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# **UNEQUAL PEACE\***

## BY ALI KAMRANZADEH AND CHARLES Z. ZHENG

### Koç University, Türkiye; University of Western Ontario, Canada

A mediator proposes a settlement between two contestants to avoid a conflict where the cost each contestant bears is inversely related to the contestant's privately known strength. Their strength levels are identically distributed, and their welfares weigh equally in the mediator's objective. However, the optimal proposal offers one contestant much more than it does the other so that the former accepts it always, whereas the latter only occasionally. This unequal treatment improves the prospect of peace by making one contestant willing to settle without fearing that the action signals his weakness that his opponent can exploit should conflict occur.

# 1. INTRODUCTION

In conflict mediation, being fair to the conflicting parties is often regarded as a basic principle. Although the notion of fairness may be subjective as an informal lingo, and complicated as a formal criterion, it might appear natural in situations where the conflicting parties are ex ante equal: the mediator should treat equal parties equally.<sup>1</sup> This article examines such an equal treatment criterion in a stylized model. Two potential adversaries are ex ante identical, and they weigh equally in the mediator's objective. Yet we find that the mediator's optimal proposal to broker peace between them does not treat them equally.

In this model, two players contest a good. Each player's type, either strong or weak, determines the player's relative strength when he is engaged in a conflict to fight for the good. A mediator proposes a split of the good and each player, privately informed of his own type, chooses whether to accept or reject it. Unless both players accept the proposal, conflict ensues as an all-pay auction, where the good is won by the player who bids higher, and the cost of bidding is borne by each, inversely related to the player's strength. The mediator's objective is to maximize the simple sum of the two players' expected payoffs. And their types are drawn independently from the same distribution. Thus, they are ex ante symmetric in our model.

Even though the two players are ex ante symmetric, we find that the mediator's optimal proposal does not treat them equally. The proposal splits the good between the two so unequally that the player who is offered the larger share accepts it whether he is weak or strong. Thus, in accepting the proposal, he does not signal his weakness that the opponent may exploit if conflict occurs. With the favored player accepting the proposal for sure, conflict will not occur if the unfavored player also accepts it. Consequently, while the unfavored player

\*Manuscript received July 2022; revised June 2024.

We thank Benjamin Balzer, Tim Conley, Maria Goltsman, Scott Kominers, Annie Liang, Martin Luccioni, Al Slivinski, Peter Streufert, Rakesh Vohra, and the referees for comments. We acknowledge financial support from the Social Science and Humanities Research Council of Canada, with Insight Grant R4809A04 for Zheng and Doctoral Fellowships Award 752-2018-2196 for Kamranzadeh. Please address correspondence to: Charles Z. Zheng, Department of Economics, University of Western Ontario, 1151 Richmond St., London, ON N6A 5C2, Canada. E-mail: *charles.zheng@uwo.ca*.

<sup>1</sup> The notion of equal split between equal claimants dates back to the Talmud (cf. Aumann and Maschler 1985). Yaari and Bar-Hillel (1984) suggest several ways to justify the equal split between contesting claimants (including the equal treatment property in general equilibrium). Recently, Keniston et al. (2021) provide a rationale for, and conduct an experimental study of, the equal split of the perceived surplus between two bargainers in a dynamic game.

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may reveal his weakness in accepting the proposal (as his share according to the proposal is so small that he would reject it if his type is strong), the revelation is harmless because conflict is preempted if he also accepts the proposal.

The insight conveyed by our finding is that a peace proposal biased toward one side may, counterintuitively, achieve better social welfare than an unbiased one because the favored side is willing to accept the peace deal without fearing being viewed to be weak and taken advantage of later, so the prospect of a peace settlement is improved. Thus, it should not be taken for granted that a peace proposal should offer a fair share to each contestant even from the viewpoint of a benevolent mediator.<sup>2</sup> This insight suggests a new light to interpret the announcement by the United States in 2018 that it was to relocate its embassy to Jerusalem. That can be viewed as a proposal for a new status quo that recognizes Israel's full ownership of Jerusalem. Soon after the announcement, the number of Arab League countries that agreed to establish diplomatic relations with Israel jumped from two to six. Another example of an unequal peace proposal is the Vatican mediation of the Beagle Channel Dispute between Argentina and Chile. In the shadow of a war between the two countries, the Pope issued a proposal that awarded Chile all of the disputed islands, granting Argentina only the navigation rights in the area waters and a shared resource right in a part of the sea. Chile immediately accepted the proposal whereas Argentina was initially reluctant but eventually accepted it (cf. Garrett, 1985; and Greig and Diehl, 2012).<sup>3</sup>

This article belongs to a series of papers on mediation and conflict prevention. With a similar model that allows for the general class of type distributions and communication mechanisms, Zheng (2019b) characterizes the set of all prior distributions given which there exists a mechanism for the mediator to fully preempt conflict. In this article, we consider a polar opposite case of those primitives, a case in which conflict cannot ever be prevented with probability one. In our model, the prior probability of either party being weak is large enough to make conflict appealing to a player who happens to be strong; therefore, no peace proposal can rule out a conflict with certainty. That is, the primitives in our model preclude the existence of any peace proposal that is acceptable to both parties for sure.

This departure from the literature brings to light a different approach to find an optimal peace proposal. In the literature, when the prior probability of either party being weak is small, conflict is costly to each player and hence can be fully prevented. All the mediator needs to do is proposing a split that offers each player a payoff above what the player gets should he veto the proposal thereby triggering conflict. And what the player gets from triggering conflict is below the proposed payoff thanks to a posterior that penalizes the vetoer most. This posterior is available because rejecting the proposal is an off-path action in the equilibrium where both sides accept the proposal for sure. In our model, by contrast, any peace proposal is rejected in some on-path event of positive probability. Thus, the posterior mentioned before is not necessarily available. To find an optimal proposal, the mediator needs to calculate how every proposal determines the corresponding posterior via Bayes's rule, and how the posterior determines the players' expected payoffs in the event of conflict, which in turn determine the likelihood that they accept the proposal.

Now that conflict occurs with a positive probability in any equilibrium, a contestant's welfare includes his payoffs not only in the event of peace, but also in the event of conflict. That is why we assume that the mediator is to maximize the sum of the two players' ex ante expected payoffs, taking both events into account. This design objective is distinct and novel rel-

<sup>&</sup>lt;sup>2</sup> Although the inequality in the optimal proposal could be mitigated, in theory, if the mediator randomizes which player is offered the larger share, in practice such ex ante randomizations in large stake disputes are rare. Moreover, randomization or not, the ensuing split remains unequal in the same degree.

<sup>&</sup>lt;sup>3</sup> If we view trade unions as settlements among countries to avoid trade conflict, the Maastricht Treaty for the United Kingdom to join the European Union is another episode of unequal proposals. The treaty offered the United Kingdom the opt-outs from the single currency mandate and the Social Chapter of employment regulations, whereas none of the other member nations were offered such opt-outs (cf. Baun, 1995–96; and Chapter 5 in Burton, 2021).

ative to the literature (e.g., Hörner et al., 2015; and Balzer and Schneider, 2021b), in which the mediator's objective is to maximize the probability of peaceful conflict resolution. The two objectives coincide in models where full prevention of conflict is possible (e.g., Celik and Peters, 2011; Zheng, 2019b; and Balzer and Schneider 2021a), where a peace proposal always acceptable to both parties exists and maximizes both the probability of peace and the contestants' total welfare. When full prevention of conflict is impossible as in our model, by contrast, the objective of maximizing the probability of peace would lead the mediator to enlarge the chance of peace resolution at the expense of the contestants' payoffs in the event of conflict. That may hurt their overall welfare as the probability of conflict cannot be eliminated.

Most papers on conflict mediation assume that the outcome of bargaining failure is exogenous in the sense that once the peace proposal is rejected, no more action is available to the players, and their eventual payoffs are determined by an exogenous lottery that depends on their types, as in Bester and Wärneryd (2006), Compte and Jehiel (2009), Fey and Ramsay (2010, 2011), Hörner et al. (2015), Meirowitz et al. (2019), and Spier (1994). This article treats what happens after bargaining failure as a continuation game thereby deriving the outcome of bargaining failure endogenously. In the continuation game, the players choose their actions conditional on their commonly observed responses to the peace proposal (at least one of which is rejection). Thus, the players can signal to each other through their responses to the peace proposal even though a proposal per se conveys no signal from either player.

A peace proposal by itself conveys no signal from either player because we assume that the mediator does not condition her proposal on any signal from the contestants and instead makes a single unconditional proposal for a resolution. This assumption is different from much of the literature, which treats peace proposals as outputs of a communication mechanism à la Myerson (1986). In addition to facilitating tractability, this assumption helps us to focus on what the mediator can accomplish despite the restriction. As explained above, the players' responses to a peace proposal play the role of signals in our model. The mediator, by adjusting the proposed shares between the players, can manipulate what these signals reveal. Accepting a small share may reveal that the player is unlikely to be strong enough to expect large payoffs in the event of conflict, whereas rejecting a large share may reveal the opposite. Such revelations, in the form of posteriors, determine the outcome of bargaining failure as such outcomes are endogenous in our model. Thus, even though she cannot receive signals from the players before making a proposal, the mediator in proposing an appropriate split can indirectly influence the players' beliefs and hence their total welfare. By contrast, in the aforementioned models that treat the outcome of conflict exogenously, the players' responses to the peace proposal merely determine whether bargaining failure occurs or not and have no effect on their welfare when bargaining failure occurs. Then the communication mechanism becomes the only channel for the players to send any signal.<sup>4 5</sup>

The other papers that also treat the outcome of bargaining failure endogenously are Balzer and Schneider (2021b), Celik and Peters (2011), Lu et al. (2023), and Zheng (2019a, 2019b). Our article differs from them except Balzer and Schneider (2021b) by considering a case where full prevention of conflict is impossible. Balzer and Schneider (2021b) have also considered such a case. They consider communication mechanisms aimed at a different objective, to maximize the probability of peaceful resolution, and they focus on the case where the designer is an arbitrator with full commitment power. While they also consider a mediation case,

<sup>&</sup>lt;sup>4</sup> For example, in Hörner et al.'s (2015) model with exogenous conflict, the prediction that mediation outperforms unmediated negotiation relies on the mediator's capability to collect confidential information from the contestants. Another assumption their prediction requires is that a contestant's payoff from the conflict lottery depends on both contestants' types, as Fey and Ramsay (2010) show that mediation cannot outperform unmediated negotiation when the said payoff depends only on the contestant's own type.

<sup>&</sup>lt;sup>5</sup> The signal-independent proposals, albeit restrictive, have a transparency appeal that makes them relevant to situations where a mediator cannot fully control the communication mechanism, say due to the likelihood of leaks (e.g., Feerick, 2003) or the doubts about the mediator's commitment to truthful conveyance of communications (cf. Kydd, 2003; Smith and Stam, 2003; and Rauchhaus, 2006).

the mediator is assumed able to communicate separately and confidentially to the contestants and able to condition such communications on the negotiation outcome. In our model, by contrast, a mediator maximizes the total welfare of both parties, taking into account that conflict is unavoidable, and can only indirectly influence the posterior system through a messageindependent proposal.

Baliga and Sjöström (2004, 2020) model conflict as a sequential game and focus on the dynamic interactions between the two adversaries during the conflict. Our article complements their perspective by focusing on the negotiation before the occurrence of conflict. Considering mediated negotiation and a mediator's optimal decision, the article also complements the work by Lu et al. (2023), who consider unmediated negotiation between the two players within the aforementioned framework where full prevention of conflict is possible.

The next section defines the model. Section 3 derives the players' interim and ex ante expected payoffs and describes how equilibria vary with the peace proposal. Section 5 presents the result. Section 6 concludes and suggests a couple of possible extensions. The Appendix contains all omitted details.

# 2. THE MODEL

Two players, named 1 and 2, contest a prize of size one. Each player's type is independently drawn from the same binary distribution, and the realization is either w ("weak") with probability  $\theta$ , or s ("strong") with probability  $1 - \theta$ , such that  $0 < \theta < 1$  and s > w > 0.<sup>6</sup> Let

$$\alpha := 1 - w/s.$$

Thus,  $0 < \alpha < 1$ . The players' types determine their relative strength in a conflict for the prize. A neutral mediator, uninformed of the players' types, makes a *peace proposal*, which proposes a split of the prize:

$$(x_1, x_2) \in [0, 1]^2$$
 such that  $x_1 + x_2 = 1$ .

Then each player, privately informed of his own type, independently and publicly announces whether to accept (A) or reject (R) the proposal. If both choose A, the game ends with player *i* getting a payoff equal to  $x_i$  ( $\forall i = 1, 2$ ). If at least one player chooses R, then conflict ensues in the form of an all-pay auction: Each player *i*, after observing each other's actions (A or R), submits a sealed bid  $b_i \in \mathbb{R}_+$ ; the higher bidder wins the prize, and ties are broken randomly with equal probabilities; the payoff for player *i* of type  $t_i$  is equal to  $\frac{1}{\alpha}(1 - b_i/t_i)$ if *i* wins, and equal to  $\frac{1}{\alpha}(-b_i/t_i)$  otherwise. Then the game ends. A player's bid represents the player's total amount of warring efforts in the conflict, and  $1/t_i$  represents a type- $t_i$  player's marginal cost of warring efforts in the conflict.<sup>7</sup>

Any peace proposal  $(x_1, x_2)$  determines a two-stage game of asymmetric information. The solution concept we use for this game is perfect Bayesian equilibrium (PBE). Any pair of a peace proposal and the corresponding PBE is called *proposal-PBE pair*, or *solution* for short.

We measure the social welfare achieved by a proposal-PBE pair by the total welfare generated on path of the PBE. By *total welfare*, we mean the sum of the two players' ex ante expected payoffs (before realization of types). A peace proposal of particular interest is the *equal split* (1/2, 1/2), treating the two ex ante identical players equally. Another proposal of

<sup>&</sup>lt;sup>6</sup> Our assumption of binary types is in line with much of the conflict resolution literature such as Balzer and Schneider (2021a, 2021b), Hörner et al. (2015), and Meirowitz et al. (2019).

<sup>&</sup>lt;sup>7</sup> We scale the payoff from the conflict by the parameter  $1/\alpha$  purely for notational convenience. That is because  $\alpha$  emerges as a multiple of each player's expected payoff from any equilibrium of the conflict continuation game (Subsection 3.1), and our scalar  $1/\alpha$  cancels out the multiple. Without the scalar  $1/\alpha$  to cancel out  $\alpha$ ,  $\alpha$  would appear in most expressions in the article thereby complicating them, though all our results remain true.

interest is  $(\theta, 1 - \theta)$ , splitting the prize according to the prior probabilities assigned to the weak and strong types.

Throughout the article, we maintain the following assumption, which constitutes our major point of departure from the previous conflict mediation literature:

(1) 
$$\theta > 1/2.$$

This inequality is the necessary and sufficient condition for nonexistence of any communication mechanism à la Myerson (1986) (containing peace proposals as special cases) given which there is a PBE where conflict occurs with zero probability.<sup>8</sup> That is, due to (1), full preemption of conflict is impossible, and conflict is necessarily an on-path event. Thus, it is appropriate for a mediator to adopt an objective—such as the total welfare considered in this article—that incorporates the players' welfare in both peace and conflict.

#### 3. INTERIM PAYOFFS AND POSTERIOR BELIEFS

3.1. The Post-Mediation Payoff in the Conflict. Let us start by considering the continuation game where conflict ensues (due to at least one player having chosen R at the proposal stage). The belief about a rival is updated conditional on the rival's response to the proposal. For each player  $i \in \{1, 2\}$ , let  $p_i$  denote the posterior probability of player i being type s(strong). This, together with the players' private information of their own types  $t_i$ , defines a Bayesian game.

