Bargaining as a Struggle Between Competing Attempts at Commitment *

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Abstract

The strategic importance of commitment in bargaining is widely acknowledged. Yet disentangling its role from key features of canonical models, such as proposal power and reputational concerns, is difficult. This paper introduces a model of bargaining with strategic commitment at its core. Following Schelling (1956), commitment ability stems from the costly nature of concession and is endogenously determined by players' demands. Agreement is immediate for familiar bargainers, modelled via renegotiation-proofness. The unique prediction at the high concession cost limit provides a strategic foundation for the Kalai bargaining solution. Equilibria with delay feature a form of gradualism in demands.

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1 Introduction

If two agents seek to divide some surplus, what division will they agree on and when and how? This set of questions, that I collectively label the bargaining problem, is key to a vast range of economic interactions. Economic models rely on the strategic theory of bargaining to resolve it, either directly or indirectly by informing the appropriate choice of a bargaining solution.

Strategic models of bargaining that allow negotiations to unfold over time typically have at their core either the alternating-offers model of Rubinstein (1982) or the reputation model of Abreu and Gul (2000). Schelling (1956, 1960) proposed a third approach. As summarized in Crawford (1982), Schelling views the bargaining process as a struggle between players to commit themselves to —that is, to convince their opponent of their inability to retreat from —advantageous bargaining positions. Schelling's own treatment of his approach was impressionistic and by way of examples. Subsequent work has either developed the theory in static environments or focused on evaluating the role of commitment while relying on one of the two canonical models mentioned above to resolve the underlying bargaining problem.[1]

This paper presents a formalization of Schelling's theory with an infinite-horizon model of bargaining with complete information. The objective is to characterize the extent to which this theory, built on the use of strategic commitments, resolves the bargaining problem and how, and furthermore establish conditions under which the model's predictions are adequately summarized by some bargaining solution.

The model builds on two key elements of Schelling's theory. First, a bargainer may find it costly to back down from a stated demand and this is the source of her commitment ability. Second, the commitment ability is nevertheless endogenous, in that it depends on the demands. A less aggressive demand weakens the opponent's commitment ability by allowing more room for her to

[1]See for example, Crawford (1982), Muthoo (1996), Ellingsen and Miettinen (2008) and Dutta (2012) for the first and Fershtman and Seidmann (1993), Compte and Jehiel (2004), Wolitzky (2012) and Basak and Deb (2020) for the second. Ellingsen and Miettinen (2014) consider a dynamic model of a hybrid nature that I discuss in detail in section 5.

back down. By contrast, a demand that leaves an opponent's back against the wall only ensures the latter's commitment.

In the model, the bargainers simultaneously announce demands. If the demands are compatible, bargaining ends on those terms. If incompatible, the players decide whether to stick to their demand or concede to the opponent's offer. Concession incurs an additional cost which is increasing in the conceded amount. If neither player concedes, then the current period of bargaining ends and the next period begins with a fresh round of demands. The game proceeds in this manner until either compatible demands or a concession following incompatible demands. The bargainers are impatient, as captured by constant discount factors. I focus on subgame perfect equilibria with pure strategies in the demand stage (henceforth SPE).

The model can be seen as a variant of the infinite horizon version of the Nash Demand Game (henceforth IH-NDG). While in the latter, incompatible demands end the current round of bargaining, in the present model bargainers get a chance to concede. Indeed, if the concession costs are made arbitrarily high, then concession is effectively ruled out and the IH-NDG obtains at the limit.

The model predictions depend on two sets of parameters, namely the discount factors and concession cost functions. In any SPE outcome, the bargainers eventually agree upon an efficient division of the surplus, following some delay, if any. In contrast to common dynamic bargaining models, the range of efficient divisions of the surplus that can arise in equilibrium is linked to the maximum delay the equilibrium accommodates following any history. Delay, while permitted under SPE, has an upper bound.

Renegotiation-proof SPE, used to model familiar bargainers, feature no delay and an exact characterization obtains for the corresponding set of surplus divisions. This leads to a key finding of the paper. As the marginal concession costs are made arbitrarily high, the set of renegotiation-proof SPE outcomes converges to selecting a unique efficient outcome in the limiting IH-NDG. This outcome is identical to that of the Kalai bargaining solution (see Kalai (1977)) with its proportion determined by the discount factors and a limit ratio of the concession cost functions. Therefore, not only does the formalization of Schelling's theory fully resolve the bargaining problem, it also provides a strategic foundation for the Kalai bargaining solution. Furthermore the parameters of the non-cooperative model select the appropriate bargaining solution from the family of solutions characterized in Kalai (1977).

Markov perfect equilibria (which may violate renegotiation-proofness) can exhibit delay. In a natural way, such equilibria with delay yield a form of gradualism, the feature in which bargainers start with extreme demands that soften over time. Finally (and surprisingly), the set of stationary Markov perfect equilibrium *outcomes* coincides with the set of renegotiation-proof SPE outcomes, despite the latter allowing arbitrarily history dependent strategies.

As Binmore, Osborne and Rubinstein (1992) states, *The ultimate aim of what is now called the "Nash program" (see Nash 1953) is to classify the various institutional frameworks within which negotiation takes place and to provide a suitable "bargaining solution" for each class.* This paper contributes to this literature by making a case for the Kalai bargaining solution in environments in which commitment ability due to concession costs is salient.[2] Binmore, Rubinstein and Wolinsky (1986) establish a robust connection between the alternating-offers model and the Nash bargaining solution. Studies on commitment that rely on the alternating-offers model, such as Muthoo (1996), find similar support for the (asymmetric) Nash bargaining solution. Relying on the struggle to commit itself to resolve the bargaining problem, as the current paper shows, leads instead to the Kalai bargaining solution. This is an important distinction. The appropriate choice of a bargaining solution is not merely a game-theoretic curiosity. Aruoba, Rocheteau and Waller (2007), for instance, show that the choice of bargaining solution matters both qualitatively and quantitatively for questions of first-order importance in monetary economics.

To the best of my knowledge, Dutta (2012) and Hu and Rocheteau (2020) are the only other papers that provide strategic bargaining foundations for the Kalai bargaining solution. Hu and Rocheteau (2020) rely on the alternating-

[2]Examples of such environments are described in section 5.1.

offers model. They show that if the surplus is divided into N parts and in each of N rounds players engage in Rubinstein bargaining over one of these parts, then the outcome corresponds to the Kalai bargaining solution as N tends to infinity. While theoretically insightful, the procedure with large N is difficult to descriptively align with typical bargaining narratives.

Dutta (2012) is the static (one-period) version of the current model and captures a qualitatively similar role for the concession costs, in that higher costs benefit the bargainer. It shares the unrealistic feature of the Nash demand game in ruling out future negotiations following a single round of disagreement, and as a result has no role for discount factors.

Given the limit uniqueness result of the static model in Dutta (2012), it is natural to expect (with some work) a similar result in the dynamic model under stationary strategies. A novel and surprising finding in the current paper is that the Kalai solution arises as the unique limit even under the assumption of renegotiation-proofness, which allows for arbitrarily history dependent strategies.[3] There is no reason to expect renegotiation-proof outcomes to coincide with stationary ones in dynamic bargaining games. Indeed, as discussed in 3.1, the acute multiplicity (of surplus division outcomes) in the multilateral version of the Rubinstein bargaining game persists unabated under the assumption of renegotiation-poofness, while stationarity delivers a unique result. The finding that the two sets of outcomes coincide in the current model is the result of the specific structure of its SPE, discussed in section 2.1.

The rest of the paper is as follows. In section 2, I introduce the general model and show how all SPE have a simple structure. In section 3, I focus on a linear specification, which allows for closed form characterizations of outcomes under SPE and those with the further restrictions of renegotiation-proofness and Markov perfection. In section 4, I return to the general model, characterize SPE outcomes categorized by maximum permissible delay, including the set of renegotiation-proof SPE outcomes and establish the link with the Kalai solution. A discussion of the intuition and some applications follow. In section 5, I discuss some key features of the model and other related literature.

[3] The intuition behind this is discussed in section 4.2.

2 The Model

Two players, 1 and 2, play an infinite horizon game to split a pie of size 1. In period $t \in \mathbb{N} \equiv \{1, 2, 3, \ldots\}$, if the bargaining problem is still unresolved, each player $i \in \{1, 2\}$ announces a demand $z_i \in [0, 1]$. The announcements are simultaneous. For a given demand profile $z = (z_1, z_2)$, let $d(z) = z_1 + z_2 - 1$. If the demands are compatible ($d(z) \leq 0$) then the game ends with both players receiving their own demands. The resulting payoff profile is $(u_1(z_1), u_2(z_2))$, where u_i is the payoff function for player i.

Following incompatible demands ($d(z) > 0$), the bargainers enter a concession stage. Here the players simultaneously decide whether to stick to their demands or back down and accept the other's offer. Backing down comes at a cost which is a function of the conceded amount, the difference between the initial demand and the accepted amount, $z_i - (1 - z_{-i}) = d(z)$, and is captured by the *concession cost function* c_i. If both players stick to their demand then the bargaining problem remains unresolved and moves to the next period. This concession stage game is represented in the table below.

Table 1: Concession Stage following Incompatible Demand Profile z

	Accept (A)	*Stick (S)*
A	$u_1(1 - z_2) - c_1(d(z)), u_2(1 - z_1) - c_2(d(z))$	$u_1(1 - z_2) - c_1(d(z)), u_2(z_2)$
S	$u_1(z_1), u_2(1 - z_1) - c_2(d(z))$	$u_1(0), u_2(0)$

As long as some player chooses A the game ends this period with the associated payoffs in the table, otherwise it moves to period $t + 1$.[4] The following assumptions hold throughout the paper.

Assumption 1 *For $i \in \{1, 2\}$, $u_i : [0, 1] \to \mathbb{R}_+$ is a strictly increasing, concave and continuously differentiable function with $u_i(0) = 0$.*

Assumption 2 *For $i \in \{1, 2\}$, $c_i : \mathbb{R}_+ \to \mathbb{R}_+$ is a strictly increasing, unbounded above and continuously differentiable function with $c_i(0) = 0$.*

[4] Following AA in the concession stage, $d(z)$ is left on the table. Alternative specifications of this outcome that additionally split $d(z)$ between the bargainers in some way leave all results unchanged.

A history of play that leads to the beginning of period $t+1$ with $t \in \mathbb{N}$, denoted as h^t, is a sequence of t incompatible demand profiles with (S,S) in the corresponding concession stages, $(z^1, SS, z^2, SS, \ldots, z^t, SS)$. Let H^t be the set of all such t-period histories, with the null history $H^0 = \{h^0\}$ and $H = \cup_{t=0}^{\infty} H^t$. A history of play that leads to the concession stage in period t, denoted as $h^{t'}$, is an element of H^{t-1} followed by an incompatible demand profile z^t. Let $H^{t'}$ be the set of all such t-period histories and $H' = \cup_{t=1}^{\infty} H^{t'}$. A pure strategy for player i is a function $\sigma_i : H \cup H' \to [0,1] \cup \{A, S\}$ such that $\sigma_i(h) \in [0,1]$ for $h \in H$ and $\sigma_i(h) \in \{A, S\}$ for $h \in H'$. The subgame following history $h \in H \cup H'$ is labeled $g(h)$.

Given a history $h^t \in H$, a strategy profile $\sigma = (\sigma_1, \sigma_2)$ determines the period $n > t$ when bargaining ends in the subgame $g(h^t)$, with payoffs in that period of $y = (y_1, y_2)$, where $y = (0,0)$ if $n = \infty$. Call $(y, n-t)$ the outcome of the game $g(h^t)$ under σ. A strategy profile σ with outcome $(y, n-t)$ in the subgame $g(h^t)$ yields the discounted payoff of $\delta_i^{n-t-1} y_i$ to player i at the beginning of the subgame, where $\delta_i \in (0,1)$ is player i's discount factor.

2.1 Subgame Perfect Equilibria

To analyze its content, I focus on *pure strategy subgame perfect equilibria* of the model. Subsequently, for expositional ease, I will refer to these simply as subgame perfect equilibria or SPE. Infinite horizon games with simultaneous moves typically feature a vast multiplicity of SPE with a sense of *anything goes*. The current model features multiplicity too. Nevertheless, the following straightforward yet useful lemma shows that all such equilibria have a simple structure. Exactly compatible demands imply $d(z) = 0$.

Lemma 1 *A subgame perfect equilibrium at any period must feature either*
(a) exactly compatible demands, or
(b) incompatible demands followed by both players choosing Stick*.*

Proof. Consider a period in which incompatible demands (z) are followed by some action profile other than (S,S) in the concession stage. Then, as the payoff matrix in table 1 shows, there must be some player i who receives a payoff strictly less than $u_i(1 - z_{-i})$ and is strictly better off by deviating to the compatible demand $1 - z_{-i}$ instead of the original z_i.

Next, given a period with compatible demands that add up to less than 1, the player with the lower demand, say i, is strictly better off demanding $1 - z_{-i}$ instead.

■

In other words, any SPE involves some rounds of delay, if any, via incompatible demands, followed by an agreement on an efficient division of the surplus.

Dynamic bargaining games featuring multiple SPE typically have the following feature.[5] The range of efficient SPE outcomes constitutes the first-order multiplicity. These rely on history-dependent strategies but do not require strategy profiles involving delay. This first-order multiplicity is used, through appropriate history-dependent strategies, to generate varying lengths of delay, the second-order multiplicity. In the current model, delay is on a more equal footing with the set of efficient SPE outcomes. Limiting the length of delay permissible in an SPE limits the range of efficient outcomes that can arise in equilibrium. The following classification of SPEs allows for characterization results that capture this feature.

Definition 1 *An SPE σ is called an SPE with maximum delay m if for any subgame $g(h^t)$, $h^t \in H$, it generates an outcome $(y, n-t)$ where $n-t-1 \leq m$.*

The characterization results that follow rely on the stationary structure of the model. To this end, for any $h \in H$, let $O^m(h)$ denote the set of outcomes of SPE with maximum delay m in the subgame $g(h)$. Now define

$$B^m \equiv \left\{ z | (u(z), t) \in O^m(h^0) \right\}$$

to be the set of all surplus divisions that can arise as the outcome of some SPE with maximum delay m in the bargaining game. Due to the stationary

[5]See, for instance, Sutton (1986), Avery and Zemsky (1994) and Merlo and Wilson (1995).

structure of the game and definition 1, it follows that

$$B^m = \{z | (u(z), t) \in O^m(h)\}, \qquad \textit{for all } h \in H.$$

Finally observe that by lemma 1, $z \in B^m \Rightarrow z_1 + z_2 = 1$.

3 The Linear Model

In this section, I analyze the following specification of the bargaining model.

$$\forall i \in \{1, 2\}, \qquad u_i(z_i) = z_i \qquad \textit{and} \qquad c_i(d(z)) = k_i d(z) \textit{ for some } k_i > 0.$$

This linear specification retains the strategic tradeoffs of the general model while allowing for closed-form characterizations of equilibrium outcomes.

The following lemma captures the key restriction that strategic considerations about commitment impose on the set of compatible demands in equilibrium. In particular, the compatible demands must be such that neither bargainer can raise her own demand and force her opponent to back down in the resulting concession stage. "Forcing" here requires the (unique) dominance solvable outcome of the concession game to have the deviator stick to her demand while her opponent concedes, irrespective of the equilibrium continuation play. This restriction, while stated below for SPE with maximum delay n^*, plays an essential role in characterizing all the classes of SPE considered in this paper.

Lemma 2 *Suppose σ is a pure strategy profile with $\sigma(h^{t-1}) = z$ and $\sum_{i=1}^{2} z_i = 1$ for some $h^{t-1} \in H$. If for some $i \in \{1, 2\}$, there exists $z_{-i} < \hat{z}_{-i} \leq 1$ such that*

$$1 - z_i - k_{-i}(z_i + \hat{z}_{-i} - 1) < \delta_{-i}^n \tilde{z}_{-i} \tag{1}$$

and

$$1 - \hat{z}_{-i} - k_i(z_i + \hat{z}_{-i} - 1) > \delta_i^n \tilde{z}_i \tag{2}$$

for all $\tilde{z} \in B^{n^*}$ and $1 \leq n \leq n^* + 1$, then σ is not an SPE with maximum delay n^*.

