repeated quick settlements, many others involve a failure to settle. These are situations where the creditor cannot get much out of an acceptable plan and thus prefers to wait to recover more in future, while keeping current collection efforts.32 Two types of conditions must concur to prevent settlement immediately upon entering bargaining.

One condition is that lending be highly risky thereby making bargaining plans that include new debt too costly. This will be the case when the level of assets is small relative to the level of debt defaulted. Figure 5 displays the schedule of debt prices and shows how having less assets makes it harder to raise resources in the credit market.33 Figure 6 shows that the expected recovery rates conditional on settlement are indeed closely related to this pattern for debt prices. For many households, especially with low income, these can be quite low. It conveys the trade-off that, when delaying, the bank faces between recovering

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31 Appendix B shows that, with random proposer’s identity, some positive informal default survives when formal default is ruled out.
32 Note that with the creditor offering the plan it is down to them to decide whether a plan will be acceptable.
33 Note for households outside the default set, this value is close to one, so that the spread is low. But the debt prices fall very quickly as we move towards positions associated with default.
more on the expectation that the household may have a good income realization later on, and recovering less as the household runs assets down in the meantime.

Figure 5: Debt price schedules

The second condition for delay is that bankruptcy is not an imminent threat so the creditor can afford to wait for his bargaining position to improve later. This condition requires the household asset holdings to be large enough. Otherwise, with little assets, the lender cannot expect its negotiating position to improve later and will be driven to settle in spite of the low recovery rates. In conclusion, the no-settlement situations must be found in an intermediate region of assets with high enough debt.

6 Main results

We first assess the quantitative performance of the model regarding observable household characteristics, and, second, discuss the significance of default for consumption risk sharing and welfare.
6.1 Quantitative evaluation

We want to see if the model’s basic implications find support in the data. The model has implications for the type of households that take either type of default decision and so we focus on the portfolio mix and net worth of households making different default decisions. We have already used the SCF 2007 to distinguish between formal and informal defaulters. As discussed in the calibration section, the definition of late payers corresponds to the notion of informal defaulters in the model. Now we want to draw the financial characteristics of late and timely payers based on the SCF 2007 in terms of portfolio mix and income.

For a meaningful comparison with the model, we have to understand what the figures obtained from the data mean. When interviewed, households reporting a recent bankruptcy filing do also report data on debts, assets and income which may refer to periods preceding and periods following the filing decision to a degree that depends on when exactly bankruptcy was declared. For late payers, the financial data reported do similarly reflect the state at the time of failing payments as well as the resulting financial choices once in
the delinquent state. Since the data does not allow a finer classification, the figures calculated from the data will be compared to model’s figures on assets and debts both as the state when defaulting and as the new positions following the default decision. If the model performs well, the data should be in a range close to these two numbers.

Table 6: Data (SCF) and model (inc-normalised)

<table>
<thead>
<tr>
<th></th>
<th>assets</th>
<th>debt</th>
<th>inc</th>
</tr>
</thead>
<tbody>
<tr>
<td>late payers data</td>
<td>1.10</td>
<td>0.161</td>
<td>0.71</td>
</tr>
<tr>
<td>informal defaulters current</td>
<td>1.50</td>
<td>0.63</td>
<td>0.26</td>
</tr>
<tr>
<td>informal defaulters next</td>
<td>1.09</td>
<td>0.48</td>
<td>0.85</td>
</tr>
<tr>
<td>bankrupt data</td>
<td>0.46</td>
<td>0.110</td>
<td>0.76</td>
</tr>
<tr>
<td>formal defaulters current</td>
<td>0.55</td>
<td>0.15</td>
<td>0.51</td>
</tr>
<tr>
<td>formal defaulters next</td>
<td>0.43</td>
<td>0.00</td>
<td>0.90</td>
</tr>
</tbody>
</table>

Table 6 reproduces figures for measures of assets, debt and income for the two groups of households. In the top section of the table, the data on late payers is to be compared with model’s measures for informal defaulters (i.e., bargainers) of debt and assets both in the current period and in the next period. In the bottom half of the table, data on households with a bankruptcy record is similarly to be compared with model’s implied variables for formal defaulters. We can conclude that the model captures well observed differences across individuals. In the data, like in the model, late payers have more debts and assets than formal defaulters, the difference in assets being more marked than the difference in debts. Also late payers have lower income. Quantitatively, the model produces values of assets close to the data for both types of defaulters. The level of debt for formal defaulters is also perfectly consistent with the data on bankrupt households. The model though overestimates the absolute level of debt of informal defaulters since in the data debt held by late payers is less than twice as large as debt held by bankrupts. In any event, and more importantly, the model produces about the right relative levels of net worth for the two groups. This model therefore does a remarkably good quantitative job given its parsimonious nature and the sparse nature of the exercise. This quantitative performance is robust to alternative reasonable calibration choices.

6.2 Informal vs formal default and risk sharing

We assess here the significance of allowing default in its different forms for economic outcomes. We can take up this question since the model allows to disentangle the role of for-
mal bankruptcy, already studied in the literature, and the role of informal delinquency, new to this study. The main trade off is between direct insurance effects and general equilibrium effects on debt prices and credit constraints. We find that the presence of informal default improves welfare and risk sharing through direct insurance opportunities, in spite of its adverse consequences for the cost of credit. In contrast, the presence of formal default is detrimental to welfare as it worsens access to credit. However the presence of bargaining weakens the case against formal default. We will then argue that distinguishing between the two forms of default is relevant for policy.

To make the main point in the starkest way, we first compare the benchmark economy with another economy where the possibility of informal default is switched off (for example, by making its cost prohibitively large), so there is default only via formal bankruptcy. We will also consider the case where formal bankruptcy is similarly ruled out. Table 7 presents values of variables for these cases as well as for the benchmark. Each column corresponds to a different default regime. The variables included are default rates, wealth to income, debt to income, and average loan price in the stationary equilibrium. It also contains the average level of consumption and the time variance of the log of consumption. Finally, it presents the proportional change in welfare starting from the benchmark economy, with welfare measured in consumption equivalent units.