Given any pair  $(p_1, p_2) \in [0, 1]^2$  of posterior probabilities, one can show that there is a unique Bayesian Nash equilibrium (BNE) of the continuation game. Both players randomly select their bids from an interval  $[0, \overline{b}]$  for some  $\overline{b}$  endogenous to the equilibrium. The strong type of a player selects his bid from an upper subinterval of  $[0, \overline{b}]$ , and the weak type of the player, from the complement of the upper subinterval. The player with the smaller  $p_i$  bids zero with a positive probability when his type is weak, whereas the other player bids zero with zero probability and hence enjoys a positive probability of winning even by bidding zero. For each player  $i \in \{1, 2\}$  and each type  $t \in \{s, w\}$ , let  $U_i^t(p_i, p_{-i})$  denote the expected payoff for player i of type t in this BNE. One can show (Appendix A.1):

(2) 
$$U_i^s(p_i, p_{-i}) = 1 - \min\{p_i, p_{-i}\},\$$

(3) 
$$U_i^w(p_i, p_{-i}) = p_i - \min\{p_i, p_{-i}\}.$$

The functions  $U_i^s(p_i, \cdot)$  and  $U_i^w(p_i, \cdot)$  are graphed in Figure 1. These conflict payoffs play a similar role as the ex post payoff that a designer would like to concavify in the information design framework, except that in our game concavification need not bring about larger total welfare, as  $U_i^s$  and  $U_i^w$  represent only the payoffs in the event of conflict.

Much of the trade-off faced by the mediator grows out of the following observation.

REMARK 1. An increase in  $p_i$  hurts the strong type of player *i* and benefits the weak type of *i*. In other words, a strong type would like to reduce, and a weak type would like to enlarge, the posterior probability that his rival assigns to the event that his type is strong.

<sup>&</sup>lt;sup>8</sup> To see that, apply Example 4 in Zheng (2019b). Since we have scaled up the payoff in the conflict to  $1/\alpha$  times the quantity assumed in Zheng (2019b), the peace-implementability threshold  $c_* = \alpha \theta$  there becomes  $(1/\alpha)c_* = \theta$ . Thus, the necessary and sufficient condition for peace implementability becomes  $2\theta \le 1$ . If  $2\theta \le 1$ , one can split the prize such that each player gets a share at least as large as  $\theta$ , and it is an equilibrium for both to accept any such splits, the equal split (1/2, 1/2) being one of them.



#### FIGURE 1

PAYOFF IN THE CONFLICT AS A FUNCTION OF THE OPPONENT'S POSTERIOR

Remark 1 can be observed from Figure 1, as an increase in  $p_i$  corresponds to a downward shift of the graph of  $U_i^s(p_i, \cdot)$  and an upward shift of the graph of  $U_i^w(p_i, \cdot)$ . Intuitively speaking, to the strong type of, say, player 1, the issue is not whether he can win the prize but rather how much he has to pay to win. When the rival player 2 is complacent, believing that player 1 is unlikely to be strong, player 2's bid (which is costly, win or lose) becomes low stochastically, and so the strong type of player 1 can win at a low cost in expectation. To the weak type of player 1, by contrast, the issue is whether he can win at all, and he gets a positive expected payoff only when player 2 bids zero. The more often is player 1 believed to be weak, the less often would player 2 bid zero (as he sees little need to concede to a weak player 1), and the less expected payoff the weak type of player 1 gets.

3.2. Interim Payoffs in Mediation. Given any proposal-PBE pair, for each player  $i \in \{1, 2\}$  and each type  $t \in \{w, s\}$ , let  $\sigma_i(t)$  denote the probability with which player *i* of type *t* chooses *R* at the proposal stage, let  $q_i$  denote player *i*'s ex ante probability (before realization of *i*'s type) of choosing *R*, namely,

(4) 
$$q_i := \theta \sigma_i(w) + (1 - \theta) \sigma_i(s),$$

and let  $p_i^A$  (respectively,  $p_i^R$ ) denote the posterior probability of player *i* being type *s* conditional on *i* having chosen *A* (respectively, *R*) in response to the peace proposal. Given type  $t \in \{w, s\}$  and anticipating the continuation payoff  $U_i^t$  in the event of conflict, player *i*'s expected payoff from choosing *A* is equal to

(5) 
$$V_i^A(t) := q_{-i} U_i^t (p_i^A, p_{-i}^R) + (1 - q_{-i}) x_i,$$

and that from choosing R is equal to

(6) 
$$V_i^R(t) := q_{-i}U_i^t(p_i^R, p_{-i}^R) + (1 - q_{-i})U_i^t(p_i^R, p_{-i}^A)$$

One can derive from Bayes's rule the next condition, called Bayesian plausibility in the information design literature:

(7) 
$$q_i p_i^R + (1 - q_i) p_i^A = 1 - \theta.$$

Thus, the point  $(1 - \theta, V_i^R(t))$  is the convex combination between the two points on the graph of  $U_i^t(p_i^R, \cdot)$  whose horizontal coordinates are  $p_{-i}^R$  and  $p_{-i}^A$ .



FIGURE 2

INTERIM EXPECTED PAYOFFS AS CONVEX COMBINATIONS

This is illustrated by Figure 2, where  $p_{-i}^A$ ,  $1 - \theta$ , and  $p_{-i}^R$  are positioned according to an intuitive Lemma A.2 (Appendix A.2):

(8) 
$$\forall i \in \{1, 2\} : p_i^A \le 1 - \theta \le p_i^R$$

That is, R (rejecting the peace proposal) signals one's strength more than A does.

REMARK 2. Figure 2 reveals the following: (a) The interim payoff for type w (weak) in the conflict is bounded from above by  $\theta$  and attains this upper bound when  $p_i^R = 1$ . (b) The interim payoff for type s (strong) in the conflict is bounded from below by  $\theta$  and attains this lower bound when  $p_i^R \ge p_{-i}^R$ . (c) It follows from (b) that, in any proposal-PBE pair, the strong type of each player can always secure an interim payoff no less than  $\theta$  through choosing R.

## 4. EQUILIBRIUM AND TOTAL WELFARE

4.1. The Equilibria. There is always a trivial PBE where conflict occurs for sure regardless of the peace proposal: each player always chooses R because he expects the same from the opponent. The other PBEs are determined by the peace proposal. For each  $i \in \{1, 2\}$ and each  $t \in \{w, s\}$ , as defined earlier,  $\sigma_i(t)$  denotes the probability with which player i of type t chooses R at the proposal stage, and  $p_i^A$  (respectively,  $p_i^R$ ) denotes the posterior probability of player i being type s conditional on having chosen A (respectively, R). The posteriors  $(p_i^A, p_i^R)_{i=1}^2$  determine the players' post-mediation payoffs  $(U_i^t(p_i, p_{-i})_{i=1}^2)_{t=w}^s$  (Subsection 3.1), where  $p_i$  is either  $p_i^A$  if player i has chosen A, or  $p_i^R$  if i has chosen R, and likewise for  $p_{-i}$ . The post-mediation payoffs in turn determine the interim expected payoffs at the proposal stage (Subsection 3.2). Then  $(\sigma_1, \sigma_2)$  is determined by the mutual best response condition based on the interim expected payoffs.

Without loss of generality, suppose that the larger share in the proposed split  $(x_1, x_2)$  is offered to player 1, namely,  $x_1 \ge x_2$ . When  $x_1$  varies in [1/2, 1) (which is the entire range of  $x_1$  except  $x_1 = 1$ , where the trivial PBE prevails), the nontrivial PBEs change in the manner listed below. We assume  $2/3 \le \theta \le 3/4$ .

1.  $x_1 \in [\theta, 1)$  is the necessary and sufficient condition for any PBE of the following form to exist:  $\sigma_1(s) = \sigma_1(w) = 0$ ,  $\sigma_2(s) = 1$ , and  $\sigma_2(w) \in (0, 1)$ . In any such a PBE, the share  $x_1 \in [\theta, 1)$  offered to player 1 is so large that both types of player 1 choose *A* for sure, and the

strong type of player 2 chooses R for sure, leaving only his weak type to mix between A and R. We call any PBE in this format *lopsided equilibrium*.

- 2.  $x_1 \in [2(1 \theta), \theta)$  is a necessary condition for any PBE of the following form to exist:  $\sigma_1(s), \sigma_1(w), \sigma_2(w) \in (0, 1), \sigma_2(s) = 1$ , and  $p_1^R \ge p_2^R$ . Now that the share  $x_1$  offered to player 1 falls below the threshold  $\theta$ , he no longer chooses A for sure. Player 2's strategy remains similar to that in the previous case.
- 3. There exists a unique  $\xi \in [1/2, 2(1 \theta)]$  such that:
  - (a)  $\xi < x_1 < 2(1 \theta)$  is a necessary condition for any PBE of the following form to exist:  $\sigma_1(s), \sigma_1(w), \sigma_2(w) \in (0, 1), \sigma_2(s) = 1$ , and  $p_1^R < p_2^R$ . With the share  $x_1$  offered to him lower than before, player 1 is willing to reject the offer more often than he does in the previous case even if his type is weak, and hence the posterior  $p_1^R$  of his type being strong signaled by *R* drops below  $p_2^R$ .
  - (b) Given any x<sub>1</sub> ∈ [1/2, ξ], a PBE of the following form may exist: σ<sub>i</sub>(t) ∈ (0, 1) for all i ∈ {1, 2} and all t ∈ {w, s}. The proposal is so near to the equal split that the two players behave similarly, each type mixing between A and R.
- 4. If  $\theta = 3/4$  and  $x_1 = 1/2$  (= 2(1  $\theta$ )), there is also a PBE for which  $\sigma_1(w), \sigma_2(w) \in (0, 1)$ , and  $\sigma_1(s) = \sigma_2(s) = 1$ . Under the equal-split proposal, the strong type of both players chooses *R* for sure, and the weak type of each player mixes between *A* and *R*.

The above list covers all the possible nontrivial PBEs (Lemma A.1). Among them and the trivial (always conflict) equilibria, the lopsided equilibrium associated with the proposal  $x_1 = \theta$  will be shown to be optimal (Section 5). The rest of this section will prove the existence of lopsided equilibria and the necessity of  $x_1 \ge \theta$  for their existence.<sup>9</sup>

Construction of Lopsided Equilibria For any  $x_1 \in [\theta, 1)$  as in Case 1, define  $p_2^R := 2 - \theta - x_1$ . We will see at the end of the construction that this  $p_2^R$  rationalizes player 2's strategy. Define the off-path posterior  $p_1^R := p_2^R$  for player 1.<sup>10</sup>

Being offered the share  $x_1 \ge \theta$ , player 1 chooses A for sure whether his type is strong or weak. The strong type chooses A because deviation leads to the off-path posterior  $p_1^R$  that is greater than or equal to its counterpart  $p_2$  for the rival player 2, whether  $p_2 = p_2^A$  when the rival chooses A ( $p_1^R \ge 1 - \theta \ge p_2^A$  by (8)), or  $p_2 = p_2^R$  when the rival chooses R ( $p_1^R = p_2^R$  by the previous definition of  $p_1^R$ ). That reduces the strong player 1's expected payoff  $U_1^s(p_1^R, p_2)$ in the event of conflict to its minimum  $\theta$  (Remark 2.b). By contrast, his expected payoff is at least  $\theta$  from choosing A: If the rival chooses A, player 1 gets the share  $x_1 \ge \theta$ ; if the rival chooses R, player 1 gets  $U_1^s(p_1^A, p_2^R)$ , which is equal to  $\theta$  because  $p_1^A = 1 - \theta$  at any lopsided equilibrium and  $1 - \theta \le p_2^R$  by (8).

To see that the weak type of player 1 chooses A for sure, note that his expected payoff is zero conditional on the rival choosing R. This follows from the fact  $p_1 \le p_2 \Rightarrow U_1^w(p_1, p_2) = 0$ (Equation (3)). With the rival choosing R,  $p_2 = p_2^R \ge 1 - \theta$  by (8). If player 1 chooses A as expected on path,  $p_1 = p_1^A = 1 - \theta$ ; if he deviates to R, the off-path posterior is  $p_1^R = p_2^R$ . Thus,  $p_1 \le p_2$  either way and so  $U_1^w(p_1, p_2) = 0$ . It follows that the choice between A and R for the weak type of player 1 is conditional on the rival choosing A. In that event, player 1 gets the offered share  $x_1 \ge \theta$  from choosing A. That is better than R, which gets him into the conflict and yields at most  $\theta$  (Remark 2.a). Thus, the weak player 1 chooses A.

Meanwhile, the strong type of player 2 chooses R for sure because the share  $x_2 = 1 - x_1$  offered to him is no more than  $1 - \theta$ , which is less than  $\theta$  by (1), whereas he can secure an expected payoff at least  $\theta$  in conflict (Remark 2.b). To see why the weak type of player 2

<sup>&</sup>lt;sup>9</sup> The necessity of  $x_1 \in [2(1 - \theta), \theta)$  for the existence of the PBE in Case 2 is deferred to Lemma A.6, and that of  $x_1 \in (\xi, 2(1 - \theta))$  for Case 3a, deferred to Lemma A.9, where  $\hat{x}_2$  is equal to  $1 - \xi$  for the cutoff  $\xi$ . For Case 3b (detailed in Appendix A.9) and Case 4 (detailed in Appendix A.10), we do not bother to show the existence of the corresponding PBEs or the set of  $x_1$  necessary for their existence, because we will show that any PBE in either case is suboptimal (Claims 3 and 4, Section 5).

<sup>&</sup>lt;sup>10</sup> Any other  $p_1^R \ge p_2^R$  works as well, with slightly longer calculations.

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mixes between A and R, note that he gets  $x_2$  from choosing A, and  $U_2^w(p_2^R, p_1^A)$  from choosing R, as player 1 chooses A for sure. Since the on-path action of player 1 signals no news,  $p_1^A = 1 - \theta$ . Since the strong type of player 2 chooses R for sure,  $p_2^R \ge 1 - \theta$  by Bayes's rule. Thus,  $U_2^w(p_2^R, p_1^A) = p_2^R - (1 - \theta)$  by (3). Consequently, the weak type of player 2 is willing to mix between A and R because  $p_2^R - (1 - \theta) = x_2$  due to the definition of  $p_2^R$  at the start of this construction ( $\sigma_2$  is then derived from  $p_2^R$  via Bayes's rule).

*Necessity of*  $x_1 \ge \theta$  *for any Lopsided Equilibrium* Suppose, to the contrary, that a lopsided equilibrium exists despite  $x_1 < \theta$ . Conditional on such an equilibrium, the strong type of player 1 would deviate to R, which secures for him an expected payoff at least  $\theta$  (Remark 2.b), while choosing A gives him less than  $\theta$ : he would get  $x_1 < \theta$  if the opponent chooses A, and  $\theta$  if the opponent chooses R (shown in the construction of lopsided equilibria). Consequently, the strong type of player 1 deviates to R, contradiction.

The Incentive for a Weak Type to Mix Let us illustrate the incentive for a weak type to mix between A and R with the player 2 in Case 2. Within that case, observe that the weak player 2's expected payoff conditional on the opponent choosing R is zero regardless of player 2's choice: If player 2 chooses A, the posterior system is  $(p_1^R, p_2^A)$  and we have  $p_1^R \ge 1 - \theta \ge p_2^A$  by (8); if player 2 chooses R, the posterior system  $(p_1^R, p_2^R)$  is such that  $p_1^R \ge p_2^R$  as defined in Case 2. Thus, whichever he chooses,  $p_1 \ge p_2$  holds and hence  $U_2^w(p_2, p_1) = 0$  by (3). Thus, the decision of the weak type of player 2 is purely based on the event where the opponent chooses A. The weak player 2 therefore mixes between A and R if  $x_2 = U_2^w(p_2^R, p_1^A)$ , which by (3) and (8) is equivalent to  $x_2 = p_2^R - p_1^A$ . This indifference is valid because one can show that a solution of  $(p_1^A, p_2^R)$  for this equation exists.

4.2. The Total Welfare. As defined before, the total welfare is the sum of the ex ante expected payoffs (before realization of types) across the two players. The total welfare of the lopsided equilibrium is easy to calculate (Lemma A.4). For the equilibrium in the other cases in Subsection 4.1, R is chosen with positive probabilities by both types of each player, and hence the total welfare is equal to  $\sum_{i=1}^{2} (\theta V_i^R(w) + (1 - \theta)V_i^R(s))$ . The next lemma provides a formula for this sum.

LEMMA 1. Let  $(\sigma_i, p_i^A, p_i^R)_{i=1}^2$  represent any PBE that is not lopsided and define  $q_i$  by (4) for each i = 1, 2. Relabel the players if necessary so that  $p_1^R \ge p_2^R$ . Then the total welfare at this PBE is equal to  $2\theta p_1^R + (q_1 - \theta) (p_1^R - p_2^R)$ .