Proof. Without loss of generality, set $i = 1$. Now note that bargaining failure in period t leads to $g(h^t)$ beginning in the next period. Since σ is an SPE with maximum delay n^*, and by lemma 1, the outcome (x, m) of this subgame must satisfy $x \in B^{n^*}$ and $m \leq n^* + 1$. Suppose one such continuation outcome is given by $(\tilde{z}, n)$. Now consider a deviation $\hat{z}_2$ from the compatible profile z which satisfies both inequalities 1 and 2 for this continuation profile.

Table 2: Augmented Concession Game following deviation $\hat{z}_2$ from Profile z

	A	S
A	$1 - \hat{z}_2 - k_1(z_1 + \hat{z}_2 - 1), 1 - z_1 - k_2(z_1 + \hat{z}_2 - 1)$	$1 - \hat{z}_2 - k_1(z_1 + \hat{z}_2 - 1), \hat{z}_2$
S	$z_1, 1 - z_1 - k_2(z_1 + \hat{z}_2 - 1)$	$\delta_1^n \tilde{z}_1, \delta_2^n \tilde{z}_2$

The deviation leads to the augmented game above in the concession stage, with (S, S) yielding a discounted payoff consistent with the continuation outcome $(\tilde{z}, n)$. Due to inequality 1, in this concession stage S strictly dominates A for player 2. Inequality 2 in turn ensures that given player 2's choice of S, player 1 strictly prefers to play A. In other words, the unique dominance solvable outcome in the augmented concession game is (A, S). Furthermore this outcome gives player 2 a strictly higher payoff than z_2. So, if there exists a $\hat{z}_2$ such that no matter what the continuation profile (consistent with σ being an SPE with maximum delay n^*) the two inequalities above are always satisfied, then $\hat{z}_2$ is a profitable deviation from z and therefore σ is not an SPE. ■

To see how the constraint identified in lemma 2 has bite, consider the compatible demand profile $(1, 0)$. Fix any set of discount factors and marginal concession costs. Notice that the highest payoff player 1 gets if bargaining fails this period is δ_1. By choosing a $\hat{z}_2 > 0$ close enough to 0, player 2 can ensure that conditional on 2 choosing S, 1 would rather concede and get a payoff arbitrarily close to 1 rather than settle for the lower amount of δ_1. By contrast, player 2 has no room for backing down since any concession leads to a negative payoff. So irrespective of the continuation strategy, following a deviation to $\hat{z}_2$, her dominant strategy would be S. In summary, player 2's deviation from $(1, 0)$ guarantees her a positive payoff. This rules out $(1, 0)$ as an equilibrium outcome.

To obtain a more complete characterization, the exercise above is extended to identify the most favourable equilibrium surplus division (element of B^{n^*}) for player i.[6] The restriction to SPE with maximum delay n^* ensures that in any continuation game the eventually agreed upon division must also belong to B^{n^*}. This recursive structure yields the following characterization.

Proposition 1 *If (z,t) is the outcome of a subgame perfect equilibrium with maximum delay n^*, then*

$$\frac{1-\delta_1}{1-\delta_2^{n^*+1}}\frac{k_2}{1+k_1} \leq \frac{z_2}{z_1} \leq \frac{1-\delta_1^{n^*+1}}{1-\delta_2}\frac{1+k_2}{k_1}. \tag{3}$$

Proof.

Let $z_i^* = \sup_{z \in B^{n^*}} z_i$. Now suppose for some exactly compatible demand profile z, there exists $\hat{z}_2$ such that

$$1 - z_1 - k_2(z_1 + \hat{z}_2 - 1) < \delta_2^{n^*+1}(1 - z_1^*) \tag{4}$$

and

$$1 - \hat{z}_2 - k_1(z_1 + \hat{z}_2 - 1) > \delta_1 z_1^*. \tag{5}$$

Then such a $\hat{z}_2$ also satisfies inequalities 1 and 2 for all $\tilde{z} \in B^{n^*}$ and $1 \leq n \leq n^* + 1$, since for any such $\tilde{z}$ and n it follows that $\delta_2^{n^*+1}(1 - z_1^*) \leq \delta_2^n \tilde{z}_2$ and $\delta_1 z_1^* \geq \delta_1^n \tilde{z}_1$. Therefore, by lemma 2, z cannot arise in any SPE (i.e., $z \notin B^{n^*}$).

Now since $z_i^* = \sup_{z \in B^{n^*}} z_i$ it must be that we cannot find such a $\hat{z}_2$ for the compatible profile $z = (z_1^*, 1 - z_1^*)$. So there cannot be a $\hat{z}_2 > 1 - z_1^*$ which satisfies both

$$1 - z_1^* - k_2(z_1^* + \hat{z}_2 - 1) < \delta_2^{n^*+1}(1 - z_1^*), \qquad \textit{and}$$

$$1 - \hat{z}_2 - k_1(z_1^* + \hat{z}_2 - 1) > \delta_1 z_1^*.$$

[6] This step is similar in spirit to the approach taken in Shaked and Sutton (1984) to solve the alternating-offers model.

These inequalities simplify to

$$\hat{z}_2 > \frac{(1-z_1^*)(1+k_2-\delta_2^{n^*+1})}{k_2} \qquad and \qquad \hat{z}_2 < 1 - \frac{(k_1+\delta_1)z_1^*}{1+k_1}.$$

Therefore such a $\hat{z}_2$ cannot exist only if

$$\begin{aligned}
&\frac{(1-z_1^*)(1+k_2-\delta_2^{n^*+1})}{k_2} \geq 1 - \frac{(k_1+\delta_1)z_1^*}{1+k_1} \\
\Rightarrow &\frac{(1-z_1^*)(1-\delta_2^{n^*+1})}{k_2} \geq \frac{z_1^*(1-\delta_1)}{1+k_1} \\
\Rightarrow &\frac{1-\delta_1}{1-\delta_2^{n^*+1}}\frac{k_2}{1+k_1} \leq \frac{1-z_1^*}{z_1^*}
\end{aligned}$$

A symmetric argument establishes

$$\frac{1-\delta_2}{1-\delta_1^{n^*+1}}\frac{k_1}{1+k_2} \leq \frac{1-z_2^*}{z_2^*}$$

which transforms to

$$\frac{z_2^*}{1-z_2^*} \leq \frac{1-\delta_1^{n^*+1}}{1-\delta_2}\frac{1+k_2}{k_1}.$$

To conclude the proof note that

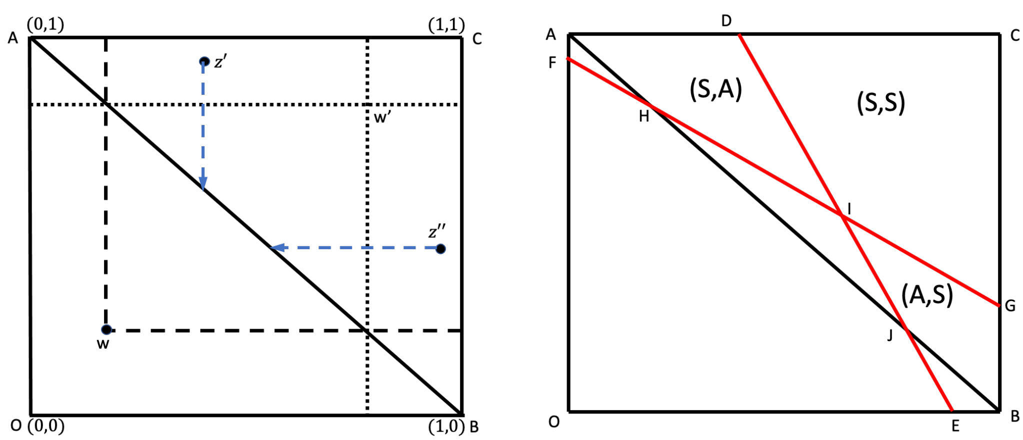
$$z \in B^{n^*} \Rightarrow \frac{1-z_1^*}{z_1^*} \leq \frac{z_2}{z_1} \leq \frac{z_2^*}{1-z_2^*}.$$

■

The result confirms Schelling's insight about weakness being a strength, in that higher marginal concession costs generate better equilibrium outcomes for the bargainer. Greater patience is similarly beneficial. This preserves a key implication of the canonical bargaining models. The result also delivers a simple way to classify equilibrium surplus divisions on the basis of the maximum anticipated delay following any history. This in turn clarifies how limiting the maximum delay permissible in an SPE restricts the range of equilibrium efficient outcomes.

Some important questions about SPEs remain unanswered. Is the amount of delay permitted in an SPE bounded? Can a sharper necessary condition be obtained for all SPEs? Is there a general sufficiency condition/existence result for SPEs? The next three results answer these in the affirmative.

Delay requires incompatible demands on the equilibrium path. A bargainer may aim to do better in two ways. Deviate to a compatible demand, or make a milder but still incompatible demand which forces a concession from the opponent. In an SPE such deviations must be unprofitable. This is precisely what bounds the amount of delay. Figures 1 and 2 describe these strategic considerations. The figures depict both the space of demands and payoffs, with player 1's demand (payoff) on the x-axis and player 2's on the y-axis. AB is the set of efficient demands (payoffs). Due to discounting, all payoffs must lie on or in the triangle AOB.



(a) (b)

Figure 1

Now consider an SPE with delay and payoff profile $w = (\delta_1^n z_1, \delta_2^n z_2)$ as in figure 1a. By lemma 1 this SPE involves n rounds of incompatible demand profiles followed by the exactly compatible demand profile z. In this SPE, the period 1 incompatible demand profile must lie within the rectangle with w' and C at opposite corners. For all other incompatible demand profiles some player is strictly better off by deviating to a compatible demand. The arrows from demand profiles z' and z'' describe such profitable deviations.

Figure 1b describes the dominance solvable outcome of the (augmented) concession game in a given period *with continuation payoff fixed at some* w (see table 3 below), for all incompatible demand profiles. For instance, following a demand profile in AHID, in the resulting concession game with continuation payoff profile w, the dominance solvable outcome is (S, A). Both (S, A) and (A, S) are Nash equilibria following demand profiles in HJI. DE and FG are the indifference lines that graph $1 - z_1 - k_2(z_1 + z_2 - 1) = w_2$ and $1 - z_2 - k_1(z_1 + z_2 - 1) = w_1$, respectively.

Table 3: Augmented Concession Game following Incompatible Profile z

	A	S
A	$1 - z_2 - k_1(z_1 + z_2 - 1), 1 - z_1 - k_2(z_1 + z_2 - 1)$	$1 - z_2 - k_1(z_1 + z_2 - 1), z_2$
S	$z_1, 1 - z_1 - k_2(z_1 + z_2 - 1)$	w_1, w_2

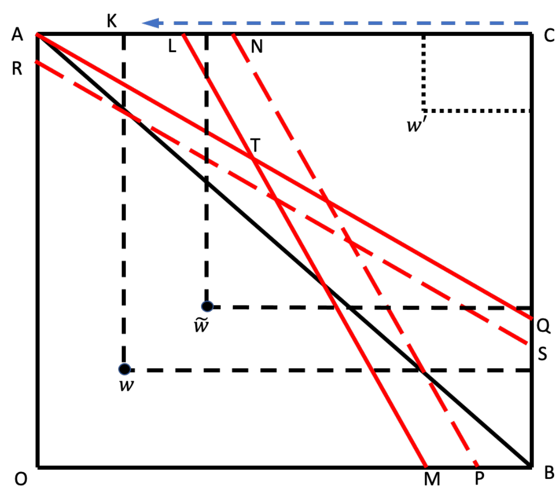


Figure 2

In figure 2 LM represents the indifference line for player 2 if she faces her best equilibrium continuation payoff. AQ represents the indifference line for player 1 if she faces her worst possible continuation payoff. This means that any given equilibrium continuation payoff would generate indifference lines for players 1 and 2 to the right of LM and to the left of AQ, respectively, such as RS and NP (from continuation payoff w). Therefore incompatible demands in the region ATL lead to (S, A) irrespective of equilibrium continuation play.

Recall the strategy profile with delay and payoff profile $w = (\delta_1^n z_1, \delta_2^n z_2)$ and suppose both players demand 1 in the first period. Player 1 can deviate from $(1, 1)$ to some point on KL and force 2 to concede, for a higher payoff.[7] Notice that such a profitable deviation exists as long as the continuation payoff to player 1 is less than the amount at L. It does not exist, for instance, for continuation profiles such as $\tilde{w}$, which involves less delay. This is the feature that bounds the amount of delay in an SPE.

Let O^{SPE} be the set of all SPE outcomes and

$$B^* \equiv \{z | (z, t) \in O^{SPE}\}$$

the set of all surplus divisions in such outcomes.

Proposition 2 *Suppose $k_j(k_{-j} - 1) > 1$ for $j \in \{1, 2\}$. If (x, t) is an SPE outcome with $t > 1$, then $x \in B^*$ and for $i \in \{1, 2\}$*

$$\delta_i^{t-1} x_i \geq \frac{1 - \delta_{-i} z_{-i}^*}{1 + k_{-i}} \tag{6}$$

where $z_{-i}^* = \sup_{z \in B^*} z_{-i}$.

It is simple to compute a bound from the result without any further knowledge about the set of SPE outcomes, B^*. For instance, there can be no SPE with delay $\tilde{t} - 1$ if $\delta_i^{\tilde{t}-1} < (1 - \delta_{-i})/(1 + k_{-i})$ for some $i \in \{1, 2\}$. Of course, knowledge of bounds on the set of efficient SPE outcomes delivers a tighter bound on SPE delay.

Proposition 1 describes how the length of delay permissible in an SPE bounds the set of efficient outcomes that can arise in equilibrium. Proposition 2 captures how the set of efficient equilibrium outcomes limits the maximum delay in an SPE. The next result combines these two results to obtain a necessary condition for SPE outcomes.

[7] Similar profitable deviations exist from any point in the $w'C$ rectangle.

Proposition 3 *Suppose $k_i(k_{-i}-1) > 1$ for $i \in \{1,2\}$. If (z,t) is a subgame perfect equilibrium outcome, then*

$$\min\left\{\frac{1-\delta_1}{1-\delta_2}\frac{k_2}{1+k_1}, \frac{1+k_2}{k_1}(1-\delta_1)\right\} \le \frac{z_2}{z_1} \le \max\left\{\frac{1-\delta_1}{1-\delta_2}\frac{1+k_2}{k_1}, \frac{k_2}{1+k_1}\frac{1}{1-\delta_2}\right\}.$$

Next, I describe an algorithm that characterizes a set of SPE outcomes. It has two parts that feed each other in a recursive manner. The first constructs a set of efficient SPE outcomes from a given set of continuation payoffs. Fixing any such payoff profile as one arising from equilibrium continuation play following impasse in the first period, it characterizes the efficient outcomes in the first period that can be sustained in equilibrium. It uses the following pair of equations.

$$\begin{aligned} 1 - y_2 - k_1(y_1 + y_2 - 1) &= w_1^c \\ 1 - y_1 - k_2(y_1 + y_2 - 1) &= w_2^c \end{aligned} \tag{7}$$

The second constructs a set of SPEs with delay from a given set of efficient SPE outcomes. Fixing an efficient SPE outcome as the one eventually agreed on, it characterizes how many periods of disagreement can precede it in equilibrium and uses the following inequality.

$$1 - \delta_1^{n-1}y_1 - \delta_2^{n-1}y_2 \le \min\{k_1\delta_2^{n-1}y_2, k_2\delta^{n-1}y_1\}. \tag{8}$$

Let E^1 be the set of all outcomes of SPE with maximum delay 0. Now, for $t \ge 2$ define the following recursively,

- Let $\zeta^t = \{(w_1, w_2)|w_i = \delta_i^n y_i, (y,n) \in E^{t-1}\}$.
- For each $w \in \zeta^t$ let y^{tw} solve 7 with $w^c = w$.
- Let Z_w^t be the set of all exactly compatible demands z such that $z_i \le y_i^{tw}$ for $i \in \{1,2\}$ and $Z^t = \cup_{w\in\zeta^t} Z_w^t$.
- Let $\tilde{E}^t = \{(y,n) \in Z^t \times \mathbb{N}|(y,n)$ *satisfies inequality* $8\}$.
- Let $E^t = E^{t-1} \cup \tilde{E}^t$.

