Interactive Blocking in Arrow-Debreu Economies*

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Abstract

Competitive behaviors such as outbidding one's rivals may be countered by the rivals' threat of mutually destructive objections. In an Arrow-Debreu model of production economies with firms privatized by property rights, we model such hindered competitive behaviors as a coalition's attempt to block a status quo given the threat that the outsiders of the coalition, especially those with whom the coalition shares ownership of firms, may resort to production-ruining secession. We introduce new concepts of the core such that a coalition's blocking plan is feasible only if it is not blocked by the outsiders with such secession. Based on such notions, we prove core equivalence theorems in the replication framework.

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1 Introduction

What distinguishes competitive markets from alternative systems? This was the central question during the 1930s debate among Lange, Lerner, von Mises and Hayek on whether competitive equilibria can be replicated by socialist central planning. The question is echoed recently by Rubinstein [16, pp. 880–881], who argues that all the welfare and existence properties of competitive equilibria are also satisfied by the equilibria in a hypothetical jungle of brute forces disregarding property rights.[1] To answer this question, Hayek [11, 12] anticipated the modern economics of information by suggesting that the essence of markets is to handle decentralized information. In this paper, we pursue a different, core equivalence approach, which relates competitive equilibria to a decentralized model of property rights. Our result is core equivalence theorems based on new concepts of the core in production economies carved up by property rights.

The idea of core equivalence is to think of a competitive equilibrium as merely the description of the path of a more completely specified equilibrium whose off-path scenario is that any other allocation whose deviation from the competitive conditions is sufficiently large would be blocked by some coalition of individuals with their own resources. In other words, although Hayek and Rubinstein are right to point out the fact that the path of such a more completely specified equilibrium does not distinguish itself from socialist central planning or jungles of brute forces, the off-path scenario of the equilibrium provides a distinction for competitive markets because coalitional blocking is based on property rights for individuals.

However, the traditional literature of core equivalence (Edgeworth [8], Debreu and Scarf [7] Anderson [4], etc.) does not have a model where coalitions take as given the property rights on the firms.[2] The main unsolved problem is that a firm may be owned only partially by members of a coalition that attempts to block an allocation, with the remaining ownership lying outside the coalition. Then how should one define what blocking plans can

[1] With the land privatization in China happening at the same time as the massive financial bailout in the United States and Western Europe, the question about the distinction between capitalism and socialism is not irrelevant to current events. For example, Ralph Nader was reported to claim that the "[US] bailout was socialism rescuing capitalism" (Pankratz [15]).

[2] Debreu and Scarf [7] in their case with production assume that technologies exhibit constant returns to scale and are available to all coalitions. Hildenbrand [13, Chapter 4] and Boehm [6] assume that every possible coalition is endowed with a technology. Allingham [2, pp. 52–53] and Aliprantis, Brown and Burkinshaw [1] assume that technologies are divisble and a shareholder controls a fraction of the firm's production set.

be made by a coalition?[3]

Haller [10] proposes a notion of blocking with property rights on the firms. A coalition may direct a firm to supply any technologically feasible production bundle as long as the coalition owns a majority share of the firm. The production bundle supplied by any firm is distributed to its shareholders in proportion to their shares. The problem is that such blocking plans may be infeasible for those outside the coalition, say leaving a negative quantity of some good for the outsiders to consume. Such a problem arises because the outsiders still need to fulfill their obligations to the firms where they hold shares, while they cannot balance their resources through trading goods with the coalition.

In a recent paper [17], we fix this feasibility problem by proposing a notion *considerate blocking*. The idea is to add a requirement that the total bundle a blocking coalition leaves for its outsiders should be nonnegative for every good, so that the outsiders can consume what is left for them feasibly. Based on considerate blocking, that paper presents a core equivalence theorem for privatized production economies.

However, considerate blocking still has a problem. Even if feasible, a considerate blocking plan may be too draconian for the outsiders to bear. In that case, the outsiders might want to default on their obligations to the firms and walk away with their endowments. If they manage to secede as such, the plan of the blocking coalition may be upset, as part of its plan may depend on the production of the firms that it shares with the outsiders. In Section 3, we illustrate this problem with an example.

This paper presents a solution. Here we formulate the outside option for any group of individuals as *secession*, forfeiting all their shares in the firms and pooling together only the goods that they are endowed with. A group of individuals can block an allocation with secession if secession makes each of them better-off than the allocation. Then we add a participation constraint to the notion of considerate blocking: the blocking plan should contain an allocation for the outsiders which the outsiders as a whole cannot block with secession. We also add the participation constraint to any core allocation: no group of individuals can block a core allocation with secession. The solution concept thereof is *fair*

[3] The recent club-formation literature such as Allouch and Wooders [3] and Ellickson, Grodal, Scotchmer and Zame [9] consider the formation of firms as part of the equilibrium or core allocation. These authors take the approach of Hildenbrand and Boehm that endows every possible coalition with a set of technologies, so that any coalition is assumed to use some production technologies from which outsiders are excluded.

core. And we prove a core equivalence theorem of fair cores (Theorem 1). This result does not require any more assumption than our recent paper [17].

In a fair core, the participation constraint on a blocking plan is collective in the sense that the outsiders need to act as a single group to block it with secession. We strengthen this participation constraint by requiring that a blocking plan should not be blocked with secession by any group of outsiders. The solution concept thereof is *sup-fair core*. Adding an assumption about the preferences, we prove a core equivalence theorem of sup-fair cores (Theorem 2).[4]

Perhaps the nearest to our theory is the concept of bargaining set introduced by Aumann and Maschler [5]. They require that an objection to an allocation should itself survive counterobjections, and we require that a blocking plan of an allocation should itself survive blocking with secession. Behind this parallel, however, is a main difference. A counterobjection to an objection is to get some of the coalition members betray the coalition. As a coalition of traitors from a previous coalition may be vulnerable to betrayal itself, the notion of counterobjection naturally leads to more complicated notions such as counterobjections to counterobjections, ad infinitum. By contrast, secession-blocking is not to estrange the original blocking coalition. Rather, a secession group merely walks away with their own endowed goods, hence there is no need to involve anyone outside the secession group. It affects the original blocking plan just because the secession disrupts the production of the firms in which the secession group holds shares. The fact that core equivalence is proved for fair and sup-fair cores suggests that our concepts may have an advantage of tractability.

2 The Primitives

There are a finite set I of individuals, a finite set J of firms, and a finite number l kinds of goods. Let i be the index for individuals and j for firms. Let θ_{ij} be i's share of firm j, with $\sum_{i \in I} \theta_{ij} = 1$ for all j. Let $\mathbb{R}^l_+$ be the consumption set of each individual, $\succeq_i$ individual i's preference relation on $\mathbb{R}^l_+$ (with strict preference $\succ_i$), e_i ($\in \mathbb{R}^l_+$) his endowment, and Y_j ($\subseteq \mathbb{R}^l$) the production set of firm j. An allocation is denoted by $(x, y) := ((x_i)_{i \in I}, (y_j)_{j \in J})$,

[4] While sup-fair blocking is mathematically stronger than fair blocking, fair core based on fair blocking is still interesting in its own right, because secession is often such a divisive issue that individuals in a society may be torn between the two camps by the "you are either with us or against us" binary mindset.

meaning that individual i consumes the bundle x_i and firm j's production plan is y_j. An allocation (x, y) is *feasible* if $x_i \in \mathbb{R}^l_+$ and $y_j \in Y_j$ for each individual i and each firm j and $\sum_{i\in I} x_i = \sum_{i\in I} e_i + \sum_{j\in J} y_j$. A *coalition* S is a nonempty set of individuals.

A *replica economy* of size r, denoted by $\mathcal{E}^r$, consists of r *units*, each of which has exactly the same composition of individuals, firms, endowments, and corporate ownership. We name an individual by an integer-pair $i := (i_1, i_2)$, and a firm by $j := (j_1, j_2)$, meaning—

$$\begin{aligned} i_1 &:= \text{the type of the individual (exactly one such individual in each unit);} \\ i_2 &:= \text{the unit to which the individual belongs;} \\ j_1 &:= \text{the type of the firm (exactly one such firm in each unit);} \\ j_2 &:= \text{the unit to which the firm belongs.} \end{aligned}$$

Individuals or firms of the same type have the same characteristics:

$$\begin{array}{lcl} \succeq_{(i_1,i_2)} & = \succeq_{(i_1,i_2')} & =: \succeq_{i_1}; \\ e_{(i_1,i_2)} & = e_{(i_1,i_2')} & =: e_{i_1}; \\ Y_{(j_1,j_2)} & = Y_{(j_1,j_2')} & =: Y_{j_1}; \\ \theta_{(i_1,k),(j_1,k)} & = \theta_{(i_1,k'),(j_1,k')} & =: \theta_{i_1 j_1}. \end{array}$$

Individuals in one unit have zero share of firms in other units:

$$\theta_{(i_1,i_2),(j_1,j_2)} = \begin{cases} \theta_{i_1 j_1} & \text{if } i_2 = j_2 \\ 0 & \text{if } i_2 \neq j_2. \end{cases}$$

Let

$$\begin{aligned} I_1 &:= \text{the index set for individual-types;} \\ J_1 &:= \text{the index set for firm-types.} \end{aligned}$$

An allocation $((x_i)_{i\in I}, (y_j)_{j\in J})$ has the *equal treatment property (ETP)* if individuals and firms of the same type have the same consumption and production plan, i.e.,

$$\begin{array}{lcl} x_{(i_1,i_2)} & = x_{(i_1,i_2')} & =: x_{i_1}; \\ y_{(j_1,j_2)} & = y_{(j_1,j_2')} & =: y_{j_1}. \end{array}$$

The following Assumptions 1–4 are usual, with the last one ensuring the ETP for firms.