**PROOF.** This lemma is based on (6) and (7), or the convex combination observation about a player's interim expected payoff in Figure 2. The upper solid graph in that figure represents a strong type's expected payoff from choosing R' as a function of the rival's posterior probability of being strong, and the lower solid graph represents the counterpart for the weak type. The two graphs are reproduced separately for player 1 in Figure 3 (for the strong type) and Figure 4 (for the weak type).

Since the lemma labels the players so that  $p_1^R \ge p_2^R$ , player 1's expected payoff  $U_1^t(p_1^R, p_2^R)$  from choosing R in the event where the rival also chooses R corresponds to the point J in Figure 3 if player 1's type is strong, or the point G in Figure 4 if player 1's type is weak. It then follows from (6) and (7) that player 1's interim expected payoff from choosing R corresponds to the point L' in Figure 3 if his type is strong, and the point B' in Figure 4 if his type is weak. That is,

(9) 
$$V_1^R(s) = \theta,$$

(10) 
$$V_1^R(w) = p_1^R - 1 + \theta_1$$





Strong player 1's payoff: L' as a convex combination between N and J



FIGURE 4

WEAK PLAYER 1'S PAYOFF: B' as a convex combination between B and G

Taking the weighted sum of (9) and (10) according to the prior distribution ( $Pr\{s\} = 1 - \theta$ ), we see that the ex ante expected payoff for player 1 is equal to  $\theta p_1^R$ .

Comparing Figure 3 with its counterpart for the strong type of player 2, and comparing Figure 4 with its counterpart for the weak type of player 2, one can show (Appendix A.3):

(11) 
$$V_2^R(s) - V_1^R(s) = q_1(p_1^R - p_2^R),$$

(12) 
$$V_2^R(w) - V_1^R(w) = -(1-q_1)(p_1^R - p_2^R).$$

The weighted sum of (11) and (12) according to the prior  $Pr\{s\} = 1 - \theta$  yields the difference in the ex ante expected payoffs between players 2 and 1:  $(q_1 - \theta)(p_1^R - p_2^R)$ . Consequently, the total welfare  $\sum_{i=1}^{2} (\theta V_i^R(w) + (1 - \theta)V_i^R(s))$  is equal to

$$\theta p_1^R + \theta p_1^R + (q_1 - \theta)(p_1^R - p_2^R) = 2\theta p_1^R + (q_1 - \theta)(p_1^R - p_2^R).$$

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REMARK 3. The proof of Lemma 1 reveals which player gets the larger share of the total welfare in a nonlopsided equilibrium: It is the player with the larger posterior  $p_i^R$ , provided that his ex ante probability  $q_i$  of choosing R is less than  $\theta$ . Since the lemma labels the players so that  $p_1^R \ge p_2^R$ , it is player 1 who gets the larger share of the total welfare provided that  $q_1 - \theta < 0$ , for then  $(q_1 - \theta)(p_1^R - p_2^R)$ , the amount by which the rival's ex ante expected payoff exceeds player 1's, is nonpositive. In other words, when rejecting a peace proposal is an on-path action for both players, the player who is perceived to become stronger conditional on having rejected the proposal gets the larger share of the total welfare, provided that he does not reject the proposal too often from the ex ante viewpoint. Roughly speaking, showing off strength through aggression pays off provided that one is rarely aggressive.

## 5. THE OPTIMALITY OF A LOPSIDED PROPOSAL

A lopsided equilibrium (Case 1, Subsection 4.1) has the advantage that one of the players chooses A independently of his own type. That is, the player accepts the peace proposal without fearing that his acceptance may betray some information that the opponent may use against him later. Among the peace proposals whose associated equilibria are lopsided, the mediator prefers those that offer more shares to the unfavored player thereby having a larger probability for him to accept the proposal as well, as long as acceptance from the favored player is still guaranteed. Since the strong type of a player can always secure an expected payoff no less than  $\theta$  by choosing R (Remark 2), the share offered to the favored player cannot fall below  $\theta$  and still guarantee his acceptance. The threshold  $\theta$  constitutes the optimal split to offer:

# **PROPOSITION 1.** If $2/3 \le \theta \le 3/4$ , the proposal that maximizes total welfare among all peace proposals is to offer $\theta$ to one player and $1 - \theta$ to the other player.

To appreciate Proposition 1, recall that a received insight in the literature (e.g., Zheng 2019b) is to adopt a posterior belief that whoever vetoes a peace proposal has the strongest possible type. As proved in Zheng (2019b), this posterior penalizes the vetoer most and represents the boundary for guaranteeing acceptance of any peace proposal. In our model, the posterior means maximizing  $p_i^R$  to one. Then any player *i* who chooses *R* gets an interim payoff equal to  $\theta$  for each type (Figure 2). That would have constituted a peace-guaranteeing solution (and hence optimal) should each player be offered a share at least as large as  $\theta$  for each to be willing to accept the proposal. Given our assumption  $\theta > 1/2$ , however, such proposals do not exist, as any split of the prize (of size one) renders the share for some player below  $\theta$ . Thus, any PBE of any proposal sees some player reject the proposal sometimes. Consequently, a player's interim payoff from choosing *R* is part of the total welfare. This, coupled with the fact that an increase in  $p_i^R$  benefits the weak and hurts the strong (Remark 1), means that the calculus of  $p_i^R$  is more involved than that in the existing literature.

Nonetheless, there are two intuitive reasons why the previous insight of achieving optimality through maximizing  $p_i^R$  might still work. First, since a strong type incurs less marginal cost in conflict than a weak type does, one would expect that a strong type is more inclined than a weak type to reject a peace proposal. Thus, if we are to pick a type to deter it from choosing *R*, it would be the strong type, and so we would reduce its interim payoff from *R* through enlarging  $p_i^R$ . Second, from the ex ante viewpoint, any quantity of payoff for a weak type does, due to the assumption  $\theta > 1/2$ . Thus, one would expect that an increase in  $p_i^R$ , benefiting the weak at the expense of the strong, enlarges the total welfare.

It is therefore conceivable that the less constrained is  $p_i^R$ , the more can  $p_i^R$  be maxed out and hence the larger the total welfare. That is where lopsided equilibria have an advantage over nonlopsided ones. In a nonlopsided equilibrium, both A and R being on path for each player, each component of the posterior system  $(p_i^A, p_i^R)_{i=1}^2$  is constrained by Bayes's rule. In a



FIGURE 5

The lopsided proposal  $(\theta, 1 - \theta)$  as the global optimum

lopsided equilibrium, by contrast, R is off path for the favored player, say player 1; hence the posterior probability  $p_1^R$  is unconstrained by Bayes's rule.

**PROOF OF PROPOSITION 1.** Relabel the players if necessary so that player 1 is offered the larger share in the peace proposal, namely,  $x_1 \ge x_2$ . A peace proposal is then represented by  $x_1$ , whose entire range is [1/2, 1]. We shall prove that  $x_1 = \theta$  maximizes the total welfare among all  $x_1 \in [1/2, 1]$ . We do that by establishing four claims, illustrated by Figure 5.

CLAIM 1.  $x_1 = \theta$  maximizes the total welfare among all lopsided equilibria associated with any  $x_1 \in [\theta, 1)$ .

Let us observe that the total welfare based on the lopsided equilibrium given any  $x_1 \in [\theta, 1)$ (Case 1, Subsection 4.1) is a strictly increasing function of  $p_2^R$ . This follows from the construction of any such equilibrium (Subsection 4.1): The ex ante expected payoff for player 2 is strictly increasing in  $p_2^R$  because her on-path interim expected payoff is equal to  $p_2^R - 1 + \theta$  when her type is weak, and  $\theta$  when her type is strong. To see the same monotonicity property for player 1, first note that player 1 prefers smaller  $q_2$  (ex ante probability of player 2 choosing R) to larger  $q_2$ : If player 2 chooses A, player 1 (who chooses A for sure) gets  $x_1 \ge \theta$ ; else player 1 gets  $\theta$  if his type is strong, and zero if his type is weak. Thus, smaller  $q_2$  makes player 1's ex ante expected payoff strictly larger. Then apply Bayes's rule to see that  $q_2p_2^R = 1 - \theta$ : smaller  $q_2$  means bigger  $p_2^R$ . Thus, both players considered, the total welfare is maximized among all lopsided equilibria when the  $p_2^R$  in the equilibrium is maximized among all such equilibria when the  $p_2^R$  in the construction of such equilibria (Subsection 4.1), maximizing  $p_2^R$  is equivalent to minimizing  $x_1$ . Thus,  $x_1 = \theta$  maximizes the total welfare among all such equilibria.

CLAIM 2. When  $x_1$  increases in  $[2(1 - \theta), \theta)$ , the total welfare of the PBE in the form of Case 2 in Subsection 4.1 increases; as  $x_1$  converges to  $\theta$  from below, the total welfare of the PBE converges to the total welfare of the lopsided equilibrium associated with  $x_1 = \theta$ .

Any PBE in the form of Case 2 is characterized by

(13) 
$$\sigma_1(s), \sigma_1(w), \sigma_2(w) \in (0, 1), \quad \sigma_2(s) = 1, \text{ and } p_1^R \ge p_2^R$$

Accordingly, one can calculate the PBE (Lemma A.6, Appendix A.8.1) and obtain

$$p_1^R = \frac{3 - 2\theta - x_2}{2}$$

$$p_2^R = 2 - 2\theta,$$
  
 $q_1 = \frac{2(\theta - 1 + x_2)}{2\theta + x_2 - 1}$ 

Since the equilibrium is nonlopsided, Lemma 1 implies that the total welfare is equal to

$$S(x_2) = 2\theta p_1^R + (q_1 - \theta) (p_1^R - p_2^R) = \theta p_1^R + q_1 (p_1^R - p_2^R) + \theta p_2^R,$$

where the total welfare is denoted as a function of  $x_2$  (= 1 -  $x_1$ ) because the variables on the right-hand side are each a function of  $x_2$  according to the equations displayed above.

As stated in Case 2, a PBE of this form exists only when  $x_1 \in [2(1-\theta), \theta)$ , namely,  $x_2 \in (1-\theta, 1-2(1-\theta)]$ . To prove the monotonicity claim, note from the above formula that  $S(x_2)$  is determined by the values of  $p_1^R$ ,  $p_2^R$ , and  $q_1$  when the PBE varies with  $x_2$ . As displayed above,  $p_2^R$  is constant. Thus, the total welfare of the PBE is determined solely by the values of  $p_1^R$  and  $q_1$ . By the above-displayed formulas of  $p_1^R$  and  $q_1$ , an increase of  $x_2$  has two opposite effects. First, it increases  $q_1$  (player 1 choosing R more often as the share  $x_1$  offered to him shrinks). Second, it decreases  $p_1^R$  (with the weak type of player 1 more willing to reject the shrinking  $x_1$ , R signals less about the strength of player 1). The above formula of  $S(x_2)$ says that the total welfare is enlarged by the first effect, and reduced by the second effect. One can show (Lemma A.8, with the assumption  $\theta \leq 3/4$ ) that the second effect outweighs the first, <sup>11</sup> and hence  $S(x_2)$  is strictly decreasing when  $x_2$  increases.

To prove the convergence part of the claim, simply use the four equations displayed above to show that  $\lim_{x_2 \downarrow 1-\theta} S(x_2)$  is equal to the total welfare of the lopsided equilibrium associated with  $x_1 = \theta$  (Lemma A.7, Appendix A.8.1).

CLAIM 3. Any PBE in the form of Case 3a or 3b in Subsection 4.1 generates less total welfare than the lopsided equilibrium associated with  $x_1 = \theta$  does.

According to Subsection 4.1, Cases 3a and 3b correspond to

(14) either 
$$\sigma_1(s), \sigma_1(w), \sigma_2(w) \in (0, 1), \quad \sigma_2(s) = 1, \text{ and } p_1^R < p_2^R$$

(15) or 
$$\forall i \in \{1, 2\}$$
:  $\sigma_i(w), \sigma_i(s) \in (0, 1)$ .

To prove the claim, we first establish that in any such a PBE,  $p_2^R \ge p_1^R$  and  $q_2 < \theta$ . If the PBE is in the form of (14),  $p_2^R \ge p_1^R$  is part of the definition, and  $q_2 < \theta$  is derived (Lemma A.10, Appendix A.8.2) from the assumption  $2/3 \le \theta \le 3/4$  and the fact  $x_1 \ge \xi > 1/2$  (a necessary condition for the form (14), stated in Case 3a, Subsection 4.1). Else, the PBE is in the form of (15), which satisfies  $p_2^R \ge p_1^R$  and  $q_2 < \theta$  due to (A.32) and Lemma A.15. Second, apply Lemma 1 to the nonlopsided equilibrium, switching the roles between players 1 and 2 in the lemma because  $p_2^R \ge p_1^R$  here. It then follows that the total welfare of the equilibrium is less than  $2\theta p_2^R$ . This quantity is less than the total welfare generated by the lopsided equilibrium given  $x_1 = \theta$ , due to the assumption  $\theta \ge 2/3$  (Lemmas A.11 and A.16).

CLAIM 4. If  $\theta \leq 3/4$  then any PBE of the following form generates less total welfare than the lopsided equilibrium associated with  $x_1 = \theta$ :

(16) 
$$\sigma_1(w), \sigma_2(w) \in (0, 1) \text{ and } \sigma_1(s) = \sigma_2(s) = 1.$$

Among all the PBE in the form of (16), the total welfare maximum is attained by the one in Case 4 in Subsection 4.1, associated with the equal-split proposal,  $x_1 = 1/2$  (Lemma A.17,

<sup>&</sup>lt;sup>11</sup> This is in line with the previous int/uition that an increase in  $p_1^R$  could improve the total welfare.

Appendix A.10). Then we show that the total welfare generated by this local maximum is still less than the one generated by the lopsided equilibrium under the proposal  $x_1 = \theta$  (last paragraph, Appendix A.10), where the assumption  $\theta \le 3/4$  is used.

Finally, it is easy to show that the lopsided equilibrium converges to a trivial (always conflict) equilibrium when  $x_1 \rightarrow 1$ . As the total welfare is decreasing when  $x_1$  increases in  $[\theta, 1)$  (Claim 1), the trivial equilibrium is suboptimal. Thus, all other trivial equilibria are suboptimal because they have the same posterior system (Lemma A.1.a) and hence generate the same total welfare. Now that all possible equilibria when  $x_1$  varies in its entire range [1/2, 1] have been covered, the optimality of  $x_1 = \theta$  is proved.

REMARK 4. The assumption  $2/3 \le \theta \le 3/4$  in Proposition 1, though partially relaxable with more calculations, reflects an intuition that the equal-split proposal  $(x_1 = 1/2)$  could be optimal when  $\theta$  is close to 1/2 or 1. Since the equal-split proposal fully prevents conflict when  $\theta \le$ 1/2 (cf. Footnote 8), it might remain optimal when  $\theta$  is just slightly above 1/2. When  $\theta \approx 1$ , the total welfare puts a heavy weight on the weak type, and one can show that the total expected payoff for the weak type of both players under the equal-split proposal is almost equal to the full size of the prize.<sup>12</sup>

REMARK 5. While a lopsided equilibrium involves an off-path posterior, the equilibrium under the optimal (lopsided) proposal  $x_1 = \theta$  satisfies both the Intuitive and D1 criteria of refinement (Appendix A.4).

REMARK 6. Even though the player who is offered the larger share of the good is "favored" at face value by a peace proposal, he need not end with a larger share of the total welfare at equilibrium. In any nonlopsided equilibrium, for instance, it is the player *i* for whom  $p_i^R > p_{-i}^R$ and  $q_i < \theta$  that has a larger share of the total welfare (Remark 3). As shown by Claim 3 in the above proof, when player 1 is offered the larger share of the good and the equilibrium takes the form of (14) or (15),  $p_1^R \le p_2^R$  and  $q_2 < \theta$ . That is, the player who is offered less by the proposed split ends with a larger share of the total welfare. Nonetheless, in the lopsided equilibrium given the optimal proposal  $x_1 = \theta$ , the two senses of favoritism coincide. Here player 1 is offered a larger share in the proposed split; meanwhile, as shown in Appendix A.6, player 1's equilibrium ex ante expected payoff  $1 - \theta/2$  is no less than its counterpart  $2 - 2\theta$  for player 2 due to the assumption  $\theta > 2/3$ . The alternative of giving player 2 a larger share of the total welfare turns out to be suboptimal as shown in Claim 3. Intuitively speaking, player 2 is offered a smaller share of the good and hence his rejecting the proposal may be attributed to the smaller offer instead of his strength. Thus, it is inefficient to raise  $p_2^R$  thereby to enlarge his ex ante payoff advantage  $|q_2 - \theta|(p_2^R - p_1^R)$  over player 1. In other words, it is inefficient to enlarge the total welfare through transferring welfare from the player favored by the peace proposal to his opponent.

