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Core equivalence theorem with production

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Abstract

In production economies, the extent to which non-equilibria are blocked depends on the allocation of control rights among shareholders, because a blocking coalition's resources are affected by the firms it jointly owns with outsiders. We formulate a notion of blocking that takes such interdependency problem into account, and we prove an analog of the Debreu–Scarf theorem for replica production economies. Our theorem differs from theirs in using an additional assumption, which we argue is indispensable and is driven by the interdependency problem.

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1. Introduction

Built upon Edgeworth's [6] elegant idea that the outcome of decentralized markets should be robust against any multilateral deviation, the core equivalence theorem has provided a justification for Walras (competitive) equilibrium. But the theorem is confined to either exchange economies or production economies with publicly accessible technologies. This paper extends the theorem to production economies where technologies are controlled by individuals according to corporate shares. It is non-trivial to introduce production with such private ownership. Without it, multilateral deviation from a status quo can be viewed simply as forming a self-sufficient blocking coalition such that each member is better-off than the status quo. With production, by contrast, it is not clear what it means for a coalition to be "self-sufficient." Can the coalition change the action of a firm not entirely owned by the members of the coalition? If Yes, what conditions does it need

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to respect for the shareholders outside the coalition? If No, does the coalition still have to carry out the part of the status quo production plan assigned to its members who are shareholders of the firm?

For example, suppose a blocking coalition owns only $\frac{1}{3}$ of a firm but is endowed with a good needed by outsiders. If the coalition blocks the supply of the good, the outsiders cannot obtain feasible consumptions unless they change the action of the firm, which they can control via majority shares. Once the firm's action is changed, the blocking coalition either carries out $\frac{1}{3}$ of the new action or forfeits its shares of the firm if default is allowed. Either way would change the resources within the coalition.

In this paper, we formulate a notion of blocking that takes the above-illustrated *interdependency problem* into account. The novelty of this notion is that it adds to the definition of blocking a requirement that the blocking plan of a coalition should allow the outsiders to have a feasible consumption plan. This *outsiders' feasibility condition*, while vacuous in exchange economies, imposes a constraint on the kind of actions that a blocking coalition can implement through a firm even if the firm is under its control.

Then we prove a core equivalence theorem (Theorem 11) for a general class of replica economies with preference- and endowment-assumptions similar to those in Debreu and Scarf [5], except that we also use a new assumption, boundary aversion.

This new assumption of boundary aversion turns out to be indispensable with production: if the interdependency problem is captured by the outsiders' feasibility condition, Example 12 shows that the assumption is indispensable for our core equivalence theorem. In contrast, had we ignored the outsiders' feasibility condition and still assumed that shareholders cannot default, Proposition 7 says that core equivalence follows without the assumption of boundary aversion. Furthermore, Proposition 19 says that, even if the interdependency problem is disentangled by allowing default, core equivalence fails in an economy that violates the boundary-aversion assumption.

Theorem 20 generalizes our core equivalence theorem when the notion of blocking is slightly strengthened by a fairness condition.

We will review the related works in Section 5.

2. The model

2.1. The primitives

There are a finite set I of individuals, a finite set J of firms, and a finite number l of goods. Let i be the index for individuals and j for firms. Let θ_{ij} be i's share of firm j, with $\sum_{i\in I}\theta_{ij}=1$ for all j. Let $\mathbb{R}^l_+$ be the consumption set of each individual, $\succcurlyeq_i$ individual i's preference relation on $\mathbb{R}^l_+$ (with strict preference $\succ_i$), e_i ($\in \mathbb{R}^l_+$) his endowment, and Y_j ($\subseteq \mathbb{R}^l$) the production set of firm j. An allocation is denoted by $(x, y) := ((x_i)_{i\in I}, (y_j)_{j\in J})$, i.e., individual i consumes the bundle x_i and firm j's production plan is y_j. An allocation (x, y) is *feasible* if $x_i \in \mathbb{R}^l_+$ and $y_j \in Y_j$ for each individual i and each firm j and $\sum_{i\in I} x_i = \sum_{i\in I} e_i + \sum_{j\in J} y_j$. A *coalition S* is a non-empty set of individuals. [1]

[1] Note that a firm in this model is not a player but is a joint venture of its shareholders. Formulating firms as players would require specification of their objectives. Profits would be an inadequate objective because they depend on market prices, not taken as given a priori in the context of the core equivalence theorem.

The following *assumptions* are made throughout the paper without being mentioned in theorems or lemmas. Every individual i's preference relation $\succsim_i$ is strongly monotone, strictly convex (convex and strongly convex), and *lower* semicontinuous on $\mathbb{R}^l_+$. The total endowment $\sum_{i\in I} e_i \gg \mathbf{0}$, i.e., it has positive quantity of each good. Every individual's endowment is a point in $\mathbb{R}^l_+ \setminus \{\mathbf{0}\}$ (i.e., $e_i \gneqq \mathbf{0}$ for all $i \in I$). For every firm j, $\mathbf{0} \in Y_j$ and Y_j is convex. To ensure the equal treatment property (ETP) for firms, we assume that, for each firm j, the production possibility frontier is *strictly concave* in the sense that no point on this frontier is the midpoint of any two distinct points of the production set Y_j.

2.2. Replica economies

A *replica economy* of size r, denoted by $\mathcal{E}^r$, consists of r *units*, each of which has exactly the same composition of individuals, firms, endowments, and corporate ownership. We name an individual by an integer-pair $i := (i_1, i_2)$, and a firm by $j := (j_1, j_2)$, meaning:

i_1 := the type of the individual (exactly one such individual in each unit);
i_2 := the unit to which the individual belongs;
j_1 := the type of the firm (exactly one such firm in each unit);
j_2 := the unit to which the firm belongs.

Individuals or firms of the same type have the same characteristics:

$$\begin{aligned}
\succsim_{(i_1,i_2)} &= \succsim_{(i_1,i_2')} =: \succsim_{i_1};\\
e_{(i_1,i_2)} &= e_{(i_1,i_2')} =: e_{i_1};\\
Y_{(j_1,j_2)} &= Y_{(j_1,j_2')} =: Y_{j_1};\\
\theta_{(i_1,k),(j_1,k)} &= \theta_{(i_1,k'),(j_1,k')} =: \theta_{i_1 j_1}.
\end{aligned}$$

Individuals in one unit have zero share of firms in other units:

$$\theta_{(i_1,i_2),(j_1,j_2)} = \begin{cases} \theta_{i_1 j_1} & \text{if } i_2 = j_2,\\ 0 & \text{if } i_2 \neq j_2. \end{cases}$$

The following notations will be useful:

I_1 := the index set for individual-types;
J_1 := the index set for firm-types;
$I_2 := J_2 := \{1, \ldots, r\}$.

An allocation $((x_i)_{i\in I}, (y_j)_{j\in J})$ has the *ETP* if individuals and firms of the same type have the same consumption and production plan, i.e.,

$$\begin{aligned}
x_{(i_1,i_2)} &= x_{(i_1,i_2')} := x_{i_1};\\
y_{(j_1,j_2)} &= y_{(j_1,j_2')} := y_{j_1}.
\end{aligned}$$

2.3. The meaning of corporate shares

As the resources for a blocking coalition partially come from the firms that they share with outsiders, the notion of blocking depends on (i) how a firm allocates its production tasks and (ii) how a firm chooses production tasks. As we consider a purely private ownership economy, both issues should be settled according to the initial allocation of shares, $((\theta_{ij})_{i\in I})_{j\in J}$.

For issue (i), we assume that shareholders are bound by their shares unless default is allowed: once a firm j's production bundle say $y_j := ((y_j)_1, \ldots, (y_j)_l)$ has been determined, each shareholder i is responsible to carry out a fraction θ_{ij} of the plan, contributing (resp. receiving) $\theta_{ij}|(y_j)_k|$ units of good k if $(y_j)_k < 0$ (resp. if $(y_j)_k > 0$). However, if *default* is allowed, an individual can default on his share of the firm so that his net receipt from the firm is null and all his share of the firm is distributed to other shareholders.

For issue (ii), we assume that there is a systematic *corporate decision rule* that uniquely determines, for every initial assignment $((\theta_{ij})_{i\in I})_{j\in J}$ of shares and every coalition S, the set $\tilde{J}(S)$ of all the firms that switch to the proposal of coalition S if S is blocking the status quo. We say a firm j is *controlled* by coalition S if and only if $j \in \tilde{J}(S)$. We assume that the function $\tilde{J}$ is exogenous and satisfies the following innocuous assumptions:

$$\sum_{i\in S}\theta_{ij} = 1 \Rightarrow j \in \tilde{J}(S); \quad \sum_{i\in S}\theta_{ij} = 0 \Rightarrow j \notin \tilde{J}(S). \tag{1}$$

An example for such corporate governance is that every firm holds a referendum with all its shareholders and it switches to the blocking coalition's proposal if and only if the proposal gets a vote greater than a predetermined threshold (with the assumption that all shareholders outside the coalition oppose the proposal).

2.4. *The notion of blocking*

To motivate our notion of blocking, we start with a notion that does not make sense:

Definition 1 (*Wishful blocking*). A feasible allocation $((x_i)_{i\in I}, (y_j)_{j\in J})$ is *wishfully blocked* by a coalition S if:

(a) for every $i \in S$ there exists an $x'_i \in \mathbb{R}^l_+$ such that $x'_i \succ_i x_i$, and
(b) for every $j \in J$ there exists a $y'_j \in Y_j$ such that

$$\sum_{i\in S} x'_i = \sum_{i\in S} e_i + \sum_{j\in J}\sum_{i\in S}\theta_{ij}y'_j. \tag{2}$$

Eq. (2) implies that the resources within a blocking coalition consist of the endowments of its members and its share of the production plan of every firm. The problem of this equation is that it also implies that a firm may do whatever the coalition wishes, whether the firm is controlled by the coalition or not. Thus, this notion of blocking fails to capture the structure of corporate controls in a private ownership economy.