Finally let $E = E^t$ if and only if $E^t = E^{t+1}$.

Proposition 4 *Every outcome in E is an SPE outcome.*

It is clear from proposition 1 that additional conditions that restrict the maximum delay allowed in an SPE, as a result, also shrink the set of compatible demand profiles that can obtain in equilibrium. I study three such conditions in the following subsections.

3.1 Renegotiation-Proofness

Negotiators who are familiar with each other should, in the presence of multiple equilibria, be able to avoid the strictly Pareto dominated ones. This is especially so, if the Pareto dominating equilibrium is one they anticipate to play following some history. Since the game is identical following any history $h \in H$, the negotiators would see the incongruence of taking an efficient path following one such history and an inefficient one following another. Given their familiarity they need not take their cues from some possibly inefficient norm, but rather count on renegotiating away from such inefficient equilibria. The notions of *weak renegotiation proofness* in Farrell and Maskin (1989) and *internal consistency* in Bernheim and Ray (1989) capture this idea in the context of repeated games. While not a repeated game, the present model shares its key feature that following any number of rounds (of failed bargaining), the continuation game looks the same. Relying on this stationarity, I import an appropriate notion of renegotiation-proofness for the current setting.

Let $\psi(\sigma; h^t)$ be the continuation payoff (profile) implied by σ given history $h^t \in H$ and let

$$\Psi(\sigma) = \cup_{h^t \in H} \psi(\sigma; h^t)$$

be the set of all continuation payoffs under σ.

Definition 2 *An SPE σ is renegotiation-proof if for no $x, y \in \Psi(\sigma)$ is $x \gg y$.*

Renegotiation-proofness is routinely studied in equilibrium analyses for a variety of economic questions, and with important implications. See, for instance, Barrett (1994) on international environmental agreements, Matsuyama

(1990) on trade liberalization, Kletzer and Wright (2000) on sovereign debt. Nonetheless, it is not the only "reasonable" description of behaviour. It does however facilitate a natural separation of all pairs of bargainers into those who always coordinate on efficient outcomes on the equilibrium path and anticipate the same off it, and others. Renegotiation-proofness delivers a sharper characterization of the behaviour of the former group.

Note that renegotiation-proofness does not rule out history dependent strategies. Consider, for instance, the construction due to Avner Shaked reported in Sutton (1986). It supports any efficient division of the surplus as a subgame perfect equilibrium outcome of a 3-person Rubinstein bargaining game for high enough discount factors. The construction relies heavily on the history-dependence of the strategy profile. Imposing an appropriate version of renegotiation-proofness has no effect on the result since all continuation outcomes are efficient. The severe multiplicity persists. In the current model, however, renegotiation-proofness sharply restricts the set of equilibrium outcomes.

Proposition 5 (z, t) *is the outcome of a renegotiation-proof subgame perfect equilibrium if and only if* $t = 1$ *and*

$$\frac{1-\delta_1}{1-\delta_2}\frac{k_2}{1+k_1} \leq \frac{z_2}{z_1} \leq \frac{1-\delta_1}{1-\delta_2}\frac{1+k_2}{k_1}. \tag{9}$$

I now sketch the argument behind this result. The detailed proof is in the appendix. Given the structure of SPE identified in lemma 1, renegotiation-proofness simply rules out any delay. The necessity of inequality 9 then follows immediately from proposition 1. To establish sufficiency, I construct the following stationary strategy profile, which I show to be subgame perfect for any z satisfying inequality 9 in lemma 4 in the appendix.

Construction 1 *Consider the following stationary strategy profile,* σ*. Fix* z *such that* $d(z) = 0$*. For all* $h^t \in H$*, set* $\sigma_i(h^t) = z_i$*. If player* i*, for some* $i \in \{1, 2\}$*, in period* t *deviates to a higher demand,* $\hat{z}_i > z_i$*, then in the concession stage game* (S, S) *is played if it is a Nash equilibrium and otherwise*

(A_i, S_{-i}) is played. For all other $h \in H'$ some pure strategy Nash equilibrium of the concession stage game is played.

The strategy profile σ above satisfies renegotiation-proofness, since following any history $h \in H$ the continuation outcome is efficient and consists of agreeing on the compatible demand profile z.

Proposition 5 offers a preview of the limit uniqueness result in section 4.2. Consider a sequence of these linear bargaining games parametrized by marginal concession costs $\{k_1^n, k_2^n\}_{n=1}^{\infty}$ such that $k_1^n = \gamma k_2^n$ for all n and $k_2^n \to \infty$ as $n \to \infty$. Observe first that at the limit, it is too costly for any bargainer to concede following any incompatible demand. The model therefore reduces to the IH-NDG. However, in contrast to the acute multiplicity of SPE in the IH-NDG, the set of renegotiation-proof SPE as characterized in proposition 5 converges to a singleton at the limit. At this unique limit outcome, the bargainers agree on the compatible profile z with

$$\frac{z_2}{z_1} = \frac{1-\delta_1}{1-\delta_2}\frac{1}{\gamma}.$$

Collard-Wexler et al. (2019) use a criteria called *no-delay* to refine the set of SPE in their bargaining model. The criteria is identical to the notion of *SPE with maximum delay 0*, in that it requires no delay following any history. All results requiring renegotiation proofness in this paper would remain unchanged if the no-delay criteria was used instead. This however is a result that follows from proposition 5. In the very closely related IH-NDG, by contrast, perpetual disagreement satisfies renegotiation proofness while obviously violating no-delay.

3.2 Markov Perfect Equilibria and Gradualism

Negotiations often take place between strangers or relatively inexperienced bargainers. The assumption of renegotiation-proofness may not be appropriate in such cases. A different assumption, routinely made in applied work, requires players to use Markov strategies. Maskin and Tirole (2001) discusses some of the theoretical considerations that support its use. Vespa (2020) finds experimental evidence of it as the modal behaviour in the dynamic common pool game. In this section I focus on SPE in Markov strategies. Similar to the previous section, the agenda is not to propose the Markov restriction as the only "sensible" one. Instead, it is to obtain a sharper characterization of the behaviour of an empirically large and relevant group of people in this setting.[8]

Definition 3 *σ_i is a Markov strategy for player i if for all $h, \tilde{h} \in H^t$*
(i) $\sigma_i(h) = \sigma_i(\tilde{h})$ and
(ii) $\sigma_i(h, z^{t+1}) = \sigma_i(\tilde{h}, z^{t+1})$.

In words, under the Markov requirement, player i's demand in period t must be invariant to the specific $t-1$ demand profiles rejected in the past. Further, the concession stage decision in period t should depend upon the period t demand profile alone. Note, however, that it allows demands and concession stage behaviour to depend on calendar time. For instance, a strategy in which the demands get less and less extreme over the first m periods of bargaining is permitted. Indeed, such strategies can generate delay in equilibrium. Let $\lfloor x \rfloor$ denote the greatest integer less than or equal to x.

Proposition 6 *Let $j \in \{1, 2\}$ such that $\delta_j \geq \delta_{-j}$. If (z, t) is a Markov perfect equilibrium outcome then $t \leq n^*$ and*

$$\frac{1-\delta_{-j}^{n^*}}{1-\delta_j^{n^*}} \frac{k_j}{1+k_{-j}} \leq \frac{z_j}{z_{-j}} \leq \frac{1-\delta_{-j}}{1-\delta_j} \frac{1+k_j}{k_{-j}},$$

with

$$n^* - 1 = \left\lfloor \frac{\ln \frac{k_1+k_2}{k_1+k_2+k_1k_2}}{\ln \delta_j} \right\rfloor.$$

As in the case of regular SPE, to sustain delay in MPE the bargainers must make incompatible demands that neither wishes to deviate from. To ensure that a unilateral deviation to a compatible profile is not profitable, the demands simply need to be sufficiently aggressive, exactly as in the case of SPE.

[8]Note that unlike in the repeated prisoner's dilemma, Markov strategies do not preclude efficiency in the current bargaining game.

It is less demanding to rule out profitable deviations where a bargainer makes a lower but still incompatible demand, which forces her opponent to concede in the subsequent concession game. Due to the Markov restriction, the continuation play cannot change following such a deviation. Long expected delay makes such deviations feasible since it lowers the payoff from disagreement and makes concession more palatable. This feature further limits the amount of delay that can arise in an MPE.

The upper bound to the length of delay, restricts the set of equilibrium continuation outcomes. The remaining argument, similar to that of proposition 5, characterizes the best compatible demand profile that can arise for each player, relying on the recursive structure of the game.

As for the result itself, observe first that if the two bargainers are equally impatient, then the set of MPE outcomes coincides with the renegotiation-proof SPE outcomes. So not only is the larger set of equilibrium surplus agreements dependent on the delay allowed by MPE, it relies on bargainers having different degrees of impatience. Second, suppose the maximum delay under MPE is *derived* to be m under proposition 6, the bounds on MPE surplus divisions are tighter than the bounds that arise for SPE with maximum delay m, as derived in proposition 1. The lack of history-dependent strategies delivers a sharper prediction.

A final interesting feature of the Markov environment is the nature of incompatible demands in an MPE with delay. Gradualism is a commonly observed feature of bargaining in which players gradually lower their demands, starting with very aggressive ones and ending with a compatible profile.[9] MPEs with delay yield gradualism in a natural way. The following proposition characterizes this feature.

Proposition 7 *If (y, m) is the outcome of a Markov perfect equilibrium with a delay of $m - 1 > 0$ periods then the incompatible demand profiles z^t for*

[9] See, for instance, Backus, Blake, Larsen and Tadelis (2020).

$1 \leq t \leq m-1$ *must satisfy*

$$z_i^t \geq \frac{(1-\delta_{-i}^{m-t} y_{-i})(1+k_i) - \delta_i^{m-t} y_i}{k_i}.$$

In words, the smallest (incompatible) demand that can arise in an MPE is higher the further away (in periods) it is made from the eventual agreement. Two separate features contribute to this. The obvious one is that for neither player to want to deviate to simply accepting the others implicit offer (by making a compatible demand) it must be that the offers are worse than accepting the delayed agreement. The longer the delay the worse the offers need to be, and therefore higher demands. The less obvious feature is that a bargainer may find it profitable to deviate to a lower but still incompatible demand profile that forces the other player to concede. To rule out such a deviation, the incompatible demands need to be even higher than the level required to rule out deviations to compatible profiles. Further, this threshold is higher the more periods that remain to agreement.

I end this section by characterizing the set of stationary MPE outcomes. Stationarity does not allow strategies to depend on calendar time. It requires

$$\sigma_i(h) = \sigma_i(\tilde{h}) \qquad \forall h, \tilde{h} \in H.$$

Proposition 8 (z,t) *is a stationary Markov perfect equilibrium outcome if and only if* $t=1$ *and*

$$\frac{1-\delta_1}{1-\delta_2}\frac{k_2}{1+k_1} \leq \frac{z_2}{z_1} \leq \frac{1-\delta_1}{1-\delta_2}\frac{1+k_2}{k_1}.$$

Notice that the set of stationary MPE outcomes coincides exactly with the set of renegotiation-proof SPE outcomes. In general dynamic games where both concepts apply the two are typically not the same. Take an infinitely repeated prisoner's dilemma game with high enough discount factors, for instance. The unique MPE outcome involves both parties defecting forever. On the other hand, cooperation can be sustained as a weak renegotiation-proof

SPE, as shown in Farrell and Maskin (1989).[10]

4 General Model

The qualitative results obtained in propositions 1 and 5 do not rely on the assumption of linearity. Let $\mathcal{U}$ and $\mathcal{C}$ be the set of all (pairs of) functions that satisfy assumptions 1 and 2, respectively. Fix some $u \in \mathcal{U}$ and $c \in \mathcal{C}$ and $n^* \in \mathbb{N}$. For $i \in \{1,2\}$, define the function $\tilde{z}_{-i}^{n^*}(z_i)$ implicitly as the solution to the equation

$$u_{-i}(1-z_i) - c_{-i}(z_i + \tilde{z}_{-i}^{n^*}(z_i) - 1) = \delta_{-i}^{n^*+1} u_{-i}(1-z_i). \tag{10}$$

Similarly define the function $\tilde{\tilde{z}}_{-i}^{n^*}(z_i)$ implicitly as the solution to the equation

$$u_i(1 - \tilde{\tilde{z}}_{-i}^{n^*}(z_i)) - c_i(z_i + \tilde{\tilde{z}}_{-i}^{n^*}(z_i) - 1) = \delta_i u_i(z_i). \tag{11}$$

It turns out that there is a unique z_i, which I denote as $z_i^{Mn^*}$, that solves

$$\tilde{\tilde{z}}_{-i}^{n^*}(z_i) = \tilde{z}_{-i}^{n^*}(z_i). \tag{12}$$

Proposition 9 *In the general model, if (y,t) is the outcome of an SPE with maximum delay n^*, with $y = u(z)$, then $d(z) = 0$ and*

$$\frac{1 - z_1^{Mn^*}}{z_1^{Mn^*}} \leq \frac{z_2}{z_1} \leq \frac{z_2^{Mn^*}}{1 - z_2^{Mn^*}}. \tag{13}$$

Relabel $z_i^{Mn^*}$ simply as z_i^M when $n^* = 0$, and the following generalization of Proposition 5 obtains.

Proposition 10 *In the general model, (y,t) is the outcome of a renegotiation-proof SPE with $y = u(z)$, if and only if $t = 1$, $d(z) = 0$ and*

$$\frac{1 - z_1^M}{z_1^M} \leq \frac{z_2}{z_1} \leq \frac{z_2^M}{1 - z_2^M}. \tag{14}$$

[10] See also van Damme (1989).

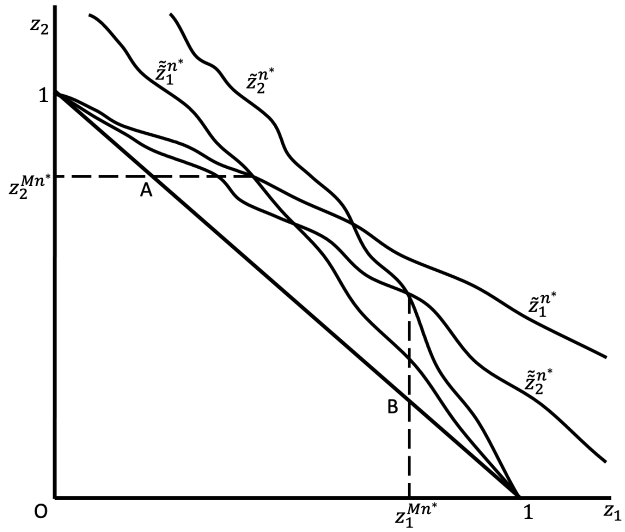


Figure 3

Figure 3 clarifies the content of these results. For a given n^*, equations 10 and 11 generate four functions. The function $\tilde{z}_2^{n*}(z_1)$ returns the smallest player 2 demand such that following any pair of incompatible demands z_1 and $z_2 > \tilde{z}_2^{n*}(z_1)$ and a continuation payoff of $\delta_2^{n^*+1}(1-z_1)$, player 2 strictly prefers S. The function $\tilde{\tilde{z}}_2^{n*}$ returns the largest player 2 demand such that following any pair of incompatible demands z_1 and $z_2 < \tilde{\tilde{z}}_2^{n*}(z_1)$ and a continuation payoff $\delta_1 u_1(z_1)$, player 1 strictly prefers concession, if 2 chooses S.