<table>
<thead>
<tr>
<th>variable</th>
<th>benchmark</th>
<th>no barg</th>
<th>no barg fix q</th>
<th>no bankr fix q</th>
</tr>
</thead>
<tbody>
<tr>
<td>Total $d$</td>
<td>0.0656</td>
<td>0.0106</td>
<td>0.0205</td>
<td>0.0000</td>
</tr>
<tr>
<td>Bankr $d^7$</td>
<td>0.0085</td>
<td>0.0106</td>
<td>0.0205</td>
<td>0.0000</td>
</tr>
<tr>
<td>Barg $d^B$</td>
<td>0.0570</td>
<td>0.0000</td>
<td>0.0000</td>
<td>0.0000</td>
</tr>
<tr>
<td>assets $a$</td>
<td>2.013</td>
<td>2.176</td>
<td>2.031</td>
<td>1.175</td>
</tr>
<tr>
<td>debt $b$</td>
<td>0.137</td>
<td>0.109</td>
<td>0.053</td>
<td>0.204</td>
</tr>
<tr>
<td>prices $q$</td>
<td>0.849</td>
<td>0.848</td>
<td>0.860</td>
<td>0.931</td>
</tr>
<tr>
<td>aver $c_t$</td>
<td>0.7260</td>
<td>0.7286</td>
<td>0.7229</td>
<td>0.7101</td>
</tr>
<tr>
<td>var log $c_t$</td>
<td>0.0753</td>
<td>0.0878</td>
<td>0.0928</td>
<td>0.0886</td>
</tr>
<tr>
<td>welfare %</td>
<td>–</td>
<td>-1.94</td>
<td>-2.78</td>
<td>+ 3.52</td>
</tr>
</tbody>
</table>

Removing bargaining has a substantial negative welfare effect, of nearly 2 per cent. Graphically, Figure 7 shows the percentage fall in welfare by type from eliminating bargaining.

34Like Chatterjee, Corbae, Nakajima, and Ríos-Rull (2007), Athreya, Tam, and Young (2009), Livshits, MacGee, and Tertilt (2007).
35Subsequently, Chatterjee and Gordon (2012) have studied informal default without bargaining, but do not carry out a similar exercise.
The utility levels have decreased uniformly but specially at intermediate asset positions with substantial debt, including the region occupied by those who default informally in the benchmark economy with bargaining. The losses are larger for low-income households.

In looking for an explanation, we turn to the changes in assets, debt, and consumption. Asset accumulation increases by about 7 per cent, and levels of debt have decreased by over 30 per cent. Along the transition, consumption levels do not change noticeably.\footnote{This can be illustrated through simulations not reported here.} Long run average consumption does not decrease and even rises slightly. So the explanation for the fall in welfare must rest on the considerable rise in the volatility of consumption. This is further supported by Figure 8 which allows for comparison of simulated consumption and debt paths in the benchmark economy and in the economy without informal bargaining.

Now, what accounts for this increased consumption volatility? The effect recorded must compound the direct insurance effect of restricting default possibilities, a partial equilib-
Figure 8: Stationary time series of consumption and debt, no bargaining

rium effect, and the general equilibrium effect of the response of the risk-adjusted debt prices. Interest rates generally decline notably, and so credit access improves, when bargaining is not available. To evaluate the importance of these two channels we have calculated in the second column of Table 7 outcomes for an economy without bargaining but with the debt prices of the benchmark economy. The welfare loss with fixed prices is much larger. This demonstrates that removing informal default decreases welfare in spite of the more favorable borrowing conditions. Therefore, the direct effect of limited insurance drives the adverse consequences from banning informal default.

We consider now the experiment that restricts formal default via bankruptcy. Note in Table 7 that, as pointed out in section 5.4, eliminating the option to declare formal bankruptcy also eliminates informal default. Restricting bankruptcy improves welfare to a large degree, by over 3 per cent. Figure 9 shows utility changes over the state space confirming improvements in most situations, with the exception of very high debt positions. Consumption volatility increases substantially, which is also illustrated in the simulation of Figure 10. This is associated with a notable shift in financial choices leading to lower
savings and larger levels of borrowing. Households hold less net worth so that long run average consumption decreases, and they use more borrowing to deal with hardship. The benefits to preventing bankruptcy are related to this change in savings behaviour. The change in environment removes default risk and improves access to credit via lower interest rates. As a consequence, households can smooth consumption by borrowing and the demand for precautionary savings as a buffer stock declines. In the process of disaving, they afford higher levels of consumption in the short run which drives the registered net utility gains. This is illustrated by the simulated consumption paths for the benchmark case and the no-bankruptcy case in Figure 11. In the latter case, this typical household saves much less which buys higher consumption over the early periods of the transition.

Since this experiment eliminates both formal and informal default, it is important to understand to significance of the latter. One way to attempt this is by considering the effect of restricting formal default in an economy without informal default. In Table 7, this can be done by considering the change from the no-bargaining case in the second column to the no-bankruptcy case in the fourth column. In contrast with the benchmark experiment with informal default, when informal default is absent eliminating formal default causes hardly any rise in the volatility of consumption and the drop in welfare is much
sharper. This is a clear indication that the rise in volatility should be attributed mainly to the endogenous elimination of informal default which, for that reason, lowers the gains to restricting formal default.\textsuperscript{37} That in models without bargaining the existence of formal default reduces the ability to insure is a point already made in previous literature like Athreya, Tam, and Young (2009) or Chatterjee and Gordon (2012). Our analysis here indicates that the existence of informal default with bargaining mitigates those negative consequences of bankruptcy.\textsuperscript{38}

In sum, restricting or punishing informal default seems a bad idea as it considerably reduces insurance opportunities. This is in contrast with the effect of eliminating formal default which expands the ability to insure and relieves the need to hold a buffer stock of savings.

Changing the cost parameters more gradually produces results in the same general di-

\textsuperscript{37} Another explicit demonstration is in Appendix B. The example there is one where informal default remains positive even after eliminating formal default. In that situation restricting (informal) default is again detrimental. Having positive default improves welfare.