Assumption 1 *For any individual i, $\succeq_i$ is strongly monotone, strictly convex (convex and strongly convex), and lower semicontinuous on $\mathbb{R}^l_+$.*

Assumption 2 *The total endowment* $\sum_{i \in I} e_i \gg \mathbf{0}$*, i.e., it has positive quantity of each good. Every individual's endowment is a point in* $\mathbb{R}^l_+ \setminus \{\mathbf{0}\}$ *(i.e.,* $e_i \gneqq \mathbf{0}$ *for all* $i \in I$*).*

Assumption 3 *For every firm* j*,* $\mathbf{0} \in Y_j$ *and* Y_j *is convex.*

Assumption 4 *For each firm* j*, no point on the production possibility frontier is the midpoint of any two distinct points of the production set* Y_j*.*

3 A Motivating Example

Let us start with a production economy with two consumers, two goods, and one firm:[5]

$$\begin{array}{ll} \text{Firm}: & \left\{(y_1, y_2) \in \mathbb{R}^2 : y_1 \leq 0; y_2 \leq \sqrt{|y_1|}\right\}; \\ \text{Ms. } 1: & e_1 = (0,1), \quad u_1(x_{11}, x_{12}) = x_{11}x_{12}, \quad \theta_1 = 1/2; \\ \text{Mr. } 2: & e_2 = (1,0), \quad u_2(x_{21}, x_{22}) = x_{21}x_{22}, \quad \theta_2 = 1/2. \end{array}$$

Consider the status quo allocation:

$$y = (-1/9, 1/3), \quad x_1 = (2/3, 1), \quad x_2 = (2/9, 1/3). \tag{1}$$

This allocation is aggregate feasible, as $x_1 + x_2 = (2/3, 1) + (2/9, 1/3) = (8/9, 4/3)$ and $e_1 + e_2 + y = (0,1) + (1,0) + (-1/9, 1/3) = (8/9, 4/3)$. The allocation is also Pareto optimal, with marginal rate of substitution and technical rate of substitution equal to each other:

$$\frac{x_{12}}{x_{11}} = \frac{1}{2/3} = \frac{3}{2}, \quad \frac{x_{22}}{x_{21}} = \frac{1/3}{2/9} = \frac{3}{2}, \quad \frac{1}{2\sqrt{|y_1|}} = \frac{1}{2\sqrt{1/9}} = \frac{3}{2}.$$

Hence (1) is a price equilibrium with transfers, supported by a price vector $(3,2)$. At (1), Ms. 1's utility is $2/3$, and Mr. 2's utility is $2/27$.

In (1), Mr. 2 is arguably exploited by Ms. 1, who gains an amount of good 1 without giving up any bit of good 2. Evaluated by the price vector $(3,2)$, the value for Mr.2 to trade with Ms. 1 is negative:

$$(3,2) \cdot (2/9, 1/3) - (3,2) \cdot (1,0) - \frac{1}{2}((3,2) \cdot (-1/9, 1/3) = \frac{4}{3} - 3 - \frac{1}{6} < 0.$$

[5] The Cobb-Douglas utility function in this example can be easily modified to satisfy all the assumptions postulated above, including strong monotonicity and strict convexity on the boundary of $\mathbb{R}^2_+$. For example, let $u(z_1, z_2) := z_1 z_2$ if $z_1 > 0$ and $z_2 > 0$, $u(z_1, z_2) := -e^{-z_1 - z_2}$ if $z_1 = 0$ or $z_2 = 0$.

The question is Can Mr. 2 somehow block the allocation (1)?

Coming from the notion of blocking in exchange economies, an idea is that Mr. 2 may want to cut off his trade with Ms. 1 and forfeit his share in the firm so that he can consume his own endowment $(1, 0)$. Generating only $1 \times 0 = 0$ utility, however, this blocking scheme does not make Mr. 2 better-off.

What about refraining from trading with Ms. 1 and yet still honoring the share in the firm? To pursue this idea, first we need to specify what it means by honoring one's share in the firm. In the Arrow-Debrue model, one's share in a firm merely means his share of the profit of the firm with a price taken as given. As blocking occurs off the path of price-taking behavior, we need to enrich the meaning of corporate shares that defines the property rights in the off-path events of blocking. Here we assume that, in case of blocking, the production bundle of a firm is distributed to its shareholders in the proportion of their shares of the firm. In this example, if the firm carries out the production plan $(-1/9, 1/3)$, half of this bundle is distributed to Mr. 2. Then Mr. 2 can consume

$$e_2 + \frac{1}{2}y = (1, 0) + \frac{1}{2}(-1/9, 1/3) = (17/18, 1/6),$$

which yields a utility 17/108, greater than the utility 2/27 that Mr. 2 gets in (1).

Thus, Mr. 2 would be better-off if he could follow the above blocking plan. But the blocking plan is infeasible because it would leave Ms. 1 the infeasible bundle

$$e_1 + \frac{1}{2}y = (0, 1) + \frac{1}{2}(-1/9, 1/3) \notin \mathbb{R}^2_+.$$

This problem was identified by our recent paper [17]. The solution provided by that paper is that the bundles left for those outside a blocking coalition may become feasible if the economy is replicated. In this example, let us make one copy of the two-consumer economy and combine the two economies into one. Now let us revisit the original Mr. 2's blocking plan of not trading with the others while honoring his shares in the firm. As before, this plan, if implemented, would make him better-off. The question is Can it be implemented?

In this replica economy, excluded by the blocking plan of the original Mr. 2, the outsiders are the original Ms. 1, the copy of Ms. 1 called Ms. 1*, and the copy of Mr. 2 called Mr. 2*. Given Mr. 2's blocking plan, the bundle of the outsiders' total resources is

$$e_1 + e_1 + e_2 + \frac{1}{2}y + y = (0, 1) + (0, 1) + (1, 0) + \frac{1}{2}(-1/9, 1/3) + (-1/9, 1/3) = (5/6, 5/2),$$

which is nonnegative in each good and hence can be split into feasible consumption bundles and distributed among Ms. 1, Ms. 1^*, and Mr. 2^*.

While the feasibility problem is resolved, the incentive issue needs to be dealt with. Would the original Mr. 2's blocking plan make the other individuals so worse-off that they may try to upset the blocking plan?

For instance, suppose the bundle $(5/6, 5/2)$, which is what is left for the outsiders given Mr. 2's blocking plan, is divided into three consumption bundles, $(1/12, 2)$, $(3/8, 1/4)$ and $(3/8, 1/4)$, one for each of the three outsiders. Then their utilities are respectively 1/6, 3/32 and 3/32. But then the three outsiders could do better if they could pool together only their endowments and forfeit their shares of the firms. If they manage to do so, their total resource is the bundle $e_1 + e_1 + e_3 = (1, 2)$, which they can divide into three identical bundles $(1/3, 2/3)$ so that each could have a utility equal to 2/9, larger than 1/6 and 3/32.

Such manner for the outsiders to upset the blocking plan of Mr. 2 we shall generalize into the notion of secession-blocking, as if the outsiders secede from the existing corporate ownership system and walk away with only their endowments. If the outsiders do that, Mr. 2's blocking plan cannot be implemented, because it needs the production bundle $(-1/9, 1/3)$, which in turn requires the endowments from the outsiders.

To solve this problem, we modify Mr. 2's blocking plan. For instance, suppose the bundle $(5/6, 5/2)$, which is what is left for the outsiders given Mr. 2's blocking plan, is evenly distributed among the outsiders Ms. 1, Ms. 1^*, and Mr. 2^*, so that each of them consumes $\frac{1}{3}(5/6, 5/2) = (5/18, 5/6)$ and gets a utility 25/108. Then the outsiders as a whole cannot benefit from secession-blocking Mr. 2's blocking plan. To see that, note that the total endowment among the three is $(0, 1) + (0, 1) + (1, 0) = (1, 2)$. Since the three agents have the same, convex preference relation, one of them must get a utility (weakly) less than $\frac{2}{9}$, which is the utility generated by $\frac{1}{3}(1, 2) = (\frac{1}{3}, \frac{2}{3})$ and is less than 25/108.[6] Therefore, the outsiders as a whole cannot upset the modified blocking plan by secession.

It follows that Mr. 2's modified blocking plan supplemented with the expectation that, in response to his blocking action, every outsider consumes the bundle $(5/18, 5/6)$, is self-consistent. It is self-consistent because the blocking plan can be carried out and make Mr. 2 better-off given such expectation of the outsiders' response, and because the outsiders as a

[6] Suppose $(1, 2)$ is distributed to the outsiders as z_1, z_2, z_3 with $z_1 + z_2 + z_3 = (1, 2)$. Without loss of generality, suppose $z_1 \preceq z_2 \preceq z_3$, by convexity $\frac{1}{3}(z_1 + z_2 + z_3) = \frac{1}{3}(1, 2) \succeq z_1$.

whole cannot benefit from deviating from such expected response given the blocking action of Mr. 2. Such notion of blocking we shall formulate as fair blocking in Section 4.

4 Fair Blocking with Collective Secession

For every initial assignment $((\theta_{ij})_{i\in I})_{j\in J}$ of shares and every coalition S, let $\tilde{J}(S)$ be the set of all the firms that switch to the proposal of coalition S if S is blocking the status quo. (For example, every firm holds a referendum with all its shareholders and it switches to the blocking coalition's proposal if and only if the proposal gets a vote greater than a predetermined threshold.) We assume that the function $\tilde{J}$ is exogenous.

For linguistic convenience, let us rephrase the definition of considerate blocking in [17]:

Definition 1 (considerate blocking) *A coalition S* considerately blocks *a feasible allocation $(x, y) := ((x_i)_{i\in I}, (y_j)_{j\in J})$ with an alternative feasible allocation $((x'_i)_{i\in I}, (y'_j)_{j\in J})$ if*

i. $x'_i \in \mathbb{R}^l_+$ *for all* $i \in I$, *and* $y'_j \in Y_j$ *for all* $j \in J$,

ii. *for each* $i \in S$, $x'_i \succ_i x_i$,

iii. *if* $j \notin \tilde{J}(S)$ *then* $y'_j = y_j$,

iv. $\sum_{i\in S} x'_i = \sum_{i\in S} e_i + \sum_{i\in S} \sum_{j\in J} \theta_{ij} y'_j$,

v. $\sum_{i\in \neg S} x'_i = \sum_{i\in \neg S} e_i + \sum_{i\in \neg S} \sum_{j\in J} \theta_{ij} y'_j$.