Notice that $z_1 > z_1^{Mn^*}$ implies $\tilde{\tilde{z}}_2^{n*}(z_1) > \tilde{z}_2^{n*}(z_1)$. Such a z_1 cannot be the best equilibrium efficient split for player 1, because player 2 could then deviate to a demand $z_2 \in (\tilde{z}_2^{n*}(z_1), \tilde{\tilde{z}}_2^{n*}(z_1))$. This would force a concession from player 1 in the resulting concession game, *for any equilibrium continuation payoff*. This is the argument behind the bound $z_1^{Mn^*}$. A symmetric argument applies to player 2's bound of $z_2^{Mn^*}$.

Increasing player 1's cost function, holding all else fixed, leaves functions $\tilde{\tilde{z}}_1^{n*}$ and $\tilde{z}_2^{n*}$ unchanged while moving $\tilde{\tilde{z}}_2^{n*}$ and $\tilde{z}_1^{n*}$ closer to the efficient frontier. Figure 3 shows that this would increase $z_1^{Mn^*}$ and lower $z_2^{Mn^*}$, confirming Schelling's insight about weakness being a strength in this more general setting. Increasing player 1's patience, δ_1, has a qualitatively similar effect to increasing her cost function. Finally observe that lowering n^*, leaves $\tilde{\tilde{z}}_i^{n*}$ unchanged for $i \in \{1,2\}$ while moving both $\tilde{z}_1^{n*}$ and $\tilde{z}_2^{n*}$ closer towards the diagonal. This means that $z_i^{Mn^*}$ is increasing in n^*, as expected.

4.1 Kalai Bargaining Solution

Kalai (1977) introduces a family of bargaining solutions parametrized by a single variable, a proportion. Any bargaining solution that is monotonic, in that increasing the set of feasible bargaining outcomes never hurts either bargainer (formally defined below), is a Kalai (or proportional) bargaining solution (KBS) and vice versa. The family of solutions is exactly characterized by the axioms of independence of irrelevant alternatives, individual monotonicity and continuity. In addition to being compelling theoretically, the solutions are used extensively and in a variety of fields. Recently, for instance, it is used increasingly in the field of monetary economics.[11]

I now introduce some notation in order to define KBS. Let $\Pi(u) = \{y|y_i = u_i(z_i), z_i \geq 0, \forall i \in \{1,2\} \ and \ z_1+z_2 \leq 1\}$ denote the set of feasible payoffs that can arise from some allocation of the surplus. Set $u^d = (u_1(0), u_2(0)) = (0,0)$ to be the disagreement point. Combined, $(\Pi(u), u^d)$ represents a bargaining problem. Finally let $\mathcal{B} = \{(\Pi(u), u^d)|u \in \mathcal{U}\}$ be the set of all bargaining problems that can arise from payoff functions that satisfy assumption 1. A bargaining solution is a function $\phi : \mathcal{B} \to \mathbb{R}^2$ such that $\phi(B) \in B$ for all $B \in \mathcal{B}$. It is monotonic if for any $A, B \in \mathcal{B}$, $A \subset B$ implies $\phi(B) \geq \phi(A)$.

The Kalai Bargaining Solution with proportions $(\theta, 1)$, denoted by $\mathcal{K}_\theta$, is defined as

$$\mathcal{K}_\theta(\Pi, u^d) = \lambda(\Pi, u^d) \cdot (\theta, 1), \forall \Pi \in \mathcal{B}$$

where $\lambda(\Pi, u^d) = \max\{q \in \mathbb{R}|q \cdot (\theta, 1) \in \Pi\}$.[12] In words, the proportion parameter, θ, fixes a unique ray in the utility space passing through $(0,0)$. For any bargaining problem, the KBS with proportion θ then simply picks the point where the ray meets the efficient frontier of the bargaining problem.

[11]See, for instance, Lagos, Rocheteau and Wright (2017). Duffy, Lebeau and Puzzello (2021) find that KBS better fits the behaviour of bargainers in the laboratory facing liquidity constraints.

[12]Note that $(\theta, 1)$ is a vector and since q is a scalar, $q \cdot (\theta, 1) = (q\theta, q)$.

4.2 Strategic Foundation

Return now to the general non-cooperative bargaining model. Making the concession cost functions steeper makes it progressively harder for the bargainers to back down from their demands. At the limit, with arbitrarily high marginal concession costs, the infinite horizon version of the Nash demand game obtains. Neither player can back down from incompatible demands. Binmore (1987) points out that any efficient payoff profile can be supported as an SPE outcome of the IH-NDG. Infinite delay can also be supported in SPE by each bargainer always demanding the entire surplus. Chatterjee and Samuelson (1990) show that this acute multiplicity further survives trembling hand perfection (see Selten (1975)). The limit set of renegotiation-proof SPE outcomes, in sharp contrast, is a singleton.

For any $u \in \mathcal{U}$ and $c \in \mathcal{C}$, let $g^c(u)$ denote the game described in section 2, where u_i and c_i are player i's payoff and concession cost functions, respectively, for $i \in \{1, 2\}$. Denote the corresponding set of renegotiation-proof SPE payoff profiles by $\xi(g^c(u))$. g^c therefore maps any pair of payoff functions in $\mathcal{U}$ to its corresponding infinite horizon bargaining game. Consider a sequence of such mappings $\{g^{c^n}\}_{n=1}^{\infty}$ with $c^n \in \mathcal{C}$ for all n, such that as $n \to \infty$, $c_i^{n'}(0+) \to \infty$ (the right derivative of the concession cost functions at 0 becomes arbitrarily large). Next, assume that there exists some integer N such that $\forall m, n > N$,

$$0 < \lim_{d \searrow 0} c_1^m(d)/c_2^m(d) = \lim_{d \searrow 0} c_1^n(d)/c_2^n(d) < \infty. \tag{15}$$

In words, the assumption requires that sufficiently far along the sequence, the ratio of the concession costs for vanishingly small concessions is the same, positive and bounded. The assumption is satisfied by the linear specification in section 3, but it does not require linearity of either the individual cost functions or even their ratio.[13]

Finally let

$$\xi^*_\gamma(u) = \lim_{n \to \infty} \xi(g^{c^n}(u)), \qquad \textit{where } \gamma = \lim_{n \to \infty} c_1^n(0+)/c_2^n(0+).$$

[13]Consider, for example, $c_1^n(d) = n(d + 2d^2)$ and $c_2^n(d) = n(4d + d^2)$.

The limit set of renegotiation-proof SPE is therefore captured by $\xi^*_\gamma(u)$. It is parameterized by γ, which is the ratio of the concession cost functions evaluated at the limit as the concessions become vanishingly small. The assumption described in 15 ensures that γ is well defined.

Proposition 11 *For all $u \in \mathcal{U}$, $\xi^*_\gamma(u) = \mathcal{K}_\theta(\Pi(u), u^d)$ where $\theta = \gamma(1-\delta_2)/(1-\delta_1)$.*

Intuition

The intuition for the result can be split into three key arguments. First, for any set of cost and payoff functions, the two distinct best renegotiation proof SPE outcomes (henceforth labeled extreme RP-SPE outcomes), one for each player, admit a stationary strategy profile each. This equilibrium feature of the current model does not hold for general dynamic bargaining games.[14] It then follows that an agent's best renegotiation proof SPE outcome is identical to her best stationary SPE outcome.

Second, the two extreme RP-SPE outcomes approach each other as the marginal concession cost increases. This can be seen in figure 3, setting $n^* = 0$. The extreme RP-SPE outcomes are $(z_1^M, 1 - z_1^M)$ and $(1 - z_2^M, z_2^M)$. As the marginal concession cost increases, all the functions $\tilde{z}_i^0$ and $\tilde{\tilde{z}}_i^0$ for $i \in \{1, 2\}$, swing towards the efficient demand frontier. In turn, the two extreme RP-SPE outcomes and, as a result all RP-SPE outcomes, converge. Very importantly, such convergence does not occur for the two best SPE outcomes in general. Higher marginal concession costs permit longer equilibrium delay, which can be used to construct equilibria that keep the two outcomes apart.

Finally, at the limit the ratio of payoffs to the two bargainers equals a constant. The constant is itself independent of the payoff functions and depends solely on the discount factors and a limit ratio of the concession cost functions. The additive separability of concession costs and their dependence on the distance between "physical" demands instead of payoff levels allow this limit characterization, which coincides with the Kalai bargaining solution. Since

[14] For instance, the extreme RP-SPE outcomes in the 3-person Rubinstein bargaining game reported in Sutton (1986) do not admit a stationary strategy profile.

the limit is defined in terms of arbitrarily steep concession cost functions, the payoff functions u_i for $i \in \{1, 2\}$ alone represent the preferences of the two bargainers (at this limit). Given the characterization, it is clear that the limit outcome, just like the Kalai solution, is not scale invariant.

The strategic foundation for KBS in Dutta (2012), which studies the single period version of the current model, is similarly obtained at the high concession cost limit. The convergence argument in that paper is substituted by the second and third arguments above, to fit the infinite horizon environment and the requirements of subgame perfection.

High concession cost limit

The high concession cost setting is best interpreted as a perturbation of the perfect commitment implicit in the IH-NDG. In the latter, agents are fully committed to their demands in that incompatible demands directly lead to an impasse in that period. The current analysis shows that allowing agents even the smallest room for concession opens up strategic considerations that substantially shrink the set of equilibrium outcomes. That perturbing the commitment structure selects the KBS, stands in sharp contrast to other equilibrium selection arguments for the NDG that deliver the Nash bargaining solution (see Nash (1953), Binmore (1987a), Carlsson (1991)) . These latter arguments perturb the information structure in a way that effectively smooths the NDG payoff function, in that the payoff following incompatible demands smoothly tapers off to zero as a function of the demands. The IH-NDG with a similar information structure perturbation is obtained as a special case of a more general bargaining model studied in Harstad (2021). Here too, the argument selects the asymmetric Nash bargaining solution. The commitment structure perturbation in the current paper does not smooth the (effective) payoff functions. They remain discontinuous at the efficient demand frontier.

The Nash program

A narrow reading of the Nash program simply calls for strategic games whose equilibria align with a given cooperative solution concept. A broader interpretation requires further that the strategic models capture key features of some class of institutional frameworks.[15] This facilitates the use of the corresponding bargaining solution for negotiations that take place in such institutional settings. The preceding analysis makes such a case for the Kalai solution in environments where commitment ability by way of concession costs is prevalent. Section 5.1 lists some such institutional and social environments.

It is clear in Kalai (1977) that while the family of bargaining solutions is a compelling one, finding the relevant proportion needs information beyond what is modelled in a standard bargaining problem (an element of $\mathcal{B}$). In proposition 11 the degree of impatience of the bargainers and their relative concession costs constitute this information. So not only does this formalization of Schelling's theory provide a strategic foundation for the Kalai bargaining solution, it also selects the appropriate proportion.

Implications

The characterization in proposition 11 is easy to interpret. The physical split of surplus must be efficient ($z_1 + z_2 = 1$) and the resulting ratio of utilities must satisfy

$$\frac{u_1(z_1)}{u_2(z_2)} = \gamma \frac{1-\delta_2}{1-\delta_1}.$$

Greater patience leads to higher payoff, as in canonical models of bargaining. In essence, despite its definition as a limit, γ captures the relative concession costs faced by the players. Higher concession costs translate into better bargaining outcomes. By virtue of its equivalence to a KBS, the limit solution inherits monotonicity while failing scale invariance. An important implication is that scaling up a specific agent's payoff function brings that agent a higher payoff (as required by monotonicity) but also a lower physical split of the surplus. Consider the following example, borrowed from Kalai (1977) that involves two scenarios of splitting one hundred chips. In both, the bargainers have the same linear utility for money, the same discount factors and $\gamma = 1$. In the first scenario either player can cash in each chip for 1 dollar. In the second, player 2 can continue to cash in each chip for one dollar while player 1 can cash in each chip for three dollars. In the limit solution the players gets 50

[15]See quoted passage from Binmore, Osborne and Rubinstein (1992) in the introduction.

dollars each from a 50-50 split in the first scenario and 75 dollars each from a 25-75 split in the second scenario. The Nash bargaining solution, by contrast, calls for a 50-50 split in both scenarios. Note that the difference arises even in a wholly linear specification.

An application in monetary economics

The observation above has implications in monetary economics. Hu and Rocheteau (2020) discuss a game where two players bargain over the sale of a divisible commodity in return for monetary payment subject to a liquidity constraint. They point out how the Nash bargaining solution and KBS diverge when the constraint binds.[16] The following example shows how the two may diverge even when the liquidity constraint does not bind.

A buyer (she) bargains with a seller (he) over the quantity of a good produced by the latter, $y \in \mathbb{R}_+$, she wishes to purchase, and its price $p \in \mathbb{R}_+$. Preferences over the bargaining outcomes $(y, p) \in \mathbb{R}^2_+$ are represented by the utility functions

$$u^b = \alpha(u(y) - p), \qquad \textit{and}$$

$$u^s = -v(y) + p$$

for the buyer and seller respectively. u is strictly increasing, strictly concave, $u'(0) = \infty$ and $u(0) = 0$ while v is strictly increasing, strictly convex with $v(0) = 0$. Further, $u'(y^*) = v'(y^*)$ for some $y^* > 0$. Finally $\alpha \in \mathbb{R}_+$.[17]

Hu and Rocheteau (2020) study the case of $\alpha = 1$ and show that if a liquidity constraint ($p \leq z$) binds then the NBS and KBS solutions differ, but otherwise coincide. The NBS and KBS are assumed to have the same weights, namely θ for the buyer and $1 - \theta$ for the seller. The result suggests that the non-linearity of the utility possibility frontier due to the liquidity constraint is responsible for the different outcomes under the two solutions.

Assume away the liquidity constraint. The usual computations then verify

[16] See also Duffy et al. (2021).

[17] Assume all transactions are carried out using chips. The buyer can exchange y units of the commodity for $u(y)$ many chips while the seller needs to spend $v(y)$ many chips to produce y. The price is also in chips. Both players have linear utility in money and the buyer can cash in each chip for α dollars while the seller can cash in each chip for 1 dollar.

that under either NBS or KBS, the level of output is y^*, characterized by $u'(y^*) = v'(y^*)$, irrespective of α. The price under NBS, again irrespective of α is given by

$$p_\theta^{NBS} = (1-\theta)u(y^*) + \theta v(y^*).$$

By contrast, under KBS, the price is

$$p_\theta^{KBS} = \frac{(1-\theta)u(y^*) + \alpha\theta v(y^*)}{(1-\theta) + \alpha\theta}.$$

The utility possibility frontier is linear for all values of α and yet the prices are different. Further $p_\theta^{KBS} > (<) p_\theta^{NBS}$ if $\alpha < (>) 1$.

An application in family economics

The intra (marital) household resource allocation problem is typically framed as one resolved by bargaining between the spouses, summarized by the Nash bargaining solution.[18] The main explanatory variables are the outside options, reflecting divorce in the early literature, and later a within-marriage non-cooperative outcome following Lundberg and Pollak (1993). Missing in this specification is how gender and cultural norms may directly impact bargaining power (instead of indirectly, through outside options).[19] The literature has responded with the more flexible asymmetric Nash bargaining solution, but without a mechanism to translate the external environment into bargaining weights.

Lundberg and Pollak (1996) state that *Norms regarding appropriate marital or parental behavior for men and women may be powerful in their ability to channel the behavior of marital partners to one equilibrium among many.* Proposition 11 offers an explicit channel through which norms influence bargaining power. For instance, in patriarchal societies, such as those studied in Andersen et al. (2018), males face relatively higher concession costs than women. The opposite is true for matriarchal societies. This would mean, assuming player 1 to be female and 2 male, a value of γ more (less) than 1 in

[18]See Doepke and Kindermann (2017) for an excellent overview of the literature.
[19]See, for instance, the experimental results of Andersen et al. (2018).

matriarchal (patriarchal) societies.

5 Discussion

5.1 Concession Costs

The commitment ability at the heart of the bargaining model is generated by the cost an agent must pay from backing down from her current incompatible demand.[20] It is therefore important to discuss the relevant features of these concession cost functions. Following the Nash program mandate, I begin by listing a few ways in which such costs arise and the forms they take.