To respect corporate controls, recall our assumption that every blocking coalition S can unilaterally change the actions of the firms in the set $\tilde{J}(S)$, determined by the initial assignment of corporate shares. The question is How would those firms outside $\tilde{J}(S)$ behave? This question is important because the firms' actions affect the resources for the coalition. To answer this question, we borrow an idea from Nash equilibrium: a deviating player expects others to stick to the status quo. That leads to the next notion, where a coalition considers changes only in the firms that they can control and expects the other firms to stick to the status quo. (In Section 3.4, we will consider cases where these other firms may change plans in response to the blocking coalition.)

Definition 2 (*Inconsiderate blocking*). A feasible allocation $((x_i)_{i\in I}, (y_j)_{j\in J})$ is said to be *inconsiderately blocked* by a coalition S if: (a) condition (a) of Definition 1 holds, and (b) for every $j \in \tilde{J}(S)$ there exists a $y'_j \in Y_j$ such that

$$\sum_{i\in S} x'_i = \sum_{i\in S} e_i + \sum_{j\in\tilde{J}(S)} \sum_{i\in S} \theta_{ij} y'_j + \sum_{j\notin\tilde{J}(S)} \sum_{i\in S} \theta_{ij} y_j. \tag{3}$$

Eq. (3) respects corporate controls by saying that the coalition can change the action of any firm it controls but expects the action of any other firm to be fixed at the status quo.

However, this notion of blocking is *inconsiderate* because such a blocking coalition gives no regard to whether the consequence of its blocking is feasible at all for outsiders. With $\mathbb{R}^l_+$ being the sum of outsiders' consumption sets, it is possible that

$$\sum_{i\notin S} e_i + \sum_{j\in\tilde{J}(S)} \sum_{i\notin S} \theta_{ij} y'_j + \sum_{j\notin\tilde{J}(S)} \sum_{i\notin S} \theta_{ij} y_j \notin \mathbb{R}^l_+;$$

then the outsiders cannot carry out their shares of the production plans, hence the right-hand side of Eq. (3) would give the wrong quantities for the resources available for the coalition. This is the *interdependency problem*.

The interdependency problem is clearly unavoidable when shareholders cannot default on their shares. When default is allowed, the resources for a blocking coalitions would again be affected by the outsiders should they default. In that case, the interdependency problem takes a different form, which we will consider in Section 3.4.

Hence comes the next notion of (considerate) blocking, assuming no default.

Definition 3 (*Blocking*). A feasible allocation $((x_i)_{i\in I}, (y_j)_{j\in J})$ is *blocked* by a coalition S if: (a) condition (a) of Definition 1 is satisfied, and (b) for every $j \in \tilde{J}(S)$ there exists a $y'_j \in Y_j$ such that Eq. (3) is satisfied and

$$\sum_{i\notin S} e_i + \sum_{j\in\tilde{J}(S)} \sum_{i\notin S} \theta_{ij} y'_j + \sum_{j\notin\tilde{J}(S)} \sum_{i\notin S} \theta_{ij} y_j \in \mathbb{R}^l_+. \tag{4}$$

We call (4) the *outsiders feasibility condition*.

The *core* (resp. *inconsiderate core*, *wishful core*) is the set of feasible allocations that are not blocked (resp. inconsiderately blocked, wishfully blocked). Within replica economy $\mathcal{E}^r$ of size r, let C^r (resp. C^r_{incon}, C^r_{wish}) denote the core (resp. inconsiderate core, wishful core). Obviously, $C^r_{\text{wish}} \subseteq C^r_{\text{incon}} \subseteq C^r$.

3. Core equivalence

One direction of the equivalence between core and equilibrium is easy:

Lemma 4. *Any Walras equilibrium allocation belongs to the core, as well as the inconsiderate and wishful cores.*

Proof. It suffices to prove that a Walras equilibrium cannot be wishfully blocked. To do that, mimic the proof of the first welfare theorem with any wishful blocking coalition. □

For the converse direction of the equivalence, we start with the ETP of core allocations. The ETP for firms follows from the uniqueness of profit-maximum for any type of firms, as each firm's production possibility frontier is assumed strictly concave. With the ETP for firms, each individual of the same type has the same bundle of resources, and then the well-known proof of ETP for consumers in exchange economies applies.

Lemma 5 (*Equal treatment*). *For any $r = 1, 2, \ldots,$ any allocation in the core C^r has the ETP. (Since $C^r_{\text{wish}} \subseteq C^r_{\text{incon}} \subseteq C^r$, ETP for C^r_{incon} and C^r_{wish} is also satisfied.)*

Proof. Appendix B.

With the ETP, the core allocations in economies of different sizes can be identified with the element of the same space, $\left(\mathbb{R}^l_+\right)^{I_1} \times \prod_{j_1 \in J_1} Y_{j_1}$. That is also true for Walras equilibrium allocations by Lemma 4. Represented by elements in $\left(\mathbb{R}^l_+\right)^{I_1} \times \prod_{j_1 \in J_1} Y_{j_1}$, the set of Walras equilibrium allocations is preserved by replication of the economy. Hence denote this set of equilibrium allocations by $\mathbb{W}$.

Obviously, $C^1 \supseteq C^2 \supseteq C^3 \supseteq \cdots$ and likewise for C^r_{incon} and C^r_{wish}. The main result of this section is that these decreasing sequences shrink to $\mathbb{W}$.

3.1. Inconsiderate core equivalence

Recall that our notion of blocking takes into account the feasibility condition for the outsiders of a coalition, while inconsiderate blocking does not. To pin down the impact of this feasibility constraint, we establish in this subsection core convergence based on inconsiderate blocking. This result will be contrasted with the core convergence theorem proved in the next subsection, which will be built upon the propositions proved here.

To get an intuition for core convergence with inconsiderate blocking, assume within this paragraph that preferences are all represented by continuously differentiable utility functions and consider a non-equilibrium allocation where everyone's consumption is strictly positive. We show that this allocation is inconsiderately blocked: first, if the allocation is not Pareto efficient, it is blocked by the grand coalition. Second, if the allocation is Pareto efficient, by the second welfare theorem (Corollary 22, Appendix A), it is a price equilibrium with transfers under some price $p \gg \mathbf{0}$, i.e., given p, the allocation maximizes each firm's profit and optimizes for every individual if his wealth is equal to the market value of his allocated consumption. Third, if this equilibrium with transfers is not a Walras equilibrium, someone must get a positive transfer, then the allocation is inconsiderately blocked by a plan similar to that of Mas-Colell et al. [12, p. 658]. Just replace $x_1 - e_1$ in their proof by $x_1 - e_1 - \sum_{j_1} \theta_{1 j_1} y_{j_1}$, where j_1 indexes the types of firms and y_{j_1} is a type-j_1 firm's production plan in the allocation.

To prove inconsiderate core convergence for the general case without assuming differentiability or interior allocations, we start with wishful cores:

Proposition 6. $\mathbb{W} = \cap_{r=1}^{\infty} C^r_{\text{wish}}$.

Proof. Mimic Debreu and Scarf [5]. See Appendix C.

By Proposition 6, a non-equilibrium allocation is wishfully blocked in sufficiently large economies. Is it also inconsiderately blocked? The answer is Yes if the economy is large enough.

To see the intuition, suppose an individual i prefers a consumption x'_i to the status quo x_i, but to consume x'_i he needs to change a firm j's action into y'_j from the status quo y_j. The problem is that his share θ_{ij} of firm j is too small for him to make that change. Hence i only wishfully blocks the status quo. To pass that into inconsiderate blocking, make another copy of the economy and label it by $\mathcal{E}'$. Form a coalition consisting of this i and everyone in $\mathcal{E}'$. Then they can inconsiderately block the status quo: i receives the status quo production bundle $\theta_{ij} y_j$ assigned to him from firm j; the counterpart of i in $\mathcal{E}'$, however, receives the bundle $\theta_{ij} y'_j$ that i wishes for—the counterpart can do that because all the firms in $\mathcal{E}'$ are controlled by the enlarged coalition; then the counterpart swaps the bundle $\theta_{ij} y'_j$ with i's bundle $\theta_{ij} y_j$. With this plan, i gets to consume his preferred x'_i, and everyone in $\mathcal{E}'$ can still get the status quo consumption. Thus, i can make all coalition members better-off by distributing part of his preferred bundle to them. Hence we have:

Proposition 7 (*Inconsiderate core equivalence*). $\mathbb{W} = \cap_{r=1}^{\infty} C^r_{\text{incon}}$.

Proof. See Appendix C.

Note that Proposition 7 would be the counterpart of the Debreu–Scarf theorem in production economies if we ignore the feasibility constraint for the outsiders of a coalition.

3.2. *Considerate core equivalence*

The only hurdle between inconsiderate versus considerate core convergence is that the latter needs to take into account the outsiders' feasibility condition. The main idea to overcome this hurdle is embodied by the following Lemma 8, which says that a blocking coalition's disturbance on feasible consumptions for outsiders diminishes as the replica economy enlarges.

An allocation (x, y) is *edgy* if the aggregate consumption bundle $\sum_{i \in I} x_i$ lies on $\partial \mathbb{R}^l_+$, the boundary of the consumption set $\mathbb{R}^l_+$. If (x, y) has the ETP, then we can write it as $((x_{i_1})_{i_1 \in I_1}, (y_{j_1})_{j_1 \in J_1})$, and it is edgy if only if $\sum_{i_1 \in I_1} x_{i_1} \in \partial \mathbb{R}^l_+$. Note that, if an allocation with ETP is feasible and not edgy, then

$$\sum_{i_1 \in I_1} e_{i_1} + \sum_{i_1 \in I_1} \sum_{j_1 \in J_1} \theta_{i_1 j_1} y_{j_1} \geqslant \mathbf{0}. \tag{5}$$

Lemma 8 (*Diminishing externality of blocking coalitions*). *For any feasible allocation with ETP, if it is inconsiderately blocked and is not edgy, then it is blocked* (*in the sense of Definition* 3) *for all replica economies with sufficiently large size*.