Conditions (iv) and (v) calculate the resources available for a coalition based on the coalition members' property rights on the goods and the firms. In particular, in the event of blocking, if a firm j is to receive a production plan y_j, a shareholder i is assumed to receive a bundle $\theta_{ij} y_j$ in the proportion of his ownership θ_{ij}. Since blocking is an off-path scenario, an observer outside the economy might not even be able to observe or falsify the assumption.

Provisions (i) and (v) constitute the outsiders' feasibility condition we proposed in a recent paper [17], which improves upon the traditional notion of blocking. However, the condition does not rule out a blocking plan that, through the predetermined obligation to joint ventures, takes away so many resources from the set $\neg S$ of outsiders that $\neg S$ would rather secede from the joint ventures, taking with them only their endowed goods. Hence we formalize the notion of secession as follows.

Definition 2 (blocking with secession) *An allocation* (x', y') *is* blocked by coalition S with secession *if: for each* $i \in S$ *there is* $x''_i \in \mathbb{R}^l_+$ *such that* $x''_i \succ_i x'_i$ *and* $\sum_{i \in S} x''_i = \sum_{i \in S} e_i$.

The assumption that a seceding coalition can take with them only their endowed goods is to capture the detrimental effect of secession in the simplest possible manner. In extreme cases, secession may ruin the social infrastructure so much that neither side of the conflict can produce much. In our production economy, secession disrupts production because a contractual relationship that the shareholders have agreed upon is being torn apart without mutual agreement—the blocking coalition counts on the contractual relationship to obtain its resources while its complement wants to secede from the contract.[7] [8]

Definition 3 (fair blocking) *A feasible allocation* (x, y) *is* fairly blocked *by coalition* S *if:*

a. either S *blocks* (x, y) *with secession,*

b. or S *considerately blocks* (x, y) *with an alternative feasible allocation* $((x'_i)_{i \in I}, (y'_j)_{j \in J})$ *that is not blocked by* $\neg S$ *with secession.*

Case (a) in the above definition is included so that the option of secession is even-handedly available to both sides of a blocking plan. It plays a role in the proof of the equal treatment property of core allocations.

To illustrate case (b), suppose S considerately blocks (x, y) with a blocking plan (x', y'), but

$$\sum_{\neg S} \sum_{\tilde{J}(S)} \theta_{ij} y'_j + \sum_{\neg S} \sum_{\neg \tilde{J}(S)} \theta_{ij} y_j \lneqq \mathbf{0}.$$

Then it is not a fair blocking plan, because $\neg S$ blocks (x', y') with secession: The above inequality implies that there is a bundle $v \gneqq \mathbf{0}$ such that

$$\sum_{\neg S} e_i = v + \sum_{\neg S} e_i + \sum_{\neg S} \sum_{\tilde{J}(S)} \theta_{ij} y'_j + \sum_{\neg S} \sum_{\neg \tilde{J}(S)} \theta_{ij} y_j = v + \sum_{\neg S} x'_i = \sum_{\neg S} \left(x'_i + \frac{1}{|\neg S|} v \right);$$

[7] To dissolve the joint ownership in a manner that may be mutually agreeable, one possible procedure is that the seceding party gives up its shares to the other party. But such arrangements would make core equivalence easy to fail, as pointed out in our recent paper [17, p. 256], because then a coalition may want to pillage the corporate shares from the outsiders by forcing them to secede.

[8] An alternative formulation for less detrimental secession is to allow a seceding group to keep the firms where it has full ownership. Such a notion may be less tractable, as our preliminary investigations indicate that the equal treatment property may fail for the core based on such notion of secession.

by monotone preferences, $x'_i + \frac{1}{|\neg S|} v \succ_i x'_i$ for all $i \in \neg S$.

The *fair core* of an economy is the set of all feasible allocations that are not fairly blocked.

Lemma 1 *If every individual i's preference relation $\succeq_i$ is locally non-satiable, then any Walras equilibrium allocation belongs to the fair core.*

Proof If an equilibrium (x, y, p) is blocked under case (b) in the definition of fair blocking, derive a contradiction by mimicking the standard proof. If it is blocked under case (a), i.e., (x, y) is secession-blocked by some S and a plan $(x''_i)_{i \in S}$, then

$$\sum_{i \in S} p \cdot x''_i > \sum_{i \in S} p \cdot e_i + \sum_{i \in S} \sum_{j \in J} \theta_{ij} p \cdot y_j \geq \sum_{i \in S} p \cdot e_i = \sum_{i \in S} p \cdot x''_i,$$

contradiction. (The first inequality follows from consumer-optimization at (x, y, p), the second one follows from profit maximization at (x, y, p) and $\mathbf{0} \in Y_j$ for all j, and the equality due to the resource-feasibility condition of secession-blocking.) ■

4.1 The Equal Treatment Property

Lemma 2 *By Assumptions 1–4, any fair core allocation has the ETP.*

Proof Let (x, y) be a fair core allocation of an r-replica economy. As in the proof of ETP in Xiong and Zheng [17], (x, y) is Pareto optimal and hence it has the ETP for firms.

To prove that (x, y) has the ETP for consumers, suppose otherwise.

For each consumer-type $i_1 \in I_1$, let $\iota_2(i_1)$ be a unit where the type-i_1 individual does the worst among type-i_1 individuals at (x, y). Let $S := \{(i_1, \iota_2(i_1) : i_1 \in I_1\}$.

If S blocks (x, y) with secession, then by case (a) of the definition of fair blocking, we are done. For the rest of the proof, suppose that S cannot block (x, y) with secession.

Consider an alternative allocation (x', y'): every firm j produces y_j as in the status quo (x, y), i.e., $y'_j = y_j$ for every j; every individual $i := (i_1, i_2)$ consumes

$$x'_{(i_1, i_2)} := \frac{1}{r} \sum_{i_2 = 1}^{r} x_{(i_1, i_2)}.$$

Obviously, (x', y') is a feasible allocation. As in the proof of the ETP lemma in Xiong and Zheng [17], it is also feasible within S and satisfies the outsiders' feasibility condition; it is also an improvement within S.

Thus, if $\neg S$ cannot block (x', y') with secession, then S fairly blocks the status quo (x, y) and we are done. Hence suppose that $\neg S$ blocks (x', y') with secession, i.e., there is $(x''_i)_{i\in\neg S}$ such that $x''_i \succ_i x'_i$ for all $i \in \neg S$ and $\sum_{\neg S} x''_i = \sum_{\neg S} e_i$. Let

$$x'''_{(i_1,\iota_2(i_1))} := \frac{1}{r-1} \sum_{i_2 \neq \iota_2(i_1)} x''_{(i_1,i_2)}.$$

By $\sum_{\neg S} x''_i = \sum_{\neg S} e_i$, we have $\sum_S x'''_i = \sum_S e_i$ because

$$(r-1) \sum_{i_1\in I_1} x'''_{(i_1,\iota_2(i_1))} = \sum_{i_1\in I_1} \sum_{i_2\neq\iota_2(i_1)} x''_{(i_1,i_2)} = \sum_{\neg S} x''_i = \sum_{\neg S} e_i = \sum_{i_1\in I_1} \sum_{i_2\neq\iota_2(i_1)} e_{i_1} = (r-1) \sum_{i_1\in I_1} e_{(i_1,\iota_2(i_1))}.$$

Since $x''_i \succ_i x'_i$ for all $i \in \neg S$, $x'''_{i_1} \succ_{i_1} x'_{i_1}$ for all $i_1 \in I_1$ by convexity of preferences. Then for each $i \in S$, $x'''_i \succ_i x'_i \succeq_i x_i$. Hence S blocks (x, y) with secession, a contradiction. ■

With the ETP, fair core allocations in economies of different sizes can be identified with the element of the same space, $\left(\mathbb{R}^l_+\right)^{I_1} \times \prod_{j_1\in J_1} Y_{j_1}$. Denote C^r_{fair} for the subset of this space that represents the fair core of replica economy of size r. Since Walras equilibrium allocations are in the fair core for any r, their ETP is preserved by replication. Hence denote $\mathbb{W}$ for the subset of $\left(\mathbb{R}^l_+\right)^{I_1} \times \prod_{j_1\in J_1} Y_{j_1}$ that represents the set of Walras equilibrium allocations.

4.2 The Core Equivalence Theorem

Why are non-equilibrium allocations eventually blocked when the economy is replicated? To answer this question, we first separate allocations into two categories. An allocation (x, y) is called *pure exchange* if $y_j = \mathbf{0}$ for every firm $j \in J$; denote such allocation by $(x, y \mid y = \mathbf{0})$. If $y_j \neq \mathbf{0}$ for some firm $j \in J$, we say it is *productive* and denote (x, y) by $(x, y \mid y \neq \mathbf{0})$.

For a pure exchange allocation, the argument is an extension of the standard proof of core equivalence for exchange economies.

For a productive allocation, assume without loss of generality that it is Pareto optimal. Given a blocking coalition, an individual from the outsiders is said *involved* if he belongs to the same unit of some member of the coalition and else is called *uninvolved*. As the allocation is Pareto optimal, it is costly for the uninvolved outsider to give up the production and participate in secession-blocking. Hence they need to be compensated by the involved outsiders. The more uninvolved outsiders, the bigger this compensation is needed. Thus, when the number of units is sufficiently large, the involved outsiders cannot gain enough to compensate for the sacrifice of the uninvolved outsiders from blocking with secession.

In our recent paper, Xiong and Zheng [17], we have shown that the next assumption is indispensable for core equivalence based on considerate blocking.

Assumption 5 (boundary aversion) *Every individual strictly prefers any interior point of* $\mathbb{R}^l_+$ *to any boundary point of* $\mathbb{R}^l_+$.