Examples

(i) *Audience Costs*: Elected representatives negotiating on behalf of their constituents are punished with a dimmer re-election prospect (the cost) for backing down from a publicly announced demand. In international negotiations this cost is generated by the domestic political audience, and has been studied in some detail following the work of Fearon (1994) and Martin (1993). Tomz (2007) provides direct evidence of these audience costs through experiments embedded in public opinion surveys.

In domestic negotiations between rival political parties the level of public support generated by each competing demand determines its audience cost. The greater the support for an announced demand the higher the cost of backing away from it. Leventoglu and Tarar (2005) and Basak and Deb (2020) study such concession costs in a Rubinstein bargaining model, where each player gets only one attempt at commitment.

(ii) *Delegated Bargaining*: Negotiations between two entities are often carried out by representatives (delegates) armed with appropriate incentives. A penalty for backing down from an announced demand is one such incentive. Indeed, the example in Schelling (1956) of a union official bargaining on behalf of the members has this feature; concession raises the odds of the official getting fired. Under this interpretation of concession costs, the form of delegation is exogenous in this paper. For other forms of (endogenous) delegation in

[20] By contrast, bargainers are not held to historical demands that led to an impasse.

bargaining see Crawford and Varian (1979), Jones (1989), Segendorff (1998) and Harstad (2008).

(iii) *Face*: Perhaps the most pervasive form of concession costs, but the least studied in economics, consists of *losing face*. Carefully detailed in Ho (1976), the concept of face, Chinese in origin, corresponds to a notion of social standing that is distinct from status, prestige, dignity, and the like. Unlike a binary variable, it can vary quantitatively in a gradual manner. Furthermore, the relevant quantity of face for an individual depends on the social situation of the interaction. As Ho points out, *It is the extent to which a particular person's social functioning is adversely affected that constitutes the true measure of what losing face means to him.* In the current setting, concession leads to losing face. This form of concession costs allows for a variety of social, political and historical features to translate into bargaining power in the model. For example, concession may lead to greater (or lesser) loss of face for a man compared to a woman, depending on gender norms.

Structure of the cost function

The qualitative results of the bargaining model, as in section 4, require only that the concession cost functions satisfy assumption 2. This is consistent with the examples above that suggest little structure for the functions other than it be increasing in the conceded amount. In particular, the cost functions need not be linear (as in section 3) or even convex. In many models of economic decision making, the cost from taking some productive action is assumed to be convex to ensure the overall objective function remains concave and admits an interior optimal solution. In the current setting, concession costs are incurred only off the equilibrium path. Therefore, despite their key role in the model, the curvature of these functions plays no role.

In line with the examples above, an agent's concession cost in the model is, in a sense, independent of how much she cares about the surplus. For instance, the audience cost faced by an elected representative is determined by how much the domestic audience cares to punish the agent for different degrees of concession. The cost is not directly related to how much the agent herself values different surplus splits. This feature is captured in two ways by the model. First, the size of the concession depends on the agents' "physical" demands and not payoff levels. Second, the concession cost is additively separable. These in turn make the (large marginal cost) limit ratio of the commitment costs independent of the payoff functions, which is a key component of proposition 11. Like the Kalai solution, the equilibrium prediction at this limit varies with the payoff function.

Relation to renegotiation proofness

The choice to concede is an individual one (made in equilibrium) and arises *after* incompatible demands. It depends on how far apart the incompatible demands are, the cost function and anticipated future play. Renegotiation-proofness rules out incompatible demands *before* they are made, by requiring agents to coordinate away from Pareto dominated equilibria. Once incompatible demands are made, however, concession (in that period) is costly. Higher concession cost functions reflect sharper incentives from sources like in the examples above, and are unrelated to the agents' coordination ability.

5.2 Simultaneous versus sequential demands

The importance of simultaneous versus sequential *demands* is, in a sense, a superficial one in the current setting. Instead, the two key feature are the following. First, the demand made by one agent is not just met by a decision to accept or reject, but by a competing demand from the other agent. Second, once two incompatible demands arrive at the table, the decision to accept or reject (stick) is made simultaneously. The latter concession stage is where the agents's concerns about concession costs and patience combine to determine her bargaining strength, and simultaneity here is important. By contrast, requiring the demands to be made in some arbitrarily fixed order, has a lot less impact. For instance, all RP (or no-delay) SPE outcomes identified in propositions 8 and 5 continue to be supported by SPE, with sequential *demands* that follow some fixed order. The specification matches descriptive accounts of bilateral negotiations that associate a round of bargaining with two competing positions. See for instance, the evolution EU-UK positions on citizen's rights during Brexit negotiations in 2017 listed in Department for Exiting the European Union and Home Office (2017).

5.3 Other Related Literature

Ellingsen and Miettinen (2014) (henceforth EM) extend the static model of Ellingsen and Miettinen (2008) to a fairly involved dynamic model. Formalizations of Schelling's ideas are usually closely related to the Nash demand game. The EM model has elements of both the Nash demand game (simultaneous demands) and the generalized Rubinstein bargaining framework. As examples of the latter, (a) following demands that are more than compatible, a single responder is selected randomly to accept or reject the other's offer and (b) following a choice of flexibility by both bargainers, a single player is randomly selected to make an offer for that period. The key difference with the current formalization, however, is that in EM (as well as Ellingsen and Miettinen (2008)) commitment ability is exogenous and independent of the actual demands made by the players. It does not matter whether a bargainer is offered a lot of room to back down or none at all, her commitment ability is pinned down by an exogenous randomization device. This distinction is critical, since in the current study the strategic feature that resolves the bargaining problem, is precisely the ability of bargainers to affect each other's commitment ability by choosing appropriate demands.

The delay obtained in Markov perfect equilibrium in section 3.2 is neither the result of money burning as in Avery and Zemsky (1994) nor due to strategic uncertainty as in Friedenberg (2019). In a sense, as Sakovics (1993) puts it, the delay is wholly ritualistic and can be expected in settings where bargainers take their cues from norms or traditions that are perhaps optimal in some larger context but offer an inefficient prescription in the specific bargaining instance. Similar equilibria also arise in Perry and Reny (1993) and Sakovics (1993), who study a generalization of the Rubinstein model with less restriction on when offers can be made and responded to. A key finding in both is that allowing for simultaneous demands generates an acute multiplicity of equilibria including those with delay. While not their focus, the SPE with delay in these models feature a milder form of the gradualism that appears in the current study. The further away the anticipated agreement, the further apart the incompatible demands need to be to deter deviation to a compatible profile. As stated earlier, in the current study the incompatible demands need to be even further apart to rule out deviations to incompatible profiles. Compte and Jehiel (2004) provides a wholly different rationale for gradualism. Players always have access to outside options whose values depend on past offers. If more favourable offers increase the value of the opponent's outside option, then bargainers find it optimal to lower their demand gradually in equilibrium.

A Appendix

Lemma 3 *σ cannot be an SPE in the general model, if for some $h \in H$, $\sigma(h) = z$ such that $z_i = 1$ and $z_{-i} = 0$ for some $i \in \{1, 2\}$.*

Proof. Suppose under σ, in the subgame $g(h)$, the two players make the compatible demands $z_i = 1$ and $z_{-i} = 0$, and player $-i$ obtains a payoff of $u_{-i}(0) = 0$. The highest payoff player i could get if bargaining broke down this period is $\delta_i u_i(1)$. Notice that $u_i(1 - \hat{z}_{-i}) - c_i(z_i + \hat{z}_{-i} - 1)$ is a continuous (decreasing) function of $\hat{z}_{-i}$. It takes a value of $u_i(1)$ at $\hat{z}_{-i} = 0$, which is strictly greater than $\delta_i u_i(1)$. Therefore there exists $\hat{z}_{-i} > 0$ such that $u_i(1 - \hat{z}_{-i}) - c_i(z_i + \hat{z}_{-i} - 1) > \delta_i u_i(1)$. Now, if player $-i$ were to deviate to this $\hat{z}_{-i}$ instead of demanding 0, then in the subsequent concession game the dominance solvable outcome would involve player i playing A and $-i$ playing S. Since this is a profitable deviation, the strategy profile σ cannot be an SPE.

■

Proof for Proposition 2.

The necessity of $x \in B^*$ follows by definition. By way of contradiction, suppose that (x, t) is an SPE outcome with $t > 1$, $x \in B^*$ and

$$\delta_1^{t-1} x_1 < \frac{1 - \delta_2 z_2^*}{1 + k_2}$$

where $z_2^* = \sup_{z \in B^*} z_2$. It then suffices to show that player 1 is better off deviating from her first period incompatible demand.

Consider the first period incompatible demand profile, z^1 in such an SPE. It must be that $z_i^1 \geq 1 - \delta^{t-1} x_{-i}$ for $i \in \{1, 2\}$. Otherwise player i could profitably deviate to making the compatible demand $1 - z_{-i}^1$ in period 1.

Fix some continuation payoff profile w. Then the set of incompatible demand profiles y for which player 2 is indifferent between A and S, conditional on player 1 choosing S, is given by the the equation $1 - y_1 - k_2(y_1 + y_2 - 1) = w_2$. Rewrite this as $y_1 = 1 - \frac{k_2}{1+k_2} y_2 - \frac{w_2}{1+k_2}$. The best continuation payoff for player 2 is $\delta_2 z_2^*$. Let

$$y_1^*(y_2) = 1 - \frac{k_2}{1 + k_2} y_2 - \frac{\delta_2 z_2^*}{1 + k_2}.$$

Notice that for any incompatible demand profile y, with $y_1 < y_1^*(y_2)$, player 2 strictly prefers A to S, conditional on 1 choosing S, if her continuation payoff is $\delta_2 z_2^*$. Further, since any SPE continuation payoff w_2 is no greater than $\delta_2 z_2^*$, following incompatible profile y with $y_1 < y_1^*(y_2)$, player 2 strictly prefers A to S, conditional on 1 choosing S, *for any SPE continuation profile.*

Given continuation payoff profile w, the set of incompatible demand profiles y for which player 1 is indifferent between A and S, conditional on player 2 choosing S, satisfies the equation $1 - y_2 - k_1(y_1 + y_2 - 1) = w_1$. Rewrite this as $y_1 = (1 - y_2)\frac{1+k_1}{k_1} - w_1 \frac{1+k_1}{k_1}$. Let

$$y_1^{**}(y_2) = (1 - y_2)\frac{1 + k_1}{k_1}.$$

Then for any incompatible demand profile y with $y_1 > y_1^{**}(y_2)$, player 1 strictly prefers to S to A, *for any SPE continuation profile.*

Return to the premise of player 1's SPE payoff $\delta_1^{t-1} x_1$ and first period incompatible profile z^1. By the inequalities derived above, if for all $z_2^1 \geq 1 - \delta^{t-1} x_1$, the inequalities $y_1^*(z_2^1) > \delta_1^{t-1} x_1$ and $y_1^*(z_2^1) > y_1^{**}(z_2^1)$ hold, then a contradiction obtains. Player 1 could then profitably deviate in period 1 to making an incompatible demand $y_1^*(z_2^1) > \hat{z}_1^1 > \max\{y_1^{**}(z_2^1), \delta_1^{t-1} x_1\}$ and force player 2 to concede, no matter the SPE continuation profile.

Next observe that $y_1^*(1) = \frac{1-\delta_2 z_2^*}{1+k_2}$. Since $\delta_1^{t-1} x_1 < \frac{1-\delta_2 z_2^*}{1+k_2}$ (by assumption) and y_1^* is a decreasing function, it follows that $y_1^*(z_2^1) > \delta_1^{t-1} x_1$ for all $z_2^1 \geq 1 -$

$\delta^{t-1}x_1$. Since y_1^{**} is also a decreasing linear function and $y_1^{**}(1) = 0$, to obtain the contradiction, it suffices to show that $y_1^*(1 - \delta_1^{t-1}x_1) > y_1^{**}(1 - \delta_1^{t-1}x_1)$. Some computation shows that $y_1^*(1 - v) \leq y_1^{**}(1 - v)$ requires

$$v \geq \frac{k_1(1 - \delta_2 z_2^*)}{1 + k_1 + k_2}.$$

Since $\delta_1^{t-1}x_1 < \frac{1-\delta_2 z_2^*}{1+k_2}$, the contradiction would obtain if

$$\frac{1 - \delta_2 z_2^*}{1 + k_2} < \frac{k_1(1 - \delta_2 z_2^*)}{1 + k_1 + k_2}.$$

The inequality indeed follows from the assumption of $k_2(k_1 - 1) > 1$. This concludes the proof for $i = 1$. A symmetric argument works for $i = 2$.

■

Proof for Proposition 3.

Let $O^{SPEd} = \{(z,t) \in O^{SPE} | t > 1\}$ collect all SPE outcomes that feature delay and $D = \{(w_1, w_2) | w_i = \delta^t z_i \textit{ for } i \in \{1,2\} \textit{ and } (z,t) \in O^{SPEd}\}$ be the set of continuation payoffs such SPE with delay generate. Let $w_i^m = \inf_{w \in D} w_i$. Recall that $B^* = \{z | (z,t) \in O^{SPE}\}$. Let $z_i^* = \sup_{z \in B^*} z_i$.

I first show that there cannot be a deviation $\hat{z}_2 > 1 - z_1^*$ such that

$$1 - z_1^* - k_2(z_1^* + \hat{z}_2 - 1) < \min\{w_2^m, \delta_2(1 - z_1^*)\} \tag{16}$$

and

$$1 - \hat{z}_2 - k_1(z_1^* + \hat{z}_2 - 1) > \delta_1 z_1^*. \tag{17}$$

The right hand side (RHS) of inequality 16 gives the worst SPE continuation payoff for player 2, while the RHS of inequality 17 gives the best SPE continuation payoff for player 1. The existence of such a $\hat{z}_2$ means that player 2 has a profitable deviation from the efficient profile $(z_1^*, 1 - z_1^*)$. This is because following such a deviation, in the resulting concession game player 2's choice of S strictly dominates A, *irrespective of the SPE continuation payoff*, due to inequality 16. Further, due to inequality 17, player 1 strictly prefers A over S, in the face of 2 choosing S. In other words, following the deviation to $\hat{z}_2$, the dominance solvable outcome of the concession game is (A, S) and brings 2 the higher payoff of $\hat{z}_2$, no matter the continuation SPE profile. Such a $\hat{z}_2$ rules out $(z_1^*, 1 - z_1^*) \in B^*$ and by continuity rules out $z_1^* = \sup_{z \in B^*} z_1$.