\textsuperscript{38} In a model of informal default without bargaining, Chatterjee and Gordon (2012) remains silent on this question.
Table 8 contains examples illustrating this point. One exception is some non-monotonicity in the response of consumption volatility which may decline initially as the cost of formal default is raised from its benchmark value. In coherence with our discussion above, this coincides with an informal default rate that has not yet declined too much. In any case, the response of welfare to higher default costs is always monotonic, positive to the bankruptcy cost and negative to the bargaining cost.

Thinking about implications for policy, we could consider now a reduction in the non-pecuniary costs of informal default. There are costs that affect only this form of default. For instance, the Fair Debt Collection Practices Act (FDCPA) 1978 seeks to eliminate abusive practices in the collection of consumer debts. Another case in point is the HAMP programme designed to facilitate negotiations deemed unfeasible due to collective action issues caused by securitisation.

Figure 11: Transitional time series, no bankruptcy.

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39These include contacting consumers by telephone outside of the prescribed hours, failure to cease communication upon request, communicating with consumers in any way (other than litigation) after receiving written notice that said consumer wishes no further communication or refuses to pay the alleged debt, causing a telephone to ring or engaging any person in telephone conversation repeatedly or continuously with intent to annoy, abuse, or harass any person at the called number; communicating with consumers at their place of employment after having been advised that this is unacceptable or prohibited by the employer.
Table 8: The effect of varying default costs

<table>
<thead>
<tr>
<th>$\chi^B$</th>
<th>$\chi'$</th>
<th>$d'$</th>
<th>$d^B$</th>
<th>wealth</th>
<th>debt</th>
<th>indebt</th>
<th>var $c_t$</th>
<th>% welf</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.20</td>
<td>0.20</td>
<td>0.0085</td>
<td>0.0570</td>
<td>2.01</td>
<td>0.137</td>
<td>0.35</td>
<td>0.0753</td>
<td>–</td>
</tr>
<tr>
<td>0.40</td>
<td>0.20</td>
<td>0.0067</td>
<td>0.0507</td>
<td>2.07</td>
<td>0.161</td>
<td>0.38</td>
<td>0.0713</td>
<td>-0.33</td>
</tr>
<tr>
<td>0.60</td>
<td>0.20</td>
<td>0.0060</td>
<td>0.0372</td>
<td>2.12</td>
<td>0.161</td>
<td>0.38</td>
<td>0.0719</td>
<td>-0.69</td>
</tr>
<tr>
<td>1.00</td>
<td>0.20</td>
<td>0.0096</td>
<td>0.0151</td>
<td>2.15</td>
<td>0.130</td>
<td>0.33</td>
<td>0.0773</td>
<td>-1.20</td>
</tr>
<tr>
<td>1.50</td>
<td>0.20</td>
<td>0.0104</td>
<td>0.0045</td>
<td>2.17</td>
<td>0.109</td>
<td>0.30</td>
<td>0.0848</td>
<td>-1.60</td>
</tr>
<tr>
<td>$+\infty$</td>
<td>0.20</td>
<td>0.0106</td>
<td>0.0000</td>
<td>2.17</td>
<td>0.109</td>
<td>0.32</td>
<td>0.0878</td>
<td>-1.94</td>
</tr>
<tr>
<td>0.20</td>
<td>0.40</td>
<td>0.0054</td>
<td>0.0580</td>
<td>1.93</td>
<td>0.165</td>
<td>0.38</td>
<td>0.0732</td>
<td>+0.56</td>
</tr>
<tr>
<td>0.20</td>
<td>1.00</td>
<td>0.0019</td>
<td>0.0574</td>
<td>1.76</td>
<td>0.248</td>
<td>0.47</td>
<td>0.0712</td>
<td>+1.69</td>
</tr>
<tr>
<td>0.20</td>
<td>1.50</td>
<td>0.0001</td>
<td>0.0423</td>
<td>1.59</td>
<td>0.250</td>
<td>0.46</td>
<td>0.0742</td>
<td>+2.40</td>
</tr>
<tr>
<td>0.20</td>
<td>$+\infty$</td>
<td>0.0000</td>
<td>0.0000</td>
<td>1.17</td>
<td>0.204</td>
<td>0.43</td>
<td>0.0886</td>
<td>+3.51</td>
</tr>
</tbody>
</table>

6.3 Informal default, with and without bargaining

In the baseline model, informal default consists of bargaining accompanied by debt collection activities. However, informal default could have been modeled as collection only in a way similar to Chatterjee and Gordon (2012). This subsection demonstrates that adding, as we have done, bargaining is important and may change conclusions in important ways.

We first return to study the effects of switching off the option to bargain. In section 6.2, we had removed all informal default at once, including not only the bargaining option but also the collection option, with results reported in Table 7. What if the collection option is maintained while removing the opportunity to bargain? To inform the conclusions, we also want to have a more flexible collection process. To this effect, we define a level of debt $b^*$ below which collection procedures stop and the household gains a clean credit record (our baseline model has $b^* = 0$). A larger $b^*$ means a less costly collection from the debtor’s perspective.\footnote{These features require a slight adaptation of the model. For the situation with both bargaining and garnishment, when $b < b^*$, a settlement is forced as follows. Having assumed the lender proposes always, in (21) we set $W^B_{lo,ac} = 0$, and the policy functions are determined by the household as in (11) but with no recovery for the lender so (12) becomes $T(.) + b'q(.) = 0$. In order to remove bargaining, when $b \geq b^*$, in (21) we set $W^B_{lo,ac} = -\infty$.}

Table 9 reports outcomes of the experiment that removes bargaining, for different values of the forgiveness threshold debt $b^*$. In the first pair of columns, the bargaining equilibrium corresponds to the baseline of the paper shown also in the first column of Table 7,
but the no-bargaining equilibrium differs from the second column in Table 7 in that here the household still has the option of informal default under collection. When banning bargaining, debt declines, the volatility of consumption rises, and welfare declines, just as in the baseline experiment. Furthermore, quantitatively the changes appear to be larger than the in baseline experiment. The collection option does not mitigate the negative effects of restricting bargaining. On the contrary, it appears that keeping the option to default under collection worsens access to credit and undermines risk sharing and welfare. We also note the sharp drop in informal default, an indication that collection accounts for very little informal default. In the remaining pairs of columns in Table 9 we have repeated this experiment for different $b^*$'s to a similar effect.