We can show that the fair cores shrink as we enlarge the economy, i.e., $C^1_{\text{fair}} \supseteq C^2_{\text{fair}} \supseteq C^3_{\text{fair}} \ldots$. First, suppose an allocation (x, y) is blocked with secession in the economy $\mathcal{E}^r$ by a coalition S with the plan $(x'_i)_{i\in E}$. Then, (x, y) is blocked with secession in the economy $\mathcal{E}^{r+1}$ by the same coalition S with the same plan $(x'_i)_{i\in E}$. Second, suppose an allocation (x, y) is considerately blocked in the economy $\mathcal{E}^r$ by a coalition S with the plan (x', y') such that no subset of $\mathcal{E}^r\backslash S$ can block (x', y'). Consider $\mathcal{E}^{r+1} := \mathcal{E}^r \cup \mathcal{E}'$ and the coalition $S' := S \cup \mathcal{E}'$, i.e., the union of S and the $(r + 1)$th copy $\mathcal{E}'$ of the prototype economy. We claim that S' fairly blocks (x, y) with the plan

$$\left((x'_i, y'_j)_{i\in I(\mathcal{E}^r), j\in J(\mathcal{E}^r)}, (x_i, y_j)_{i\in I(\mathcal{E}'), j\in J(\mathcal{E}')}\right),$$

i.e., the original economy $\mathcal{E}^r$ adopts the plan (x', y') and the additional economy $\mathcal{E}'$ stays with the plan (x, y). Clearly, no subset of $\mathcal{E}^{r+1}\backslash S' = \mathcal{E}^r \backslash S$ can block this plan with secession. Also, in the coalition $S' = S \cup \mathcal{E}'$, the members of $\mathcal{E}'$ get at least as well off as (x, y) and the members of S get strictly better-off than (x, y). Then by lower semicontinuity of preferences we can transfer a sufficiently small bundle of goods from S to the rest of the coalition, so that everyone in the coalition gets strictly better off. Therefore, $C^1_{\text{fair}} \supseteq C^2_{\text{fair}} \supseteq C^3_{\text{fair}} \ldots$. Theorem 1 shows that the sequence converges to the set of Walrus equilibria.

Theorem 1 *By Assumptions 1–5,* $\mathbb{W} = \cap_{r=1}^{\infty} C^r_{\text{fair}}$.

Proof First, by Lemmas 1 and 2, $\mathbb{W} \subseteq \cap_{r=1}^{\infty} C^r_{\text{fair}}$.

We still need to prove $\cap_{r=1}^{\infty} C^r_{\text{fair}} \subseteq \mathbb{W}$.

First, we claim that any non-equilibrium allocation can be fairly blocked if it is a pure exchange allocation, i.e., if it is of the form $(x, y \mid y = \mathbf{0})$.

We may assume without loss that $(x, y \mid y = \mathbf{0})$ is Pareto optimal (otherwise it is blocked by the grand coalition). Thus, $(x, y \mid y = \mathbf{0})$ is a price equilibrium with nonzero transfer under some price $p \in \mathbb{R}^l$. Note that dividends at this equilibrium are zero. Since transfer is nonzero, $p \cdot x_i \neq p \cdot e_i$ for some consumer i.

Consider the pure exchange economy that is the same as the original production economy except that all production sets are $\{\mathbf{0}\}$. In this economy, x is again a price equilibrium with nonzero transfer under the price p, because $p \cdot x_i \neq p \cdot e_i$ for some consumer i. By core convergence in pure exchange economies, x is blocked by some blocking plan $(x'_i)_{i \in S}$. Then in the original economy, S considerately blocks $(x, y \mid y = \mathbf{0})$ with the plan $((x'_i)_{i \in S}, \mathbf{0})$.

Given this blocking plan, which conducts zero production, condition (iv) of considerate blocking says that the aggregate consumption for the set $\neg S$ of outsiders is equal to $\sum_{i \in \neg S} e_i$, which can be turned into any Pareto optimal allocation $(x'_i)_{i \in \neg S}$ within $\neg S$ subject to $\sum_{\neg S} x'_i = \sum_{\neg S} e_i$. Then the alternative allocation $((x'_i)_{i \in S}, (x'_i)_{i \in \neg S}, \mathbf{0})$ cannot be blocked by $\neg S$ with secession. Hence S fairly blocks $(x, y \mid y = \mathbf{0})$ in the original production economy.

Second, we claim that any non-equilibrium allocation can be fairly blocked if it is a productive allocation, i.e., if it is of the form $(x, y \mid y \neq \mathbf{0})$.

By the ETP of fair core allocations, we may assume without loss that $(x, y \mid y \neq \mathbf{0})$ satisfies ETP. As $y \neq \mathbf{0}$, $y_{j_1^*} \neq \mathbf{0}$ for some firm-type $j_1^* \in J_1$.

We may also assume without loss that $(x, y \mid y \neq \mathbf{0})$ is Pareto optimal (otherwise it is blocked by the grand coalition). Then it is a price equilibrium with nonzero transfer under some price p.

By the considerate core convergence theorem of Xiong and Zheng [17], $(x, y \mid y \neq \mathbf{0})$ is considerately blocked by some coalition S within some r-replica economy $\mathcal{E}^r$, with an alternative allocation

$$\left((x'_i)_{i \in S}, (x'_i)_{i \in I^{(r)} \setminus S}, (y'_j)_{j \in J^{(r)}}\right).$$

Here $I^{(r)}$ and $J^{(r)}$ denote the sets of individuals and firms in $\mathcal{E}^r$.

Now add N copies of the unit economy. Let $I^{(N)}$ and $J^{(N)}$ denote the sets of individuals and firms in these N units, and let $\mathcal{E}^{r+N}$ denote the enlarged, $(r+N)$-replica economy. Note, in $\mathcal{E}^{r+N}$, the coalition S in the original $\mathcal{E}^r$ considerately blocks $(x, y \mid y \neq \mathbf{0})$ with the plan

$$\left((x'_i)_{i \in S}, (x'_i)_{i \in I^{(r)} \setminus S}, (x_i)_{i \in I^{(N)}}, (y'_j)_{j \in J^{(r)}}, (y_j)_{j \in J^{(N)}}\right), \tag{2}$$

i.e., the agents in the original $\mathcal{E}^r$ follow $\left((x'_i)_{i \in S}, (x'_i)_{i \in I^{(r)} \setminus S}, (y'_j)_{j \in J^{(r)}}\right)$ and the agents in the new N replica follow $(x, y \mid y \neq \mathbf{0})$. Here the set of outsiders becomes the disjoint union between $I^{(r)} \setminus S$ and the agents in the N additional replica, $I^{(N)}$, with a consumption plan $((x'_i)_{i \in I^{(r)} \setminus S}, (x_i)_{i \in I^{(N)}})$.

To complete the proof, we claim: the outsiders $(I^{(r)} \setminus S) \cup I^{(N)}$ cannot block the alternative allocation (2) with secession when N is sufficiently large.

Suppose the outsiders can block (2) with secession, with a plan $\left((\tilde{x}_i)_{i\in I^{(r)}\setminus S}, (\hat{x}_i)_{i\in I^{(N)}}\right)$. We may assume without loss that the $(\hat{x}_i)_{i\in I^{(N)}})$ here satisfies ETP (otherwise, replace every $\hat{x}_i$ by $\sum_{i_2'=1}^{N} \hat{x}_{(i_1,i_2')}/N$ for every $i \in I^{(N)}$). Thus, the secession-blocking plan is

$$\left((\tilde{x}_i)_{i\in I^{(r)}\setminus S}, \underbrace{(\hat{x}_{i_1})_{i_1\in I_1}, \ldots, (\hat{x}_{i_1})_{i_1\in I_1}}_{N}\right). \tag{3}$$

Let

$$V := \left\{\sum_{i_1\in I_1} t_{i_1} : t_{i_1} \in \mathbb{R}^l_+ \text{ and } t_{i_1} \succeq_{i_1} x_{i_1} \text{ for all } i_1 \in I_1\right\}.$$

V is convex, because it is the sum of upper contour sets, which are convex. Thus, $\overline{V}$, the closure of V, is both closed and convex.

Also, $\sum_{i_1\in I_1} e_{i_1} \notin \overline{V}$. Suppose $\sum_{i_1\in I_1} e_{i_1} \in \overline{V}$, i.e., there is $\left(t^n_{i_1}\right)_{i_1\in I_1}$ such that $\sum_{i_1\in I_1} t^n_{i_1} \to_n \sum_{i_1\in I_1} e_{i_1}$ and $t^n_{i_1} \succeq_{i_1} x_{i_1}$ for all $i_1 \in I_1$ and for all n. Recall $(x, y \mid y \neq \mathbf{0})$ is a price equilibrium with transfer under price p. By local non-satiation, $t^n_{i_1} \succeq_{i_1} x_{i_1}$ implies $p \cdot t^n_{i_1} \geq p \cdot x_{i_1}$. Hence

$$p \cdot \sum_{i_1\in I_1} e_{i_1} = \lim_{n\to\infty} p \cdot \sum_{i_1\in I_1} t^n_{i_1} \geq p \cdot \sum_{i_1\in I_1} x_{i_1} = p \cdot \left(\sum_{i_1\in I_1} e_{i_1} + \sum_{j_1\in J_1} y_{j_1}\right).$$

Thus, for the firm-type j_1^* such that $y_{j_1^*} \neq \mathbf{0}$, both $\mathbf{0}$ and $y_{j_1^*}$ are profit maximizers under p, contradicting the uniqueness of profit maximum given Assumption 3.

Since $\overline{V}$ is closed and convex and $\sum_{i\in I_1} e_i \notin \overline{V}$, by the Separating Hyperplane Theorem, there exist $q \in \mathbb{R}^l \setminus \{\mathbf{0}\}$ and $c \in \mathbb{R}$ such that $q \cdot \sum_{i_1\in I_1} e_{i_1} < c$ and $q \cdot v > c$ for all $v \in \overline{V}$.