Proof. Suppose allocation (x, y), as specified by the hypothesis, is inconsiderately blocked by coalition S with a plan $((x'_i)_{i \in S}, (y'_j)_{j \in \tilde{J}(S)})$. It suffices to prove Eq. (4) for sufficiently large size r. (Inconsiderate blocking is preserved when the replica economy enlarges.) Let

$$r^* := \max\{i_2 : (i_1, i_2) \in S \text{ for some } i_1\}.$$

Thus, for any $j_2 = r^* + 1, \ldots, r$ and for any firm-type j_1, none of the coalition members hold any share of firm (j_1, j_2). Hence the left-hand side of Eq. (4) is equal to a fixed vector plus

$$(r - r^*) \left[\sum_{i_1 \in I_1} e_{i_1} + \sum_{i_1 \in I_1} \sum_{j_1 \in J_1} \theta_{i_1 j_1} y_{j_1} \right]. \tag{6}$$

As the allocation is feasible and not edgy, the vector in the square bracket $[\cdots]$ is positive in every component by (5); when $r \to \infty$, r^* is unchanged, so the vector (6) goes to infinity in every component. Thus, the left-hand side of Eq. (4) belongs to $\mathbb{R}^l_+$ when r is sufficiently large, as desired. □

To ensure the non-edgyness for Lemma 8, within the rest of this subsection we assume:

Assumption 9 (*Boundary aversion*). Every individual strictly prefers any interior point of $\mathbb{R}^l_+$ to any boundary point of $\mathbb{R}^l_+$.

Lemma 10. *If Assumption* 9 *holds, then no element in the core* C^r *is edgy.*

Proof. If an allocation (x, y) in the core C^r is edgy, then $x_i \in \partial\mathbb{R}^l_+$ for all individuals $i \in I$ (since $x_i \in \mathbb{R}^l_+$ for all individuals i). Then the grand coalition I blocks the allocation by having every firm produce $\mathbf{0}$ and equally dividing the total endowment $\sum_{i \in I} e_i$ among all individuals. Since $\sum_{i \in I} e_i \gg \mathbf{0}$ by assumption, everyone is better-off by the assumption of boundary aversion. The outsiders' feasibility condition Eq. (4) is vacuously satisfied. □

Theorem 11 (*Considerate core equivalence*). *If Assumption* 9 *holds, then* $\mathbb{W} = \cap_{r=1}^{\infty} C^r$.

Proof. Lemma 4 implies $\mathbb{W} \subseteq \cap_{r=1}^{\infty} C^r$. For the converse, pick any allocation in $\cap_{r=1}^{\infty} C^r$. Then the allocation is not edgy (Lemma 10 and Assumption 9) and consequently it is not inconsiderately blocked no matter how large the replica economy is (otherwise it is blocked for any sufficiently large r, by Lemma 8 and non-edginess). Then Proposition 7 implies that the allocation is an equilibrium allocation. □

Except Assumption 9, the preference- and endowment-assumptions of Theorem 11 are similar to those in Debreu and Scarf [5] (detailed in Section 5). Since Assumption 9 is shown indispensable in Section 3.3 and is indispensable even for some other notions of blocking by the impossibility result in Section 3.4, Theorem 11 is a general version of the production-counterpart of the Debreu–Scarf theorem.

3.3. The indispensability of boundary aversion

Here is an example showing that the boundary-aversion assumption in Theorem 11 is indispensable. In this example, preferences are not boundary-averse and there is a non-equilibrium that cannot be blocked, though it is inconsiderately blocked. Since boundary aversion is not needed for core equivalence had we ignored the outsiders' feasibility condition (4), as in Proposition 7, this example shows the effect of incorporating the outsiders' feasibility condition into the analysis of cores. [2]

Example 12 (*Edgy core*). There are two goods, two individual-types, and one firm-type. The endowment for each type-1 individual is $e_1 = (1, 0)$, and that for type-2 is $e_2 = (0, 1)$. For each type, the utility from consumption bundle (x_{i1}, x_{i2}) is

$$u(x_{i1}, x_{i2}) := x_{i1} + 10\sqrt{x_{i2}}.$$

Each firm is equally shared by a type-1 and a type-2 individuals, its decision is determined by unanimity rule ($(j_1, j_2) \in \tilde{J}(S) \Leftrightarrow \{(1, j_2), (2, j_2)\} \subseteq S$), and its production set is

$$Y := \left\{(y_{j1}, y_{j2}) \in \mathbb{R}^2 : y_{j2} \leqslant \sqrt{-y_{j1}};\, y_{j1} \leqslant 0\right\}.$$

In any replica of the economy, there is a unique equilibrium allocation [3]:

$$x_1^* = (0, 3/4), \quad x_2^* = (0, 5/4), \quad y^* = (-1, 1), \quad p^* = (1, 2).$$

But the following allocation belongs to the core in any replica of the economy:

$$x_1^{\mathrm{o}} = (0, 1/2), \quad x_2^{\mathrm{o}} = (0, 3/2), \quad y^{\mathrm{o}} = (-1, 1). \tag{7}$$

However, it is inconsiderately blocked in sufficiently large economies, by Proposition 7.

Let us prove that allocation (7) belongs to the core in any replica economy.

Note that (7) constitutes a price equilibrium with transfers, resulting from the following procedure: first, under the price

$$p^{\mathrm{o}} = (1, 2),$$

each firm maximizes its profit and distributes it equally to the two shareholders, each getting $\frac{1}{2}$ dollar as dividend, so that a type-i_1 individual's wealth at this point is

$$w_{i_1}^* := p^{\mathrm{o}} \cdot e_{i_1} + (1/2)p^{\mathrm{o}} \cdot y^{\mathrm{o}} = \begin{cases} 3/2 & \text{if } i_1 = 1, \\ 5/2 & \text{if } i_1 = 2. \end{cases} \tag{8}$$

[2] As our intention is just to identify the effect of production to core equivalence, we do not argue whether boundary aversion is a strong or weak assumption. Nevertheless, we should point out that the assumption is incompatible with strongly monotone and upper semicontinuous preferences. (Note, however, that Theorem 11 does not need upper semicontinuity.) This assumption also rules out quasilinear preferences. (William Zame pointed out that it also rules out commodities that serve purely as intermediary products.)

[3] To prove that, note that an individual's consumption of good 2 cannot exceed 2 units. This follows from the ETP of equilibrium and the fact that there are at most two units of good 2 in each unit economy, due to the technology and endowments. Thus, the marginal rate of substitution, in absolute value, is at most $\frac{\sqrt{2}}{5}$. Since the slope of the production possibility frontier (the northeastern boundary of the set $\mathbb{R}^2_+ \cap (e_1 + e_2 + Y)$), in absolute value, is at least $\frac{1}{2}$, any Pareto efficient allocation would use all the good 1 to produce good 2, i.e., an efficient production plan is necessarily $(-1, 1)$. By the first welfare theorem, the equilibrium price ratio is $p_1^*/p_2^* = \frac{1}{2}$, the slope of the frontier at $(-1, 1)$. Then the equilibrium consumption bundles are easily derived.

Second, each type-1 individual transfers his dividend, $\frac{1}{2}$ dollar, to the type-2 individual in the same unit, so that a type-i_1 individual's wealth becomes

$$w^{\mathrm{o}}_{i_1} := \begin{cases} w^*_1 - 1/2 = 1 & \text{if } i_1 = 1, \\ w^*_2 + 1/2 = 3 & \text{if } i_1 = 2. \end{cases} \tag{9}$$

Third, given the wealth w^{o}_i and price p^{o}, each individual i buys an optimal consumption, x^{o}_i.

Now suppose a coalition S blocks the allocation $(x^{\mathrm{o}}, y^{\mathrm{o}})$ with a plan (x', y'). Since x^{o}_i is optimal for person i given $(p^{\mathrm{o}}, w^{\mathrm{o}}_i)$, $\sum_{i\in S} p^{\mathrm{o}} \cdot x'_i > \sum_{i\in S} w^{\mathrm{o}}_i$. Thus (by Eq. (3)), the market value of the resources for the coalition needs to exceed $\sum_{i\in S} w^{\mathrm{o}}_i$, i.e.,

$$\sum_{i\in S}\left[p^{\mathrm{o}} \cdot e_i + \sum_{j\in\tilde{J}(S)} \theta_{ij} p^{\mathrm{o}} \cdot y'_j + \sum_{j\notin\tilde{J}(S)} \theta_{ij} p^{\mathrm{o}} \cdot y^{\mathrm{o}}\right] > \sum_{i\in S} w^{\mathrm{o}}_i. \tag{10}$$

We claim that (10) cannot hold. To prove that, recall that $p^{\mathrm{o}} \cdot y'_j \leqslant p^{\mathrm{o}} \cdot y^{\mathrm{o}}_j$ for any technologically feasible production plan y'_j, hence

$$p^{\mathrm{o}} \cdot e_i + \sum_{j\in\tilde{J}(S)} \theta_{ij} p^{\mathrm{o}} \cdot y'_j + \sum_{j\notin\tilde{J}(S)} \theta_{ij} p^{\mathrm{o}} \cdot y^{\mathrm{o}}$$

cannot exceed individual i's pre-transfer wealth w^*_i. Thus, the left-hand side of (10) is less than or equal to $\sum_{i\in S} w^*_i$. Our claim will follow if

$$\sum_{i\in S} w^*_i \leqslant \sum_{i\in S} w^{\mathrm{o}}_i.$$

To prove this inequality, partition S into two sets, S_1 and S_2:

$$S_1 := \{(i_1, i_2) \in S : i_1 = 1\},$$
$$S_2 := \{(i_1, i_2) \in S : i_1 = 2\}.$$

By (9),

$$\sum_S (w^*_i - w^{\mathrm{o}}_i) = (|S_1| - |S_2|)\,(1/2).$$

Here is where the outsiders' feasibility condition (4) plays a crucial role: (4) implies that

$$|S_1| \leqslant |S_2|.$$

To see that, we calculate the total resources for the outsiders. By the unanimity rule, every firm where an outsider holds a share would stick to the status quo production plan $(-1, 1)$. With a share of 50%, each outsider is obligated to receive the production bundle $\frac{1}{2}(-1, 1)$. In the r-replica economy, there are $r - |S_1|$ type-1 and $r - |S_2|$ type-2 outsiders. Hence the total bundle of the outsiders' resources is

$$(r - |S_1|)(1, 0) + (r - |S_2|)(0, 1) + (r - |S_1| + r - |S_2|)\,\tfrac{1}{2}(-1, 1).$$

The first component of this vector is equal to $(|S_2| - |S_1|)/2$. Thus, (4) implies that $|S_1| \leqslant |S_2|$. Then $\sum_S (w^*_i - w^{\mathrm{o}}_i) \leqslant 0$, as claimed. It follows that the allocation (7) cannot be blocked.