Inequality 16 simplifies to

$$\hat{z}_2 > \frac{(1 - z_1^*)(1 + k_2) - \min\{w_2^m, \delta_2(1 - z_1^*)\}}{k_2}$$

while 17 simplifies to

$$\hat{z}_2 < 1 - \frac{(k_1 + \delta_1)z_1^*}{1 + k_1}.$$

Therefore $z_1^* = \sup_{z \in B^*} z_1$ requires

$$\frac{(1 - z_1^*)(1 + k_2) - \min\{w_2^m, \delta_2(1 - z_1^*)\}}{k_2} \geq 1 - \frac{(k_1 + \delta_1)z_1^*}{1 + k_1}.$$

By proposition 2, $w_2^m \geq \frac{1 - \delta_1 z_1^*}{1 + k_1}$. Then the relevant inequality is

$$\frac{(1 - z_1^*)(1 + k_2) - \min\{\frac{1 - \delta_1 z_1^*}{1 + k_1}, \delta_2(1 - z_1^*)\}}{k_2} \geq 1 - \frac{(k_1 + \delta_1)z_1^*}{1 + k_1}.$$

There are two cases to consider. If $\frac{1 - \delta_1 z_1^*}{1 + k_1} \geq \delta_2(1 - z_1^*)$ then the inequality reduces to

$$\frac{z_1^*}{1 - z_1^*} \leq \frac{1 - \delta_2}{1 - \delta_1} \frac{1 + k_1}{k_2}. \tag{18}$$

Alternatively, if $\frac{1 - \delta_1 z_1^*}{1 + k_1} < \delta_2(1 - z_1^*)$ then the inequality reduces to

$$\frac{z_1^*}{1 - z_1^*} < \frac{k_1}{(1 + k_2)(1 - \delta_1)}.$$

Now, $\frac{1 - \delta_1 z_1^*}{1 + k_1} \geq \delta_2(1 - z_1^*)$ itself simplifies to

$$\frac{z_1^*}{1 - z_1^*} \geq \frac{\delta_2(1 + k_1) - 1}{1 - \delta_1}.$$

Since in this case, inequality 18 emerges, it must be that

$$\frac{1-\delta_2}{1-\delta_1}\frac{1+k_1}{k_2} \geq \frac{\delta_2(1+k_1)-1}{1-\delta_1}.$$

It turns out that this inequality is equivalent to

$$\frac{1-\delta_2}{1-\delta_1}\frac{1+k_1}{k_2} \geq \frac{k_1}{(1+k_2)(1-\delta_1)}.$$

This generates the required expression

$$\frac{z_1^*}{1-z_1^*} \leq \max\left\{\frac{1-\delta_2}{1-\delta_1}\frac{1+k_1}{k_2}, \frac{k_1}{(1+k_2)(1-\delta_1)}\right\}.$$

■

Lemma 4 *The strategy profile σ described in Construction 1 is an SPE if*

$$\frac{1-\delta_1}{1-\delta_2}\frac{k_2}{1+k_1} \leq \frac{z_2}{z_1} \leq \frac{1-\delta_1}{1-\delta_2}\frac{1+k_2}{k_1}.$$

Proof. The payoff to player i from σ at any subgame $g(h)$ with $h \in H$ is simply z_i. A lower demand would only lower the payoff. A higher demand would lead to either (S,S) and a continuation payoff of $\delta_i z_i$ or (A_i, S_{-i}) leading to a payoff strictly lower than z_i due to the resulting concession cost. Therefore no player has an incentive to deviate in the demand stage of any period.

To verify subgame perfection, therefore, it is sufficient to show that in the concession stage game following an incompatible demand profile $(\hat{z}_i, z_{-i})$, if (S,S) is not a Nash equilibrium then (A_i, S_{-i}) is. To establish this result, in turn, it is sufficient to show the following,

$$1-\hat{z}_i - k_{-i}(\hat{z}_i + z_{-i} - 1) > \delta_{-i} z_{-i} \Rightarrow 1 - z_{-i} - k_i(\hat{z}_i + z_{-i} - 1) > \delta_i z_i$$

which is equivalent to

$$1 - \frac{\delta_{-i} z_{-i} + k_{-i} z_{-i}}{1+k_{-i}} > \hat{z}_i \Rightarrow \frac{(1-z_{-i})(1+k_i-\delta_i)}{k_i} > \hat{z}_i.$$

A sufficient condition for this is simply

$$
\begin{aligned}
&\frac{(1-z_{-i})(1+k_i-\delta_i)}{k_i} > 1 - \frac{\delta_{-i}z_{-i}+k_{-i}z_{-i}}{1+k_{-i}} \\
\Leftrightarrow\ &\frac{(1-z_{-i})(1-\delta_i)}{k_i} > \frac{z_{-i}(1-\delta_{-i})}{1+k_{-i}} \\
\Leftrightarrow\ &\frac{1-\delta_i}{1-\delta_{-i}}\frac{1+k_{-i}}{k_i} > \frac{z_{-i}}{1-z_{-i}}.
\end{aligned}
$$

Requiring the above inequalities to hold for $i \in \{1,2\}$ make them equivalent to

$$\frac{1-\delta_1}{1-\delta_2}\frac{k_2}{1+k_1} \leq \frac{z_2}{z_1} \leq \frac{1-\delta_1}{1-\delta_2}\frac{1+k_2}{k_1}.$$

■

Proof for Proposition 4.

The set E^1 is a set of SPE outcomes, by definition. This set is fully characterized in proposition 5. Further, by lemma 4, any element in E^1 is the outcome of an SPE strategy profile described in construction 1. So to prove the proposition by induction it suffices to show that for any $t-1 \in \mathbb{N}$, if every outcome in E^{t-1} is an SPE outcome then the same holds for E^t.

Fix some $t-1 \in \mathbb{N}$ such that every outcome in E^{t-1} is an SPE outcome. If $E^t = E^{t-1}$, then the result follows. Suppose instead that $E^{t-1} \neq E^t$. ζ^t is simply the set of continuation payoffs generated by the outcomes in E^{t-1}. Fix some $w \in \zeta^t$ and some $z \in Z^t_w$. I will now describe an SPE profile in which z is the compatible demand profile chosen in the first period.

Construction 2 *In period 1 player i demands z_i, for $i \in \{1,2\}$. If bargaining is unresolved in period 1 then from period 2 onwards the players play the SPE strategy profile that yields the continuation profile w. In period 1, if player i deviates to a higher demand, $\tilde{z}_i > z_i$, then in the concession stage game (S,S) is played if it is a Nash equilibrium, and otherwise (A_i, S_{-i}) is played. For all other $h \in H^{1'}$ some Nash equilibrium of the concession stage game is played.*

To show that this construction is an SPE it suffices to prove two claims. First,

$z_i \geq w_i$ for $i \in \{1, 2\}$, in that neither player wishes to deviate in the first period in a way that leads to the continuation payoff w. Second, in the concession stage game following the deviation $\tilde{z}_i > z_i$, it is true that when (S, S) is not a Nash equilibrium then (A_i, S_{-i}) is. Indeed if these two claims were true, then no player i would wish to deviate from the compatible demand profile z in the first period. This is because such deviations would either bring i a lower payoff due to lower demand by her, or in the case of a higher and therefore incompatible demand by her, would bring a lower payoff of w_i or the lower payoff from backing down in the concession stage.

Recall that y^{tw} is the solution to the equations

$$\begin{aligned} 1 - y_2 - k_1(y_1 + y_2 - 1) &= w_1 \\ 1 - y_1 - k_2(y_1 + y_2 - 1) &= w_2 \end{aligned} \tag{19}$$

and must therefore satisfy $y_1^{tw} + y_2^{tw} > 1$. To see why, add the two equations in 19 to get $1 + (1 - y_1 - y_2)(1 + k_1 + k_2) = w_1 + w_2$. If $y_1 + y_2 \leq 1$ then $w_1 + w_2 \geq 1$, a contradiction, since w is the continuation payoff and $\delta_i < 1$ for $i \in \{1, 2\}$. Since $y_1^{tw} + y_2^{tw} > 1$, it follows that $w_i < 1 - y_{-i}^{tw}$ from 19. Also, $z_{-i} \leq y_{-i}^{tw}$, by construction, which means $z_i \geq 1 - y_{-i}^{tw}$. This establishes the first claim

$$z_i \geq 1 - y_{-i}^{tw} > w_i.$$

To establish the second claim it is sufficient to show that

$$1 - \tilde{z}_i - k_{-i}(\tilde{z}_i + z_{-i} - 1) > w_{-i} \Rightarrow 1 - z_{-i} - k_i(\tilde{z}_i + z_{-i} - 1) > w_i.$$

The equation $1 - y_{-i} - k_i(y_i + y_{-i} - 1) = w_i$ has a slope $\frac{\partial y_i}{\partial y_{-i}} < -1$ while the equation $1 - y_i - k_{-i}(y_i + y_{-i} - 1) = w_{-i}$ has a slope $-1 < \frac{\partial y_i}{\partial y_{-i}} < 0$. This means that for all values of y_{-i} less than at the point of intersection y_{-i}^{tw}, the line $1 - y_{-i} - k_i(y_i + y_{-i} - 1) = w_i$ lies above the line $1 - y_i - k_{-i}(y_i + y_{-i} - 1) = w_{-i}$. Since $z_{-i} \leq y_{-i}^{tw}$ by construction, if $1 - \tilde{z}_i - k_{-i}(\tilde{z}_i + z_{-i} - 1) > w_{-i}$ then it must be that $(\tilde{z}_i, z_{-i})$ lies below the line $1 - y_i - k_{-i}(y_i + y_{-i} - 1) = w_{-i}$. This in turn implies that $(\tilde{z}_i, z_{-i})$ lies below the line $1 - y_{-i} - k_i(y_i + y_{-i} - 1) = w_i$

and therefore, $1 - z_{-i} - k_i(\tilde{z}_i + z_{-i} - 1) > w_i$, as required.

This concludes the argument showing that construction 2 is an SPE. It has therefore been shown that for all $z \in Z^t$ there exists an SPE in which z is the compatible demand profile in the first period.

Fix some $(y, n) \in \tilde{E}^t$. Since $y \in Z^t$, by the argument above there exists an SPE strategy profile with the compatible demand profile y in the first period. If $n = 1$ then it has already been established that (y, n) is an SPE outcome. Suppose $n > 1$. Consider the following construction with outcome, (y, n).

Construction 3 *For periods 1 to $n-1$, both players demand 1 followed by S in the concession stage. From period n onwards, they play the SPE strategy profile that yields the outcome $(y, 1)$. For any history $h \in H^{t'}$, where $1 \leq t \leq n-1$, they play some Nash equilibrium of the corresponding concession stage game.*

To prove that this strategy profile is an SPE it is sufficient to show that neither player wishes to deviate from their demand of 1 in periods 1 to $n-1$. Consider period 1. The only way player i can change the outcome by deviating at the demand stage is by choosing a demand that is either compatible or on that forces $-i$ to concede in the subsequent concession stage. A compatible demand brings a payoff of 0 and therefore cannot be a profitable deviation. Since $-i$ is demanding 1, to make her prefer A to S in the concession stage, i must make a demand $\tilde{z}_i$ such that

$$1 - \tilde{z}_i - k_{-i}(\tilde{z}_i + 1 - 1) > \delta_{-i}^{n-1} y_{-i}.$$

This simplifies to

$$\tilde{z}_i < \frac{1 - \delta_{-i}^{n-1} y_{-i}}{1 + k_{-i}}.$$

Such a deviation brings i a payoff of $\tilde{z}_i$. To be profitable this requires $\tilde{z}_i > \delta_i^{n-1} y_i$ and therefore

$$\delta_i^{n-1} y_i < \frac{1 - \delta_{-i}^{n-1} y_{-i}}{1 + k_{-i}} \tag{20}$$

which simplifies to $1 - \delta_i^{n-1} y_i - \delta_{-i}^{n-1} y_{-i} > k_{-i} \delta_i^{n-1} y_i$. But this inequality does not hold since $(y, n) \in \tilde{E}^t$ and therefore satisfies inequality 8. Therefore no profitable deviation exists for i in period 1.

Finally notice that for a similar deviation in any period $1 \leq t \leq n-1$ to be profitable requires, by a symmetric argument,

$$\delta_i^{t-1} y_i < \frac{1-\delta_{-i}^{t-1} y_{-i}}{1+k_{-i}}.$$

Since this inequality does not hold for $t = n$ as in 20, it does not hold for the other values either. This concludes the proof of construction 3 satisfying subgame perfection. Therefore every outcome in $\tilde{E}^t$ is an SPE outcome. Since $E^t = E^{t-1} \cup \tilde{E}^t$, this concludes the proof. ■

Proof for Proposition 5. Lemma 3 above establishes that, even in the general model, compatible demand profiles in which one player demands the entire surplus cannot arise in an SPE. This combined with lemma 1 implies that if (z,t) is the outcome of an SPE then $d(z) = 0$ and $z_i \in (0,1)$ for $i \in \{1,2\}$. This in turn means that if σ is a renegotiation-proof SPE with outcome (z,t) then $t = 1$. To see why, suppose instead that $t > 1$. Then

$$\psi(\sigma; h^0) = (\delta_1^{t-1} z_1, \delta_2^{t-1} z_2) \ll (z_1, z_2) = \psi(\sigma, \tilde{h}^t),$$

where $\tilde{h}^t$ is the history that occurs on the equilibrium path with the $t-1$ periods of incompatible demands with neither player conceding in the subsequent concession games. Therefore by proposition 1, inequality 9 is a necessary condition for renegotiation-proof SPE outcomes. Lemma 4 establishes sufficiency by constructing stationary SPE strategies with outcome (z,t) for any z satisfying inequality 9 and $t = 1$. Fix one such z and its corresponding stationary SPE strategy profile, σ. Notice that σ satisfies renegotiation-proofness since by construction $\psi(\sigma; h) = (z_1, z_2)$ for all $h \in H$. ■

Lemma 5 *If (y,m) is the outcome of a Markov perfect equilibrium and $\delta_j \geq \delta_{-j}$ for some $j \in \{1,2\}$, then*

$$m - 1 \leq \left\lfloor \frac{\ln \frac{k_1+k_2}{k_1+k_2+k_1k_2}}{\ln \delta_j} \right\rfloor.$$

Proof. Suppose σ is a Markov perfect equilibrium with outcome (y, m) that features delay and so $m > 1$. Consider the demand profile $z^1 = \sigma(h^0)$, which must be incompatible, $d(z^1) > 0$. Let the continuation payoff profile following such demands be (w_1, w_2), which results from the outcome (y, m) of the subgame $g(h^0)$. In particular, $w_i = \delta_i^{m-1} y_i$. Further y is an exactly compatible demand profile by lemma 1, as in $y_1 + y_2 = 1$. $w_1 + w_2 < 1$ follows from $m > 1$.

First note that to be in equilibrium requires $z_i^1 \geq 1 - w_{-i}$ for $i \in \{1, 2\}$. Otherwise player $-i$ would be strictly better off making the compatible demand $1 - z_i^1$ in period t. Set $D = \{z | z_i \geq 1 - w_{-i}, \forall i \in \{1, 2\}\}$.

Next, observe that the equation $1 - y_i - k_{-i}(y_1 + y_2 - 1) = w_{-i}$ is satisfied at $y = (1 - w_{-i}, w_{-i})$. It follows that if $y_i \geq 1 - w_{-i}$ and $y_{-i} > w_{-i}$ then $1 - y_i - k_{-i}(y_1 + y_2 - 1) < w_{-i}$. Therefore for any $z \in D$ and any $\hat{z}_{-i} > w_{-i}$ the following inequality holds

$$1 - z_i - k_{-i}(z_i + \hat{z}_{-i} - 1) < w_{-i}.$$

This implies that for any incompatible profile in D played in period 1, player $-i$ can deviate to a demand arbitrarily close to w_{-i} and in the resulting concession stage game her action S would strictly dominate A. If following such a deviation player i preferred A to S, then $-i$ would indeed be better off with the deviation since she would obtain a higher payoff than w_{-i}.