Since $q \cdot \sum_{i_1\in I_1} e_{i_1} < c$, define

$$\xi := q \cdot \sum_{i_1\in I_1} e_{i_1} - c < 0.$$

Recall, from the fact that (3) is a secession-blocking plan of (2), that $\hat{x}_{i_1} \succ_{i_1} x_{i_1}$ for all $i_1 \in I_1$. Hence $\sum_{i_1\in I_1} \hat{x}_i \in V$. With $q \cdot v > c$ for all $v \in V$, we have

$$q \cdot \sum_{i_1\in I_1} e_{i_1} - q \cdot \sum_{i_1\in I_1} \hat{x}_{i_1} < q \cdot \sum_{i_1\in I_1} e_{i_1} - c = \xi < 0. \tag{4}$$

By the resource constraint for the secession-blocking plan (3) for the outsiders,

$$\begin{aligned}
& \sum_{i \in I^{(r)} \setminus S} \tilde{x}_i + N \sum_{i_1 \in I_1} \hat{x}_{i_1} = \sum_{i \in I^{(r)} \setminus S} e_i + N \sum_{i_1 \in I_1} e_{i_1} \\
\Rightarrow \quad & \sum_{i \in I^{(r)} \setminus S} \tilde{x}_i - \sum_{i \in I^{(r)} \setminus S} e_i = N \left(\sum_{i_1 \in I_1} e_{i_1} - \sum_{i_1 \in I_1} \hat{x}_{i_1} \right) \\
\Rightarrow \quad & q \cdot \left(\sum_{i \in I^{(r)} \setminus S} \tilde{x}_i - \sum_{i \in I^{(r)} \setminus S} e_i \right) = N \left(q \cdot \sum_{i_1 \in I_1} e_{i_1} - q \cdot \sum_{i_1 \in I_1} \hat{x}_{i_1} \right). \qquad (5)
\end{aligned}$$

The left-hand side of Eq. (5) is bounded from below by $-q \cdot \left(\sum_{i \in I^{(r)} \setminus S} e_i \right)$; the right-hand side, however, is less than $N\xi$ by Eq. (4), which goes to $-\infty$ as $N \to \infty$. Thus, Eq. (5) cannot hold when N is sufficiently large. This contradiction implies that the outsiders cannot block (2) with secession when N is sufficiently large.

Therefore, any non-equilibrium $(x, y \mid y \neq \mathbf{0})$ can be fairly blocked, as claimed. ■

5 Sup-Fair Blocking When Anyone May Secede

The notion of fair blocking requires only that the outsiders—as a whole—cannot block the coalition's alternative allocation with secession. Next we consider another notion of blocking where any proper subset of the outsiders may also block with secession.

To illustrate, let us recall the example in Section 3. There the original Mr. 2's blocking plan leaves a bundle $(5/6, 5/2)$ to be consumed by the outsiders, Ms. 1, Ms. 1*, and Mr. 2*. Part of the blocking plan is to divide the bundle evenly among the three so that each gets a utility $25/108$. Then the outsiders as a whole cannot benefit from secession.

However, Ms. 1 and Mr. 2* could be better-off if they are allowed to form a secession coalition with only the two of them. Their total endowment is $e_1 + e_2 = (1, 1)$, which can be split evenly between the two, giving each a utility $1/4$, greater than $25/108$. If Ms. 1 and Mr. 2* manage to secede in this manner, the production plan $(-1/9, 1/3)$ cannot be implemented, so the original Mr. 2's blocking plan would fail.

To make the original Mr. 2's blocking plan feasible despite such secession-blocking, the bundle $(5/6, 5/2)$ left for the other individuals needs to be allocated with some care. One such allocation is to give Mr. 2* $\left(\frac{5}{6} - \epsilon, \frac{5}{2} - \epsilon\right)$ and to give $\left(\frac{\epsilon}{2}, \frac{\epsilon}{2}\right)$ to each Ms. 1 and Ms. 1*, where $\epsilon > 0$ is chosen to be small so that $\left(\frac{5}{6} - \epsilon\right)\left(\frac{5}{2} - \epsilon\right)$, the utility for Mr. 2*, is greater than 2. (Such ϵ exists because $(5/6)(5/2) = 25/12$.)

Then no one can benefit from secession-blocking Mr. 2's blocking plan. Without involving Mr. 2^*, Ms. 1 and Ms. 1^* can only get zero utility. To involve Mr. 2^*, however, they need to give him a utility greater than two, but the bundle of the total resources available for the three of them, should they secede, is only $e_1 + e_1 + e_2 = (1, 2)$, which can generate a utility of at most 2.

In the following, we study a stronger notion of blocking, sup-fair blocking, that allows any set of outsiders to secede. Again, it can be shown that any Walras equilibrium cannot be sup-fairly blocked. However, can any non-equilibrium be sup-fairly blocked if we include sufficiently many copies of economy? The answer is again Yes (Theorem 2), if we make an additional assumption on the differentiability of agents' utility function.

Definition 4 (sup-fair blocking) *A feasible allocation* (x, y) *is* sup-fairly blocked *by coalition* S *if:*

a. either S *blocks* (x, y) *with secession,*

b. or S *considerately blocks* (x, y) *with an alternative feasible allocation* $((x'_i)_{i \in I}, (y'_j)_{j \in J})$ *that is not blocked by any subset of* $\neg S$ *with secession.*

5.1 The Equal Treatment Property

Lemma 3 *By Assumptions 1–4, any sup-fair core allocation has the ETP.*

Proof Let (x, y) be a fair core allocation of an r-replica economy. As in the proof of ETP in Xiong and Zheng [17], (x, y) is Pareto optimal and hence it has the ETP for firms.

To prove that (x, y) has the ETP for consumers, suppose otherwise.

If (x, y) can be blocked by some coalition with secession, then we are done. For the rest of the proof, suppose that (x, y) cannot be blocked by any coalition with secession.

For each consumer-type $i_1 \in I_1$, we can find a one-to-one function $\iota_{i_1} : \{1, 2, ..., r\} \to \{1, 2, ..., r\}$ such that the r type-i_1 individuals are listed as $x_{i_1, \iota_{i_1}(1))}, \ldots, x_{(i_1, \iota_{i_1}(r))}$ such that,

$$x_{(i_1, \iota_{i_1}(1))} \preceq_{i_1} x_{(i_1, \iota_{i_1}(2))} \preceq_{i_1} \cdots \preceq_{i_1} x_{(i_1, \iota_{i_1}(r))}.$$

Define a new allocation, (x', y) as follows. For each consumer-type $i_1 \in I_1$,

$$\begin{aligned} x'_{(i_1,\iota_{i_1}(r))} &:= \frac{1}{r}\sum_{k=1}^{r} x_{(i_1,\iota_{i_1}(k))}, \\ x'_{(i_1,\iota_{i_1}(h))} &:= \frac{r-1}{r}x_{(i_1,\iota_{i_1}(h))} + \frac{1}{r}x_{(i_1,\iota_{i_1}(r))} \quad \forall h \in \{1,2,...,r-1\}. \end{aligned}$$

Consider the coalition $S := \{(i_1, \iota_{i_1}(h)) : i_1 \in I_1 \text{ and } h \in \{1,2,...,r-1\}\}$. We will show S considerately blocks (x, y) such that no subset of $\neg S$ can block (x', y) with secession.

First, note that (x', y) is a feasible allocation within $\neg S$:

$$\begin{aligned} \sum_{i\in\neg S} x'_i &= \sum_{i_1\in I_1} x'_{(i_1,\iota_{i_1}(r))} = \sum_{i_1\in I_1}\frac{1}{r}\sum_{k=1}^{r} x_{(i_1,\iota_{i_1}(k))} = \frac{1}{r}\sum_{i_1\in I_1}\sum_{i_2=1}^{r} x_{(i_1,i_2)} \\ &= \frac{1}{r}\sum_{i_1\in I_1}\sum_{i_2=1}^{r}\left(e_{i_1} + \sum_{j_1\in J_1}\theta_{i_1j_1}y_{j_1}\right) = \sum_{i\in\neg S}\left(e_{i_1} + \sum_{j_1\in J_1}\theta_{i_1j_1}y_{j_1}\right), \end{aligned}$$

where the fourth equality follows from the feasibility of the allocation (x, y).

Second, (x', y) is a feasible allocation for the grand coalition I:

$$\begin{aligned} &\sum_{i_1\in I_1}\left(x'_{(i_1,\iota_{i_1}(r))} + \sum_{k\neq r} x'_{(i_1,\iota_{i_1}(k))}\right) \\ =& \sum_{i_1\in I_1}\left(\frac{1}{r}\sum_{h=1}^{r} x_{(i_1,\iota_{i_1}(h))} + \sum_{k\neq r}\left(\frac{r-1}{r}x_{(i_1,\iota_{i_1}(k))} + \frac{1}{r}x_{(i_1,\iota_{i_1}(r))}\right)\right) \\ =& \sum_{i_1\in I_1}\left(\frac{1}{r}x_{(i_1,\iota_{i_1}(r))} + \frac{1}{r}\sum_{k\neq r} x_{(i_1,\iota_{i_1}(k))} + \frac{r-1}{r}\sum_{k\neq r} x_{(i_1,\iota_{i_1}(k))} + \frac{1}{r}\sum_{k\neq r} x_{(i_1,\iota_{i_1}(r))}\right) \\ =& \sum_{i_1\in I_1}\left(\sum_{k\neq r} x_{(i_1,\iota_{i_1}(k))} + x_{(i_1,\iota_{i_1}(r))}\right) \\ =& \sum_{i_1\in I_1}\sum_{k=1}^{r} x_{(i_1,\iota_{i_1}(k))}. \end{aligned}$$

Third, (x', y) is a feasible allocation within S. That follows from the fact that (x', y) is a feasible allocation within $\neg S$ and within I.

For all $h \in \{1,2,...,r-1\}$, since $x_{(i_1,\iota_{i_1}(h))} \preceq_{i_1} x_{(i_1,\iota_{i_1}(r))}$, strict convexity of preferences implies $x'_{(i_1,\iota_{i_1}(h))} \succ_{i_1} x_{(i_1,\iota_{i_1}(h))}$. Hence, every member of S gets strictly better-off. Therefore, S considerately blocks (x, y) with the plan (x', y).