Although this non-equilibrium is never blocked, it is inconsiderately blocked in large enough economies by Proposition 7. (Independent of Proposition 7, Appendix D constructs inconsiderate blocking plans for all non-equilibria in this example.) Thus, the example illustrates the effect of the outsiders' feasibility coalition.

3.4. Allowing default

So far we have been handling the interdependency between a blocking coalition and its outsiders through requiring that a blocking plan satisfy the outsiders' feasibility condition (4). An alternative is to disentangle the interdependency by allowing shareholders to default on their shares. Then the outsiders can at least retain their endowments, thereby securing a feasible allocation among them, hence a blocking plan feasible within the coalition may be automatically feasible for outsiders. Would this approach obtain a reasonably general core equivalence theorem without the restriction of boundary aversion?

We give a negative answer by proving that core equivalence fails in Example 12 for a class of blocking notions that allow default (Proposition 19). Thus, boundary aversion is indispensable for core equivalence not only when the interdependence problem takes the form of outsiders' feasibility condition, but also when it is disentangled by the possibility of default. Since the interdependence problem is inherent given the private ownership of production technologies, boundary aversion appears to be a necessary price for an extension of core equivalence to the Arrow–Debreu production economies.

3.4.1. A set of axioms for blocking

Recall that $\tilde{J}(S)$ is the set of firms controlled by a blocking coalition S. Let Θ_j denote the set of shareholders of firm j: $\Theta_j := \{i \in I : \theta_{ij} > 0\}$. Let us assume:

Axiom 13 (*Triviality*). $\tilde{J}(\emptyset) = \emptyset$.

Axiom 14 (*Unanimity*). $j \in \tilde{J}(\Theta_j)$.

Axiom 15 (*Irrelevance of non-shareholders*). $j \in \tilde{J}(S) \Leftrightarrow j \in \tilde{J}(S \cap \Theta_j)$.

All three axioms are self-evident. Axiom 15 says that adding non-shareholders of a firm to the coalition does not affect whether the firm is controlled by the coalition or not. Coupled with Axiom 13, it implies that non-shareholders of a firm cannot control the firm.

Now replace the outsiders' feasibility condition (4) with the possibility of default:

Axiom 16 (*Default*). For any firm, any individual can choose to default on all his shares of the firm (so that he gives **0** to and gets **0** from the firm). If he does so, his shares are divided among other individuals in the proportion of their previous shares of the firm.

However, merely allowing default would make core equivalence easier to fail, because even a Walras equilibrium may be blocked with default. For instance, suppose that 51% of a firm is owned by Ms. Big and the other 49% is owned by Mr. Small, and the firm is ruled by majority of share. Then Ms. Big can choose a production plan such that it is infeasible for Mr. Small to carry out his share of that plan with his individual endowment, so he has to default and the firm becomes entirely owned by Ms. Big. Thus, a transfer of properties occurs and results in an outcome different from any Walras equilibrium.

Thus, if one still wishes to expand the scope of core equivalence, the other conditions required by the blocking notions in previous subsections need to be modified. Let us relax a previous assumption that the firms outside $\tilde{J}(S)$ cannot deviate from the status quo.

Let $\tilde{K}(S)$ be the set of all the firms that are not controlled by the coalition S and whose productions may be changed in response to the blocking coalition—either to restore outsiders' feasibility or to retaliate the coalition—and such changes are expected by the coalition when it plots the blocking plan. Call $\tilde{K}$ *response coordination*.

We shall restrict the response coordination by the next axiom. Let

$$\mathrm{prj}_2(S) := \text{the set of units each of which contains some member of } S.$$

Given a blocking coalition S, the units in $\mathrm{prj}_2(S)$ are called *involved* with S. The other units are called *uninvolved* with S. The next axiom implies that only firms in the involved units may change their production plans in response to the blocking coalition.

Axiom 17 (*Irrelevance of uninvolved units*). $\tilde{K}(S) = \left(J_1 \times \mathrm{prj}_2(S)\right) \setminus \tilde{J}(S)$.

Without the "$\subseteq$" part of Axiom 17, the feasibility problem for outsiders could be resolved trivially by altering production in the uninvolved units. To see this, suppose in an N-replica economy, an allocation (x, y) is inconsiderately blocked by coalition S with a plan $((x'_i)_{i\in S}, (y'_j)_{j\in\tilde{J}(S)})$ infeasible for outsiders:

$$\sum_{i\notin S} e_i + \sum_{j\in\tilde{J}(S)}\sum_{i\notin S}\theta_{ij}y'_j + \sum_{j\notin\tilde{J}(S)}\sum_{i\notin S}\theta_{ij}y_j =: v \notin \mathbb{R}^l_+.$$

Consider the $(N+m)$-replica economy and the same coalition S with the same blocking plan. Let all firms in the uninvolved units produce $\mathbf{0}$. Then the total resource for the outsiders is $v+m\sum_{i_1} e_{i_1}$. Since $\sum_{i_1} e_{i_1} \gg \mathbf{0}$, for all sufficiently large m, $v + m\sum_{i_1} e_{i_1}$ is positive in each coordinate and hence S can "block" (x, y) in the $(N+m)$-replica economy.

However, it is not reasonable to expect that an uninvolved unit would change its action in response to the blocking coalition. That is because an uninvolved unit can completely insulate itself from the effect of the blocking coalition by sticking to any core allocation—due to its ETP, any core allocation can be achieved within the unit, as each unit is self-sufficient. Whereas, the non-coalition members in the involved units are directly affected by the blocking plan, so it is more reasonable to expect that they would adjust productions to recover their feasibility condition instead of hoping the people in the uninvolved units to do that for them. Thus, we postulate Axiom 17.

To specify the response of outsiders, the next axiom assumes that they try to deter any blocking coalition that excludes them if such deterrence plan exists.

Axiom 18 (*Aversion to being left behind*). The outsiders of S choose actions for the firms in $\tilde{K}(S)$ so that S cannot block the status quo, if such actions exist.

Axiom 18 is needed only if a blocking coalition does not include everyone. The outsider who cannot be included is worse-off if the blocking plan is implemented. Naturally such an individual would deter the blocking plan if he can. Hence this axiom is reasonable.

3.4.2. An impossibility result

The proposition proved next says that there is a sequence of replica economies such that core equivalence is impossible for any notion of blocking that satisfies Axioms 13–18.

To see the intuition, imagine a situation where there is an individual-type who owns some share of a firm and is endowed with all the inputs needed by the firm. If this monopolist type is the decision maker of the firm, he can freely take over the stocks from the other shareholders by telling them: "I am having the firm take an action infeasible for you unless you default and give all your shares to me. Your default cannot undermine my plan because you are endowed with no input for the firm." To avoid this problem, one has to exclude the monopolist from decision making. But then the core would allow a transfer from the monopolist to the other shareholders: Having no control of the firm, the monopolist alone cannot block this transfer; nor can he block it with some other shareholders, who gain from this transfer. Thus, one way or another, someone is victimized.

Proposition 19 (*Impossibility*). *There exists a replica economy $\mathcal{E}^r$ that satisfies all the assumptions of Theorem* 11 *except Assumption* 9 *and yet, for any notion of blocking that satisfies Axioms* 13–18, *the core does not converge to Walras equilibrium allocations as $r \to \infty$.*

Proof. We shall prove that Example 12 in Section 3.3 is such a replica economy (without relying on its specific unanimity rule of corporate controls). Recall in that example that type-1 individuals are endowed with all of good 1, the input for production. Also recall that the unique Walras equilibrium allocation is $[x_1^* = (0, \frac{3}{4}), x_2^* = (0, \frac{5}{4}), y^* = (-1, 1)]$.

If, in some unit, the type-1 individual alone controls the firm in that unit, then he alone can block a neighborhood of the Walras equilibrium allocation, hence the core is bounded away from the equilibrium allocation for any size of the economy. His blocking plan is to consume the bundle (0, 1) and have the firm produce $(-1, 1)$. By the axiom of irrelevance of uninvolved units, only the action of the firm in the involved unit is changeable but this firm is already controlled by the blocking individual. Thus, given the blocking plan, the type-2 individual in the same unit must default to have a feasible consumption, leaving the entire output $(-1, 1)$ to the type-1 individual and so making the blocking plan feasible.

Thus, we assume without loss that there is no unit where the type-1 individual alone controls the firm in that unit. This, by the axiom of irrelevance of non-shareholders, implies:

$$\text{If the type-2 individual in a unit does not belong to a coalition } S, \text{ then the firm in this unit does not belong to } \tilde{J}(S), \text{ i.e., is not controlled by } S. \tag{11}$$

We claim: *The allocation where every type*-1 *individual consumes* $x_1^{\mathrm{o}} = (0, 1/2)$, *every type*-2 *individual consumes* $x_2^{\mathrm{o}} = (0, 3/2)$, *and every firm produces* $y^o = (-1, 1)$, *cannot be blocked*. If this claim is true, then the conclusion of the proposition follows.

It suffices to prove that the above allocation cannot be blocked under an additional *shutdown assumption*: for any blocking coalition S, any firm j that belongs to $\tilde{K}(S)$ (controlled by outsiders of S) produces $\mathbf{0}$. If the allocation cannot be blocked under this additional assumption, then the axiom of aversion to being left behind implies that the firms in $\tilde{K}(S)$ do produce in such a way that deters blocking.

Recall that the allocation $[x_1^{\mathrm{o}} = \left(0, \frac{1}{2}\right), x_2^{\mathrm{o}} = \left(0, \frac{3}{2}\right), y^{\mathrm{o}} = (-1, 1)]$, coupled with price $p^{\mathrm{o}} = (1, 2)$, constitutes a price equilibrium with transfers such that every type-1 individual transfers all his dividend, $\frac{1}{2}$ dollar, to the type-2 individual in the same unit, i.e., the post-transfer wealth w_i^{o} is equal to 1 dollar for type 1 and 3 dollars for type 2.