# 6. CONCLUSION

Humanity is often trapped in conflict situations where conflict cannot be fully avoided. In such situations, it is inadequate for a benevolent social planner to aim merely at minimizing the likelihood of conflict, as the social welfare in both the event of peace and the event of conflict should be taken into account. This article contributes to the conflict mediation literature by incorporating both conflict and peace into maximization of total welfare and presenting an explicit solution for the maximization problem. In our model, a mediator is restricted in instruments so that she cannot effect any information structure deemed desirable with tailor-made communication mechanisms, but rather can only indirectly influence the outcome through simple mechanisms the integrity of which is easy to trust. Thus, techniques in

<sup>&</sup>lt;sup>12</sup> In a PBE under the equal-split proposal,  $p_1^R = p_2^R = 1/2$  (Lemma A.17, Appendix A.10) and hence each player's weak type gets  $p_1^R - (1 - \theta) = \theta - 1/2$  (Figure A.3). Thus, the total expected payoff for them,  $2\theta - 1$ , converges to one as  $\theta \to 1$ .

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the information-design literature are not readily available, and this article contributes an explicit analysis on how a mediator can nonetheless achieve a constrained optimal posterior information structure given simple, message-independent mechanisms.

Our solution produces a surprising implication: Even though the adversaries are ex ante identical, and are assigned equal welfare weights, the socially optimal peace proposal favors one adversary against the other so much that the favored party always accepts the proposal. Thus, it should not be taken for granted that a peace proposal should offer a fair share to each contestant even from the viewpoint of a benevolent mediator. The insight conveyed by our result is that a peace proposal biased toward one side may, counterintuitively, achieve better social welfare than an unbiased one because the favored side is willing to accept the peace deal without fearing being viewed to be weak and getting exploited later, so that the mediator can devote more resources to compensate the unfavored side.

Whereas the design objective we consider is to maximize the total welfare, the optimality of a lopsided peace proposal demonstrated by our result is extendable to models where the design objective is to minimize the probability of conflict. With the same intermediate range of the weak-type probability  $\theta$  given which the lopsided proposal maximizes total welfare, one can show that the lopsided proposal also minimizes the probability of conflict. In addition, the equal-split proposal minimizes the probability of conflict when the probability of being weak is very high or when it is low enough to be near to the region where peace can be guaranteed. This is similar to the pattern with respect to the objective that we consider.

An open question is what happens if a contestant can renege on its acceptance of a peace deal. After Iran accepted the nuclear deal in 2015, the United States withdrew from the agreement in 2018 thereby resuming the hostile relationship. It is conceivable that Iran, in retrospect, would attribute the U.S. withdrawal to Iran's acceptance of the deal in 2015, which might have revealed Iran's weak position in the conflict. That taken into account, Iran will be more reluctant to accept any nuclear deal in the future than before, for fear of its weakness being further revealed and exploited. Thus, we conjecture that the inscrutability of a contestant's response to a peace proposal can only become more important when contestants may renege. In the sense that a lopsided solution guarantees the same action from both types of the favored side thereby keeping its type inscrutable, the optimality of lopsided solutions may be robust to such limited commitment situations. See Chapter 4 in Kamranzadeh (2022) for details.

For tractability, and for a clear contrast with the lopsided solution, we assume that the two contestants are ex ante identical and that the contested prize is of common value. A natural question is to what extent a lopsided solution may remain optimal when ex ante asymmetry or private values are considered. While we conjecture that the inscrutability advantage that a lopsided solution provides for the favored party remains crucial, extension in either direction is likely to bring about new questions.

DATA AVAILABILITY STATEMENT Data sharing not applicable to this article as no data sets were generated or analyzed during the current study.

## APPENDIX A

A.1. Derivation of (2) and (3) for the All-Pay Auction. Consider any Bayesian Nash equilibrium (BNE) of the all-pay auction where  $p_i$  denotes the probability with which player *i*'s type is *s* (strong) for each i = 1, 2. If player *i*'s type is  $t_i$  ( $t_i \in \{w, s\}$ ) and if  $G_{-i}$  is the c.d.f. of the bid from the rival -i at equilibrium, then *i*'s expected payoff from bidding *b* is equal to

$$\frac{1}{\alpha} \left( G_{-i}(b) - \frac{b}{t_i} \right)$$



FIGURE A.1

THE EQUILIBRIUM IN THE ALL-PAY AUCTION

unless b is an atom of  $G_{-i}$ . According to the all-pay auction literature, there exists a unique equilibrium and  $(G_1, G_2)$  is characterized by the first-order condition

$$G'_{i}(b) = \begin{cases} 1/s & \text{if } G_{-i}(b) > 1 - p_{-i} \\ 1/w & \text{if } G_{-i}(b) < 1 - p_{-i} \end{cases}$$

for each  $i \in \{1, 2\}$ . Assume for now that  $p_1 \ge p_2$ . Coupled with the equilibrium boundary condition that  $G_i(0) = 0$  for at least one player, this differential equation system admits a unique solution. <sup>13</sup> One way to solve it is to start with the endogenous maximum bid  $\overline{b}$ , common to both players, and trace the graphs of  $G_1$  and  $G_2$  according to the differential equation system when the bid decreases from  $\overline{b}$  to zero. As in Figure A.1, both graphs start by decreasing at the rate equal to 1/s. Then the graph of  $G_1$  changes to the steeper slope 1/w at the bid b for which  $G_2(b) = 1 - p_2$ , whereas  $G_2$  remains decreasing at the rate 1/s until  $G_1(b) = 1 - p_1$  (because  $p_1 \ge p_2$ ). Thus, when the bid decreases down to zero,  $G_2(0) \ge G_1(0)$ . Since the zero bid cannot be an atom for both bidders (or an equilibrium condition is violated),  $G_1(0) = 0$ . That pins down  $\overline{b}$  and  $G_2(0)$ :

$$ar{b}/s = 1 - (1 - w/s)(1 - p_2) = 1 - lpha(1 - p_2),$$
  
 $G_2(0) = (1 - w/s)(p_1 - p_2) = lpha(p_1 - p_2),$ 

where we have used the notation  $\alpha := 1 - w/s$ . Thus, for each player *i*, the expected payoff for the strong type in the equilibrium is equal to

$$U_i^s(p_1, p_2) = \frac{1}{\alpha} \left( 1 - \frac{b}{s} \right) = 1 - p_2 = 1 - \min\{p_1, p_2\}.$$

<sup>13</sup> Since  $G_i$  and  $G_{-i}$  need not be differentiable, the differential equation system holds only for almost all b in their common support. However, one can prove that  $G_i$  and  $G_{-i}$  are each absolutely continuous on  $\begin{bmatrix} 0, \overline{b} \end{bmatrix}$  and hence the system coupled with a boundary condition admits a unique solution. See Zheng (2019b) for details.

The expected payoffs for the weak type of the two players are:

$$U_1^w(p_1, p_2) = \frac{1}{\alpha} (G_2(0) - 0/w) = p_1 - p_2 = p_1 - \min\{p_1, p_2\},$$
  
$$U_2^w(p_2, p_1) = 0 = p_2 - \min\{p_2, p_1\}.$$

Then remove the assumption  $p_1 \ge p_2$  to generalize the above to (2) and (3).

A.2. *Categorization of All Equilibria.* The next lemma classifies all the possible cases of proposal-PBE (perfect Bayesian equilibrium) pairs, called solutions for short. Case (a) corresponds to the trivial (always conflict) equilibria, Case (b) corresponds to lopsided equilibria, Case (c) the PBEs that satisfy (16), Case (d) those satisfying (15), and Case (e) those satisfying (13) or (14).

LEMMA A.1. For any solution  $(x_i, \sigma_i, p_i^A, p_i^R, q_i)_{i=1}^2$ , exactly one of the following is true:

- a.  $q_i = 1$  for some player *i*, and the on-path posterior is equal to the prior for both players; b. for some  $i \in \{1, 2\}$ ,  $\sigma_i(w) = \sigma_i(s) = 0$  and  $0 < \sigma_{-i}(w) < 1 = \sigma_{-i}(s)$ ; c. for each  $i \in \{1, 2\}$ ,  $0 < \sigma_i(w) < 1 = \sigma_i(s)$ ;
- *d.* for each  $i \in \{1, 2\}$ ,  $\sigma_i(w)$ ,  $\sigma_i(s) \in (0, 1)$ ;
- *e.* for some  $i \in \{1, 2\}$ ,  $\sigma_i(w)$ ,  $\sigma_i(s)$ ,  $\sigma_{-i}(w) \in (0, 1)$ , and  $\sigma_{-i}(s) = 1$ .

**PROOF.** First, we observe that the lemma follows from the following claims:

- 1. If  $q_i = 1$  for some player *i*, the on-path posterior is equal to the prior for each player.
- 2. If  $q_i < 1$  for each player *i*, then there does not exist any  $i \in \{1, 2\}$  for whom:
  - i.  $\sigma_i(w) = 0 < \sigma_i(s) \le 1$ ; or
  - ii.  $\sigma_i(s) = 0 < \sigma_i(w) \le 1$ ; or
  - iii.  $0 < \sigma_i(s) < 1 = \sigma_i(w)$ ; or
  - iv.  $\sigma_i(w) = \sigma_i(s) = 0$  and  $\sigma_{-i}(w), \sigma_{-i}(s) \in (0, 1)$ .

To see why the claims suffice, note that Claims 2.i and 2.ii together imply  $\sigma_i(w) = 0 \Leftrightarrow \sigma_i(s) = 0$ , and Claim 2.iii implies  $0 < \sigma_i(s) < 1 \Rightarrow \sigma_i(w) < 1$ . This coupled with Claim 2.i implies  $0 < \sigma_i(s) < 1 \Rightarrow 0 < \sigma_i(w) < 1$ . In sum, for each player  $i \in \{1, 2\}$ , if  $q_i < 1$  then there are only three possibilities: either  $\sigma_i(w) = \sigma_i(s) = 0$ , or " $0 < \sigma_i(w) < 1$  and  $0 < \sigma_i(s) < 1$ ," or " $0 < \sigma_i(w) < 1$  and  $\sigma_i(s) = 1$ " (where  $\sigma_i(s) \neq 0$  because of the first implication). Thus, in any equilibrium where  $q_i < 1$  for both players *i* (i.e., outside Case (a) in the lemma), there are only nine combinations for  $(\sigma_1, \sigma_2)$ , as in the following table:

	$\sigma_2(w) = \sigma_2(s) = 0$	$\sigma_2(w), \sigma_2(s) \in (0,1)$	$0 < \sigma_2(w) < 1 = \sigma_2(s)$
$ \begin{aligned} \sigma_1(w) &= \sigma_1(s) = 0 \\ \sigma_1(w), \sigma_1(s) &\in (0, 1) \\ 0 &< \sigma_1(w) &< 1 = \sigma_1(s) \end{aligned} $	impossible	impossible	case (b)
	impossible	case (d)	case (e)
	case (b)	case (e)	case (c)

In this table, the cell (1, 1)  $(\sigma_1(w) = \sigma_1(s) = 0 = \sigma_2(w) = \sigma_2(s))$  is impossible because our assumption  $\theta > 1/2$  implies that it is impossible to have  $\sigma_i(s) = \sigma_i(w) = 0$  for both players *i* (Footnote 8). Claim 2.iv says that the cells (1, 2) and (2, 1) (one player's strategy is totally mixed and the other chooses *A* for sure) are each impossible. The other cells are the possible ones, filled in with the corresponding cases in the lemma.

The rest of the proof establishes the claims listed above.

Claim 1 Let  $q_i = 1$  for some player *i*. Then the on-path posterior about *i* is  $p_i^R = 1 - \theta$ . For player -i, suppose that the action *A* is on path and  $p_{-i}^A$  is not equal to the prior  $1 - \theta$ . Then Bayes's rule requires that the other action *R* be on path as well so that  $p_{-i}^R \neq 1 - \theta$  and (7) be satisfied. Thus, one of  $p_{-i}^A$  and  $p_{-i}^R$  is above  $1 - \theta$ , and the other below  $1 - \theta$ . If  $p_{-i}^A > 1 - \theta > p_{-i}^R$ , then by (2) and (3) (or simply Figure 1),

$$egin{aligned} &U^s_{-i}(p^R_{-i},1- heta)= heta<1-p^R_{-i}=U^s_{-i}(p^R_{-i},1- heta),\ &U^w_{-i}(p^R_{-i},1- heta)=0< p^A_{-i}- heta+1=U^w_{-i}(p^A_{-i},1- heta). \end{aligned}$$

Thus, player -i of type *s* would choose *R* for sure, and -i of type *w*, *A* for sure. That implies  $p_{-i}^{R} = 1$  and  $p_{-i}^{A} = 0$ , contradicting  $p_{-i}^{A} > 1 - \theta > p_{-i}^{R}$ . The other case, where  $p_{-i}^{A} < 1 - \theta < p_{-i}^{R}$ , is self-contradicting analogously. This proves Claim 1.

*Claim 2.i* Suppose, to the contrary, that  $\sigma_i(w) = 0 < \sigma_i(s) \le 1$  for some player *i*. By Bayes's rule,  $\sigma_i(w) = 0$  implies  $p_i^R = 1$ . Then the two graphs in Figure 1 coincide, with  $p_i$  there equal to  $p_i^R = 1$ , and hence  $V_i^R(s) = V_i^R(w) = 1 - (1 - \theta) = \theta$  by (6)—simply put, the dashed segment in Figure 2 coincides with the solid thick line because any  $p_{-i}^A$  and  $p_{-i}^R$  are less than or equal to  $1 = p_i^R$ . Recall from (5) that  $V_i^A(t)$  denotes *i*'s expected payoff from choosing *A* given type  $t \in \{s, w\}$ . By the best response condition,

$$\sigma_i(w) = 0 \Rightarrow V_i^A(w) \ge V_i^R(w) = \theta,$$
  
$$\sigma_i(s) > 0 \Rightarrow V_i^A(s) \le V_i^R(s) = \theta.$$

Thus,  $V_i^A(w) \ge V_i^A(s)$ . Meanwhile, (5) implies that  $V_i^A(w) \le V_i^A(s)$ , as  $U_i^w(p_i^A, \cdot) \le U_i^s(p_i^A, \cdot)$ for any  $p_i^A \in [0, 1]$ . Consequently,  $V_i^A(w) = V_i^A(s)$ . Then (5) coupled with  $q_{-i} > 0$  implies that  $U_i^w(p_i^A, p_{-i}^R) = U_i^s(p_i^A, p_{-i}^R)$ . Compare (2) with (3)—or simply inspect Figure 1—to see that the equation is possible only if  $p_i^A = 1$ . But that violates Bayes's rule given that  $\sigma_i(w) < 1$ . Thus, Claim 2.i follows.

Claim 2.ii Suppose, to the contrary, that  $q_i < 1$  for both players *i*, and  $\sigma_i(s) = 0 < \sigma_i(w) \le 1$  for some player *i*. By Bayes's rule,  $\sigma_i(s) = 0$  implies  $p_i^R = 0$ . By (2) and (3),  $U_i^s(p_i^R, \cdot) = 1$  and  $U_i^w(p_i^R, \cdot) = 0$ . It follows from (6) that  $V_i^R(s) = 1$  and  $V_i^R(w) = 0$ . By the best response condition for  $\sigma_i(w) > 0$ ,

$$0 = V_i^R(w) \ge V_i^A(w) \stackrel{(5)}{=} q_{-i}U_i^w(p_i^A, p_{-i}^R) + (1 - q_{-i})x_i \ge (1 - q_{-i})x_i$$

and hence  $x_i = 0$  (since  $1 - q_{-i} > 0$ ). This coupled with the best response condition for  $\sigma_i(s) = 0$  implies

 $1 = V_i^R(s) \le V_i^A(s) = 0 + q_{-i}U_i^s(p_i^A, p_{-i}^R) \stackrel{(2)}{=} q_{-i}(1 - \min\{p_i^A, p_{-i}^R\}).$ 

Thus,  $q_{-i} = 1$ , contradiction.