The above discussion indicates that collection does not help risk sharing and welfare. We now look directly into the role of collection by considering an economy where bargaining is switched off so, like in the model of Chatterjee and Gordon (2012), informal default consists only of the collection or garnishment option. We now effect an increase in the cost of informal default represented by the parameter $\chi^B$. Table 10 reports results. The first pair of columns corresponds to the case with the baseline parameter values $\tau_b = 0.08$ and $b^* = 0.00$. The increase in the cost of informal default via the collection option registers a reduction in the volatility of consumption and a substantial rise in welfare. This is robust to changes in the cost parameter. The second pair of columns, which repeats the exercise for an economy with a positive $b^*$, shows the same result. For larger collection rate, the third pair of columns shows this result is qualitatively preserved. Thus informal default in the form of collection has adverse welfare effects, contrary to the outcome when informal default involves also negotiations reported earlier in Table 8. The sign of implications for other main variables is also different.

In sum, the two forms of informal default may have very different implications. Infor-
Table 10: **Cost of informal default without bargaining**

<table>
<thead>
<tr>
<th>Cost informal def</th>
<th>$\tau = 0.08, b^* = 0.00$</th>
<th>$\tau = 0.08, b^* = 0.20$</th>
<th>$\tau = 0.10, b^* = 0.20$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Bankruptcy</td>
<td>0.0053</td>
<td>0.0082</td>
<td>0.0138</td>
</tr>
<tr>
<td>Informal default</td>
<td>0.0083</td>
<td>0.0095</td>
<td>0.0084</td>
</tr>
<tr>
<td>wealth $a$</td>
<td>1.826</td>
<td>2.173</td>
<td>1.843</td>
</tr>
<tr>
<td>debt $b$</td>
<td>0.028</td>
<td>0.114</td>
<td>0.021</td>
</tr>
<tr>
<td>var $c_t$</td>
<td>0.1010</td>
<td>0.0827</td>
<td>0.0974</td>
</tr>
<tr>
<td>% welfare</td>
<td>–</td>
<td>+0.844</td>
<td>+1.110</td>
</tr>
</tbody>
</table>

mal default via collection has adverse consequences for access to credit and welfare in the present baseline. The positive consequences of informal default are therefore entirely driven by the possibility of bargaining rather than the collection option.\(^{41}\)

### 7 Two policy experiments

#### 7.1 Exemption

Exemption levels differ across states in the U.S. (see Pavan (2008) and Lefgren and McIntyre (2009)). There has also been Federal legislation - in the form of the bankruptcy reform act 2005 - to limit the number of assets and labor income which can be protected from collection or seizure in bankruptcy.\(^{42}\) We consider the effects of adjusting the exemption level. We illustrate the consequences with an experiment the reduces by 20 percent the amount of assets that the household can protect from seizure under bankruptcy. We report the results from this experiment on various variables in Table 11.

Reducing the level of assets exempt causes an increase in informal default and a reduction in formal default. The proportion of households in debt rises, average levels of debt also increase, and levels of asset holdings decrease. The stationary level of consumption hardly varies but there is a visible reduction in consumption volatility by about 3 per cent.

\(^{41}\)It is true that with a high enough collection rate, the higher recovery rate might plausibly confer collection a positive role, and there could be something like an optimal collection rate. This will ultimately depend on the exogenous specification of the detailed garnishment technology, something well beyond the scope of this paper. The welfare enhancing consequences of the endogenous terms arising from bargaining appear to be robust though.

\(^{42}\)Under the recent Bankruptcy Abuse Prevention and Consumer Protection Act 2005, the homestead exemption, which allows bankruptcy filers in some states to exempt the value of their homes from creditors, is limited in various ways.
As for welfare, tightening the asset exemption leads to an overall welfare gain of 0.54 per cent in equivalent consumption units. Reduced wealth accumulation an lower variance of consumption point to the enhanced opportunities for insurance as the main explanation. Lowering the exemption level increases creditor recovery which makes it easier to borrow. In effect, with a tighter exemption the price of debt increases substantially near the exemption level of assets and, particularly, at positions where debt is moderately risky. Changes in debt prices from lowering the exemption are shown in Figure 12.

![Figure 12: Change in prices, exemption a^7 reduced](image)

On the other hand, however, a tighter exemption must also have a direct negative effect in that formal bankruptcy procedures provide now less insurance to households near the exemption limit, and also weakens generally the outside option for the household in negotiations.

<table>
<thead>
<tr>
<th>Description</th>
<th>d%</th>
<th>d7%</th>
<th>dB%</th>
<th>wealth</th>
<th>debt</th>
<th>indebt</th>
<th>averc_t</th>
<th>varc_t</th>
<th>% welf</th>
</tr>
</thead>
<tbody>
<tr>
<td>benchmark</td>
<td>6.56</td>
<td>0.85</td>
<td>5.70</td>
<td>2.01</td>
<td>0.137</td>
<td>0.35</td>
<td>0.726</td>
<td>0.0753</td>
<td>–</td>
</tr>
<tr>
<td>(\bar{a}^7) down .40</td>
<td>7.14</td>
<td>0.54</td>
<td>6.60</td>
<td>1.94</td>
<td>0.171</td>
<td>0.41</td>
<td>0.727</td>
<td>0.0731</td>
<td>+0.54</td>
</tr>
</tbody>
</table>

Table 11: Reduction in exemption level
Figure 13 shows net percentage utility gains over the state space. Welfare loses happen for households with high debts, and specially for those who get to find themselves above the exemption level as a direct consequence of the policy change. The direct insurance loss dominates here. On the other hand, at the other positions, including those with small debts where most population stand, there are gains. This is due to easier access to credit.