Suppose a coalition S blocks this allocation $(x^{\mathrm{o}}, y^{\mathrm{o}})$ with a blocking plan (x', y'). By the shutdown assumption, S gets the bundle **0** from any firm outside $\tilde{J}(S)$. Denote $\theta^*_{ij} := 1$ if the outsiders default on firm j and otherwise $\theta^*_{ij} := \theta_{ij}$. Then feasibility within S says

$$\sum_{i\in S} e_i + \sum_{i\in S} \sum_{j\in\tilde{J}(S)} \theta^*_{ij} y'_j = \sum_{i\in S} x'_i. \tag{12}$$

As x^{o}_i is optimal given p^{o} and w^{o}_i, we have $\sum_{i\in S} p^{\mathrm{o}} \cdot x'_i > \sum_{i\in S} w^{\mathrm{o}}_i$. Thus, Eq. (12) implies

$$\sum_{i\in S} p^{\mathrm{o}} \cdot e_i + \sum_{i\in S} \sum_{j\in\tilde{J}(S)} \theta^*_{ij} p^{\mathrm{o}} \cdot y'_j > \sum_{i\in S} w^{\mathrm{o}}_i.$$

We shall prove that this inequality does not hold. Since y^{o} maximizes profits given p^{o}, it suffices to prove that

$$\sum_{i\in S} \left(p^{\mathrm{o}} \cdot e_i + \sum_{j\in\tilde{J}(S)} \theta^*_{ij} p^{\mathrm{o}} \cdot y^{\mathrm{o}}_j \right) \leqslant \sum_{i\in S} w^{\mathrm{o}}_i. \tag{13}$$

I.e., with each firm producing y^{o}, the coalition cannot have more wealth from blocking the dividend transfer than it would have from abiding to the transfer.

To prove (13), partition the coalition S into three parts: (i) n unit economies (i.e., n pairs of type-1 and type-2 individuals in the same unit), (ii) g type-1 individuals from g separate units (where the type-2 individuals are not in S), and (iii) h type-2 individuals from h units (where the type-1 individuals are not in S).

The total wealth of each of the n unit economies remains the same when the unit joins S, as the dividend transfer is a pure transfer between the two individuals in each unit.

The wealth of each of the g single type-1 individuals remains the same when he joins S: if he abides to the transfer, he loses his $\frac{1}{2}$ dollar dividend. If he joins S, the firm in his unit does not belong to $\tilde{J}(S)$ by fact (11) and hence it belongs to $\tilde{K}(S)$ by the assumption of irrelevance of uninvolved units; so this firm produces the bundle **0** by the shutdown assumption and hence this individual again loses the $\frac{1}{2}$ dollar dividend.

For each of the h single type-2 individuals, in joining S, her wealth cannot increase: if she abides to the transfer, she gains the $\frac{1}{2}$ dollar dividend from the type-1 individual in the same unit. If she joins S, she cannot gain more than this $\frac{1}{2}$ dollar, because the total profit in her unit is only 1 dollar and she already has half of it without the transfer.

Thus, (13) follows. Hence the non-equilibrium allocation $(x^{\mathrm{o}}, y^{\mathrm{o}})$ cannot be blocked. □

Although the above proof and Example 12 both use the fact that a coalition cannot gather sufficient wealth to support its blocking plan, the forces that drive this fact are different. In Example 12, every type-1 member, by blocking the status quo transfer, brings to the coalition a net gain $\frac{1}{2}$ dollar; but this contribution is offset by the type-2 members because the outsiders' feasibility condition requires that a coalition contain at least as many type-2 as type-1 members. In Proposition 19, by contrast, a type-1 member contributes no net gain to the coalition, because in blocking the status quo, he loses control of the firm and gets zero dividend from it.

4. Discussions and further research

One innovation of this paper is to address the interdependency problem in production economies. This problem is inherent given the possibility that firms are jointly owned by individuals. Our notion of considerate blocking addresses the interdependency problem by requiring that a blocking plan satisfy the outsiders' feasibility condition. This notion of blocking is appropriate if individuals cannot secede from the arrangement on their property rights of the firms: a blocking coalition may expect that it can carry out any blocking plan that takes resources from the outsiders via the obligation and entitlement implied by the property rights arrangement, no matter how draconian it may appear, as long as the blocking plan admits feasible consumptions for the outsiders.

This notion of blocking is worthwhile considering because it has the appealing property of admitting a general core equivalence theorem. The no-secession assumption implicit in this notion makes sense because the notion of Walras equilibrium, which a core equivalence is supposed to justify, also assumes that shareholders are bound by their dividend shares.

However, it may be interesting to consider other notions of blocking that allow individuals to withdraw from the existing property rights arrangement.

An alternative setup has been suggested to us, where an individual can secede from the production economy and, if someone secedes, no one can use the production technologies. Then a blocking plan needs to satisfy the participation constraints for the outsiders. Specifically, it has been suggested to us to add the next condition to Definition 3:

$$\sum_{i\in\neg S}\sum_{j\in\tilde{J}(S)}\theta_{ij}y'_j+\sum_{i\in\neg S}\sum_{j\in\neg\tilde{J}(S)}\theta_{ij}y_j\in\mathbb{R}^l_+. \tag{14}$$

This is equivalent to saying that the total bundle for the set $\neg S$ of outsiders should be at least as large as their total endowment. Otherwise, the outsiders may secede, making the production technologies useless to the blocking coalition. Let us call this strengthened notion of blocking *R(espectful)-blocking*, and the corresponding core *R-core*.

The problem of R-core is that it does not work for core equivalence, nor even the ETP. To see that, consider any economy where the technology is to use good 1 to produce good 2, zero production is Pareto inferior, and every individual has a positive share of the firm in his unit. Then any non-zero production plan would have a negative component at the coordinate for good 1, so condition (14) cannot be satisfied unless the blocking coalition S is the grand coalition. Therefore, *the R-core is equal to the set of Pareto efficient allocations*. That has three consequences:

First, the ETP is not satisfied. Consider the allocation of giving all resources to individual 1, making his consumption maximal, and having everyone else consume $\mathbf{0}$. Obviously it is Pareto efficient and hence is in the R-core, but it violates the ETP.

Second, since the ETP does not hold, it is hard to compare the R-cores in different replica economies because they cannot be represented in the same space. Thus, a core convergence result à la Debreu and Scarf is impossible.

Third, even if the ETP issue were set aside, the R-core does not shrink when an economy is replicated. That is because *if an allocation* (x, y) *with ETP is Pareto dominated in an r-replica economy* $\mathcal{E}^r$, *then the allocation is Pareto dominated in the* 1*-replica economy* $\mathcal{E}^1$: the fact that (x, y) is Pareto dominated in $\mathcal{E}^r$ means that, for each individual (i_1, i_2) there is an $x'_{(i_1,i_2)} \succ_{i_1} x_{(i_1,i_2)}$, and for each firm (j_1, j_2) there is a $y'_{(j_1,j_2)} \in Y_{j_1}$, such that

$$\sum_{(i_1,i_2)\in I} x'_{(i_1,i_2)} = \sum_{(i_1,i_2)\in I} e_{(i_1,i_2)} + \sum_{(j_1,j_2)\in J} y'_{(j_1,j_2)},$$

i.e.,

$$\sum_{i_1\in I_1} \frac{1}{r}\sum_{i_2=1}^{r} x'_{(i_1,i_2)} = \sum_{i_1\in I_1} e_{i_1} + \sum_{j_1\in J_1} \frac{1}{r}\sum_{j_2=1}^{r} y'_{(j_1,j_2)}.$$

By convexity of preferences and technologies, $\frac{1}{r}\sum_{i_2=1}^{r} x'_{(i_1,i_2)} \succ_{i_1} x_{(i_1,i_2)}$ and $\frac{1}{r}\sum_{j_2=1}^{r} y'_{(j_1,j_2)} \in Y_{j_1}$ for all consumer- and firm-types i_1 and j_1. Hence (x, y) is Pareto dominated even in $\mathcal{E}^1$.

Although the particular notion of R-core does not work, the idea of adding a participation constraint to the notion of blocking is interesting. The question is How to formulate the "preferences" for the entire group of outsiders? Condition (14), which resorts to comparing the quantities of the bundles, is not a compelling participation constraint, because the outsiders do not necessarily opt for secession even when (14) is violated: by trading some goods away for other goods, the outsiders may be better-off than retaining their endowments.

Alternatively, consider the following condition:

$$\sum_{i\in\neg S}\sum_{j\in\tilde{J}(S)} \theta_{ij} y'_j + \sum_{i\in\neg S}\sum_{j\in\neg\tilde{J}(S)} \theta_{ij} y_j \notin \left(-\mathbb{R}^l_+\right)\Big\backslash \{\mathbf{0}\}. \tag{15}$$

I.e., a blocking coalition should not take some good from the outsiders without returning something. This condition is compelling because the outsiders would opt for secession if (15) is violated: for any consumption plan $(x'_i)_{i\in\neg S}$ for the outsiders feasible given the blocking plan of coalition S, if (15) is violated then there is a bundle $v \in \mathbb{R}^l_+ \setminus \{\mathbf{0}\}$ such that

$$\sum_{\neg S} e_i = v + \sum_{\neg S} e_i + \sum_{\neg S}\sum_{\tilde{J}(S)} \theta_{ij} y'_j + \sum_{\neg S}\sum_{\neg\tilde{J}(S)} \theta_{ij} y_j = v + \sum_{\neg S} x'_i = \sum_{\neg S}\left(x'_i + \frac{1}{|\neg S|} v\right);$$

by strongly monotone preferences, $x'_i + \frac{1}{|\neg S|} v \succ_i x'_i$ for all $i \in \neg S$; hence the outsiders would unanimously opt for keeping only their endowments.

Call a considerate blocking plan that satisfies (15) *superconsiderate blocking*, and the corresponding core the *superconsiderate core*, denoted by $C^r_{\sup}$ for the r-replica economy. Then a core equivalence theorem based on superconsiderate blocking is at hand.

Theorem 20. *Any $C^r_{\sup}$-allocation has the ETP. If Assumption* 9 *holds, then* $\mathbb{W} = \cap_{r=1}^{\infty} C^r_{\sup}$.