Claim 2.iii Suppose, to the contrary, that  $q_i < 1$  for both players *i*, and  $0 < \sigma_i(s) < 1 = \sigma_i(w)$  for some player *i*. By Bayes's rule,  $\sigma_i(w) = 1$  implies  $p_i^A = 1$ . It then follows from (2) and (3) that  $U_i^s(p_i^A, \cdot) = U_i^w(p_i^A, \cdot)$  and hence, by (5),  $V_i^A(s) = V_i^A(w)$ . By the best response condition,  $0 < \sigma_i(s) < 1$  implies  $V_i^R(s) = V_i^A(s)$ , and  $\sigma_i(w) > 0$  implies  $V_i^R(w) \ge V_i^A(w)$ . Thus,  $V_i^R(w) \ge V_i^R(s)$ . This, by inspection of Figure 2—or (6)—is possible only if  $p_i^R = 1$ . But  $p_i^R = 1$  violates Bayes's rule since  $\sigma_i(w) > 0$ , contradiction.

Claim 2.iv Suppose, to the contrary, that for each player *i* we have  $q_i < 1$  and  $\sigma_i(w) = \sigma_i(s) = 0$ ,  $0 < \sigma_{-i}(w) < 1$  and  $0 < \sigma_{-i}(s) < 1$ . With  $\sigma_i(w) = \sigma_i(s) = 0$ , we have  $q_i = 0$  and





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 $p_i^A = 1 - \theta$ . Plug them into (6)—or simply noting that the convex combination in Figure 2 degenerates to the point  $1 - \theta$ —to see that  $V_{-i}^R(w) = p_{-i}^R - (1 - \theta)$  and  $V_{-i}^R(s) = 1 - (1 - \theta) = \theta$ . Since  $\sigma_{-i}(w) > 0$ ,  $p_{-i}^R < 1$  and hence  $p_{-i}^R - (1 - \theta) < \theta$ . Consequently,  $V_{-i}^R(w) < V_{-i}^R(s)$ . Meanwhile, by the best response condition and  $q_i = 0$ ,

$$0 < \sigma_{-i}(w) < 1 \Rightarrow x_{-i} = V_{-i}^{A}(w) = V_{-i}^{R}(w),$$
  
$$0 < \sigma_{-i}(s) < 1 \Rightarrow x_{-i} = V_{-i}^{A}(s) = V_{-i}^{R}(s).$$

Thus,  $V_{-i}^R(w) = V_{-i}^R(s)$ , contradiction.

An implication of Lemma A.1 is that the condition  $p_i^R \ge 1 - \theta \ge p_i^A$  in Figures 2–A.2 is indeed satisfied:

LEMMA A.2. For any solution  $(x_i, \sigma_i, p_i^A, p_i^R, q_i)_{i=1}^2$ , either  $q_i = 1$  for some player *i* and the on-path posterior is equal to the prior for both players, or  $q_i < 1$  for both players *i* and, for each player *i*,  $q_i > 0 \Rightarrow p_i^R > 1 - \theta > p_i^A$ .

PROOF. By Lemma A.1, either Case (a) is true, which means the on-path posterior is equal to the prior for both players, or (a) is not true and hence  $q_i < 1$  for both players *i*. In the latter alternative, if  $q_i > 0$  then we have either (I)  $\sigma_i(w), \sigma_i(s) \in (0, 1)$ —which is true for both players in case (d), and player *i* in case (e), in Lemma A.1—or (II)  $\sigma_i(s) > \sigma_i(w)$  (which is true for player -i in case (b), both players in case (c), and player -i in case (e)). In (I), the best response condition implies

$$V_i^R(s) - V_i^A(s) = 0 = V_i^R(w) - V_i^A(w),$$

which, by (A.1), simplifies to  $1 - p_i^R = q_{-i}(1 - p_i^A)$ . This coupled with  $q_{-i} < 1$  implies  $1 - p_i^R < 1 - p_i^A$ , that is,  $p_i^R > p_i^A$ . In (II), by Bayes's rule  $\sigma_i(s) = q_i p_i^R / (1 - \theta)$  and  $\sigma_i(w) = q_i(1 - p_i^R)/\theta$ , and by  $q_i > 0$ , we have  $p_i^R / (1 - \theta) > (1 - p_i^R)/\theta$ , namely,  $p_i^R > 1 - \theta$ . Both cases considered, we have shown that  $q_i > 0$  implies  $p_i^R > p_i^A$  or  $p_i^R > 1 - \theta$ . In either case, the Bayesian plausibility condition (7) implies  $p_i^R > 1 - \theta > p_i^A$ .

A.3. Proof of (11) and (12) for Lemma 1. Figure A.2 depicts the expected payoff from choosing R for the strong type of each player, with curve ILM for player 1, and curve IJK



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for player 2. Curve *ILM* lies below curve *IJK* because the lemma labels the players so that  $p_1^R \ge p_2^R$ . Similarly, Figure A.3 depicts the expected payoff from choosing *R* for the weak type of each player, with curve *DEH* for player 1, and curve *FGH* for player 2. Curve *DEH* lies above curve *FGH* again by  $p_1^R \ge p_2^R$ .

To prove (11), note in Figure A.2 that  $\Delta NK'L'$  and  $\Delta NKL$  are similar triangles. Thus,

$$\frac{|K'L'|}{|KL|} = \frac{1 - \theta - p_1^A}{p_1^R - p_1^A}$$

Consequently, since  $|K'L'| = V_2^R(s) - V_1^R(s)$  and  $|KL| = p_1^R - p_2^R$ , we have

$$V_2^R(s) - V_1^R(s) = \frac{1 - \theta - p_1^A}{p_1^R - p_1^A} (p_1^R - p_2^R) = q_1(p_1^R - p_2^R)$$

with the second equality due to the Bayesian plausibility condition (7). Thus (11) follows.

Analogously, in Figure A.3,  $\Delta EB'C'$  and  $\Delta EBC$  are similar triangles. Thus,

$$\frac{|B'C'|}{|BC|} = \frac{p_1^R - (1 - \theta)}{p_1^R - p_1^A} = 1 - q_1,$$

with the second equality again due to (7). Consequently, since  $V_2^R(w) - V_1^R(w) = -|B'C'|$  and  $|BC| = p_1^R - p_2^R$ , Equation (12) follows.

A.4. Verification of the Intuitive and D1 Criteria. It is easy to derive from the construction of lopsided equilibria (Subsection 4.1) that  $q_2 = 1/2$  and  $p_2^R = 2(1 - \theta)$  in the lopsided equilibrium when the proposal is  $x_1 = \theta$ . Note that the only observable deviation from the equilibrium is player 1 choosing R. Also note that player 1's expected payoff from this equilibrium is equal to  $V_1^A(s) = \theta$  when his type is s, and  $V_1^A(w) = \theta/2$  when the type is w. For each  $t \in \{s, w\}$  and any  $p_1^R \in [0, 1]$ , let  $\tilde{V}_1^R(t, p_1^R)$  denote type-t player 1's expected payoff from the deviation provided that the posterior probability of him being strong is  $p_1^R$  (together with the on-path posterior probability  $p_2^R = 2(1 - \theta)$  of player 2 being strong).

Intuitive Criterion Denote J for the set of player 1's types whose equilibrium payoff is higher than any payoff he could get by playing R, as long as player 2's action is rationalizable. That is,

$$J := \left\{ t \in \{s, w\} \; \middle| \; V_1^A(t) > \max_{p_1^R \in [0,1]} \tilde{V}_1^R(t, p_1^R) \right\}.$$

Observe that  $J = \emptyset$ :  $s \notin J$  because the equilibrium payoff  $\theta$  is the minimum payoff that a strong type *s* can achieve from playing *R* (Remark 2);  $w \notin J$  because the equilibrium payoff  $\theta/2$  is less than  $\theta$ , which is equal to  $\tilde{V}_1^R(w, 1)$  because  $p_1^R = 1 > 2(1 - \theta) = p_2^R$  implies via (10) that  $\tilde{V}_1^R(w, 1) = 1 - (1 - \theta) = \theta$ . Now that  $J = \emptyset$ , the set of distributions of player 1's type whose supports exclude *J* (the empty set) contains the posterior distribution that supports the lopsided equilibrium. Thus, the equilibrium satisfies the Intuitive Criterion.

D1 Criterion It suffices to falsify the following inequality for each  $t \in \{s, w\}$  (and  $\{t'\} := \{s, w\} \setminus \{t\}$ ):

$$\left\{p_1^R \in [0,1] \; \middle| \; V_1^A(t) \le \tilde{V}_1^R(t,p_1^R)\right\} \subsetneq \left\{p_1^R \in [0,1] \; \middle| \; V_1^A(t') < \tilde{V}_1^R(t',p_1^R)\right\}.$$

To that end, consider first t = s (so t' = w). Since  $V_1^A(s) = \theta$  is the minimum payoff that a strong type s can achieve from playing R (Remark 2), the left-hand side is equal to [0, 1] and hence the (strict) inequality cannot hold. Next consider t = w (and so t' = s). Note that  $p_1^R = 1$  belongs to the left-hand side, as  $V_1^A(w) = \theta/2 < \theta = \tilde{V}_1^R(w, 1)$ , shown in the previous paragraph. However,  $p_1^R = 1$  does not belong to the right-hand side, because  $V_1^A(s) = \theta$  and  $\tilde{V}_1^R(s, 1) = \theta$  by (9). Thus again the inequality displayed above does not hold. Both cases considered, the D1 Criterion is satisfied.

## A.5. Three Useful Equations.

LEMMA A.3. In any solution  $(x_i, \sigma_i, p_i^A, p_i^R, q_i)_{i=1}^2$ ,

(A.1) 
$$V_i^R(s) - V_i^A(s) - \left(V_i^R(w) - V_i^A(w)\right) = 1 - p_i^R - q_{-i}(1 - p_i^A)$$

for each player *i*, and if  $p_i^R \ge p_{-i}^R \ge 1 - \theta$ , then

(A.2) 
$$V_i^R(w) - V_i^A(w) = p_i^R - (1 - q_{-i})x_i - 1 + \theta,$$

(A.3) 
$$V_{-i}^{R}(w) - V_{-i}^{A}(w) = (1 - q_{i})(p_{-i}^{R} - p_{i}^{A} - x_{-i}).$$

PROOF. To prove (A.1), note from (5) and (6) that the left-hand side is equal to

$$\begin{split} q_{-i} & \left( U_i^s(p_i^R, p_{-i}^R) - U_i^s(p_i^A, p_{-i}^R) - U_i^w(p_i^R, p_{-i}^R) + U_i^w(p_i^A, p_{-i}^R) \right) \\ & + (1 - q_{-i}) \left( U_i^s(p_i^R, p_{-i}^A) - U_i^w(p_i^R, p_{-i}^A) \right) \\ \stackrel{(2),(3)}{=} & q_{-i} \left( 1 - \min\{p_i^R, p_{-i}^R\} - 1 + \min\{p_i^A, p_{-i}^R\} - p_i^R + \min\{p_i^R, p_{-i}^R\} + p_i^A - \min\{p_i^A, p_{-i}^R\} \right) \\ & + (1 - q_{-i}) \left( 1 - \min\{p_i^R, p_{-i}^A\} - p_i^R + \min\{p_i^R, p_{-i}^A\} \right) \\ & = q_{-i} \left( -p_i^R + p_i^A \right) + (1 - q_{-i}) \left( 1 - p_i^R \right), \end{split}$$

which is equal to the right-hand side. To prove (A.2), assume without loss that  $p_1^R \ge p_2^R$ . Thus for each player *i*,  $p_i^R \ge 1 - \theta$  and hence, by the Bayesian plausibility condition (7),  $p_i^R \le 1 - \theta$ .

Use (5) and (6) to obtain

$$\begin{aligned} V_1^R(w) - V_1^A(w) &= q_2 \left( U_1^w(p_1^R, p_2^R) - U_1^w(p_1^A, p_2^R) \right) + (1 - q_2) \left( U_1^w(p_1^R, p_2^A) - x_1 \right) \\ &\stackrel{(3)}{=} q_2 \left( p_1^R - \min\{p_1^R, p_2^R\} - p_1^A + \min\{p_1^A, p_2^R\} \right) + (1 - q_2) \left( p_1^R - \min\{p_1^R, p_2^A\} - x_1 \right) \\ &= q_2 \left( p_1^R - p_2^R - p_1^A + p_1^A \right) + (1 - q_2) \left( p_1^R - p_2^A - x_1 \right) \\ &= p_1^R - q_2 p_2^R - (1 - q_2) p_2^A - (1 - q_2) x_1 \\ &= p_1^R - (1 - \theta) - (1 - q_2) x_1, \end{aligned}$$

with the third line due to  $p_1^R \ge p_2^R \ge 1 - \theta \ge p_j^A$  for each player *j*, and the last line due to the Bayesian plausibility condition (7). Thus (A.2) is true. Analogously, (A.3) follows from

$$V_2^R(w) - V_2^A(w) = q_1 (p_2^R - p_2^R - p_2^A + p_2^A) + (1 - q_1) (p_2^R - p_1^A - x_2)$$
  
=  $(1 - q_1) (p_2^R - p_1^A - x_2).$ 

## A.6. The Total Welfare of the Optimal Lopsided Solution.

LEMMA A.4. The total welfare generated by the lopsided equilibrium associated with the proposal  $(\theta, 1 - \theta)$  is equal to  $\theta(3 - 5\theta/2)$ .

**PROOF.** By definition of any lopsided equilibrium,  $q_1 = 0$  and  $0 < \sigma_2(w) < 1 = \sigma_1(s)$ . Thus, the total welfare from  $(\theta, 1 - \theta)$  is equal to

$$\underbrace{(1-q_2)\theta + q_2 \left[\theta U_1^w(p_1^A, p_2^R) + (1-\theta)U_1^s(p_1^A, p_2^R)\right]}_{\text{player 1}} + \underbrace{\theta U_2^w(p_2^R, p_1^A) + (1-\theta)U_2^s(p_2^R, p_1^A)}_{\text{player 2}}.$$

By Bayes's rule,  $p_1^A = 1 - \theta$ ,  $p_2^A = 0$  and  $q_2 = (1 - \theta)/p_2^R$ . As explained in the construction of lopsided equilibria (Subsection 4.1),  $p_2^R = 1 - \theta + x_2 = 2(1 - \theta)$ . Combine them with (2) and (3) to calculate the above-displayed sum:

$$(1 - q_2)\theta + q_2(\theta \cdot 0 + (1 - \theta)(1 - 1 + \theta)) + \theta(p_2^R - 1 + \theta) + (1 - \theta)(1 - 1 + \theta)$$
  
=  $\left(1 - \frac{1 - \theta}{p_2^R}\right)\theta + \frac{1 - \theta}{p_2^R}(1 - \theta)\theta + \theta(p_2^R - 1 + \theta) + (1 - \theta)\theta$   
=  $\left(1 - \frac{1 - \theta}{2(1 - \theta)}\right)\theta + \frac{1 - \theta}{2(1 - \theta)}(1 - \theta)\theta + \theta(2(1 - \theta) - 1 + \theta) + (1 - \theta)\theta$   
=  $\underbrace{(2 - \theta)\theta/2}_{\text{player 1}} + \underbrace{(2 - 2\theta)\theta}_{\text{player 2}}_{\text{player 2}}$   
=  $\theta(3 - 5\theta/2).$ 

A.7. Suboptimality of Any Trivial Equilibrium. By Claim 1 in the proof of Lemma A.1, any trivial PBE, namely, any Case-(a) solution, has the on-path posterior equal to the prior for each player. Since  $q_i = 1$  for some player *i*, conflict takes place for sure and hence each player's ex ante payoff from the PBE is equal to

$$\theta U_i^w (1-\theta, 1-\theta) + (1-\theta) U_i^s (1-\theta, 1-\theta) = 0 + (1-\theta)(1-(1-\theta)) = \theta(1-\theta).$$

Thus, the total welfare generated by the PBE is equal to  $2\theta(1-\theta)$ , which is less than  $\theta(3-5\theta/2)$ , the total welfare generated by the lopsided proposal  $(\theta, 1-\theta)$  (Lemma A.4). Thus, any PBE that belongs to Case (a) is suboptimal.