To assess the importance of bargaining for the effect of the exemption, we perform a similar policy experiment on the economy without bargaining. Table 12 reports the main result that tightening the exemption produces a rise of welfare of about 0.27 per cent, much milder than in the economy with bargaining. The notable difference is a flatter response of utility levels as shown in Figure 14. Specifically, the economy without bargaining does not feature the improved utility at low levels of debt typical of the economy with bargaining. The upwards response of prices in this case is similar to that shown in Figure 12 and cannot therefore be the reason for the differences. The explanation is that the economy without bargaining features households resorting less to borrowing in the first place and

\footnote{It may be useful to remind the reader that Figure 12 is drawn over tomorrow’s states whereas Figure 13 is over current states.}
will consequently benefit less from the lower interest rates brought about by a tighter exemption. On the other hand, in this economy households do not have the option to default informally when the lower exemption makes formal default more costly, an option seen to be used in the economy with bargaining in Table 11. Without bargaining option, there is less room to temper the direct loss of ability to insure via default.

The weaker effect in this no-bargaining case is in line with recent work in Pavan (2008) and Athreya (2006) which, having only formal default and no adjustment costs, find that the direct loss-of-insurance effect is strong and results in an at most muted response of welfare to tightening the level of exemptions. Hintermaier and Koeniger (2009) also find that, quantitatively, that type of impact must be small.

Table 12: Reduction in exemption level. No-bargaining case

<table>
<thead>
<tr>
<th>Description</th>
<th>d%</th>
<th>d%</th>
<th>d%</th>
<th>wealth</th>
<th>debt</th>
<th>indebt</th>
<th>averc_t</th>
<th>varc_t</th>
<th>% welf</th>
</tr>
</thead>
<tbody>
<tr>
<td>benchmark</td>
<td>1.06</td>
<td>1.06</td>
<td>—</td>
<td>2.18</td>
<td>0.109</td>
<td>0.32</td>
<td>0.729</td>
<td>0.0878</td>
<td>—</td>
</tr>
<tr>
<td>π^7 down .40</td>
<td>0.83</td>
<td>0.83</td>
<td>—</td>
<td>2.17</td>
<td>0.125</td>
<td>0.34</td>
<td>0.730</td>
<td>0.0872</td>
<td>+0.273</td>
</tr>
</tbody>
</table>

Figure 14: Welfare change in no-bargaining case, exemption $\pi^7$ reduced

Summing up, the existence of informal default mitigates the loss of insurance and ampli-
fies the improvement in access to credit following a tighter credit exemption. Ignoring bargaining will underestimate the benefits to reducing the exemption level in a quantitatively substantial way.

### 7.2 Collection policy

Many individuals face collection efforts outside bankruptcy. We look now at the role of collection policy, an issue which, but for Chatterjee and Gordon (2012), has not received attention in the quantitative theoretical literature.\textsuperscript{44} Proposals to strengthen consumer’s protection against collection are common among consumer advocates as it is ordinarily believed to alleviate bankruptcy and improve household welfare. We consider the effects of reducing collection as represented by $\tau_b$.

Table 13 reports outcomes following a reduction in $\tau_b$ to $0.04$ down from the benchmark $0.08$. Default increases, while the level of assets and indebtedness decrease. The volatility of individual consumption rises notably with a lower level of collection. Overall welfare declines by a sizeable $0.7$ per cent.\textsuperscript{45}

<table>
<thead>
<tr>
<th>Description</th>
<th>$d%$</th>
<th>$d^%$</th>
<th>$d^%$</th>
<th>wealth</th>
<th>debt</th>
<th>indebt</th>
<th>aver$c_t$</th>
<th>var$c_t$</th>
<th>% welf</th>
</tr>
</thead>
<tbody>
<tr>
<td>benchmark</td>
<td>6.56</td>
<td>0.85</td>
<td>5.70</td>
<td>2.01</td>
<td>0.137</td>
<td>0.35</td>
<td>0.726</td>
<td>0.0753</td>
<td>–</td>
</tr>
<tr>
<td>$\tau_b$ down .04</td>
<td>7.36</td>
<td>0.99</td>
<td>6.36</td>
<td>1.91</td>
<td>0.071</td>
<td>0.24</td>
<td>0.723</td>
<td>0.0822</td>
<td>-0.71</td>
</tr>
</tbody>
</table>

An expected effect of this policy is an increase in the ability of consumers to use informal default as insurance as fewer of their resources are seized to repay creditors while in informal default. This effect can be seen by examining the percentage change in utility levels displayed in Figure 15. Utility gains occur for highly indebted agents with considerable assets who, we know, are most likely to choose bargaining as the default option. The reduced level of asset accumulation reflects the lower demand for precautionary savings given this insurance effect.

On the other hand, however, limiting collection reduces access to debt. The reason for these changes follows arguments presented previously. The credit constraint effect is sharp in this case because default into informal default leads to lower repayments. This effect

\textsuperscript{44}In empirical work, however, Lefgren and McIntyre (2009) argue that collection policy is an important determinant of bankruptcy rates.

\textsuperscript{45}Chatterjee and Gordon (2012) consider a zero-tolerance case but on an economy with formal bankruptcy switched off. Although not directly comparable, it seems to point in the same general direction.
is visible through examining the change in debt prices in Figure 16 which decrease when collection is reduced. This explains the state positions where Figure 15 shows declines in welfare, including the region of low debts occupied by the majority of the household population.