Proof. The ETP simply follows from the proof of Lemma 5. Recall that the blocking plan in that proof leaves the production plans $(y_j)_{j\in J}$ in the status quo unchanged. Hence the additional condition (15) for superconsiderate blocking becomes

$$\sum_{j_1\in J_1} y_{j_1} \notin \left(-\mathbb{R}^l_+\right)\Big\backslash \{\mathbf{0}\},$$

which is satisfied because any superconsiderate core allocation is Pareto efficient and preferences are strongly monotone.

With ETP, the equation $\mathbb{W} = \cap_{r=1}^{\infty} C_{\sup}^{r}$ is meaningful. The part "$\mathbb{W} \subseteq \cap_{r=1}^{\infty} C_{\sup}^{r}$" follows directly from Lemma 4.

The proof for the converse "$\mathbb{W} \supseteq \cap_{r=1}^{\infty} C_{\sup}^{r}$" is similar to that of Theorem 11. The only thing we need to change is Lemma 8. Now it says: if an allocation with ETP is inconsiderately blocked and not edgy, then it is superconsiderately blocked if the replica economy is sufficiently large. We just need to add a proof for (15):

First, if the allocation (x, y) is not Pareto efficient, it is superconsiderately blocked by the grand coalition, with (15) vacuously true. Second, if (x, y) is Pareto efficient, (15) is satisfied when the economy is sufficiently replicated: that is because the left-hand side of (15) is equal to a fixed vector plus a multiple of $\sum_{j_1 \in J_1} y_{j_1}$, and $\sum_{j_1 \in J_1} y_{j_1} \notin \left(-\mathbb{R}_{+}^{l}\right) \setminus \{\mathbf{0}\}$ by Pareto efficiency and strongly monotone preferences. Thus, (15) is satisfied when this multiple is sufficiently large. □

Although (15) is a necessary condition to keep the outsiders from seceding, it need not be sufficient. What is an appropriate formulation for the outsiders' participation constraint? More generally, how should the core be defined and how does it behave when individuals can opt out of a property rights arrangement? We leave these questions for future research.

5. Bibliography note

Debreu and Scarf [5] considered a special case with production where the technology is constant returns to scale *and* is available to all coalitions. Other than the main differences in the treatment of production and the boundary aversion assumption, the assumption-differences between our Theorem 11 and theirs are minor. Debreu and Scarf assumed insatiability and continuity, we assume strong monotonicity but only lower semicontinuity. They assumed strict positivity of individual endowments, we assume non-void individual endowments and only strict positivity of the total endowment. Assuming strict concavity of production frontiers, we do not cover constant returns to scale in this paper, but we do in a companion paper, Xiong and Zheng [13], based on an alternative notion of blocking, stochastic blocking.

The most general core convergence theorem for exchange economies is given by Anderson [3], who allowed non-convex preferences. In [13], we prove an extension of Anderson's theorem to production economies.

Other than Debreu and Scarf, there had been two approaches to include production to core equivalence, but none captured the interactions among shareholders. One group of authors such as Hildenbrand [11, Chapter 4] and Boehm [4] assumed that every possible coalition is endowed with a technology. Another group of authors, Allingham [2, pp. 52–53] and Aliprantis et al. [1], assumed that technologies are divisible and a shareholder controls a fraction of the firm's production set.

Haller [10] is the only earlier contribution to core equivalence that considers production and the ramifications of corporate control rights. Haller's notion of blocking does not take into account the constraint that the outsiders of a coalition need to have feasible consumptions, while the notions of blocking in our theorems take that feasibility constraint into account.

In addition to this main difference, there are other substantial differences between Haller's work and its counterpart in our paper. Haller's notion of blocking is a precedent of one of our intermediary notions of blocking, inconsiderate blocking. However, the two notions are different. As we will show, in his Example 2, an allocation cannot be Haller-blocked but can be inconsiderately blocked. Furthermore, Haller's core convergence theorem says that if a non-equilibrium allocation satisfies an interior condition and a liquidity condition, then the allocation is Haller-blocked.

By contrast, our Proposition 7, core convergence based on inconsiderate blocking, says that *any* non-equilibrium allocation is inconsiderately blocked. Appendix E has detailed comparison.

Ellickson et al. [7–9] prove core equivalence in their general equilibrium theory of clubs. Unlike in our model, a blocking coalition in their model is not interlocked with its outsiders in any firm, because they assume that a coalition forms clubs (firms) within itself and its members do not join any club (firm) containing outsiders.

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Appendix A. A complete second welfare theorem

The second welfare theorem proved here, Corollary 22, is used by Lemma 5. [4]

Lemma 21. *If $\left((x_i)_{i\in I}, (y_j)_{j\in J}; p\right)$ is a quasiequilibrium and $p \neq \mathbf{0}$, then $p \gg \mathbf{0}$ and the quasiequilibrium is a price equilibrium with possible transfers such that each person i's wealth is equal to $p \cdot x_i$.*

Proof. Let w_i denote individual i's wealth at this quasiequilibrium. By definition of quasiequilibrium (e.g., [12, Definition 16.D.1]), each firm's production plan is profit-maximizing given p and each consumer is quasioptimizing. Consumers' quasioptimization and strongly monotone preferences imply that the supporting price is nonnegative in each component, i.e., $p \geqq \mathbf{0}$. Then it follows from profit maximization and the assumptions "$\sum_{i\in I} e_i \gg \mathbf{0}$ and $\mathbf{0} \in \sum_{j\in J} Y_j$" that

$$\sum_{i\in I} w_i = \sum_{i\in I} p \cdot e_i + \sum_{j\in J} p \cdot y_j > 0.$$

Thus, there exists an individual i whose wealth w_i at this quasiequilibrium is positive. It follows that $p \gg \mathbf{0}$: suppose not, say $p_k = 0$ for good k, then the new bundle x_i' obtained from adding one unit of good k to individual i's original bundle x_i would cost him at most w_i. As his preference is strongly monotone, $x_i' \succ_i x_i$. Then the quasiequilibrium condition implies that $p \cdot x_i' = w_i$. By lower semicontinuity of his preference, we can scale down x_i' to $\lambda x_i'$ for some positive λ sufficiently close to one so that $p \cdot \lambda x_i' < w_i$ and $\lambda x_i' \succ_i x_i$, contradicting the quasiequilibrium condition. Thus, $p \gg \mathbf{0}$.

It follows that x_i is optimum for each individual i with wealth $w_i^* := p \cdot x_i$: If $x_i \gneqq \mathbf{0}$ then $w_i^* > 0$ (since $p \gg \mathbf{0}$), i.e., the cheaper-consumption condition is met, hence x_i is optimum by

[4] In textbooks, the second welfare theorem is usually presented in two propositions: one says Pareto efficiency implies existence of a price that supports the allocation as a quasiequilibrium, and the other gives a sufficient condition, which however is about the endogenous supporting price, for a quasiequilibrium to be an equilibrium with transfers. Unlike such versions, Corollary 22 is based on purely primitive assumptions.

lower semicontinuous preferences; else $x_i = \mathbf{0}$, then since $p \gg \mathbf{0}$, person i's budget set is the singleton $\{x_i\} = \{\mathbf{0}\}$. Hence $(x, y; p)$ is a price equilibrium with possible transfers such that each individual i's wealth is $w_i^* = p \cdot x_i$. □

Corollary 22 (*Second welfare theorem*). *If* (x, y) *is a Pareto efficient allocation, then there exists a price vector* $p \gg \mathbf{0}$ *that supports* (x, y) *as a price equilibrium with transfers.*

Proof. By the usual version of the second welfare theorem, there exists a $p \in \mathbb{R}^l_+ \backslash \{\mathbf{0}\}$ that supports the Pareto efficient allocation as a quasiequilibrium (e.g., [12, Proposition 16.D.1; Definition 16.D.1]). Then the corollary follows from Lemma 21. □

Appendix B. The proof of Lemma 5 (ETP)

First, we show the ETP for firms: since an element of the core C^r is Pareto efficient (Eq. (4) is vacuously satisfied for the grand coalition), by a modified version of the second welfare theorem (Corollary 22 in Appendix A), there exists a price vector $p \gg \mathbf{0}$ at which each firm's production plan is profit-maximizing. Then the strict concavity of the production possibility frontier implies that such production plan is unique for each type of firms.

Thus, at any core allocation (x, y), individuals of the same type i_1 have the same production bundle $\sum_{j_1 \in J_1} \theta_{i_1 j_1} y_{j_1}$ and hence the same bundle of resources $e_{i_1} + \sum_{j_1} \theta_{i_1 j_1} y_{j_1}$.

Then the ETP for consumers is proved as follows: if allocation (x, y) does not have this property, then the well-known "underdog coalition," consisting of a least favored individual of each type, blocks (x, y) by the plan of keeping production unchanged and having each member of type i_1 consume $\frac{1}{r} \sum_{i_2=1}^{r} x_{(i_1,i_2)}$. As in the standard proof (e.g., [12, Proposition 18.B.2]), this plan is an improvement for the coalition members; it is also feasible within the coalition:

$$\sum_{i \in S} x_i' = \frac{1}{r} \sum_{i_1 \in I_1} \sum_{i_2=1}^{r} x_{(i_1,i_2)} = \sum_{i_1 \in I_1} e_{i_1} + \sum_{i_1 \in I_1} \sum_{j_1 \in J_1} \theta_{i_1 j_1} y_{j_1}$$
$$= \sum_{i \in S} e_i + \sum_{i \in S} \sum_{j \in J} \theta_{ij} y_j. \tag{B.1}$$

To complete the proof, we need to verify Eq. (4), i.e., this plan is feasible for outsiders. Given this plan, the total bundle for outsiders is equal to

$$(r-1) \left[\sum_{i_1 \in I_1} e_{i_1} + \sum_{i_1 \in I_1} \sum_{j_1 \in J_1} \theta_{i_1 j_1} y_{j_1} \right] = (r-1) \sum_{i \in S} x_i',$$

where the equality follows from Eq. (B.1). Since $r - 1 \geqslant 0$ and since $\sum_{i \in S} x_i' \in \mathbb{R}^l_+$, the total bundle for outsiders also belongs to $\mathbb{R}^l_+$. Hence Eq. (4) is satisfied, as desired.