A.8. Suboptimality of Any Equilibrium in Equation (13) or (14). Equilibria in the form of Equation (13) or (14) correspond to Case (e) in Lemma A.1: exactly one of the two players is totally mixing A and R for each type. Relabeling the players if necessary, assume without loss that in any Case-(e) PBEs it is player 1 who is totally mixing, that is,

(A.4) 
$$0 < \sigma_1(w) < 1, \quad 0 < \sigma_1(s) < 1, \quad 0 < \sigma_2(w) < 1, \quad \sigma_2(s) = 1.$$

Call a Case-(e) solution *Case* (e)-*i* if  $p_2^R \le p_1^R$ , and *Case* (e)-*ii* if  $p_1^R < p_2^R$ . This labeling of the players implies  $x_1 \ge x_2$  (and hence is consistent with the labeling in Subsection 4.1), because  $x_2 \le 1/2$  according to Lemma A.6 for Subcase-(e)-i, and Lemma A.9 for Subcase-(e)-ii.

LEMMA A.5. A tuple  $(x_i, \sigma_i, p_i^A, p_i^R, q_i)_{i=1}^2$  constitutes a Case-(e)-i solution if and only if it satisfies (A.4) and all the following:

(A.5) 
$$1 - p_1^R = q_2(1 - p_1^A)$$

$$(A.6) 1-p_2^R \ge q_1,$$

$$(A.7) p_2^R \le p_1^R,$$

(A.8) 
$$p_1^R + \theta - 1 = (1 - q_2)x_1$$

(A.9) 
$$p_2^R = p_1^A + x_2.$$

A tuple  $(x_i, \sigma_i, p_i^A, p_i^R, q_i)_{i=1}^2$  constitutes a Case-(e)-ii solution if and only if it satisfies (A.4), (A.5), (A.6) and all the following:

(A.10) 
$$p_1^R < p_2^R$$

(A.11) 
$$p_1^R = x_1,$$

(A.12) 
$$p_2^R + \theta - 1 = (1 - q_1)x_2.$$

PROOF. The best response condition for (A.4) to constitute a PBE is that  $V_1^R(w) - V_1^A(w) = V_1^R(s) - V_1^A(s) = 0$  for player 1 and  $V_2^R(w) - V_2^A(w) = 0 \le V_2^R(s) - V_2^A(s)$  for player 2. By (A.1), that is equivalent to simultaneous satisfaction of  $V_1^R(w) - V_1^A(w) = V_2^R(w) - V_2^A(w) = 0$ ,  $(1 - p_1^R) = q_2 (1 - p_1^A)$  and  $1 - p_2^R \ge q_1$  (i.e., Ineq. (A.6), the derivation of which also uses the fact  $p_2^A = 0$  implied by Bayes's rule with respect to  $\sigma_2(s) = 1$ ). To write the condition  $V_1^R(w) - V_1^A(w) = V_2^R(w) - V_2^A(w) = 0$  explicitly, note for each player *i* that  $q_i < 1$  in this PBE and hence  $p_i^A < 1 - \theta < p_i^R$  by Lemma A.2. If the solution belongs to Subcsae (i) of Case (e),  $p_1^R \ge p_2^R$ , then (A.2) and (A.3) apply to the case i = 1 and hence

$$V_1^R(w) - V_1^A(w) = p_1^R - (1 - \theta) - (1 - q_2)x_1$$
  
$$V_2^R(w) - V_2^A(w) = (1 - q_1)(p_2^R - p_1^A - x_2).$$

Thus the condition  $V_1^R(w) - V_1^A(w) = 0$  becomes (A.8), and the condition  $V_2^R(w) - V_2^A(w) = 0$  becomes (A.9). Analogously, if it is Subcase (ii) of Case (e),  $p_1^R \le p_2^R$ , then (A.2) and (A.3)

apply to the case i = 2 and hence

$$V_2^R(w) - V_2^A(w) = p_2^R - (1 - \theta) - (1 - q_1)x_2,$$
  

$$V_1^R(w) - V_1^A(w) = (1 - q_2)(p_1^R - p_2^A - x_1) = (1 - q_2)(p_1^R - x_1),$$

with the last "=" due to  $p_2^A = 0$  (since  $\sigma_2(s) = 1$ ). Thus, the condition  $V_i^R(w) - V_i^A(w) = 0$  for both players *i* becomes (A.11) and (A.12).

A.8.1. Subcase (i): 
$$p_1^R \ge p_2^R$$
 (Equation (13)).

LEMMA A.6. For any  $x_2 \in [0, 1]$  there is at most one tuple  $(\sigma_i, p_i^A, p_i^R, q_i)_{i=1}^2$  that constitutes a Case-(e)-i solution, and for any such solution,  $1 - \theta < x_2 \le 2\theta - 1$ , where  $2\theta - 1 \le 1/2$  if  $\theta \le 3/4$ .

PROOF. Let  $x_2 \in [0, 1]$  and  $(x_i, \sigma_i, p_i^A, p_i^R, q_i)_{i=1}^2$  be a Case-(e)-i solution. By Lemma A.5, the tuple satisfies Equations (A.5), (A.8), and (A.9). Combine (A.5), (A.8), and (A.9) with  $q_2 = \theta \sigma_2(w) + 1 - \theta$  (definition of  $q_i$ ),  $p_2^R = (1 - \theta)/q_2$  (Bayes's rule with respect to  $\sigma_2(s) = 1$ ) and  $x_1 + x_2 = 1$  (definition of a peace proposal) to obtain

(A.13) 
$$\sigma_2(w) = 1 - \frac{1}{2\theta}.$$

Plug (A.13) into the system consisting of (A.4), (A.5), (A.8), and (A.9) to obtain a unique solution for all components of the tuple:

(A.14) 
$$q_2 = \theta \left(1 - \frac{1}{2\theta}\right) + 1 - \theta = \frac{1}{2},$$
$$p_2^R = \frac{1 - \theta}{q_2} = 2 - 2\theta,$$

(A.15) 
$$p_1^R = 1 - \theta + (1 - 1/2)(1 - x_2) = \frac{3 - 2\theta - x_2}{2},$$

R

(A.16) 
$$p_1^A = p_2^A - x_2 = 2(1-\theta) - x_2,$$
$$q_1 = \frac{1-\theta - p_1^A}{p_1^R - p_1^A} = \frac{2(\theta - 1 + x_2)}{2\theta + x_2 - 1},$$

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(A.17) 
$$\sigma_1(w) = \frac{\theta - 1 + x_2}{\theta}.$$

In particular, (A.17) follows from

$$\begin{aligned} \theta \sigma_1(w) &= q_1 - (1 - \theta) \sigma_1(s) = q_1 - p_1^{\kappa} q_1 \\ &= \frac{2(\theta - 1 + x_2)}{2\theta + x_2 - 1} \left( 1 - \frac{3 - 2\theta - x_2}{2} \right) \\ &= \theta - 1 + x_2. \end{aligned}$$

Since  $\sigma_1(w) > 0$  by definition of any Case-(e) solution, (A.17) implies  $x_2 > 1 - \theta$ .

To prove  $x_2 \le 2\theta - 1$ , plug (A.14) and (A.15) into the condition  $p_1^R \ge p_2^R$  that defines Subcase (e)-i to obtain

$$p_1^R \ge p_2^R \iff \frac{3-2\theta-x_2}{2} \ge 2-2\theta \iff x_2 \le 2\theta-1.$$

#### UNEQUAL PEACE

LEMMA A.7. When  $x_1$  converges to  $\theta$  from above, the total welfare generated by any Case-(e)i solution given proposal  $(x_1, x_2)$  converges to the total welfare generated by the lopsided equilibrium given proposal  $(\theta, 1 - \theta)$ .

**PROOF.** By Lemma A.6, any Case-(e)-i solution is uniquely determined by the  $x_2$  in the tuple, with 2 being the label for the player for whom  $p_2^R \le p_1^R$ . Thus, the total welfare generated by the solution is uniquely determined by  $x_2$ . Hence let  $S_e(x_2)$  denote the total welfare generated by a Case-(e)-i solution that offers  $x_2$  to the player -i for whom  $p_{-i}^R \le p_i^R$ . Since R is a best reply for each type of each player in any Case-(e) solution, Lemma 1 implies

(A.18) 
$$S_e(x_2) = 2\theta p_1^R + (q_1 - \theta)(p_1^R - p_2^R).$$

By Lemma A.6,  $x_2 > 1 - \theta$ . Taking the limit of (A.15) and (A.16) as  $x_2$  converges to  $1 - \theta$  from above, we have

$$\lim_{\substack{x_2 \downarrow 1-\theta}} p_1^R = \frac{2-\theta}{2},$$
$$\lim_{\substack{x_2 \downarrow 1-\theta}} q_1 = 0.$$

Combine them with the above formula of  $S_e(x_2)$  and (A.14) to obtain

$$\lim_{x_2 \downarrow 1-\theta} S_e(x_2) = 2\theta p_1^R - \theta (p_1^R - p_2^R) = \theta (p_1^R + p_2^R)$$
$$= \theta \left(\frac{2-\theta}{2} + 2 - 2\theta\right)$$
$$= \theta \left(3 - \frac{5}{2}\theta\right),$$

which by Lemma A.4 is equal to the total welfare generated by the lopsided equilibrium given proposal  $(\theta, 1 - \theta)$ .

LEMMA A.8. If  $\theta \leq 3/4$ , the lopsided equilibrium given proposal  $(\theta, 1 - \theta)$  generates larger total welfare than any Case-(e)-i solution.

**PROOF.** By Lemma A.7, it suffices to prove that  $\frac{d}{dx_2}S_e(x_2) < 0$  for all  $x_2 > 1 - \theta$ . To prove that, use (A.18) and  $dp_2^R/dx_2 = 0$  (Equation (A.14)) to obtain

(A.19) 
$$\frac{d}{dx_2}S_e(x_2) = \frac{\partial S_e}{\partial p_1^R}\frac{dp_1^R}{dx_2} + \frac{\partial S_e}{\partial q_1}\frac{dq_1}{dx_2} = (q_1 + \theta)\frac{dp_1^R}{dx_2} + (p_1^R - p_2^R)\frac{dq_1}{dx_2}$$
$$= -\frac{q_1 + \theta}{2} + (p_1^R - p_2^R)\frac{2\theta}{(2\theta + x_2 - 1)^2},$$

with the last equality due to (A.15) and (A.16). Note that the expression (A.19) is strictly decreasing in  $x_2$ : By (A.14) and (A.15),  $p_1^R - p_2^R = (2\theta - 1 - x_2)/2$ , which is strictly decreasing in  $x_2$ ; as can be seen above (due to (A.16)),

$$\frac{dq_1}{dx_2} = \frac{2\theta}{(2\theta + x_2 - 1)^2} > 0$$

and so  $-(q_1 + \theta)/2$  is strictly decreasing in  $x_2$  as well. Thus,  $\frac{d}{dx_2}S_e(x_2)$  is strictly decreasing in  $x_2$ .

Now that  $\frac{d}{dx_2}S_e(x_2)$  is strictly decreasing in  $x_2$  for all  $x_2 > 1 - \theta$ , and  $x_2 > 1 - \theta$  for any Case-(e)-i solution, to show that  $S_e(x_2)$  is strictly decreasing in  $x_2$ , we need only

$$\lim_{x_2\downarrow 1-\theta}\frac{d}{dx_2}S_e(x_2)<0.$$

To show that, take the limit of (A.19) as  $x_2$  converges to  $1 - \theta$  from above and use (A.14), (A.15), and (A.16) (so  $\lim_{x_2 \downarrow 1-\theta} q_1 = 0$  and  $\lim_{x_2 \downarrow 1-\theta} (p_1^R - p_2^R) = (3\theta - 2)/2$ ) to obtain

$$\lim_{x_2\downarrow 1-\theta} \frac{d}{dx_2} S_e(x_2) = -\frac{\theta}{2} + \frac{(3\theta-2)}{2} \cdot \frac{2}{\theta} = \frac{-\theta^2 + 6\theta - 4}{2\theta} = -\frac{1}{2\theta} \big( (\theta-3)^2 - 5 \big),$$

which is negative because the condition  $\theta \leq 3/4$  in the lemma implies  $\theta < 3 - \sqrt{5}$ . Thus, the supremum of  $\frac{d}{dx_2}S_e(x_2)$  is negative among all  $x_2 > 1 - \theta$ , so  $\lim_{x_2 \downarrow 1-\theta} S_e(x_2)$  is the supremum total welfare among all Case-(e)-i solutions. By Lemma A.7, the supremum is equal to the total welfare generated by the lopsided equilibrium given proposal  $[\theta, 1 - \theta]$ .

A.8.2. Subcase (ii): 
$$p_1^R < p_2^R$$
 (Equation (14)).

LEMMA A.9. For any  $x_2 \in [0, 1]$  there is at most one tuple  $(\sigma_i, p_i^A, p_i^R, q_i)_{i=1}^2$  that constitutes a Case-(e)-ii solution; if  $\theta \leq 3/4$  in addition, then  $2\theta - 1 < x_2 < \hat{x}_2$  for any such a solution, where  $\hat{x}_2$  is uniquely determined by  $\theta$  and belongs to  $[2\theta - 1, 1/2]$ .

**PROOF.** By Lemma A.5, the tuple satisfies Equations (A.5), (A.11), and (A.12). Plug Bayes's rule  $1 - p_1^R = \theta \sigma_1(w)/q_1$  into Equation (A.11) to obtain

(A.20) 
$$\sigma_1(w) = \frac{(1-\theta)(1-x_1)}{\theta x_1} \sigma_1(s).$$

Equation (A.11), combined with  $1 - p_1^R = \theta \sigma_1(w)/q_1$  and  $x_1 + x_2 = 1$ , also implies

(A.21) 
$$q_1 = \frac{\theta \sigma_1(w)}{x_2}$$

Thus, from Bayes's rule we have

$$1 - p_1^A = \frac{\theta(1 - \sigma_1(w))}{1 - q_1} = \frac{\theta - q_1 x_2}{1 - q_1}.$$

Plug this into (A.5), replace  $p_1^R$  via  $p_1^R = x_1$  (Equation (A.11)) and replace  $q_2$  through  $q_2 = (1 - \theta)/p_2^R$  (due to (7) and  $p_2^A = 0$ , the latter due to  $\sigma_2(s) = 1$ ), and eliminate  $p_2^R$  by (A.12). Then

$$x_2 = \frac{(1-\theta)}{1-\theta+(1-q_1)x_2} \cdot \frac{\theta-q_1x_2}{1-q_1},$$

which simplifies to a quadratic equation

$$(q_1)^2(x_2)^2 - 2q_1(x_2)^2 + (x_2)^2 + (1-\theta)(x_2-\theta) = 0,$$

namely,

$$(x_2)^2(q_1-1)^2 = (1-\theta)(\theta-x_2).$$

We claim  $\theta - x_2 > 0$ . To see that, note  $p_1^A < p_1^R$  due to Lemma A.2 and  $\sigma_1(w) < 1$  and hence  $q_1 < 1$  in any Case-(e) PBE. Then the Bayesian plausibility condition (7) implies  $p_1^R > 1 - \theta$ .

This, combined with Bayes's rule  $p_1^R = (1 - \theta)\sigma_1/q_1$  and  $1 - p_1^R = \theta\sigma_1(w)/q_1$ , implies  $\sigma_1(w) < \sigma_1(s)$ . Then (A.20) implies  $1 - x_1 < \theta$ , namely,

$$(A.22) \qquad \qquad \theta - x_2 > 0.$$

Thus, the above quadratic equation implies  $x_2(q_1 - 1) = -\sqrt{(1 - \theta)(\theta - x_2)}$ , namely,

(A.23) 
$$q_1 = 1 - \frac{1}{x_2} \sqrt{(1-\theta)(\theta - x_2)}.$$

Thus, the Case-(e) solution is uniquely determined by  $x_2$ . In particular,

(A.24) 
$$p_1^R \stackrel{(A.11)}{=} 1 - x_2,$$

(A.25) 
$$p_2^R \stackrel{(A.12)}{=} 1 - \theta + \sqrt{(1-\theta)(\theta - x_2)},$$

(A.26) 
$$\sigma_1(w) \stackrel{(A.21)}{=} \frac{x_2 - \sqrt{(1-\theta)(\theta - x_2)}}{\theta},$$

$$(A.27) q_2 = \frac{1-\theta}{p_2^R},$$

with (A.27) due to Bayes's rule with respect to  $\sigma_2(s) = 1$ .