The net aggregate welfare depends on the relationship between these two effects. Aggregate welfare is lowered in this experiment which is especially true of consumers whose financial health keeps them far away from default. However the welfare response is not uniform as delinquent consumers would benefit from this policy change. These are the consumers who benefit most from the improvement in insurance.\(^{46}\)

\(^{46}\)We note that this negative welfare effect from limiting collection is the opposite of the positive welfare effect from limiting the deadweight losses associated with informal default discussed in section 5.2. This is no contradiction since this is a change in a transfer against creditors who do therefore restrict credit. The reason is that collection is a source of repayment whose presence loosens credit constraints whereas deadweight losses are not a form of repayment. A distinction therefore worth making is between the efficiency in the collection process (welfare improving) and the possibly abusive practices in pursuing it.
Figure 16: Change in prices, collection $\tau_b$ reduced

8 Concluding remarks

In this paper we have taken a broader view of the process of personal default by introducing the possibility of informal negotiations outside of formal procedures. Default emerges as a process often featuring recurring attempts to negotiate which may eventually lead to formal bankruptcy. This fits existing descriptions of defaults as messy events. The model seems also to be quantitatively sound in portraying the observable financial profiles of formal and informal defaulters.

In this context, facilitating negotiations enhances risk-sharing opportunities and improves welfare, a result which is interesting since there are policies that specifically affect the cost of informal default for households. Limiting debt collection, another novel policy dimension that this paper can tackle, tends to be detrimental overall. The findings also indicate that results from standard models of default that ignore the flexibility inherent in informal default need to be reconsidered, including the quantitative impact of exemption policies and the bankruptcy code.

The version of the model used is deliberately parsimonious and has the virtue of keeping
the focus on the new core issues under study. Incorporating richer institutional detail and shock structures will be feasible. Interesting extensions of the analysis will involve, for example, an explicit role for housing and durable consumption, and secured credit. The type of questions we have begun to explore here thus deserve further scrutiny in that context. New questions in our agenda include the study of mortgage renegotiation programmes. The insurance implications of debt negotiation may also be relevant for the active line of research on household consumption inequality.
References


A Definition of equilibrium

Define first an equilibrium for given bargaining outcomes. Given the bargaining values for the household $V_{B}^{h_{o}}$ and $V_{B}^{l_{o}}$, and for the bank $W_{B}^{h_{o}}$ and $W_{B}^{l_{o}}$, a bargaining-conditional equilibrium is a price schedule, value functions and policy rules such that:

(i) Given the value of bankruptcy for the household $V^{7}$, the value of defaulting $V^{D}$ and the probabilities of bankruptcy, $p_{D,7}^{D, l_{o}}$ and $p_{D,7}^{D, h_{o}}$, solve (4) to (3).

(ii) Given the ex-ante value $V$ and the price schedule $q$, the value of repaying $V^{R}$ solves (8).

(iii) Given the ex-ante value $V$, the value of bankruptcy $V^{7}$ solves (9).

(iv) Given the values to repaying $V^{R}$ and defaulting $V^{D}$, the ex-ante value $V$ and default probabilities $d$ solve (2) and (1).

(v) The bank’s value under bankruptcy $W^{7}$ solves (17).

(vi) Given the probabilities of bankruptcy, $p_{D,7}^{D, l_{o}}$ and $p_{D,7}^{D, h_{o}}$, and $W^{7}$, the bank’s value to defaulting $W_{l_{o}}^{D}$, $W_{h_{o}}^{D}$ and $W^{D}$ solve (27) and (28).

(vii) Given default probabilities $d$ and bank’s default value $W^{D}$, the price schedule $q$ solves (26).

Given $q$, (i)-(vi) defines a fixed point problem in $V$, $V^{7}$, $V^{D}$, $V^{R}$, $W^{7}$ and $W^{D}$ and the associated decision rules. Adding (vii) results in another fixed point problem in $q$. These are for given bargaining values $V_{0}^{B}$ and $W_{0}^{B}$.

A bargaining equilibrium. Given non-bargaining functions $V$, $V^{7}$, $W^{7}$, and prices $q$ a bargaining equilibrium are values and decisions functions such that:

(i) Given the values of continuing bargaining for the household $V_{B}^{l_{o}}$ and $V_{B}^{h_{o}}$, the probabilities of filing for bankruptcy or bargain and the values, $p_{D,7}^{D, l_{o}}$, $p_{D,7}^{D, h_{o}}$, $V_{l_{o}}^{D}$, $V_{h_{o}}^{D}$, solve (6), (7).
(ii) Given the the values to continue bargaining for the household, \( V_{D,B}^{lo} \) and \( V_{D,B}^{ho} \), and for the bank, \( W_{D,B}^{lo} \) and \( W_{D,B}^{ho} \), the bargaining value for the household when it proposes, \( V_{ho}^{B} \), solves (10) to (13).

(iii) Given the the values to continue bargaining for the household, \( V_{D,B}^{lo} \) and \( V_{D,B}^{ho} \), the bargaining value for the household when the bank proposes, \( V_{ho}^{B} \) and \( V_{ho}^{B,noac} \), solves (14) to (15).

(iv) Given the probabilities of filing for bankruptcy or bargain and bank values, \( p_{D,B}^{7,lo} \), \( p_{D,B}^{7,ho} \), and \( W_{ho}^{B} \), the values for the bank of being in default and bargaining \( W_{D,B}^{lo} \) and \( W_{D,B}^{ho} \) solve (16).

(v) Given the values for the bank of being in default and bargaining \( W_{D,B}^{lo} \) and \( W_{D,B}^{ho} \) and the household’s behaviour if its proposal is not accepted \( V_{ho}^{B,noac} \), the banks value of entering bargaining when the household proposes \( W_{B}^{B} \), solves (19) and (20).

(vi) Given the values for the bank of being in default and bargaining \( W_{D,B}^{lo} \) and \( W_{D,B}^{ho} \) and the household’s behaviour under \( V_{ho}^{B,noac} \), the banks value of entering bargaining when the banks proposes \( W_{B}^{B} \), solves (21), (22), (23), (24), and (25).

A bargaining equilibrium if a fixed point in \( V_{ho}^{B} \) and \( W_{ho}^{B} \). It takes as given non-bargaining values \( V \), \( V^{7} \), and \( W^{7} \), and \( q \). An equilibrium is functions \( V_{ho}^{B} \), \( W_{ho}^{B} \), \( V \), \( V^{7} \), \( W^{7} \) and \( q \) that satisfy both a bargaining equilibrium and a no-bargaining equilibrium. Finding an equilibrium involves solving a number of nested fixed point problems.