For player i to prefer A to S following an incompatible demand profile y requires $1 - y_{-i} - k_i(y_1 + y_2 - 1) > w_i$. If this inequality is satisfied for y with $y_i = 1$ then it will be satisfied for all (x_i, y_{-i}) with $1 - w_{-i} \leq x_i \leq 1$. So if the inequality

$$1 - w_{-i} - k_i(1 + w_{-i} - 1) > w_i \tag{21}$$

holds then for any incompatible profile in D, player $-i$ can deviate to an appropriate demand greater than w_{-i} such that the unique dominance solvable outcome in the concession stage has i playing A and $-i$ playing S, with a payoff greater than w_{-i} to $-i$. So for D to contain some incompatible demand profile that can support the delay in period 1 under σ with continuation payoff w

requires

$$\frac{1}{k_i} \leq \frac{w_{-i}}{1 - w_1 - w_2}, \qquad \forall i \in \{1, 2\}.$$

A necessary condition for this is

$$\frac{1}{k_1} + \frac{1}{k_2} \leq \frac{w_1 + w_2}{1 - w_1 - w_2},$$

which simplifies to $w_1 + w_2 \geq (k_1 + k_2)/(k_1 + k_2 + k_1 k_2)$. Since $\delta_j \geq \delta_{-j}$, it follows that $\delta_j^{m-1} \geq (k_1 + k_2)/(k_1 + k_2 + k_1 k_2)$ is necessary in turn. The result follows. ■

Lemma 6 *If $\delta_j \geq \delta_{-j}$ then*

$$\frac{1 - \delta_j^n}{1 - \delta_{-j}^n} \leq \frac{1 - \delta_j^{n+1}}{1 - \delta_{-j}^{n+1}}.$$

Proof.

$$\begin{aligned}
\frac{1 - \delta_j^n}{1 - \delta_{-j}^n} \leq \frac{1 - \delta_j^{n+1}}{1 - \delta_{-j}^{n+1}} &\Leftrightarrow \frac{1 - \delta_{-j}^{n+1}}{1 - \delta_{-j}^n} \leq \frac{1 - \delta_j^{n+1}}{1 - \delta_j^n} \\
&\Leftrightarrow \frac{1 + \delta_{-j} + \delta_{-j}^2 + \ldots + \delta_{-j}^n}{1 + \delta_{-j} + \delta_{-j}^2 + \ldots + \delta_{-j}^{n-1}} \leq \frac{1 + \delta_j + \delta_j^2 + \ldots + \delta_j^n}{1 + \delta_j + \delta_j^2 + \ldots + \delta_j^{n-1}} \\
&\Leftrightarrow \frac{\delta_{-j}^n}{1 + \delta_{-j} + \delta_{-j}^2 + \ldots + \delta_{-j}^{n-1}} \leq \frac{\delta_j^n}{1 + \delta_j + \delta_j^2 + \ldots + \delta_j^{n-1}} \\
&\Leftrightarrow \frac{1}{\delta_j^n} + \frac{1}{\delta_j^{n-1}} + \ldots + \frac{1}{\delta_j} \leq \frac{1}{\delta_{-j}^n} + \frac{1}{\delta_{-j}^{n-1}} + \ldots + \frac{1}{\delta_{-j}} \\
&\Leftrightarrow \delta_j \geq \delta_{-j}.
\end{aligned}$$

■

Proof for Proposition 6. Let C^M denote the set of compatible demand profiles that can arise in some Markov perfect equilibrium. Let $z_i^* = \sup_{z \in C^M} z_i$. First I show that there cannot exist a $\hat{z}_2 > 1 - z_1^*$ such that

$$1 - z_1^* - k_2(z_1^* + \hat{z}_2 - 1) < \delta_2^n (1 - z_1^*) \tag{22}$$

and

$$1 - \hat{z}_2 - k_1(z_1^* + \hat{z}_2 - 1) > \delta_1^n z_1^* \tag{23}$$

for all $1 \leq n \leq n^*$ where $n^* - 1 = \left\lfloor \frac{\ln \frac{k_1+k_2}{k_1+k_2+k_1k_2}}{\ln \delta_j} \right\rfloor$. Consider a Markov perfect equilibrium in which $(z_1^*, 1 - z_1^*)$ is agreed upon in the first period. The equilibrium must specify a continuation payoff profile if the current period instead ended in an impasse. This must be some fixed $(\delta^n z_1, \delta^n z_2)$ where $z \in C^M$ and, by lemma 5, $1 \leq n \leq n^*$. It is fixed in the sense that the payoff is independent of the exact incompatible demands made, due to Markov perfection. If there exists a $\hat{z}_2$ that satisfies inequalities 22 and 23 for all $1 \leq n \leq n^*$ then it must also satisfy

$$1 - z_1^* - k_2(z_1^* + \hat{z}_2 - 1) < \delta_2^n(1 - z_1) \qquad \textit{and}$$

$$1 - \hat{z}_2 - k_1(z_1^* + \hat{z}_2 - 1) > \delta_1^n z_1$$

for any such $z \in C^M$ and $1 \leq n \leq n^*$, since $\delta_1^n z_1 \leq \delta_1^n z_1^*$ and $\delta_2^n(1 - z_1) \geq \delta_2^n(1 - z_1^*)$. This means that following the incompatible demand profile $(z_1^*, \hat{z}_2)$ the (unique) dominance solvable outcome in the concession game is $(\textit{Accept}, \textit{Stick})$, bringing player 2 the higher payoff of $\hat{z}_2$. So in this case, the compatible demand profile $(z_1^*, 1 - z_1^*)$ cannot arise in a Markov perfect equilibrium. The same argument applies to compatible demand profiles arbitrarily close to $(z_1^*, 1 - z_1^*)$. Therefore, it must be that no such $\hat{z}_2 > 1 - z_1^*$ exists that satisfies inequalities 22 and 23 for all $1 \leq n \leq n^*$.

Inequalities 22 and 23 simplify to

$$\hat{z}_2 > \frac{(1 - z_1^*)(1 + k_2 - \delta_2^n)}{k_2}, \qquad \textit{and}$$

$$\hat{z}_2 < 1 - \frac{(k_1 + \delta_1^n)z_1^*}{1 + k_1}.$$

Therefore a $\hat{z}_2$ satisfying inequalities 22 and 23 for a given $1 \leq n \leq n^*$ cannot exist only if

$$\begin{aligned}
&\frac{(1-z_1^*)(1+k_2-\delta_2^n)}{k_2} \geq 1 - \frac{(k_1+\delta_1^n)z_1^*}{1+k_1} \\
\Rightarrow &\frac{(1-z_1^*)(1-\delta_2^n)}{k_2} \geq \frac{z_1^*(1-\delta_1^n)}{1+k_1} \\
\Rightarrow &\frac{1-\delta_1^n}{1-\delta_2^n}\frac{k_2}{1+k_1} \leq \frac{1-z_1^*}{z_1^*}.
\end{aligned}$$

Finally then a $\hat{z}_2$ satisfying inequalities 22 and 23 for *all* $1 \leq n \leq n^*$ cannot exist only if

$$\min_{n\leq n^*} \frac{1-\delta_1^n}{1-\delta_2^n}\frac{k_2}{1+k_1} \leq \frac{1-z_1^*}{z_1^*}.$$

A symmetric argument establishes

$$\begin{aligned}
&\min_{n\leq n^*} \frac{1-\delta_2^n}{1-\delta_1^n}\frac{k_1}{1+k_2} \leq \frac{1-z_2^*}{z_2^*} \\
\Rightarrow &\frac{z_2^*}{1-z_2^*} \leq \max_{n\leq n^*} \frac{1-\delta_1^n}{1-\delta_2^n}\frac{1+k_2}{k_1}.
\end{aligned}$$

Since $z \in C^M$ implies that $(1-z_1^*)/z_1^* \leq z_2/z_1 \leq z_2^*/(1-z_2^*)$, it follows that

$$\min_{n\leq n^*} \frac{1-\delta_1^n}{1-\delta_2^n}\frac{k_2}{1+k_1} \leq \frac{z_2}{z_1} \leq \max_{n\leq n^*} \frac{1-\delta_1^n}{1-\delta_2^n}\frac{1+k_2}{k_1}.$$

By lemma 6, if $\delta_1 \geq \delta_2$ then

$$\frac{1-\delta_1^n}{1-\delta_2^n} \leq \frac{1-\delta_1^{n+1}}{1-\delta_2^{n+1}}.$$

So if $\delta_1 \geq \delta_2$ then

$$\min_{n\leq n^*} \frac{1-\delta_1^n}{1-\delta_2^n} = \frac{1-\delta_1}{1-\delta_2} \qquad \textit{and} \qquad \max_{n\leq n^*} \frac{1-\delta_1^n}{1-\delta_2^n} = \frac{1-\delta_1^{n^*}}{1-\delta_2^{n^*}}$$

and the result follows. A symmetric argument works for $\delta_2 \geq \delta_1$.

■

Proof for Proposition 7. Let σ be the Markov perfect equilibrium with the outcome (y, m). Let $z^t = \sigma(h^{t-1})$ for $h^{t-1} \in H$ and $1 \leq t \leq m-1$. By assumption z^t is an incompatible demand profile. In the subgame $g(h^{t-1})$, player i's payoff from following σ is $\delta_i^{m-t} y_i$. Then it must be that $z_i^t \geq 1 - \delta_{-i}^{m-t} y_{-i}$. Otherwise player $-i$ would do better by making the compatible demand $1 - z_i^t$. Set $D = \{z | z_i \geq 1 - \delta_{-i}^{m-t} y_{-i}\}$. So $z^t \in D$.

Next, there cannot exist $\hat{z}_{-i} > \delta_{-i}^{m-t} y_{-i}$ such that

$$1 - z_i^t - k_{-i}(z_i^t + \hat{z}_{-i} - 1) < \delta_{-i}^{m-t} y_{-i}$$

and

$$1 - \hat{z}_{-i} - k_i(z_i^t + \hat{z}_{-i} - 1) > \delta_i^{m-t} y_i.$$

Otherwise, player $-i$ in period t would deviate to the incompatible demand $\hat{z}_{-i}$ and the dominance solvable outcome of the resulting concession game would be (A_i, S_{-i}) with the higher payoff of $\hat{z}_{-i}$. It is already shown in the proof for lemma 5 that for all $z \in D$ there exists $\hat{z}_{-i} > \delta_{-i}^{m-t} y_{-i}$ such that $1 - z_i - k_{-i}(z_i + \hat{z}_{-i} - 1) < \delta_{-i}^{m-t} y_{-i}$.

Finally, requiring $1 - \hat{z}_{-i} - k_i(z_i^t + \hat{z}_{-i} - 1) \geq \delta_i^{m-t} y_i$ to hold for all $\hat{z}_{-i} > \delta_{-i}^{m-t} y_{-i}$ implies that

$$z_i^t \geq \frac{(1 - \delta_{-i}^{m-t} y_{-i})(1 + k_i) - \delta_i^{m-t} y_i}{k_i}.$$

■

Proof for Proposition 8. A stationary MPE must feature either immediate agreement or perpetual delay. Perpetual delay is ruled out since either player would deviate in the first period to making an arbitrarily small demand. This would either lead to a compatible demand profile, or if incompatible force the opponent to concede. Therefore stationary MPEs feature no delay. The result then follows from lemma 4 and proposition 1. ■

Lemma 7 *For $i \in \{1, 2\}$ and $n^* \in \mathbb{N}$, following equations 10, 11 and 12,*
(a) $\tilde{z}_{-i}^{n^*}$ is a well defined function with $-1 < \frac{\partial \tilde{z}_{-i}^{n^*}}{\partial z_i} < 0$,
(b) $\tilde{\tilde{z}}_{-i}^{n^*}$ is a well defined function with $\frac{\partial \tilde{\tilde{z}}_{-i}^{n^*}}{\partial z_i} < -1$,

(c) $z_i^{Mn^*}$ is well defined.

Proof. (a) Since c_{-i} is unbounded above, $\tilde{z}_{-i}^{n^*}$ is indeed well defined for all $z_i \in [0,1]$. Further by the implicit function theorem it is a decreasing function with slope

$$\frac{\partial \tilde{z}_{-i}^{n^*}}{\partial z_i} = -\frac{(1-\delta_{-i}^{n^*+1})u'_{-i}(1-z_i)}{c'_{-i}(z_i+\tilde{z}_{-i}^{n^*}-1)} - 1 < -1.$$

(b) Again by the implicit function theorem, $\tilde{\tilde{z}}_{-i}^{n^*}$ is well defined, decreasing and with slope

$$\frac{\partial \tilde{\tilde{z}}_{-i}^{n^*}(z_i)}{\partial z_i} = -\frac{\delta_i u'_i(z_i) + c'_i(z_i+\tilde{\tilde{z}}_{-i}^{n^*}-1)}{u'_i(1-\tilde{\tilde{z}}_{-i}^{n^*}) + c'_i(z_i+\tilde{\tilde{z}}_{-i}^{n^*}-1)} > -1$$

by the concavity of u_i.

(c) Note that $\tilde{z}_{-i}^{n^*}(1) = 0$ while $\tilde{\tilde{z}}_{-i}^{n^*}(1) > 0$. Also, $\tilde{z}_{-i}^{n^*}(0) > 1$ while $\tilde{\tilde{z}}_{-i}^{n^*}(0) = 1$. Therefore the function $\tilde{z}_{-i}^{n^*}(z_i) - \tilde{\tilde{z}}_{-i}^{n^*}(z_i)$ is positive at $z_i = 0$, negative at $z_i = 1$, continuous and (from the slope inequalities above) strictly decreasing over the interval $[0,1]$. By the intermediate value theorem it follows that $z_i^{Mn^*}$ is well defined and unique. ■

Proof for Proposition 9. Let $z_i^{n^*} = \sup_{z \in B^{n^*}} z_i$. Then there cannot exist a deviation $\hat{z}_2 > 1 - z_1^{n^*}$ such that

$$u_2(1-z_1^{n^*}) - c_2(z_1^{n^*}+\hat{z}_2-1) < \delta_2^{n^*+1}u_2(1-z_1^{n^*}) \tag{24}$$

and

$$u_1(1-\hat{z}_2) - c_1(z_1^{n^*}+\hat{z}_2-1) > \delta_1 u_1(z_1^{n^*}). \tag{25}$$

To see why, suppose that $\sigma(h^0) = (z_1^{n^*}, 1-z_1^{n^*})$ and there exists $\hat{z}_2$ that satisfies the inequalities above. Then it must be that

$$u_2(1-z_1^{n^*}) - c_2(z_1^{n^*}+\hat{z}_2-1) < \delta_2^{t+1}u_2(z_2)$$

and

$$u_1(1-\hat{z}_2) - c_1(z_1^{n^*}+\hat{z}_2-1) > \delta_1^{t+1}u_1(z_1)$$

for any outcome $(u(z), t)$ of an SPE with maximum delay n^*, since for all such $(u(z), t)$, it follows that $z_1 \leq z_1^{n^*}$ and $z_2 = 1 - z_1 > 1 - z_1^{n^*}$ and $1 \leq t+1 \leq n^*+1$. In other words, irrespective of the continuation strategy profile, following such a deviation, in the resulting concession stage game, the dominance solvable outcome would be (A, S), giving player 2 the payoff $u_2(\hat{z}_2)$ which is strictly greater than $u_2(1 - z_1^{n^*})$. Therefore, if such a deviation were to exist then $(z_1^{n^*}, 1 - z_1^{n^*}) \notin B^{n^*}$. The same argument ensures that $z \notin B^{n^*}$ for z arbitrarily close to $(z_1^{n^*}, 1 - z_1^{n^*})$, which in turn contradicts $z_1^{n^*} = \sup_{z \in B^{n^*}} z_1$.

Lemma 7 shows that $\tilde{z}_2^{n^*}$ and $\tilde{\tilde{z}}_2^{n^*}$ are well defined functions with $\tilde{z}_2^{n^*}(z_1) - \tilde{\tilde{z}}_2^{n^*}(z_1)$ strictly decreasing over the interval $[0, 1]$ and equal to zero at $z_1^{Mn^*}$. Now, it cannot be that $\tilde{\tilde{z}}_2^{n^*}(z_1^{n^*}) > \tilde{z}_2^{n^*}(z_1^{n^*})$ since then a deviation that satisfies inequalities 24 and 25 would exist; any $\hat{z}_2 \in (\tilde{z}_2^{n^*}(z_1^{n^*}), \tilde{\tilde{z}}_2^{n^*}(z_1^{n^*}))$ would suffice. Since $\tilde{\tilde{z}}_2^{n^*}(z_1) > \tilde{z}_2^{n^*}(z_1)$ for any $z_1 > z_1^{Mn^*}$, it must be that $z_1^{n^*} \leq z_1^{Mn^*}$. A symmetric argument establishes that $z_2^{n^*} \leq z_2^{Mn^*}$. ■

Proof for Proposition 10.

Necessity

By lemma 1, any SPE at any history $h \in H$ will involve exactly compatible demands or incompatible ones followed by (S, S). SPE that further satisfy renegotiation-proofness cannot permit delay. To see this, consider a strategy profile, σ with outcome (y, t) where $t > 1$. By lemma 1, $y_i = u_i(z_i)$ with $d(z) = 0$. By lemma 3, $y_i > 0$. Now, $c(\sigma; h^0) = (\delta_1^t y_1, \delta_2^t y_2)$ while $c(\sigma; h^t) = (y_1, y_2)$. Since $c(\sigma; h^t) \gg c(\sigma; h^0)$, σ is not renegotiation-proof. This concludes the argument for why $t = 1$ if (y, t) is the outcome of a renegotiation-proof SPE in the general model. The rest follows from Proposition 9.