Finally, we verify that  $2\theta - 1 < x_2 < 1/2$  in any Case-(e)-ii solution. Recall from the definition of Case-(e)-ii solutions that  $p_2^R > p_1^R$ . By (A.24) and (A.25),

(A.28) 
$$p_2^R > p_1^R \iff 1 - \theta + \sqrt{(1 - \theta)(\theta - x_2)} > 1 - x_2$$
$$\iff \sqrt{(1 - \theta)(\theta - x_2)} > \theta - x_2.$$

By (A.22), the inequality in (A.28) is equivalent to

$$\left(\sqrt{(1-\theta)(\theta-x_2)}\right)^2 > (\theta-x_2)^2,$$

namely,  $1 - \theta > \theta - x_2$ . Thus

(A.29)  $x_2 > 2\theta - 1.$ 

To prove  $x_2 < \hat{x}_2$ , the claim about  $\hat{x}_2$  in the lemma, recall that (A.6) holds for any Case-(e)ii solution (Lemma A.5), namely,  $q_1 \le 1 - p_2^R$ . Plug (A.23) and (A.25) into this inequality to obtain

$$\left(\frac{1}{x_2}-1\right)\sqrt{(1-\theta)(\theta-x_2)} \ge 1-\theta,$$

namely,

(A.30) 
$$(x_2)^2(1-\theta) - (\theta - x_2)(1-x_2)^2 \le 0.$$

Note that the left-hand side of (A.30) is strictly increasing in  $x_2$ . By the assumption  $\theta \le 3/4$ , the left-hand side of (A.30) is equal to  $(1 - \theta)(4\theta - 3) \le 0$  when  $x_2 = 2\theta - 1$ , and equal to  $3/8 - \theta/2 \ge 0$  when  $x_2 = 1/2$ . Thus, there exists a unique  $\hat{x}_2 \in [2\theta - 1, 1/2]$  for which (A.30) holds at equality when  $x_2 = \hat{x}_2$ , and holds strictly for all  $x_2 < \hat{x}_2$ , as asserted.

LEMMA A.10. If  $2/3 \le \theta \le 3/4$  then  $q_2 < \theta$  in any Case-(e)-ii PBE.

PROOF. By (A.25) and (A.27).

$$q_{2} < \theta \iff \frac{1-\theta}{1-\theta+\sqrt{(1-\theta)(\theta-x_{2})}} < \theta$$
$$\iff (1-\theta)^{2} \le \theta\sqrt{(1-\theta)(\theta-x_{2})}$$
$$\iff x_{2} \le \theta - \frac{(1-\theta)^{3}}{\theta^{2}}.$$

Thus, since  $x_2 < 1/2$  by Lemma A.9, it suffices to show  $1/2 \le \theta - (1 - \theta)^3/\theta^2$ , namely,

$$\frac{4\theta^3 - 7\theta^2 + 6\theta - 2}{2\theta^2} \ge 0$$

Thus, we are done if  $4\theta^3 - 7\theta^2 + 6\theta - 2 \ge 0$ . To show that, note

$$\frac{d}{d\theta} \left[ 4\theta^3 - 7\theta^2 + 6\theta - 2 \right] = 12\theta^2 - 14\theta + 6 = 6\theta(2\theta - 1) + 2(3 - 4\theta) > 0$$

because  $2\theta > 1$  by (1) and  $\theta \le 3/4$  by assumption. Thus, the term  $4\theta^3 - 7\theta^2 + 6\theta - 2$  is strictly increasing in  $\theta$ . Since it is equal to 2/27 at  $\theta = 2/3$ , it follows that  $4\theta^3 - 7\theta^2 + 6\theta - 2 > 0$  for all  $\theta \in [2/3, 3/4]$ . This proves  $q_2 < \theta$ , as desired.

LEMMA A.11. If  $2/3 \le \theta \le 3/4$ , then the lopsided equilibrium associated with proposal  $(\theta, 1 - \theta)$  generates larger total welfare than any Case-(e)-ii solution.

**PROOF.** Since any Case-(e)-ii solution corresponds to a nonlopsided equilibrium, Lemma 1 applies with the roles of players 1 and 2 switched due to  $p_2^R \ge p_1^R$  in Case-(e)-ii. Thus, the total welfare is equal to

$$S'_e := 2\theta p_2^R + (q_2 - \theta)(p_2^R - p_1^R).$$

To prove that  $S'_e$  is less than the total welfare generated by the lopsided equilibrium given proposal  $(\theta, 1 - \theta)$ , which is equal to  $\theta(3 - 5\theta/2)$  by Lemma A.4, it suffices to prove  $p_2^R < 2 - 2\theta$  for any Case-(e)-ii solution: Since  $q_2 < \theta$  by Lemma A.10, we have  $S'_e < 2\theta p_2^R$  because  $p_2^R - p_1^R > 0$  in any Case-(e)-ii solution. If, in addition,  $p_2^R < 2 - 2\theta$ , then

$$S'_e < 2\theta p_2^R < 2\theta (2 - 2\theta) \le \theta (3 - 5\theta/2),$$

with the last inequality due to the condition  $\theta \ge 2/3$  in the lemma.

Thus, we verify  $p_2^R < 2 - 2\theta$ . Note from (A.25) that  $p_2^R < 2 - 2\theta$  is equivalent to

$$1 - \theta + \sqrt{(1 - \theta)(\theta - x_2)} < 2 - 2\theta \iff \sqrt{(1 - \theta)(\theta - x_2)} < 1 - \theta$$
$$\iff 1 - \theta < \theta - x_2 \iff 2\theta - 1 < x_2,$$

where  $2\theta - 1 < x_2$  is true by Lemma A.9. Thus,  $p_2^R < 2 - 2\theta$ , as desired.

A.9. Suboptimality of Any Equilibrium in Equation (15). Equilibria in the form of Equation (15) correspond to Case (d) in Lemma A.1. In any such PBE, each type of each player is totally mixing A and R:

(A.31)  $\forall i \in \{1, 2\}: 0 < \sigma_i(w) < 1 \text{ and } 0 < \sigma_i(s) < 1.$ 

This being symmetric between the two players, let us assume without loss that

$$(A.32) p_2^R \ge p_1^R.$$

This labeling of the players will be shown to imply  $x_1 \ge x_2$  (Lemma A.14) and hence consistent with the labeling in Subsection 4.1.

LEMMA A.12. A tuple  $(x_i, \sigma_i, p_i^A, p_i^R, q_i)_{i=1}^2$  that satisfies (A.32) constitutes a Case-(d) solution if and only if it satisfies (A.31) and all the following:

(A.33) 
$$1 - p_1^R = q_2(1 - p_1^A),$$

(A.34) 
$$1 - p_2^R = q_1(1 - p_2^A),$$

(A.35) 
$$p_1^R = p_2^A + x_1$$

(A.36) 
$$p_2^R + \theta - 1 = (1 - q_1)x_2.$$

PROOF. The best response condition for (A.31) to constitute a PBE is that  $V_i^R(w) - V_i^A(w) = V_i^R(s) - V_i^A(s) = 0$  for each player *i*. By (A.1), that is equivalent to simultaneous satisfaction of  $V_1^R(w) - V_1^A(w) = V_2^R(w) - V_2^A(w) = 0$ ,  $(1 - p_1^R) = q_2 (1 - p_1^A)$ , and  $1 - p_2^R = q_1(1 - p_2^A)$ . To write the condition  $V_1^R(w) - V_1^A(w) = V_2^R(w) - V_2^A(w) = 0$  explicitly, note for each player *i* that  $q_i < 1$  in this PBE and hence  $p_i^A < 1 - \theta < p_i^R$  by Lemma A.2. This combined with (A.32) implies that (A.2) and (A.3) apply to the case i = 2 and hence

$$V_2^R(w) - V_2^A(w) = p_2^R - (1 - \theta) - (1 - q_1)x_2,$$
  
$$V_1^R(w) - V_1^A(w) = (1 - q_2)(p_1^R - p_2^A - x_1).$$

Consequently, with  $q_2 < 1$ ,

$$V_1^R(w) - V_1^A(w) = 0 \iff p_1^R = p_2^A + x_1,$$
  
$$V_2^R(w) - V_2^A(w) = 0 \iff p_2^R + \theta - 1 = (1 - q_1)x_2.$$

LEMMA A.13. If  $(x_i, \sigma_i, p_i^A, p_i^R, q_i)_{i=1}^2$  is a Case-(d) solution such that  $p_2^R \ge p_1^R$ , then

(A.37) 
$$\sigma_1(w) = \frac{\theta + x_1 - 1 + q_1(1 - 2x_1)}{\theta},$$

(A.38) 
$$\sigma_1(s) = \frac{q_1 - \theta \sigma_1(w)}{1 - \theta}$$

(A.39) 
$$\sigma_2(w) = 1 - \frac{x_2}{\theta}$$

(A.40) 
$$\sigma_2(s) = \frac{\theta - x_2}{1 - \theta} \cdot \frac{1 - \theta + x_2(1 - q_1)}{\theta + x_2(q_1 - 1)},$$

$$(A.41) x_2 < \theta, \quad and$$

(A.42) 
$$(q_1)^3 x_2(1-2x_2) + (q_1)^2 x_2(3x_2-1-\theta) + q_1(3x_2-1-\theta)(\theta-x_2) + (\theta-x_2)^2 = 0.$$

**PROOF.** Equation (A.38) follows trivially from  $q_1 = \theta \sigma_1(w) + (1 - \theta)\sigma_1(s)$ . To prove the rest, first apply Bayes's rule to  $1 - p_2^A$  and then to  $1 - p_2^R$  to obtain

$$1 - p_2^A = \frac{\theta(1 - \sigma_2(w))}{1 - q_2} = \frac{(1 - p_2^R)\theta(1 - \sigma_2(w))}{1 - p_2^R - (1 - p_2^R)q_2} = \frac{(1 - p_2^R)\theta(1 - \sigma_2(w))}{1 - p_2^R - \theta\sigma_2(w)}.$$

Then,

$$p_2^R - p_2^A = (1 - p_2^A) - (1 - p_2^R) = \frac{(1 - p_2^R)\theta(1 - \sigma_2(w))}{1 - p_2^R - \theta\sigma_2(w)} - (1 - p_2^R) = \frac{(1 - p_2^R)(\theta + p_2^R - 1)}{1 - p_2^R - \theta\sigma_2(w)}$$

By (A.34) we have  $q_1 = (1 - p_2^R)/(1 - p_2^A)$ . Plug this into (A.36) to obtain

$$(\theta + p_2^R - 1)(1 - p_2^A) = (p_2^R - p_2^A)x_2$$

Plugging into this equation the formulas of  $1 - p_2^A$  and  $p_2^R - p_2^A$  obtained above, we have

$$(\theta + p_2^R - 1) \cdot \frac{(1 - p_2^R)\theta(1 - \sigma_2(w))}{1 - p_2^R - \theta\sigma_2(w)} = \frac{(1 - p_2^R)(\theta + p_2^R - 1)}{1 - p_2^R - \theta\sigma_2(w)} \cdot x_2$$

namely,

$$\theta(1-\sigma_2(w))=x_2$$

Thus (A.39) is true. Then, Equation (A.39) coupled with  $\sigma_2(w) > 0$  implies (A.41).

Second, plug Equations. (A.34) and (A.35) into Equation (A.36) to obtain

$$1 - q_1(1 - p_1^R + x_1) = 1 - \theta + (1 - q_1)x_2.$$

Eliminate  $1 - \theta$  therein by Equation (7) and combine terms to obtain

 $(1-q_1)(1-p_1^A) = x_2 - q_1(x_2 - x_1).$ 

By Bayes's rule, the above equation is equivalent to

(A.43) 
$$\theta(1 - \sigma_1(w)) = x_2 - q_1(x_2 - x_1),$$

which in turn is equivalent to Equation (A.37).

Third, rewrite (A.34) as  $q_1 = (1 - p_2^R)/(1 - p_2^A)$  and then rewrite the right-hand side by Bayes's rule to obtain

$$q_{1} = \frac{\theta \sigma_{2}(w)}{\theta(1 - \sigma_{2}(w))} \cdot \frac{\theta(1 - \sigma_{2}(w)) + (1 - \theta)(1 - \sigma_{2}(s))}{\theta \sigma_{2}(w) + (1 - \theta)\sigma_{2}(s)} \stackrel{(A.39)}{=} \frac{\theta - x_{2}}{x_{2}} \cdot \frac{x_{2} + (1 - \theta)(1 - \sigma_{2}(s))}{\theta - x_{2} + (1 - \theta)\sigma_{2}(s)}$$

which implies Equation (A.40).

Finally, we prove Equation (A.42). Use Bayes's rule on player 2 and then use (A.39) to obtain

$$(1-q_2)(1-p_2^A) = \theta(1-\sigma_2(w)) = x_2.$$

Eliminate the  $q_2$  in this equation by (A.33), and the  $p_2^A$  by (A.35). Then, the equation displayed above becomes

$$\left(1 - \frac{1 - p_1^R}{1 - p_1^A}\right)(1 - p_1^R + x_1) = x_2$$

namely,

(A.44) 
$$(1-p_1^A)x_2 = (p_1^R - p_1^A)(1-p_1^R + x_1).$$

Meanwhile, use Bayes's rule on player 1 and then use (A.43) to obtain

$$1 - p_1^A = \frac{\theta(1 - \sigma_1(w))}{1 - q_1} = \frac{x_2 - q_1(x_2 - x_1)}{1 - q_1}.$$

Analogously, use Bayes's rule on player 1 and then use Equation (A.37) to obtain

$$1 - p_1^R = \frac{\theta \sigma_1(w)}{q_1} = \frac{\theta + x_1 - 1 + q_1(1 - 2x_1)}{q_1}.$$

From the two formulas, we get

$$p_1^R - p_1^A = \frac{x_2 - q_1(x_2 - x_1)}{1 - q_1} - \frac{\theta + x_1 - 1 + q_1(x_2 - x_1)}{q_1}$$
$$= \frac{-\theta - x_1 + 1 + q_1\theta + 2q_1x_1 - q_1}{q_1(1 - q_1)}$$
$$= \frac{x_2 - \theta - q_1(x_2 - x_1 - \theta)}{q_1(1 - q_1)} \quad (by \ x_1 + x_2 = 1).$$

Replace the  $1 - p_1^A$ ,  $1 - p_1^R$  and  $p_1^R - p_1^A$  in (A.44) with the above formulas to rewrite (A.44) as

$$\frac{x_2 - q_1(x_2 - x_1)}{1 - q_1} x_2 = \left(\frac{x_2 - \theta - q_1(x_2 - x_1 - \theta)}{(1 - q_1)q_1}\right) \left(\frac{\theta + x_1 - 1 + q_1(1 - 2x_1)}{q_1} + x_1\right)$$
$$= \left(\frac{x_2 - \theta - q_1(x_2 - x_1 - \theta)}{(1 - q_1)q_1}\right) \left(\frac{q_1x_2 + \theta - x_2}{q_1}\right),$$

with the second line due to  $x_1 + x_2 = 1$ . Simplify the above equation into

$$x_2(x_2-q_1(x_2-x_1))=\frac{x_2-\theta-q_1(x_2-x_1-\theta)}{q_1}\cdot\frac{q_1x_2+\theta-x_2}{q_1},$$

namely,

$$(q_1)^2 x_2 (q_1 x_1 + (1 - q_1) x_2) = (q_1 x_1 - (1 - q_1)(\theta - x_2))(q_1 x_2 + \theta - x_2).$$

Plug  $x_2 = 1 - x_1$  into the above displayed equation to obtain

$$\begin{aligned} (q_1)^2 x_2(q_1(1-x_2)+(1-q_1)x_2) &= (q_1(1-x_2)-(1-q_1)(\theta-x_2)) \cdot (q_1x_2+\theta-x_2) \\ \iff (q_1)^2 x_2(q_1(1-2x_2)+x_2) &= (q_1(1+\theta-2x_2)+x_2-\theta) \cdot (q_1x_2+\theta-x_2), \\ \iff (q_1)^3 x_2(1-2x_2)+(q_1)^2 (x_2)^2 &= (q_1)^2 (1+\theta-2x_2)x_2+(\theta-x_2)q_1(1+\theta-3x_2)-(\theta-x_2)^2 \\ \iff (q_1)^3 x_2(1-2x_2)+(q_1)^2 x_2(3x_2-1-\theta)+q_1(\theta-x_2)(3x_2-1-\theta)+(\theta-x_2)^2 = 0. \end{aligned}$$

Thus, Equation (A.42) is true.