### B Random proposer

In the analysis of this paper, it is assumed only the lender proposes plans, so \( \gamma_{ho} = 0 \). This appendix shows a case where both parties make proposals to illustrate the fact that the aspect of the model does not appear to be critical for the results. Specifically, assume that \( \gamma_{ho} = 0.50 \) so lenders and households stand to propose with the same probability. This change implies a large rise in default rates, so the costs of default \( \chi_{7} \) and \( \chi_{B} \) are increased from 0.20 to 0.40 so that the model matches the rate of informal default. In order to keep the exercise as transparent as possible, the remaining parameters are as in Tables 1 and 2. The following Table 14 reproduces the main aggregates for the paper’s benchmark with \( \gamma_{ho} = 0.0 \) and for present example with \( \gamma_{ho} = 0.5

<table>
<thead>
<tr>
<th>Description</th>
<th>Model ( \gamma_{ho} = 0.00 )</th>
<th>Model ( \gamma_{ho} = 0.50 )</th>
</tr>
</thead>
<tbody>
<tr>
<td>Wealth/inc</td>
<td>2.01</td>
<td>1.95</td>
</tr>
<tr>
<td>Debt/inc</td>
<td>0.137</td>
<td>0.121</td>
</tr>
<tr>
<td>proportion in debt</td>
<td>0.349</td>
<td>0.327</td>
</tr>
<tr>
<td>Formal default</td>
<td>0.0085</td>
<td>0.0057</td>
</tr>
<tr>
<td>Informal default</td>
<td>0.0570</td>
<td>0.0551</td>
</tr>
<tr>
<td>Interest rate %</td>
<td>17.8</td>
<td>18.2</td>
</tr>
</tbody>
</table>

This economy displays similar qualitative and quantitative properties as for the benchmark reported in Table 4. In this case, however, there is more detailed information that has to be supplied since there is a larger set of states associated with the identity of the proposer, \( ho \) for household or \( lo \) for lender. The figures are in Table 15 and 16.
Table 15: Summary distribution and choices with random proposer (non-normalised)

<table>
<thead>
<tr>
<th>Clean credit:</th>
<th>mass</th>
<th>debt</th>
<th>assets</th>
<th>borrowing</th>
<th>saving</th>
<th>inc</th>
</tr>
</thead>
<tbody>
<tr>
<td>All</td>
<td>0.9695</td>
<td>0.085</td>
<td>1.376</td>
<td>0.078</td>
<td>1.424</td>
<td>0.708</td>
</tr>
<tr>
<td>Formal defaulters</td>
<td>0.0057</td>
<td>0.145</td>
<td>0.288</td>
<td>0.000</td>
<td>0.161</td>
<td>0.226</td>
</tr>
<tr>
<td>Informal defaulters</td>
<td>0.0551</td>
<td>0.401</td>
<td>0.982</td>
<td>0.333</td>
<td>0.708</td>
<td>0.160</td>
</tr>
<tr>
<td>Inf. def. who settle with ho</td>
<td>0.0256</td>
<td>0.386</td>
<td>0.949</td>
<td>0.312</td>
<td>0.764</td>
<td>0.174</td>
</tr>
<tr>
<td>Inf. def. who settle with lo</td>
<td>0.0124</td>
<td>0.365</td>
<td>0.860</td>
<td>0.218</td>
<td>0.600</td>
<td>0.187</td>
</tr>
<tr>
<td>Inf. def. who delay with ho</td>
<td>0.0047</td>
<td>0.326</td>
<td>0.746</td>
<td>0.326</td>
<td>0.401</td>
<td>0.120</td>
</tr>
<tr>
<td>Inf. def. who delay with lo</td>
<td>0.0123</td>
<td>0.496</td>
<td>1.264</td>
<td>0.496</td>
<td>0.819</td>
<td>0.120</td>
</tr>
<tr>
<td>Repeat defaulters all</td>
<td>0.232</td>
<td>0.564</td>
<td></td>
<td></td>
<td></td>
<td>0.182</td>
</tr>
<tr>
<td>Repeat defaulters with ho</td>
<td>0.0103</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Repeat defaulters with lo</td>
<td>0.0049</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Failed to agree:

| All                                    | 0.0064 | 0.403 | 0.706 | 0.000 | 0.502 |
| Formal defaulters                      | 0.0001 | 0.384 | 0.570 | 0.093 | 0.607 |

Household offers

| Informal defaulters                    | 0.0064 | 0.278 | 0.486 |        | 0.618 |
| Inf. def. who settle                  | 0.0049 | 0.301 | 0.399 | 0.049 | 0.425 | 0.628 |
| Inf. def. who delay                   | 0.0015 | 0.200 | 0.775 | 0.200 | 0.763 | 0.582 |
| Repeat defaulters                     | 0.0001 | 0.169 | 0.310 |        | 0.161 |

Lender offers

| Informal defaulters                    | 0.0126 | 0.466 | 0.818 |        | 0.586 |
| Inf. def. who settle                  | 0.0123 | 0.462 | 0.806 | 0.089 | 0.655 | 0.600 |
| Inf. def. who delay                   | 0.0004 | 0.579 | 1.242 | 0.579 | 0.782 | 0.120 |
| Repeat defaulters                     | 0.0022 | 0.202 | 0.503 |        | 0.176 |

Table 16: Settlements with random proposer

<table>
<thead>
<tr>
<th></th>
<th>debt</th>
<th>transfer</th>
<th>new debt</th>
<th>saving</th>
<th>recovered</th>
</tr>
</thead>
<tbody>
<tr>
<td>clean credit history ho</td>
<td>0.386</td>
<td>-0.0903</td>
<td>0.312</td>
<td>0.764</td>
<td>0.119</td>
</tr>
<tr>
<td>clean credit history lo</td>
<td>0.365</td>
<td>0.0224</td>
<td>0.218</td>
<td>0.601</td>
<td>0.161</td>
</tr>
<tr>
<td>failed to agree in past ho</td>
<td>0.309</td>
<td>0.0675</td>
<td>0.039</td>
<td>0.405</td>
<td>0.086</td>
</tr>
<tr>
<td>failed to agree in past lo</td>
<td>0.462</td>
<td>0.2201</td>
<td>0.089</td>
<td>0.655</td>
<td>0.279</td>
</tr>
</tbody>
</table>
Comparative results are also very similar to those reported in the benchmark. The following Table 17 summarizes some of the findings. This is to be compared with Table 7 for the benchmark economy in the main text. Note there is now some informal bargaining even when bankruptcy is banned; this is associated with the household proposing. The last line illustrates that, in this case, restricting informal default continues to be suboptimal (it reduces the welfare gain over the initial situation).