Lastly, we show that no subset of $\neg S$ can block (x', y) with secession. Suppose otherwise, i.e., there is a set $E \subseteq I_1$ and a plan $(x''_{(i,\iota_i(r))})_{i\in E}$ such that

$$x''_{(i,\iota_i(r))} \succ_{i_1} x'_{(i,\iota_i(r))} \quad \forall i \in E,$$

$$\sum_{i \in E} x''_{(i,\iota_i(r))} = \sum_{i \in E} e_{(i,\iota_i(r))}.$$

Then the coalition $S' = \{(i, \iota_i(1)) : i \in E\}$ can block (x, y) with secession by the plan $\left(x''_{(i,\iota_i(1))} := x''_{(i,\iota_i(r))}\right)_{i \in E}$, because by strict convexity of preferences,

$$x''_{(i,\iota_i(1))} = x''_{(i,\iota_i(r))} \succ_{i_1} x'_{(i,\iota_i(r))} = \frac{\sum_{k=1}^{r} x_{(i_1,\iota_{i_1}(r))}}{r} \succ_{i_1} x_{(i,\iota_i(1))} \text{ for any } i \in E,$$

$$\text{and } \sum_{i \in E} x''_{(i,\iota_i(1))} = \sum_{i \in E} x''_{(i,\iota_i(r))} = \sum_{i \in E} e_{(i,\iota_i(r))} = \sum_{i \in E} e_{(i,\iota_i(1))}.$$

This contradicts to the assumption made above that no coalition can block (x, y) with secession. Thus, no subset of $\neg S$ can block (x', y) with secession, as asserted. ■

With ETP, sup-fair core allocations in economies of different sizes can be identified with the element of the same space, $\left(\mathbb{R}^l_+\right)^{I_1} \times \prod_{j_1 \in J_1} Y_{j_1}$. Let $C^r_{\text{sup-fair}}$ denote the subset of this space that represents the sup-fair core in the r-replica economy.

Since an allocation can be fairly blocked if it can be sup-fairly blocked, Lemma 3 provides another proof for the ETP of fair core allocations, Lemma 2.

5.2 Core Convergence

The proof of Theorem 1 exploits a feature in fair blocking such that the outsiders of a blocking coalition has to act as a whole in order to secession-block the coalition. This feature is no longer available in the notion of sup-fair blocking, as any set of outsiders can secession-block the coalition.

To prove core convergence based on sup-fair blocking, our construction of a blocking plan is more involved. We sketch the idea here and illustrate it with Figure 1.

i. Say (x, y) is the non-equilibrium allocation that we try to block sup-fairly. As that can be done trivially if (x, y) is not Pareto optimal, we can assume that it is Pareto optimal and hence, by the second welfare theorem, it is supported by some price p as a price equilibrium with transfers.

ii. Our construction starts with a blocking plan in a sufficiently large replica economy that blocks away only one individual whose type say i''_1 enjoys a net gain from (x, y) evaluated by the price p. In Figure 1, this individual is represented by the dark disk. To ensure that such exclusion is feasible for the excluded person, we replicate the economy

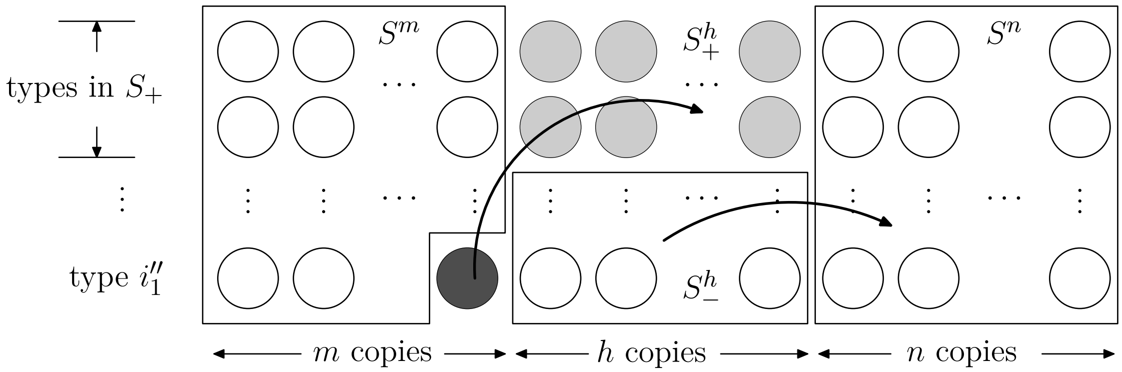

Figure 1: The blocking coalition $S^m \cup S_-^h \cup S^n$

à la our recent paper [17] so that the additional copies of the prototype economy have enough resources to be shared with the excluded person.

iii. With the economy replicated, however, uninvolved individuals are added to the economy, hence we need to ensure that none of them can benefit from forming a secession coalition with the excluded individual. To do that, we transfer a bundle z from the excluded type-i_1'' person to some of the uninvolved individuals, leaving only something for this person merely good enough so that he cannot gain from seceding by himself. The bundle z is distributed only to those types of uninvolved individuals such that their consumptions at (x, y) worth more than their endowments at the price p. These individuals are represented by the grey disks in Figure 1. Then they cannot gain from seceding with the excluded person.

iv. The uninvolved individuals who do not get to share the bundle z are those such that their consumptions at (x, y) worth less than their endowments evaluated by the price p. Hence they get better-off if they do not trade according to (x, y). To balance the resources that they need for such an action, we simply replicate the economy further.

The aforementioned construction uses the following assumption.

Assumption 6 (differentiable utility) *For any individual i, $\succeq_i$ can be represented as a utility function that is continuously differentiable in the interior of the consumption set.*

Lemma 4 *Suppose Assumption 6. If (x, y) is supported by price p as a price equilibrium with transfers, if $x_i \gg \mathbf{0}$ for all individuals i, and if $z \in \mathbb{R}^l$ such that $p \cdot z > 0$, then for all i,*

$$x_i + \frac{z}{m} \succ_i x_i$$

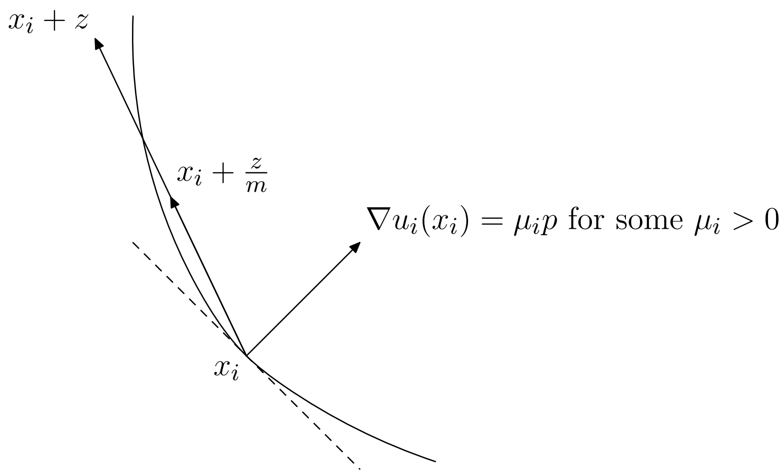

Figure 2: The bundle $x_i + \frac{z}{m}$ belongs to the upper contour set if m is sufficiently small

for any sufficiently large m.

Proof See Mas-Colell, Whinston and Green [14, p658]. (Figure 2, borrowed from their Figure 18.B.3, illustrates the intuition.) ■

Lemma 5 *By Assumptions 1, 2 and 5, in any sup-fair core allocation (x, y), $x_i \gg \mathbf{0}$ for all $i \in I$.*

Proof Let (x, y) be a sup-fair core allocation. First, for all $i \in I$, $x_i \neq \mathbf{0}$, otherwise by strong monotonicity (Assumption 1) $\{i\}$ can block (x, y) with secession, since $\mathbf{0} \neq e_i \succ_i \mathbf{0}$.

Second, recall from Xiong and Zheng [17] that an allocation (x, y) is said *edgy* if the aggregate consumption bundle $\sum_{i \in I} x_i$ lies on the boundary of the consumption set $\mathbb{R}^l_+$. As the sup-fair core allocation (x, y) has the ETP (Lemma 3), it follows from Xiong and Zheng [17, Lemma 10] and Assumption 5 that (x, y) is not edgy, otherwise (x, y) would be considerately blocked by the grand coalition and hence is also sup-fairly blocked by the grand coalition.

Third, we show that $x_i \gg \mathbf{0}$ for all $i \in I$. Suppose not. Then some individual i consumes zero quantity of some good k, i.e., $(x_i)_k = 0$. As already established, $x_i \neq \mathbf{0}$, so i consumes a positive quantity $(x_i)_{k'}$ of another good k'. Since the allocation is not edgy, another individual i' consumes a positive quantity $(x_{i'})_k$ of good k. Then change the allocation (x, y) by giving all the $(x_i)_{k'}$ units of good k' from individual i to i' (without changing production). That makes i' better-off by his strongly monotone preference; thus, by lower semicontinuity of his preference, we further have individual i' return to i a sufficiently small positive amount of both goods k and k' such that i' remains better-off than the status quo (x, y). Repeat the above procedure if there is another good for which person i consumes zero quantity. Then we eventually move individual i's consumption from the boundary to the interior of $\mathbb{R}^l_+$, hence he is better-off by the boundary aversion assumption; any other individual (such as i') involved in this process is also better-off. Finally, transfer a sufficiently small bundle $v \in \mathbb{R}^l_{++}$ from i and distribute it evenly among all individuals so that everyone is better-off than the original allocation (x, y). Then the grand coalition blocks (x, y), because the feasibility condition for the grand coalition is satisfied since the total consumption does not change. As the complement of the grand coalition is empty, this suffices a sup-fair blocking of (x, y). ■

With a similar argument for fair cores in Section 4, we can show that the sup-fair cores shrink as we enlarge the economy, i.e., $C^1_{\sup-\text{fair}} \supseteq C^2_{\sup-\text{fair}} \supseteq C^3_{\sup-\text{fair}} \cdots$. Theorem 2 shows that the sup-fair cores actually converge to Walrus equilibria as we enlarge economy.