LEMMA A.14. If  $(x_i, \sigma_i, p_i^A, p_i^R, q_i)_{i=1}^2$  is a Case-(d) solution such that  $p_2^R \ge p_1^R$ , then  $x_1 \ge 1/2 \ge x_2$ .

 $\Box$ 

PROOF. By Bayes's rule,

$$1-p_1^R = \frac{\theta \sigma_1(w)}{q_1} \stackrel{(A.37)}{=} \frac{\theta - x_2 + q_1(2x_2 - 1)}{q_1},$$

with the second "=" also due to  $x_1 + x_2 = 1$ . Meanwhile, write (A.36) into

$$1 - p_2^R = \theta + x_2(q_1 - 1).$$

Thus,

(A.4

5)  

$$p_{2}^{R} \ge p_{1}^{R} \iff \frac{\theta - x_{2} + q_{1}(2x_{2} - 1)}{q_{1}} \ge \theta + x_{2}(q_{1} - 1)$$

$$\iff (3x_{2} - 1 - \theta)q_{1} + (\theta - x_{2}) \ge (q_{1})^{2}x_{2}$$

$$\iff (3x_{2} - 1 - \theta)(\theta - x_{2})q_{1} + (\theta - x_{2})^{2} \ge (q_{1})^{2}x_{2}(\theta - x_{2})$$

with the last line due to  $\theta - x_2 > 0$  (Ineq. (A.41)). Subtract Ineq. (A.45) by Equation (A.42) and cancel some terms to see that Ineq. (A.45) is equivalent to

$$0 \ge (q_1)^3 x_2 (1 - 2x_2) + (q_1)^2 x_2 (3x_2 - 1 - \theta) + (q_1)^2 x_2 (\theta - x_2)$$

namely,

$$0 \ge (q_1)^2 x_2 (1-q_1) (2x_2-1).$$

Thus,

$$p_2^R \ge p_1^R \iff 0 \ge (q_1)^2 x_2 (1-q_1) (2x_2-1) \iff 0 \ge 2x_2-1,$$

with the second " $\iff$ " due to the fact  $q_1 < 1$  in all Case-(d) PBEs. We thus have  $2x_2 \le 1$ , which by  $x_1 + x_2 = 1$  implies  $x_1 \ge 1/2 \ge x_2$ , as claimed.

LEMMA A.15. In any Case-(d) solution,  $p_2^R < 2 - 2\theta$  and, if  $\theta \ge 2/3$  in addition, then  $x_1 < \theta$  and  $q_2 < \theta$ .

PROOF. First, observe a necessary condition for any Case-(d) proposal-PBE pair:

(A.46) 
$$q_1 > \frac{\theta - x_2}{1 - x_2}$$

This follows from plugging (A.40) into the Case-(d) condition  $\sigma_2(s) < 1$ , which gives

$$\frac{\theta - x_2}{1 - \theta} \cdot \frac{1 - \theta + x_2(1 - q_1)}{\theta - x_2 + x_2 q_1} < 1$$

Since  $\theta - x_2 > 0$  by (A.41), the above-displayed inequality simplifies to (A.46).

Next, we prove  $p_2^R < 2 - 2\theta$ . It suffices to show  $(1 - q_1)x_2 < 1 - \theta$ , as the two inequalities are equivalent by (A.36). Since  $(1 - q_1)x_2 < 1 - \theta \iff q_1 > (x_2 + \theta - 1)/x_2$ , the desired inequality follows from (A.46) if

$$\frac{\theta-x_2}{1-x_2} \ge \frac{x_2+\theta-1}{x_2},$$

which is equivalent to

$$(2x_2-1)(1-\theta) \le 0.$$

The last inequality is true because  $x_2 \le 1/2$  (Lemma A.14).

Now assume  $\theta \ge 2/3$  to prove  $x_1 < \theta$  and  $q_2 < \theta$ . By (A.37) and (A.38), the Case-(d) condition  $\sigma_1(s) < 1$  becomes

$$\frac{q_1 - \theta(\theta + x_1 - 1 + q_1(1 - 2x_1))/\theta}{1 - \theta} < 1,$$

which simplifies to  $q_1 < 1/2$ . This, coupled with (A.46), implies  $(\theta - x_2)/(1 - x_2) < 1/2$ , namely,  $x_2 > 2\theta - 1$ . Thus, since  $2\theta - 1 \ge 1 - \theta$  (assumption  $\theta \ge 2/3$ ) and  $x_2 = 1 - x_1$ ,  $x_1 < \theta$  follows.

To prove  $q_2 < \theta$ , combine the proved fact  $x_1 < \theta$  (i.e.,  $x_2 > 1 - \theta$ ) with  $\sigma_2(s) < 1$  (part of the definition of Case (d)) to obtain  $\sigma_2(s) < x_2/(1 - \theta)$ . Plug this and (A.39) into the definition  $q_2 = \theta \sigma_2(w) + (1 - \theta)\sigma_2(s)$  to obtain  $q_2 < \theta(1 - x_2/\theta) + (1 - \theta)(x_2/(1 - \theta)) = \theta$ .

LEMMA A.16. If  $\theta \ge 2/3$ , the lopsided equilibrium given proposal  $(\theta, 1 - \theta)$  generates strictly larger total welfare than any Case-(d) solution does.

PROOF. Consider any Case-(d) solution  $(x_i, \sigma_i, p_i^A, p_i^R, q_i)_{i=1}^2$ . As in (A.32), we have  $p_2^R \ge p_1^R$ . Lemma 1, with the roles between players 1 and 2 switched due to  $p_2^R \ge p_1^R$ , implies that the total welfare generated by this solution is equal to  $2\theta p_2^R + (q_2 - \theta) (p_2^R - p_1^R)$ , which by the fact  $q_2 < \theta$  (Lemma A.15) is less than  $2\theta p_2^R$ . Since the total welfare generated by the lopsided equilibrium under proposal  $(\theta, 1 - \theta)$  is equal to  $\theta(3 - 5\theta/2)$  (Lemma A.4), the proof is complete if  $\theta(3 - 5\theta/2) \ge 2\theta p_2^R$ . As in the proof of Lemma A.11, this inequality follows from  $\theta \ge 2/3$  (assumption) and  $p_2^R < 2 - 2\theta$  (Lemma A.15).

A.10. Suboptimality of Any Equilibrium in Equation (16). Equilibria in the form of (16) correspond to Case (c) in Lemma A.1. First we show that, within the Case-(c) PBEs, the one admitted by the equal-split proposal maximizes the total welfare.

LEMMAA.17 (I). The Case-(c) PBE given the equal-split proposal maximizes the total welfare among all Case-(c) solutions. (ii) At this Case-(c) optimal solution,  $p_1^R = p_2^R = 1/2$ ,  $q_2 = 2(1 - \theta)$ , and the total welfare is equal to  $\theta$ .

PROOF. As defined in Lemma A.1, a PBE belongs to Case (c) if and only if its strategy profile satisfies

(A.47) 
$$\forall i \in \{1, 2\} : 0 < \sigma_i(w) < 1 = \sigma_i(s)$$

Then, Bayes's rule implies  $p_i^A = 0$  and hence (by (7))  $q_i p_i^R = 1 - \theta$  for each player *i*. The best response condition for (A.47) to constitute a PBE is that  $V_i^R(w) - V_i^A(w) = 0$  and  $V_i^R(s) - V_i^A(s) \ge 0$  for each player *i*. Since (A.47) is symmetric between the two players, assume without loss that

$$(A.48) p_1^R \ge p_2^R.$$

We will see that  $p_1^R$  implies  $x_2 \le 1/2$  (Equation (A.52)). Thus, this assumption is consistent with the labeling of the players in Subsection 4.1. Apply (A.2) and (A.3) to the case i = 1 to obtain

$$V_1^R(w) - V_1^A(w) = p_1^R - (1 - \theta) - (1 - q_2)x_1,$$
  

$$V_2^R(w) - V_2^A(w) = (1 - q_1)(p_2^R - p_1^A - x_2) = (1 - q_1)(p_2^R - x_2),$$

with the last "=" due to  $p_i^A = 0$ . Thus, the condition  $V_i^R(w) - V_i^A(w) = 0$  for both *i* becomes

(A.49) 
$$p_1^R = 1 - \theta + (1 - q_2)x_1,$$

(A.50) 
$$p_2^R = x_2$$

Plug  $q_2 = (1 - \theta)/p_2^R$ ,  $x_1 = 1 - x_2$  and (A.50) into (A.49) to have

(A.51) 
$$p_1^R = 1 - \theta + \left(1 - \frac{1 - \theta}{x_2}\right)(1 - x_2) = \frac{\theta + x_2(1 - 2\theta) - (1 - x_2)^2}{x_2}.$$

Thus, we obtain the following equivalent forms of  $p_1^R \ge p_2^R$  (Ineq. (A.48)):

$$\begin{aligned} \theta + x_2(1 - 2\theta) - (1 - x_2)^2 &\ge x_2^2 \iff \theta(1 - 2x_2) + x_2(1 - x_2) - (1 - x_2)^2 &\ge 0 \\ &\iff \theta(1 - 2x_2) + (1 - x_2)(x_2 - 1 + x_2) &\ge 0 \\ &\iff (1 - 2x_2)(\theta - 1 + x_2) &\ge 0. \end{aligned}$$

The last inequality in the multiline displayed above is equivalent to either (i)  $1 - 2x_2 \ge 0$ and  $\theta - 1 + x_2 \ge 0$  (namely,  $1 - \theta \le x_2 \le 1/2$ ), or (ii)  $1 - 2x_2 \le 0$  and  $\theta - 1 + x_2 \le 0$  (namely,  $1/2 \le x_2 \le 1 - \theta$ ), which is impossible due to (1). Thus,

(A.52) 
$$p_1^R \ge p_2^R \iff 1 - \theta \le x_2 \le 1/2$$

Let *S* denote the total welfare generated by the PBE. By Lemma 1 (which applies directly because  $p_1^R \ge p_2^R$  here) and the fact  $q_1 p_1^R = 1 - \theta$ ,

$$S = 2\theta p_1^R + (q_1 - \theta)(p_1^R - p_2^R) = 2\theta p_1^R + \left(\frac{1 - \theta}{p_1^R} - \theta\right)(p_1^R - p_2^R).$$

Since  $p_1^R$  and  $p_2^R$  are each a function of  $x_2$  via (A.50) and (A.51), it follows that S is a function of  $x_2$ . We claim that S is strictly increasing in  $x_2$ . To prove that, first calculate:

$$\begin{aligned} \frac{\partial S}{\partial p_1^R} &= 2\theta + \frac{1-\theta}{p_1^R} - \theta - \frac{1-\theta}{\left(p_1^R\right)^2} \left(p_1^R - p_2^R\right) = \theta + \frac{(1-\theta)p_2^R}{\left(p_1^R\right)^2},\\ \frac{\partial S}{\partial p_2^R} &= \theta - \frac{1-\theta}{p_1^R}. \end{aligned}$$

Second, by (A.50) and (A.51), we have  $dp_2^R/dx_2 = 1$  and

$$\frac{dp_1^R}{dx_2} = \frac{1}{(x_2)^2} \left( ((1-2\theta) + 2(1-x_2))x_2 - \theta - x_2(1-2\theta) + (1-x_2)^2 \right) = \frac{1}{(x_2)^2} (1-\theta - (x_2)^2).$$

Then plug them into

$$\frac{d}{dx_2}S = \frac{\partial S}{\partial p_1^R}\frac{dp_1^R}{dx_2} + \frac{\partial S}{\partial p_2^R}\frac{dp_2^R}{dx_2} = \frac{\partial S}{\partial p_1^R} \cdot \frac{1 - \theta - (x_2)^2}{(x_2)^2} + \frac{\partial S}{\partial p_2^R}$$

to obtain

$$\frac{d}{dx_2}S = \left(\theta + \frac{(1-\theta)p_2^R}{(p_1^R)^2}\right) \left(\frac{1-\theta}{(x_2)^2} - 1\right) + \theta - \frac{1-\theta}{p_1^R} \\ = \left(\theta + \frac{(1-\theta)p_2^R}{(p_1^R)^2}\right) \left(\frac{1-\theta}{(p_2^R)^2} - 1\right) + \theta - \frac{1-\theta}{p_1^R} \quad \text{(since } p_2^R = x_2, \text{(A.50)})$$

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$$= \frac{\theta(1-\theta)}{(p_2^R)^2} - \theta + \frac{(1-\theta)^2}{(p_1^R)^2 p_2^R} - \frac{(1-\theta)p_2^R}{(p_1^R)^2} + \theta - \frac{(1-\theta)}{p_1^R}$$

$$= \frac{(1-\theta)}{(p_1^R)^2 (p_2^R)^2} \Big[ \theta(p_1^R)^2 + (1-\theta)p_2^R - p_2^R (p_2^R)^2 - p_1^R (p_2^R)^2 \Big]$$
(since  $q_2 p_2^R = 1-\theta$ )
$$\geq \frac{(1-\theta)}{(p_1^R)^2 (p_2^R)^2} \Big[ \theta(p_2^R)^2 + q_2 (p_2^R)^2 - p_2^R (p_2^R)^2 - p_1^R (p_2^R)^2 \Big]$$
(since  $p_1^R \ge p_2^R$ )
$$= \frac{(1-\theta)}{(p_1^R)^2 (p_2^R)^2} \Big[ \theta + q_2 - p_2^R - p_1^R \Big]$$

$$= \frac{(1-\theta)}{(p_1^R)^2} \Big[ \theta - x_2 + q_2 - p_1^R \Big]$$
(since  $p_2^R = x_2$ )

The inequality at the end holds because  $\theta - x_2 + q_2 - p_1^R > 0$ . To prove this inequality, use the fact  $q_2 = (1 - \theta)/p_2^R = (1 - \theta)/x_2$  and (A.51) to obtain

$$\theta - x_2 + q_2 - p_1^R = \theta - x_2 + \frac{1 - \theta}{x_2} - \frac{\theta + x_2(1 - 2\theta) - (1 - x_2)^2}{x_2}$$
$$= \frac{3\theta x_2 + 2 - 2\theta - 3x_2}{x_2} = \frac{(1 - \theta)(2 - 3x_2)}{x_2},$$

which is strictly positive because  $x_2 \le 1/2 < 2/3$  due to (A.48) and (A.52).

Now that S is strictly increasing in  $x_2$  and  $x_2 \le 1/2$ , S is maximized at  $x_2 = 1/2$  among all the solutions  $(x_i, \sigma_i, p_i^A, p_i^R, q_i)_{i=1}^2$  that belong to Case (c). It follows that the equal-split proposal,  $x_1 = x_2 = 1/2$ , attains the maximum of S among these solutions. Since it is easy to verify that the Case-(c) solution under this proposal does constitute a PBE, Claim (i) of the lemma is proved.

To prove Claim (ii) of the lemma, plug  $x_1 = x_2 = 1/2$  into (A.49)–(A.51) to obtain  $p_2^R = 1/2$ ,  $q_2 = (1 - \theta)/p_2^R = 2(1 - \theta)$ , and  $p_1^R = 1/2$ . By  $p_1^R = p_2^R = 1/2$  and Lemma 1, the total welfare is equal to  $\theta$ . Claim (ii) thus follows.

By Lemma A.17, the largest total welfare that any Case-(c) solution can achieve is equal to  $\theta$ . By contrast, the total welfare generated by the lopsided solution  $(\theta, 1 - \theta)$  is equal to  $\theta(3 - 5\theta/2)$  by Lemma A.4. Our assumption  $\theta \le 3/4$  in Proposition 1 implies the desired conclusion  $\theta < \theta(3 - 5\theta/2)$ .

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