Table 17: Findings with random proposer

<table>
<thead>
<tr>
<th></th>
<th>$d^I$</th>
<th>$d^B$</th>
<th>wealth</th>
<th>debt</th>
<th>in debt</th>
<th>var $c_t$</th>
<th>% welf</th>
</tr>
</thead>
<tbody>
<tr>
<td>Initial</td>
<td>0.0057</td>
<td>0.0551</td>
<td>1.95</td>
<td>0.121</td>
<td>0.33</td>
<td>0.0726</td>
<td>–</td>
</tr>
<tr>
<td>no bargaining</td>
<td>0.0098</td>
<td>0.0000</td>
<td>2.06</td>
<td>0.107</td>
<td>0.31</td>
<td>0.0872</td>
<td>-1.12</td>
</tr>
<tr>
<td>no bankruptcy</td>
<td>0.0000</td>
<td>0.0107</td>
<td>1.14</td>
<td>0.196</td>
<td>0.43</td>
<td>0.0818</td>
<td>+3.85</td>
</tr>
<tr>
<td>no bankr + high $\chi^B = 2.0$</td>
<td>0.0000</td>
<td>0.0024</td>
<td>1.22</td>
<td>0.196</td>
<td>0.43</td>
<td>0.0835</td>
<td>+3.71</td>
</tr>
</tbody>
</table>

C Data

In this brief appendix we walk you through the data and the decisions we made to split data into debt and assets. There are two general issues which we have to deal with in terms of measurement. First though our data record entry into a given chapter of default, they don’t record exit and it is hard to tell whether a household has had debt forgiven in formal bankruptcy. Second there is the issue of measuring debt to correspond to the measure of debt in the model. There is a well defined protocol for describing net worth in the data. However splitting net worth up into debt and assets is a model specific challenge. We have a baseline measure of debt which we report in the paper. Here we show its construction. We also show that the relationships between debt between the two chapters of default are constant for other plausible measures of debt. Finally in the paper we handled the issues about debt forgiveness by reporting debt before and after entering bankruptcy. But it is also possible to produce a subsample of households whose portfolios strongly suggest that they have received a clean start in bankruptcy. We show that our data measures our robust to this measurement.

So let us begin with principles. Debt in our model is unsecured, so we begin by eliminating secured debt. Second our model lacks expense or preference shocks which are the common tools for modeling medical debt and legal bills. This is actually an important consideration as our sample of bankrupt households has a higher measure of medical/legal debt than our delinquent measure. We also want to avoid considering educational debt which is non-dischargeable in bankruptcy, so we avoid this debt as well. In terms of debt we produce three samples of debt. We count consumer debt and credit card debt as a clear base. We also consider the portion of a consumer’s mortgage in excess of the value of the home and the value of the car payment in excess of the value of the car. In our baseline model of debt we exclude loans for other properties because we have no way of knowing the value of the assets they may be secured against. In our alternative measure of debt we add in the values of these loans (Residential Debt) and ignore any potential security interest which may offset them. We report data for the entire population and for two subsets of the population. First we consider individuals who carry positive credit card debt. This is a good candidate for lacking a fresh start since this type of debt is frequently forgiven. Second we consider households whose debts are greater than the excess value of the mortgage which are also a good candidate for lacking a fresh start, since mortgage debt is non-dischargeable. We report all of this data in the following table.
<table>
<thead>
<tr>
<th>Households</th>
<th>Baseline Debt Measure</th>
<th>Expanded Debt Measure</th>
<th>Baseline Assets Measure</th>
<th>Expanded Assets Measure</th>
<th>Net Worth</th>
</tr>
</thead>
<tbody>
<tr>
<td>All Households</td>
<td>0.125</td>
<td>0.171</td>
<td>2.10</td>
<td>2.24</td>
<td>1.98</td>
</tr>
<tr>
<td>Households Behind</td>
<td>0.161</td>
<td>0.197</td>
<td>1.10</td>
<td>1.06</td>
<td>0.86</td>
</tr>
<tr>
<td>Households in Bankruptcy</td>
<td>0.112</td>
<td>0.157</td>
<td>0.459</td>
<td>0.505</td>
<td>0.35</td>
</tr>
<tr>
<td>Households with Credit Card Balances</td>
<td>0.188</td>
<td>0.242</td>
<td>2.24</td>
<td>2.30</td>
<td>2.07</td>
</tr>
<tr>
<td>Behind Households w. Balances</td>
<td>0.251</td>
<td>0.296</td>
<td>1.34</td>
<td>1.38</td>
<td>1.09</td>
</tr>
<tr>
<td>Bankrupt Households w. Balances</td>
<td>0.185</td>
<td>0.292</td>
<td>-0.03</td>
<td>0.08</td>
<td>-0.22</td>
</tr>
<tr>
<td>Household with Debt in Excess of Mortgage</td>
<td>0.220</td>
<td>0.278</td>
<td>2.25</td>
<td>2.51</td>
<td>2.23</td>
</tr>
<tr>
<td>Behind Household with Excess Debt</td>
<td>0.169</td>
<td>0.207</td>
<td>0.98</td>
<td>1.02</td>
<td>0.82</td>
</tr>
<tr>
<td>Bankrupt Households with Excess Debt</td>
<td>0.136</td>
<td>0.191</td>
<td>0.191</td>
<td>0.248</td>
<td>0.06</td>
</tr>
</tbody>
</table>