Theorem 2 *By Assumptions 1–6,* $\mathbb{W} = \cap_{r=1}^{\infty} C^r_{\sup-\text{fair}}$.

Proof Since $C^r_{fair} \subseteq C^r_{\text{sup-fair}}$ and $\mathbb{W} \subseteq \cap_{r=1}^{\infty} C^r_{\text{fair}}$, we have $\mathbb{W} \subseteq \cap_{r=1}^{\infty} C^r_{\sup-\text{fair}}$.

We still need to prove $\cap_{r=1}^{\infty} C^r_{\sup-\text{fair}} \subseteq \mathbb{W}$. Pick any $(x, y) \notin \mathbb{W}$, we will prove $(x, y) \notin \cap_{r=1}^{\infty} C^r_{\text{sup-fair}}$. By Lemma 3, we may assume without loss that (x, y) satisfies ETP.

We may assume without loss that (x, y) is Pareto optimal (otherwise it is blocked by the grand coalition). Then it is a price equilibrium with nonzero transfer under some price p (Xiong and Zheng [17, Appendix A]).

If (x, y) can be blocked with secession in some $\mathcal{E}^r$, then $(x, y) \notin \cap_{r=1}^{\infty} C^r_{\sup-\text{fair}}$. For the rest of the proof, suppose that (x, y) cannot be blocked with secession in any $\mathcal{E}^r$.

There are two cases: (i) (x, y) is a pure exchange allocation, i.e., of the form $(x, y \mid y = \mathbf{0})$; (ii) (x, y) is a productive allocation, i.e., of the form $(x, y \mid y \neq \mathbf{0})$.

First, consider case (i), $y = \mathbf{0}$.

There is some $i'_1 \in I_1$ who gets strictly negative transfer, i.e., $p \cdot x_{i'_1} < p \cdot e_{i'_1}$. Note that the profit from firms is zero because $y = \mathbf{0}$.

We show that (x, y) can be blocked with secession in $\mathcal{E}^{r+1}$ for sufficiently large r.

Consider the coalition $E := \mathcal{E}^r \cup \{(i'_1, r+1)\}$ with the plan

$$\left(\left(x^*_{(i_1,h)} := x_{(i_1,h)} + \frac{e_{i'_1} - x_{i'_1}}{r \times |I_1|} \right)_{i_1 \in I_1, h \in \{1,2,\ldots,r\}}, \left(x^*_{(i'_1,r+1)} := x_{(i'_1,r+1)} \right), \left(y^*_j := y_j = \mathbf{0} \right)_{j \in J} \right).$$

Under this plan, resources are balanced, with the total bundle for E being $\sum_{i\in E} e_i$. By Lemma 4, when r is large enough, everyone in $\mathcal{E}^r$ gets strictly better-off than (x, y), and the person $(i_1', r+1)$ as well off. (Here we use Assumption 6 and the fact that $x_{(i_1,h)} \gg \mathbf{0}$ for all (i_1, h), which follows from Lemma 5.) Thus, the coalition E blocks (x, y) with secession by the plan (x^*, y^*), a contradiction.

Second, consider case (i), $y \neq \mathbf{0}$.

Since $(x, y) \notin \mathbb{W}$, there exists some $i_1'' \in I_1$ who gets strictly positive transfer, i.e.,

$$p \cdot x_{i_1''} \;>\; p \cdot e_{i_1''} + \sum_{j_1 \in J_1} \theta_{i_1'',j_1} p \cdot y_{j_1} \quad \text{and} \tag{6}$$

$$\sum_{i_1 \in I_1 \setminus \{i_1''\}} p \cdot x_{i_1} \;<\; \sum_{i_1 \in I_1 \setminus \{i_1''\}} p \cdot e_{i_1} + \sum_{i_1 \in I_1 \setminus \{i_1''\}} \sum_{j_1 \in J_1} \theta_{i_1,j_1} p \cdot y_{j_1}. \tag{7}$$

From profit maximization at (x, y) and the assumption $\mathbf{0} \in Y_{j_1}$ for all j_1, we have

$$p \cdot e_{i_1''} + \sum_{j_1 \in J_1} \theta_{i_1'',j_1} p \cdot y_{j_1} \geq p \cdot e_{i_1''}.$$

Let us start with the sub-case where

$$p \cdot e_{i_1''} + \sum_{j_1 \in J_1} \theta_{i_1'',j_1} p \cdot y_{j_1} = p \cdot e_{i_1''}.$$

As $p \cdot y_{j_1} \geq 0$ for all j_1, this sub-case implies $\theta_{i_1'' j_1} p \cdot y_{j_1} = 0$ for all j_1. Since the profit maximizer is unique (Assumption 4), we have $\theta_{i_1'',j_1} > 0$ implies $y_{j_1} = \mathbf{0}$. Therefore,

$$e_{i_1''} + \sum_{j_1 \in J_1} \theta_{i_1'',j_1} y_{j_1} = e_{i_1''}. \tag{8}$$

We show that (x, y) can be sup-fairly blocked in $\mathcal{E}^{W+1}$ for sufficiently large W. Consider the coalition $\overline{E} := I \setminus \{(i_1'', W+1)\}$ which includes everyone in $\mathcal{E}^{W+1}$ except the type-i_1'' individual in the last replica economy. The following is the desired blocking allocation.

$$\left(\left(\overline{x}_i := x_i + \frac{x_{i_1''} - e_{i_1''}}{(W+1) \times |I_1| - 1} \right)_{i \in I \setminus \{(i_1'', W+1)\}}, \left(\overline{x}_{(i_1'', W+1)} := e_{i_1''} \right), \left(\overline{y}_j := y_j \right)_{j \in J} \right).$$

Under this blocking plan, resources are balanced within $\overline{E}$ by (8), and trivially balanced within $\mathcal{E}^{W+1} \setminus \overline{E} = \{(i_1'', W+1)\}$. By Lemma 4, for sufficiently large W, everyone in $\overline{E}$ gets strictly better off, and hence $\overline{E}$ considerately blocks (x, y). Since the outsider $(i_1'', W+1)$ cannot be better-off than consuming $e_{i_1''}$ by blocking with secession, this constitutes a sup-fair blocking plan.

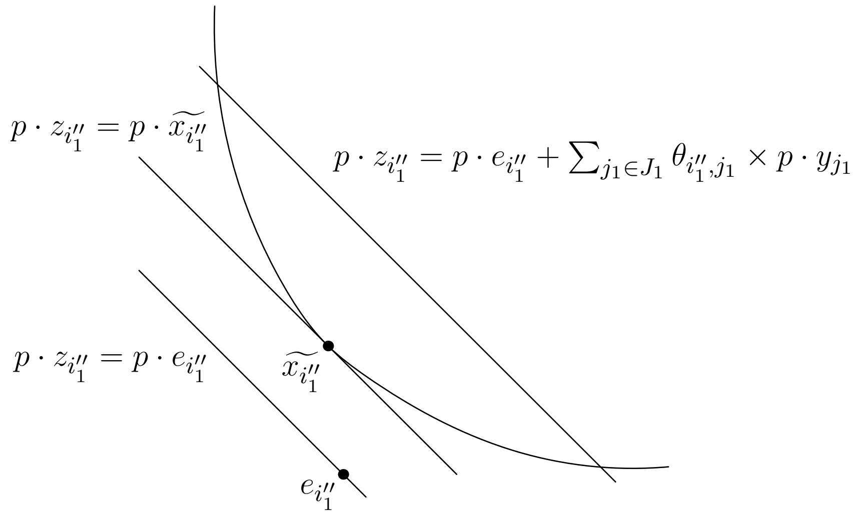

Figure 3: The bundle $\widetilde{x_{i_1''}}$

We are left with the sub-case where

$$p \cdot e_{i_1''} + \sum_{j_1 \in J_1} \theta_{i_1'',j_1} \times p \cdot y_{j_1} > p \cdot e_{i_1''}.$$

This inequality implies that there exists an $\widetilde{x_{i_1''}}$ such that

$$p \cdot x_{i_1''} \overset{(6)}{>} p \cdot e_{i_1''} + \sum_{j_1 \in J_1} \theta_{i_1'',j_1} \times p \cdot y_{j_1} > p \cdot \widetilde{x_{i_1''}} > p \cdot e_{i_1''}. \tag{9}$$

Without loss of generality, assume that $\widetilde{x_{i_1''}}$ is a utility maximizer under the price p and the wealth $p \cdot \widetilde{x_{i_1''}}$ (Figure 3). Then for any $z \succ_{i_1''} \widetilde{x_{i_1''}}$, we have

$$p \cdot z \geq p \cdot \widetilde{x_{i_1''}} > p \cdot e_{i_1''}. \tag{10}$$

We shall complete the proof by constructing an alternative allocation (x^*, y^*) that sup-fairly blocks (x, y) in a sufficiently large replica economy. The alternative production plan y^* is defined to be the same as the status quo,

$$y^*_{j_1} := y_{j_1} \quad \forall j_1 \in J_1.$$

The construction of the alternative consumption plan x^* is more involved. The blocking coalition in our construction will be some $S^m \cup S^h_- \cup S^n$. Here S^m is a sufficiently large, m-fold replica economy with only one individual excluded. The resources of the excluded individual are balanced by a subset S^h_+ of another sufficiently large h-fold replica economy.

The complement of S_+^h in this replica economy is S_-^h, and the resources of S_-^h are balanced by a third sufficiently large n-fold replica economy S^n. See Figure 1 for the illustration.