Appendix C. The proof of Propositions 6 and 7

Proof of Proposition 6. The part $\mathbb{W} \subseteq \cap_{r=1}^{\infty} C^r_{\text{wish}}$ follows from Lemma 4. To prove the converse, $\cap_{r=1}^{\infty} C^r_{\text{wish}} \subseteq \mathbb{W}$, pick any allocation from $\cap_{r=1}^{\infty} C^r_{\text{wish}}$. With ETP, denote this allocation by $\left((x_{i_1})_{i_1 \in I_1}, (y_{j_1})_{j_1 \in J_1}\right)$. We shall prove that this allocation belongs to $\mathbb{W}$ by mimicking Debreu and Scarf [5].

For each $i_1 \in I_1$, define

$$\Gamma_{i_1} := \left\{ z_{i_1} \in \mathbb{R}^l : z_{i_1} + e_{i_1} + \sum_{j_1 \in J_1} \theta_{i_1 j} y'_{j_1} \succ_{i_1} x_{i_1} \text{ for some } (y'_{j_1})_{j_1 \in J_1} \in \left(Y_{j_1}\right)_{j \in J_1} \right\},$$

which is convex. Let Γ be the set of convex combinations of these sets, i.e.,

$$\Gamma := \left\{ z = \sum_{i_1 \in I_1} \alpha_{i_1} z_{i_1} : \sum_{i_1 \in I_1} \alpha_{i_1} = 1; \forall i_1 \in I_1 \left[\alpha_{i_1} \geqslant 0; z_{i_1} \in \Gamma_{i_1}\right] \right\}.$$

Claim: $\mathbf{0} \notin \Gamma$. Suppose $\mathbf{0} \in \Gamma$. Then $\sum_{i_1 \in I_1} \alpha_{i_1} z_{i_1} = \mathbf{0}$, with $\alpha_{i_1} \geqslant 0$, $\sum_{i_1 \in I_1} \alpha_{i_1} = 1$, and $z_{i_1} + e_{i_1} + \sum_{j_1 \in J_1} \theta_{i_1 j_1} y'_{j_1} \succ_{i_1} x_{i_1}$, for some $(y'_{j_1})_{j_1 \in J_1} \in (Y_{j_1})_{j_1 \in J_1}$ and for all $i_1 \in I_1$. Consider an arbitrary large integer k. Denote $\lceil k\alpha_{i_1} \rceil$ for the smallest integer greater than or equal to $k\alpha_{i_1}$. Let S_1 be the set of $i_1 \in I_1$ for which $\alpha_{i_1} > 0$. For each $i_1 \in S_1$, let $z^k_{i_1} := \left(k\alpha_{i_1}/\lceil k\alpha_{i_1} \rceil\right) z_{i_1}$. Note that $z^k_{i_1}$ goes to z_{i_1} as k goes to infinity. By the lower semicontinuity of preference and the fact that $z_{i_1} + e_{i_1} + \sum_{j \in J} \theta_{i_1 j_1} y'_{j_1} \succ_{i_1} x_{i_1}$, there exists a large enough k such that $z^k_{i_1} + e_{i_1} + \sum_{j \in J} \theta_{i_1 j_1} y'_{j_1} \succ_{i_1} x_{i_1}$ and

$$\sum_{i_1 \in S_1} \lceil k\alpha_{i_1} \rceil z^k_{i_1} = k \sum_{i_1 \in S_1} \alpha_{i_1} z_{i_1} = \mathbf{0}.$$

In the $\sum_{i_1 \in S_1} \lceil k\alpha_{i_1} \rceil$-replica economy, let the coalition consist of $\lceil k\alpha_{i_1} \rceil$ individuals of type i_1 for each type i_1, such that none of the coalition members live in the same unit. The blocking plan is: for every $i_1 \in S_1$, each coalition member of type i_1 consumes $z^k_{i_1} + e_{i_1} + \sum_{j_1 \in J_1} \theta_{i_1 j} y'_{j_1}$, and each type-$j_1$ firm where this member is a shareholder produces y'_{j_1}. (Since no two coalition members share the same firm, they have no conflict on these production plans.) This coalition wishfully blocks $\left((x_{i_1})_{i_1 \in I_1}, (y_{j_1})_{j_1 \in J_1}\right)$, contradiction.

Hence $\Gamma \cap \{\mathbf{0}\} = \emptyset$. Also Γ is convex; by the separating hyperplane theorem, there exists a price $p \in \mathbb{R}^l \setminus \{\mathbf{0}\}$ such that $p \cdot z \geqslant 0$ for all $z \in \Gamma$.

To complete the proof, by Lemma 21, it suffices to show that (i) $((x_{i_1})_{i_1 \in I_1}, (y_{j_1})_{j_1 \in J_1}, p)$ is a quasiequilibrium and (ii) for all $i_1 \in I_1$,

$$p \cdot x_{i_1} = p \cdot e_{i_1} + p \cdot \sum_{j_1 \in J_1} \theta_{i_1 j_1} y_{j_1}, \tag{C.1}$$

because then Lemma 21 implies that it is a Walras equilibrium that requires zero transfer.

Let $\pi^*_{j_1} := \sup\{p \cdot y'_{j_1} : y'_{j_1} \in Y_{j_1}\}$ for each firm-type j_1. Since

$$x'_{i_1} \succ_{i_1} x_{i_1} \Longrightarrow x'_{i_1} - e_{i_1} - \sum_{j_1 \in J_1} \theta_{i_1 j_1} y'_{j_1} \in \Gamma_{i_1}, \quad \forall (y'_{j_1})_{j_1 \in J_1} \in (Y_{j_1})_{j_1 \in J_1},$$

the separating-hyperplane property of p implies

$$x'_{i_1} \succ_{i_1} x_{i_1} \Longrightarrow p \cdot x'_{i_1} \geqslant p \cdot e_{i_1} + \sum_{j_1 \in J_1} \theta_{i_1 j_1} \pi^*_{j_1}. \tag{C.2}$$

By local non-satiation of preference, (C.2) implies, for all $i_1 \in I_1$,

$$p \cdot x_{i_1} \geqslant p \cdot e_{i_1} + \sum_{j_1 \in J_1} \theta_{i_1 j_1} \pi^*_{j_1}. \tag{C.3}$$

Since $((x_{i_1})_{i_1\in I_1}, (y_{j_1})_{j_1\in J_1})$ is feasible, $\sum_{i_1\in I_1} x_{i_1} = \sum_{i_1\in I_1} e_{i_1} + \sum_{j_1\in J_1} y_{j_1}$. Then (C.3) implies

$$p\cdot\sum_{i_1\in I_1} e_{i_1} + p\cdot\sum_{j_1\in J_1} y_{j_1} = p\cdot\sum_{i_1\in I_1} x_{i_1} \geqslant p\cdot\sum_{i_1\in I_1} e_{i_1} + \sum_{j_1\in J_1}\pi^*_{j_1} \geqslant p\cdot\sum_{i_1\in I_1} e_{i_1} + p\cdot\sum_{j_1\in J_1} y_{j_1}, \tag{C.4}$$

where the last inequality follows from the definition of $\pi^*_{j_1}$. By (C.4), $\sum_{j_1} p\cdot y_{j_1} = \sum_{j_1}\pi^*_{j_1}$, hence every y_{j_1} maximizes profits for firms of type j_1 under the price p. From (C.4) we also have $p\cdot\sum_{i_1\in I_1} x_{i_1} = p\cdot\sum_{i_1\in I_1} e_{i_1} + \sum_{j_1\in J_1}\pi^*_{j_1}$; hence (C.3) implies

$$p\cdot x_{i_1} = p\cdot e_{i_1} + \sum_{j_1\in J_1}\theta_{i_1 j_1}\pi^*_{j_1} = p\cdot e_{i_1} + p\cdot\sum_{j_1\in J_1}\theta_{i_1 j_1} y_{j_1}$$

for all $i_1\in I_1$. This has two implications: first, Eq. (C.1) is true. Second, (C.2) implies $x'_{i_1}\succ_{i_1} x_{i_1} \Rightarrow p\cdot x'_{i_1}\geqslant p\cdot x_{i_1}$ for all $i_1\in I_1$. Hence $((x_{i_1})_{i_1\in I_1}, (y_{j_1})_{j_1\in J_1}, p)$ is a quasiequilbrium and (C.1) is true, as desired. □

Proof of Proposition 7. To prove $\mathbb{W}\subseteq\cap_{r=1}^{\infty} C^r_{\text{incon}}$, just mimic the first welfare theorem.

To prove $\cap_{r=1}^{\infty} C^r_{\text{incon}}\subseteq\mathbb{W}$, we shall prove that if an allocation is wishfully blocked in $\mathcal{E}^r$, then it is inconsiderately blocked in $\mathcal{E}^{2r}$. That means $\cap_{r=1}^{\infty} C^r_{\text{incon}}\subseteq\cap_{r=1}^{\infty} C^r_{\text{wish}}$. Then by Proposition 6 we have $\cap_{r=1}^{\infty} C^r_{\text{incon}}\subseteq\cap_{r=1}^{\infty} C^r_{\text{wish}}\subseteq\mathbb{W}$.

Suppose in an r-replica economy $\mathcal{E}^r$, a feasible allocation (x, y) is wishfully blocked by a coalition S with blocking plan $\left((x^*_i)_{i\in S}, (y^*_j)_{j\in J}\right)$. Hence

$$\sum_{i\in S} x^*_i = \sum_{i\in S} e_i + \sum_{j\in J}\sum_{i\in S}\theta_{ij} y^*_j. \tag{C.5}$$

Abusing notation, let $\mathcal{E}^{r\prime}$ denote the other r-replica economy in the $2r$-replica economy $\mathcal{E}^{2r}$, i.e., $\mathcal{E}^{2r} = \mathcal{E}^r\cup\mathcal{E}^{r\prime}$ and $\mathcal{E}^r$ is disjoint with $\mathcal{E}^{r\prime}$. Let I, J be the sets of consumers and firms in $\mathcal{E}^r$, and we use $[']$ to denote the counterpart in $\mathcal{E}^{r\prime}$, so I', J' are the sets of consumers and firms in $\mathcal{E}^{r\prime}$, and S' is the $\mathcal{E}^{r\prime}$-counterpart of S.