Sufficiency

Fix some z such that $d(z) = 0$ and $z_i \leq z_i^M$ for $i \in \{1, 2\}$. Consider the following stationary strategy profile, σ. For all $h^t \in H$, $\sigma_i(h^t) = z_i$. If player i in period t deviates to making a higher demand, $\hat{z}_i > z_i$, then in the concession stage game (S, S) is played if it is a Nash equilibrium and otherwise (A_i, S_{-i}) is played. For all other $h \in H'$ some pure strategy Nash equilibrium of the concession stage game is played.

Given the strategy profile σ, it is clear that making a lower demand at any period is never profitable. Making a higher demand for player i also yields her a lower payoff, since it either leads to (S,S) in the concession game and a continuation payoff of $\delta_i u_i(z_i)$ or (A_i, S_{-i}) with a payoff strictly less than $u_i(z_i)$ due to the concession cost. Hence no profitable deviation exists in any demand stage. To verify subgame perfection, therefore, it is sufficient to verify that following an incompatible demand profile $(z_i, \hat{z}_{-i})$, if (S,S) is not a Nash equilibrium then (S_i, A_{-i}) is. For this it is sufficient to show that $\tilde{z}^0_{-i}(z_i) \geq \tilde{\tilde{z}}^0_{-i}(z_i)$.

Recall that $\tilde{z}^0_{-i}(z_i)$ as defined in equation 10, with $n^* = 0$, corresponds to the smallest demand by $-i$ that leads to incompatibility and ensures that $-i$ prefers S over A in the subsequent concession stage game, assuming that in the next period the compatible profile z is announced. $\tilde{\tilde{z}}^0_{-i}(z_i)$, as defined in equation 11, with $n^* = 0$, in turn is the largest demand by $-i$ that leads to incompatibility and ensures that in the subsequent concession game, i prefers (A_i, S_{-i}) to (S,S), assuming that in the next period z is announced. So if $\tilde{z}^0_{-i}(z_i) \geq \tilde{\tilde{z}}^0_{-i}(z_i)$ then following any incompatible demand $\hat{z}_{-i}$, if (A_i, S_{-i}) is a Nash equilibrium, then it must be that $\hat{z}_{-i} \leq \tilde{\tilde{z}}^0_{-i}(z_i) \leq \tilde{z}^0_{-i}(z_i)$ and therefore (S_i, A_{-i}) is a Nash equilibrium too. Since (A,A) is never a Nash equilibrium, this shows that with $\tilde{z}^0_{-i}(z_i) \geq \tilde{\tilde{z}}^0_{-i}(z_i)$ if (S,S) is not a Nash equilibrium then (S_i, A_{-i}) must be.

Finally observe that $\tilde{z}^0_{-i}(z_i) \geq \tilde{\tilde{z}}^0_{-i}(z_i)$ since $z_i \leq z_i^M$. ■

Proof for Proposition 11. It follows from proposition 10 that

$$\xi(g^{c^n}) = \left\{ y = u(z) \left| \frac{1 - z_1^{Mn}}{z_1^{Mn}} \leq \frac{z_2}{z_1} \leq \frac{z_2^{Mn}}{1 - z_2^{Mn}} \text{ and } d(z) = 0 \right. \right\},$$

where the incompatible demand profile $(z_i^{Mn}, \hat{z}_{-i}^{Mn})$ for $i \in \{1,2\}$ is characterized by the equations,

$$u_{-i}(1 - z_i^{Mn}) - c^n_{-i}(z_i^{Mn} + \hat{z}^n_{-i}(z_i^{Mn}) - 1) = \delta_{-i} u_{-i}(1 - z_i^{Mn}) \tag{26}$$

$$u_i(1 - \hat{z}^n_{-i}(z_i^{Mn})) - c^n_i(z_i^{Mn} + \hat{z}^n_{-i}(z_i^{Mn}) - 1) = \delta_i u_i(z_i^{Mn}). \tag{27}$$

Set $z_i^{M*} = \lim_{n\to\infty} z_i^{Mn}$. Notice that since u_{-i} is bounded above and $c^{n'}_{-i}(0+) \to$

∞ as $n \to \infty$, it follows from equation 26 that $\lim_{n\to\infty} \hat{z}_{-i}(z_i^{Mn}) = 1 - \lim_{n\to\infty} z_i^{Mn} = 1 - z_i^{M*}$.

Now equations 26 and 27 together imply

$$\frac{(1-\delta_{-i})u_{-i}(1-z_i^{Mn})}{u_i(1-\hat{z}^n_{-i}(z_i^{Mn})) - \delta_i u_i(z_i^{Mn})} = \frac{c^n_{-i}(z_i^{Mn} + \hat{z}_{-i}(z_i^{Mn}) - 1)}{c^n_i(z_i^{Mn} + \hat{z}^n_{-i}(z_i^{Mn}) - 1)}.$$

Taking limits on both sides of this equation as $n \to \infty$ gives

$$\frac{(1-\delta_{-i})u_{-i}(1-z_i^{M*})}{(1-\delta_i)u_i(z_i^{M*})} = \lim_{n\to\infty} \frac{c^n_{-i}(z_i^{Mn} + \hat{z}_{-i}(z_i^{Mn}) - 1)}{c^n_i(z_i^{Mn} + \hat{z}^n_{-i}(z_i^{Mn}) - 1)}.$$

The right hand side is equal to γ for $i = 2$ and $1/\gamma$ for $i = 1$. Therefore,

$$\frac{(1-\delta_2)u_2(1-z_1^{M*})}{(1-\delta_1)u_1(z_1^{M*})} = \frac{1}{\gamma} \; \textit{and} \; \frac{(1-\delta_1)u_1(1-z_2^{M*})}{(1-\delta_2)u_2(z_2^{M*})} = \gamma.$$

Now $y \in \xi^*_\gamma(u)$ implies that $y = u(z)$ such that $d(z) = 0$ and

$$\frac{u_2(1-z_1^{M*})}{u_1(z_1^{M*})} \le \frac{u_2(z_2)}{u_1(z_1)} \le \frac{u_2(z_2^{M*})}{u_1(1-z_2^{M*})}$$

$$\Leftrightarrow \frac{1-\delta_1}{1-\delta_2}\frac{1}{\gamma} \le \frac{u_2(z_2)}{u_1(z_1)} \le \frac{1-\delta_1}{1-\delta_2}\frac{1}{\gamma}.$$

■

References

[1] Abreu, Dilip and Faruk Gul. (2000), "Bargaining and reputation." *Econometrica*, 68(1), 85-117.

[2] Andersen, Steffen, Seda Ertac, Uri Gneezy, John A. List and Sandra Maximiano. (2018), "On the cultural basis of gender differences in negotiation." *Experimental Economics*, 21, 757-778.

[3] Aruoba, S. Boragan, Guillaume Rocheteau, and Christopher Waller. (2007), "Bargaining and the Value of Money." *Journal of Monetary Economics*, 54(8), 2636-2655.

[4] Avery, Christopher, and Peter B. Zemsky. (1994), "Money burning and multiple equilibria in bargaining." *Games and Economic Behavior*, 7(2), 154-168.

[5] Backus, Matthew, Thomas Blake, Brad Larsen, and Steven Tadelis. (2020), "Sequential bargaining in the field: Evidence from millions of online bargaining interactions." *The Quarterly Journal of Economics*, 135(3),1319-1361.

[6] Barrett, Scott. (1994), "Self-enforcing international environmental agreements." *Oxford Economic Papers*, 46, 878-894.

[7] Basak, Deepal, and Joyee Deb. (2020), "Gambling over Public Opinion." *American Economic Review*, 110 (11), 3492-3521.

[8] Bernheim, B. Douglas, and Debraj Ray. (1989), "Collective dynamic consistency in repeated games." *Games and Economic Behavior*, 1(4), 295-326.

[9] Binmore, Ken. (1987a), "Nash bargaining theory II." In: Binmore, KG, Dasgupta, P. (eds), *Economics of Bargaining*, Basil Blackwell, Oxford.

[10] Binmore, Ken. (1987), "Nash bargaining and incomplete information." In: Binmore, KG, Dasgupta, P. (eds), *Economics of Bargaining*, Basil Blackwell. 155-192.

[11] Binmore, Ken, Martin J. Osborne, and Ariel Rubinstein. (1992), "Noncooperative models of bargaining." *Handbook of game theory with economic applications*, 1, 179-225.

[12] Binmore, Ken, Ariel Rubinstein and Asher Wolinsky. (1986), "The Nash bargaining solution in economic modelling." *The RAND Journal of Economics*, 17(2), 176-188.

[13] Carlsson, Hans. (1991), "A bargaining model where parties make errors." *Econometrica*, 59(5), 1487-1496.

[14] Chatterjee, Kalyan, and Larry Samuelson. (1990), "Perfect equilibria in simultaneous-offers bargaining." *International Journal of Game Theory*, 19(3), 237-267.

[15] Collard-Wexler, Alan, Gautam Gowrisankaran, and Robin S. Lee. (2019), ""Nash-in-Nash bargaining: A microfoundation for applied work." *Journal of Political Economy*, 127(1), 163-195.

[16] Compte, Olivier, and Philippe Jehiel. (2004), "Gradualism in bargaining and contribution games." *The Review of Economic Studies*, 71(4), 975-1000.

[17] Crawford, Vincent P. (1982), "A theory of disagreement in bargaining." *Econometrica*, 50(3), 607-637.

[18] Crawford, Vincent P., and Hal Varian. (1979), "Distortion of preferences and the Nash theory of bargaining." *Economics Letters*, 3(3), 203-206.

[19] van Damme, Eric. (1989), "Renegotiation-proof equilibria in repeated prisoners' dilemma." *Journal of Economic Theory*, 47(1), 206-217.

[20] Department for Exiting the European Union. (20 July 2017), "Joint technical note on the comparison of EU-UK positions on citizens' rights." https://www.gov.uk/government/publications/joint-technical-note-on-the-comparison-of-eu-uk-positions-on-citizens-rights

[21] Doepke, Matthias, and Fabian Kindermann. (2017), "Intrahousehold decision making and fertility." In: *Demographic Change and Long-Run Development*, MIT Press, 159-182.

[22] Duffy, John, Lucie Lebeau and Daniela Puzzello. (2021), "Bargaining under liquidity constraints: Nash vs. Kalai in the laboratory." *work in progress.*

[23] Dutta, Rohan. (2012), "Bargaining with revoking costs." *Games and Economic Behavior*, 74(1), 144-153.

[24] Ellingsen, Tore, and Topi Miettinen. (2008), "Commitment and conflict in bilateral bargaining." *American Economic Review*, 98(4), 1629-1635.

[25] Ellingsen, Tore, and Topi Miettinen. (2014), "Tough negotiations: Bilateral bargaining with durable commitments." *Games and Economic Behavior*, 87, 353-366.

[26] Farrell, Joseph, and Eric Maskin. (1989), "Renegotiation in repeated games." *Games and Economic Behavior* 1(4), 327-360.

[27] Fearon, James D. (1994), "Domestic political audiences and the escalation of international disputes." *American Political Science Review*, 88(3), 577-592.

[28] Fershtman, Chaim, and Daniel J. Seidmann. (1993), "Deadline effects and inefficient delay in bargaining with endogenous commitment." *Journal of Economic Theory*, 60(2), 306-321.

[29] Friedenberg, Amanda. (2019), "Bargaining Under Strategic Uncertainty: The Role of Second Order Optimism." *Econometrica*, 87(6), 1835-1865.

[30] Harstad, Bard. (2008), "Do side payments help? Collective decisions and strategic delegation." *Journal of the European Economic Association*, 6(2-3), 468-477.

[31] Harstad, Bard. (2021), "A theory of pledge-and-review bargaining." *working paper.*

[32] Hu, Tai-Wei, and Guillaume Rocheteau. (2020), "Bargaining under liquidity constraints: Unified strategic foundations of the Nash and Kalai solutions." *Journal of Economic Theory*, 189, 105098.

[33] Jones, Stephen. (1989), "Have your lawyer call my lawyer: Bilateral delegation in bargaining situations." *Journal of Economic Behavior and Organization*, 11(2), 159-174.

[34] Kalai, Ehud. (1977), "Proportional solutions to bargaining situations: interpersonal utility comparisons." *Econometrica*, 45(7), 1623-1630.

[35] Kletzer, Kenneth M., and Brian D. Wright. (2000), "Sovereign debt as intertemporal barter." *American Economic Review*, 90(3), 621-639.

[36] Lagos, Ricardo, Guillaume Rocheteau, and Randall Wright. (2017), "Liquidity: A new monetarist perspective." *Journal of Economic Literature*, 55(2), 371-440.

[37] Leventoglu, Bahar, and Ahmer Tarar. (2005), "Prenegotiation public commitment in domestic and international bargaining." *American Political Science Review*, 99(3), 419-433.

[38] Lundberg, Shelly, and Robert A. Pollak. (1993), Separate spheres bargaining and the marriage market." *Journal of Political Economy*, 101(6), 988-1010.

[39] Lundberg, Shelly, and Robert A. Pollak. (1996), "Bargaining and distribution in marriage." *Journal of Economic Perspectives*, 10(4), 139-158.

[40] Matsuyama, Kiminori. (1990), "Perfect equilibria in a trade liberalization game." *American Economic Review*, 480-492.

[41] Martin, Lisa L. (1993), "Credibility, Costs and Institutions: Cooperation on Economic Sanctions." *World Politics*, 45(3), 406-432.

[42] Maskin, Eric, and Jean Tirole. (2001), "Markov perfect equilibrium: I. Observable actions." *Journal of Economic Theory*, 100(2), 191-219.

[43] Merlo, Antonio, and Charles Wilson. (1995), "A stochastic model of sequential bargaining with complete information." *Econometrica*, 63(2), 371-399.

[44] Muthoo, Abhinay. (1996), "A bargaining model based on the commitment tactic." *Journal of Economic Theory*, 69(1), 134-152.

[45] Nash, John. (1953), "Two-person cooperative games." *Econometrica*, 21(1), 128-140.

[46] Perry, Motty, and Philip J. Reny. (1993), "A non-cooperative bargaining model with strategically timed offers." *Journal of Economic Theory*, 59(1), 50-77.

[47] Rubinstein, A. (1982), "Perfect equilibrium in a bargaining model." *Econometrica*, 50(1), 97-109.

[48] Sakovics, Jozsef. (1993), "Delay in bargaining games with complete information." *Journal of Economic Theory*, 59(1), 78-95.

[49] Schelling, Thomas C. (1956), "An essay on bargaining." *The American Economic Review*, 46(3), 281-306.

[50] Schelling, Thomas C. (1960), *The Strategy of Conflict.* Cambridge, MA: Harvard University Press.

[51] Segendorff, Bjorn. (1998), "Delegation and threat in bargaining." *Games and Economic Behavior*, 23(2), 266-283.

[52] Selten, Reinhard. (1975), "Reexamination of the perfectness concept for equilibrium points in extensive games." *International Journal of Game Theory*, 4(1), 25-55.

[53] Shaked, Avner, and John Sutton. (1984), "Involuntary unemployment as a perfect equilibrium in a bargaining model." *Econometrica*, 52(6), 1351-1364.

[54] Sutton, John. (1986), "Non-cooperative bargaining theory: An introduction." *The Review of Economic Studies*, 53(5), 709-724.

[55] Tomz, Michael. (2007), "Domestic audience costs in international relations: an experimental approach." *International Organization*, 61(4), 821-840.

[56] Vespa, Emanuel. (2020), "An experimental investigation of cooperation in the dynamic common pool game." *International Economic Review*, 61(1), 417-440.

[57] Wolitzky, Alexander (2012), "Reputational Bargaining with Minimal Knowledge of Rationality." *Econometrica*, 80(5) 2047-2087.