Now we construct the plan formally. First, consider the following bundle, which is the resources that would be freed up if a type-i_1'' individual is excluded:

$$x_{i_1}'' - \left(e_{i_1''} + \sum_{j_1 \in J_1} \theta_{i_1'' j_1} y_{j_1} \right).$$

By Lemma 4, Ineq. (6), and the fact that $x_{i_1} \gg \mathbf{0}$ for all i (Lemma 5), there exists a sufficiently large m such that

$$x_{i_1}^m := x_{i_1} + \frac{x_{i_1}'' - \left(e_{i_1''} + \sum_{j_1 \in J_1} \theta_{i_1'' j_1} y_{j_1} \right)}{(m+1) \times |I_1| - 1} \succ_{i_1} x_{i_1} \quad \forall i_1 \in I_1.$$

Hence define

$$x^*_{(i_1, i_2)} := x_{i_1}^m$$

for every $(i_1, i_2) \in S^m$ with

$$S^m := \{(i_1, 1) : i_1 \neq i_1''\} \cup \{(i_1, i_2) : i_1 \in I_1;\ i_2 \in \{2, \ldots, m+1\}\}.$$

I.e., exclude individual $(i_1'', 1)$ and distribute the bundle freed up thereof to individuals in S^m so that everyone in S^m is strictly better-off than (x, y). Let the consumption for $(i_1'', 1)$ be

$$x^*_{(i_1'', 1)} := \widetilde{x_{i_1''}}.$$

Then this person needs to receive a bundle

$$\widetilde{x_{i_1''}} - \left(e_{i_1''} + \sum_{j_1 \in J_1} \theta_{i_1'' j_1} y_{j_1} \right).$$

To balance that, partition I_1 into two subsets I_1^+ and I_1^-:

$$\begin{aligned} I_1^+ &= \{i_1 \in I_1 : p \cdot x_{i_1} \geq p \cdot e_{i_1}\}, \\ I_1^- &= \{i_1 \in I_1 : p \cdot x_{i_1} < p \cdot e_{i_1}\}. \end{aligned}$$

Note that $I_1^+ \neq \varnothing$, as $i_1'' \in I_1^+$. By the second inequality of (9), Lemma 4 again implies that there exists a sufficiently large h such that

$$x_{i_1}^h := x_{i_1} + \frac{e_{i_1''} + \sum_{j_1 \in J_1} \theta_{i_1'' j_1} y_{j_1} - \widetilde{x_{i_1''}}}{h \times |I_1^+|} \succ_{i_1} x_{i_1} \quad \forall i_1 \in I_1.$$

Hence define

$$x^*_{(i_1,i_2)} := x^h_{i_1}$$

for every $(i_1, i_2) \in S^h_+$ with

$$S^h_+ := I_1^+ \times \{m+2, \ldots, m+h+1\}.$$

I.e., we balance the resources of individual $(i_1'', 1)$ by having him trade exclusively with the individuals in S^h_+ so that everyone in S^h_+ is strictly better-off than (x, y). But then the individuals in S^h_-, with

$$S^h_- := I_1^- \times \{m+2, \ldots, m+h+1\},$$

are left out (by S^h_+) and so their resources need to be balanced. To stay put with their consumptions in (x, y), the individuals in S^h_- need to give away a total bundle

$$\sum_{i_1 \in I_1^-} \left(e_{i_1} + \sum_{j_1 \in J_1} \theta_{i_1 j_1} y_{j_1} - x_{i_1} \right),$$

which, by the definition of I_{-1} and the fact $p \cdot y_{j_1} \geq 0$ for all j_1, has a positive value when evaluated by the price p. Then Lemma 4 again implies that there exists a sufficiently large n such that

$$x^n_{i_1} := x_{i_1} + \frac{\sum_{i_1 \in I_1^-} \left(e_{i_1} + \sum_{j_1 \in J_1} \theta_{i_1 j_1} y_{j_1} - x_{i_1} \right)}{n \times |I_1|} \succ_{i_1} x_{i_1} \quad \forall i_1 \in I_1.$$

Thus, define

$$x^*_{(i_1,i_2)} := \begin{cases} x_{i_1} & \text{if } (i_1, i_2) \in S^h_- \\ x^n_{i_1} & \text{if } (i_1, i_2) \in S^n, \end{cases}$$

with

$$S^n := I_1 \times \{m+h+2, \ldots, m+h+n+1\}.$$

I.e., we balance the resources of the individuals in S^h_- by distributing the resources they free up to an additional n-replica economy S^n so that everyone in S^h_- is as well off as in (x, y) and everyone in S^n is strictly better-off than (x, y).

In the $(m+h+n+1)$-replica economy $\mathcal{E}^{m+h+n+1}$, consider the coalition

$$S^* := S^m \cup S^h_- \cup S^n$$

with the blocking plan (x^*, y^*) defined above. By construction, (x^*, y^*) is feasible within S^* and feasible within the outsiders, $S^h_+ \cup \{(i_1'', 1)\}$. Since the individuals in $S^m \cup S^n$ are strictly better-off than (x, y) and those in S_-^h are as well-off as (x, y), by lower semi-continuous preferences, we can transfer some resources from the former to the latter so that everyone in S^* is strictly better-off. Thus, $(S^*, (x^*, y^*))$ considerately blocks (x, y).

Finally, we show that no subset of $S_+^h \cup \{(i_1'', 1)\}$ can block (x^*, y^*) with secession. Suppose $T \subseteq S_+^h \cup \{(i_1'', 1)\}$ secession-blocks (x^*, y^*) with a consumption plan $(\hat{x}_i)_{i \in T}$ so that $\sum_{i \in T} \hat{x}_i = \sum_{i \in T} e_i$ and $\hat{x}_i \succ_i x_i^*$ for all $i \in T$. If $i = (i_1'', 1) \in T$, then $\hat{x}_i \succ_i x_i^* = \widetilde{x_{i_1''}}$ and (10) implies that $p \cdot \hat{x}_i > p \cdot e_i$. If $i \in T \setminus \{(i_1'', 1)\}$, then $i \in S_+^h$, so $\hat{x}_i \succ_i x_i^* = x_{i_1}^h \succ_i x_i$; with (x, y) being a price equilibrium with transfers under price p, we have $p \cdot \hat{x}_i > p \cdot x_i \geq p \cdot e_i$ by the definition of I_1^+. But then we have

$$\sum_{i \in T} p \cdot \hat{x}_i > \sum_{i \in T} p \cdot e_i,$$

a contradiction. Therefore, T cannot block (x^*, y^*) with secession. Hence $(S^*, (x^*, y^*))$ sup-fairly blocks the non-equlbrium allocation (x, y), as asserted. ■

6 Conclusion

The traditional core equivalence theory captures an utmost form of competition where anyone who may gain any surplus above the competitive level will have his surplus competed away by someone who slightly outbids him. The modern economics of information may be regarded as a framework to capture a friction due to asymmetric information that dampens such unhindered competition.

This paper is aimed at another kind of friction—the objection from the potential losers of competition such as workers protesting the outsourcing of their jobs. The disruptive effect of such objections forces the potential winners not to completely ignore the interests of the losers. Different from the friction driven by asymmetric information in the economics of information, such friction is due to the conflict of interests in the division of a pie.

In the language of core equivalence, the competitive behaviors such as outbidding and undercutting would be captured by the notion of blocking, with the potential winners corresponding to the blocking coalition, and the potential losers the coalition's complement. The existing core equivalence literature disregards the interests of the complement of a blocking coalition. This paper takes into account their interests by restricting blocking coalitions with a participation condition—which is driven by the property rights on the firms—that a blocking coalition should not make its complement so worse-off that the latter would rather resort to secession that disrupts production. Our theory of fair and sup-fair cores is a preliminary description of the outcomes driven by the kind of competition hampered by such threat of social unrest.

References

[1] Charalambos Aliprantis, Donald Brown, and Owen Burkinshaw. Edgeworth equilibria in production economies. *Journal of Economic Theory*, 43:252–291, 1987.

[2] Michael Allingham. *General Equilibrium*. The MacMillan Press, 1975.

[3] Nizar Allouch and Myrna Wooders. Price taking equilibrium in economies with multiple memberships in clubs and unbounded club sizes. *Journal of Economic Theory*, 141:246–278, 2008.

[4] Robert Anderson. An elementary core equivalence theorem. *Econometrica*, 46(6):1483–1487, 1978.

[5] Robert J. Aumann and Michael Maschler. The bargaining set for cooperative games. In M. Dresher, L. S. Shapley, and A. W. Tucker, editors, *Advances in Game Theory*, pages 443–447. Princeton: Princeton University Press, 1964.

[6] Volker Boehm. The core of an economy with production. *Review of Economic Studies*, 41(3):429–436, July 1974.

[7] Gerard Debreu and Herbert Scarf. A limit theorem on the core of an economy. *International Economic Review*, 4(3):235–246, September 1963.

[8] F. Y. Edgeworth. *Mathemtaical Psychics*. London: Routledge & Kegan Paul, 1881.

[9] Bryan Ellickson, Birgit Grodal, Suzanne Scotchmer, and William R. Zame. Clubs and the market. *Econometrica*, 67(5):1185–1217, September 1999.

[10] Hans Haller. Corporate production and shareholder cooperation under uncertainty. *International Economic Review*, 32(4):823–842, November 1991.

[11] Friedrich A. Hayek. Economics and knowledge. *Economica*, 4(13):33–54, February 1937.

[12] Friedrich A. Hayek. The use of knowledge in society. *American Economic Review*, 35(4):519–530, September 1945.

[13] Werner Hildenbrand. *Core and Equilibria of a Large Economy*. Princeton University Press, 1974.

[14] Andreu Mas-Colell, Michael D. Whinston, and Jerry R. Green. *Microeconomic Theory*. Oxford University Press, 1995.

[15] Howard Pankratz. Nader: Bailout was socialism rescuing capitalism. *The Denver Post*, Denver and the West, October 24, 2008.

[16] Ariel Rubinstein. Dilemmas of an economic theorist. *Econometrica*, 74(4):865–883, July 2006.

[17] Siyang Xiong and Charles Z. Zheng. Core Equivalence Theorem with Production. *Journal of Economic Theory*, 137(1):246–270, November 2007.