In $\mathcal{E}^{2r}$ $(=\mathcal{E}^r\cup\mathcal{E}^{r\prime})$, consider the coalition, $S^* := S\cup I'$, with the blocking plan:

(i) Consumption: $\left((x^*_i)_{i\in S}, (x_i)_{i\in I'}\right)$, i.e., every agent in S consumes x^*_i, and every agent in $\mathcal{E}^{r\prime}$ takes the status quo consumption.
(ii) Production: every firm in $\mathcal{E}^r$ takes the status quo production; the firms in $\mathcal{E}^{r\prime}$ takes the following aggregate production:

$$\sum_{j\in J'}\sum_{i\in S'}\theta_{ij} y^*_j + \sum_{j\in J'}\sum_{i\in I'\setminus S'}\theta_{ij} y_j,$$

which is feasible since all Y_j are convex and all the firms in $\mathcal{E}^{r\prime}$ are controlled by S^*.

To see that the above blocking plan is feasible, observe that pooling resources within this coalition amounts to a trade between S and I': I' gives S the production bundle $\sum_{j\in J'}\sum_{i\in S'}\theta_{ij} y^*_j$, and S gives I' the production bundle $\sum_{j\in J}\sum_{i\in S}\theta_{ij} y_j$. Since S' and J' are the $\mathcal{E}^{r\prime}$-counterparts of the sets S and J,

$$\sum_{j\in J'}\sum_{i\in S'}\theta_{ij}y_j^* = \sum_{j\in J}\sum_{i\in S}\theta_{ij}y_j^*,$$
$$\sum_{j\in J}\sum_{i\in S}\theta_{ij}y_j = \sum_{j\in J'}\sum_{i\in S'}\theta_{ij}y_j.$$

Thus, it is feasible for members of S to consume $(x_i^*)_{i\in S}$ (recalling (C.5)), and it is feasible for members of I' to consume $(x_i)_{i\in I'}$, as $(x_i)_{i\in I'}$ is aggregate feasible in $\mathcal{E}^{r'}$.

Since this blocking plan makes each member of S better-off and none of I' worse-off, it can be slightly adjusted to make each member of I' also better-off by lower semicontinuous preferences. Thus, (x, y) is inconsiderately blocked by the coalition S^* in $\mathcal{E}^{2r}$, as desired. □

Appendix D. An inconsiderate blocking plan for Example 12

Here is a direct construction that inconsiderately blocks all non-equilibria in Example 12 without using Proposition 7.

We shall show that if an allocation is not Walras equilibrium, then it can be inconsiderately blocked. Clearly there is no loss of generality to assume that a non-equilibrium allocation is Pareto efficient, then it belongs to

$$\{[x_1 = (0, a)\,,\ x_2 = (0, 2-a)\,,\ y = (-1, 1)] : a \in [0..2]\}.$$

There are two cases:

(i) $a < \frac{3}{4}$.

In the $(N+1)$-replica economy, let the coalition S consist of all consumers except the $(N+1)$th type-2 consumer. The blocking plan is: each firm in the first N-replica economy chooses the production plan $\left(-1-\frac{1}{2N}, \sqrt{1+\frac{1}{2N}}\right)$; the firm in the $(N+1)$th economy sticks to $(-1, 1)$; in the first N-replica economy, the consumption is $(0, a)$ for each type-1 consumer and $(0, 2-a)$ for each type-2 consumer; and the $(N+1)$th type-1 consumer takes $\left(0, N\sqrt{1+\frac{1}{2N}}-N+\frac{1}{2}\right)$. This blocking plan is feasible and inconsiderately blocks $[x_1 = (0, a)\,, x_2 = (0, 2-a)]$ for sufficient large N, because

$$\lim_{N\to\infty} N\sqrt{1+\frac{1}{2N}}-N+\frac{1}{2}$$
$$= \lim_{N\to\infty}\frac{\left(N\sqrt{1+\frac{1}{2N}}-N+\frac{1}{2}\right)\left(N\sqrt{1+\frac{1}{2N}}+N-\frac{1}{2}\right)}{N\sqrt{1+\frac{1}{2N}}+N-\frac{1}{2}}$$
$$= \lim_{N\to\infty}\frac{\frac{3}{2}N-\frac{1}{4}}{N\sqrt{1+\frac{1}{2N}}+N-\frac{1}{2}} = \frac{3}{4}.$$

(ii) $a > \frac{3}{4}$.

Hence every type-2 person consumes $(0, 2-a)$ and $(2-a) < 5/4$.

Consider the $(N+1)$-replica economy, and the coalition S consisting of all consumers except the $(N+1)$th type-1 consumer. The blocking plan is: each firm in the first N-replica economy chooses the production plan $\left(-1+\frac{1}{2N}, \sqrt{1-\frac{1}{2N}}\right)$; the firm in the $(N+1)$th economy sticks to $(-1, 1)$; the consumers in first N-replica economy take the consumption $[x_1 = (0, a), x_2 = (0, 2-a)]$, and the $(N+1)$th type-1 consumer take $\left(0, N\sqrt{1-\frac{1}{2N}} - N + \frac{3}{2}\right)$. This blocking plan is feasible and inconsiderately blocks $[x_1 = (0, a), x_2 = (0, 2-a)]$ for sufficient large N, because

$$\begin{aligned}
&\lim_{N\to\infty} N\sqrt{1-\frac{1}{2N}} - N + \frac{3}{2}\\
&= \lim_{N\to\infty} \frac{\left(N\sqrt{1-\frac{1}{2N}} - N + \frac{3}{2}\right)\left(N\sqrt{1-\frac{1}{2N}} + N - \frac{3}{2}\right)}{N\sqrt{1-\frac{1}{2N}} + N - \frac{3}{2}}\\
&= \lim_{N\to\infty} \frac{\frac{5}{2}N - \frac{9}{4}}{N\sqrt{1-\frac{1}{2N}} + N - \frac{3}{2}} = \frac{5}{4}.
\end{aligned}$$

Appendix E. Comparison with Haller [10]

The model of Haller [10] is a 2-period (0 and 1) incomplete-market economy where the only way to reallocate the resources in period 1 is to trade the stocks of the firms in period 0. An allocation in his model specifies each consumer i's share $(\theta^*_{ij})_{j\in J}$ of the firms, besides i's consumption x^*_i and each firm j's production y^*_j. Part of his feasibility condition is that i's period-1 consumption should conform to this share [10, Eq. (I4), p. 830]:

$$x^*_{i1} = e_{i1} + \sum_{j\in J} \theta^*_{ij} y^*_{j1}.$$

Hence people cannot swap their period-1 resources; they can only trade their resources available in period 0. Thus, the only resources that a coalition can trade within itself are: period-0 endowments, period-0 productions, and the corporate shares its members are endowed with. Specifically, if none of the coalition members is endowed with any corporate share, every member's period-1 consumption has to be the same as the status quo.

Unlike his model, our model is an Arrow–Debreu economy where agents can trade any bundle in the commodity space.

Different from the main focus of our paper, Haller's notion of blocking does not require that a coalition's blocking plan should allow the outsiders to come up with a feasible consumption plan. Haller allows a blocking coalition to change the production plans of the firms that it controls, but he does not require any condition about the outsiders' consumptions given the new production plans. As explained in our Introduction, if the outsiders cannot have feasible consumptions, they have to change the actions of the firms that they can control and hence may upset the blocking plan of the coalition.

One of our intermediary notions of blocking, inconsiderate blocking, does not require the feasibility condition for outsiders of a blocking coalition. Hence Haller's notion of blocking is a precedent of inconsiderate blocking. But the two notions are different because of Haller's restriction on the possible trades within a coalition, as explained at the first paragraph of this appendix. The next example illustrates this difference.

Example 23 (*Haller's Example 2*). There are 2 goods, 3 consumer-types and 1 firm-type.

$$\text{firm:} \quad Y := \left\{(y_0, y_1) \in \mathbb{R}^2 : y_0 \leqslant 0, y_1 \leqslant 1, y_0 + y_1 \leqslant 0\right\};$$

type-1 consumer: $e_1 = (0, 0), u_1(x_0, x_1) = \frac{1}{2}x_0 + x_1, \theta_1 = 1;$
type-2 consumer: $e_2 = (1, 0), u_1(x_0, x_1) = \frac{1}{2}x_0 + x_1, \theta_2 = 0;$
type-3 consumer: $e_3 = (1, 1), u_1(x_0, x_1) = x_0 x_1, \theta_3 = 0.$

The following allocation with ETP,

$y^* = (-1, 1)$ for the firm, $[x_1^* = (0, 1/2)\,, \theta_1^* = 1/2]$ for a type-1 consumer,
$[x_2^* = (0, 1/2)\,, \theta_2^* = 1/2]$ for type 2, and $[x_3^* = (1, 1), \theta_3^* = 0]$ for type 3,

cannot be blocked in Haller's model, but is inconsiderately blocked in our model.

The status quo allocation (x_i^*, y^*, θ^*) is inconsiderately blocked in our model even without replication: The blocking coalition consists of a type-2 and a type-3 individuals. The two coalition members are totally endowed with a consumption bundle (2, 1) but zero corporate share. Hence the type-2 member can consume $(\frac{1}{2}, \frac{1}{4})$ and the type-3 member can consume $(\frac{3}{2}, \frac{3}{4})$. Then the type-2 member is not hurt and the type-3 member is better-off, so the former can be made also better-off by receiving a small bundle from the latter.

In Haller's model, by contrast, this blocking plan is not feasible. That is because none of the members of this coalition is endowed with any share of a firm, hence they cannot have period-1 consumptions different from the status quo (the first paragraph of this appendix).

Even if we set aside the difference between Haller's blocking notion and its counterpart in this paper, inconsiderate blocking, the core convergence results based on these two notions have different ranges. Haller's core convergence theorem says that if a non-equilibrium allocation satisfies an interiority condition and a liquidity condition, then the allocation is Haller-blocked.[5] By contrast, our Proposition 7, core convergence based on inconsiderate blocking, says that *any* non-equilibrium allocation is inconsiderately blocked.

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[5] Haller's liquidity condition requires that every shareholder's endowment alone should be enough to cover her share of the production plan in the first period. For example, if an individual is endowed with a bundle (1, 1) and shares $\frac{1}{2}$ of a firm, the firm cannot pick any production plan $(-a, b)$ where $a > 2$.

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