Unequal Peace*

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Abstract

Two players contest a good. Each player knows privately his own type, which is either strong or weak. A mediator proposes a split of the good and the players separately choose whether to accept or reject it. Unless both players accept the proposal, conflict ensues as an all-pay auction, where the good is won by the player who bids higher, and the cost of bidding is borne by each, inversely related to the player's strength. The parameter value precludes the existence of any proposal that can prevent conflict with probability one. Nonetheless, the mediator's proposal can improve the welfare of the players because the outcome of the conflict depends on their bids chosen after they have observed each other's responses to the proposal. In the proposal that maximizes the total welfare, the good is split unequally so that one player is offered a much larger share than the other. That makes the favored player always willing to accept the proposal without fearing that his action signals weakness that may be exploited in the event of conflict. Consequently, conflict does not occur if the unfavored player also accepts the proposal, and hence he does not have that fear either.

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1 Introduction

Conflict is often unavoidable despite every effort to mediate a peace settlement. Yet mediation still has an effect, because the outcome of the conflict is determined by the actions chosen by both adversaries, and their actions are conditional on their observations about each other during the mediation stage. With a stylistic model of such situations, this paper observes such crucial role of a mediator. All that she needs to do is to propose a particular split of the object contested by the two parties.

This paper belongs to a series of papers on mediation and conflict prevention. Analyzing mediation in a two-player all-pay contest with private information, Zheng [27] has derived necessary and sufficient conditions for mediation to fully prevent bargaining failure. In this paper, we consider a polar opposite case, a contest in which conflict cannot ever be prevented with probability one. Specifically, the primitives do not satisfy the condition for full prevention of conflict derived in [27], and so there exists no peace proposal acceptable to both parties for sure. The mediator's problem is to maximize the total welfare of both parties taking into account that conflict is unavoidable. This is distinct and novel relative to earlier models (e.g., Hörner et al. [16]), in which the mediator's objective is to maximize the probability of peaceful conflict resolution.

In our model, two players contest a good. Each player knows privately his own type, which is either strong or weak. After the types are drawn, a mediator proposes a split of the good and the players separately choose whether to accept or reject it. Unless both players accept the proposal, conflict ensues as an all-pay auction, where the good is won by the player who bids higher, and the cost of bidding is borne by each, inversely related to the player's strength.

The key aspect of the mediator's optimal solution is that she can make the action of accepting the proposal by one of the players so uninformative that the action does not signal weakness that may be exploited in the event of conflict. Such helpful inscrutability is achieved by proposing a significantly unequal split of the good. The favored player is offered such a larger share that he would accept it whether he is weak or strong, and so his accepting the proposal does not signal weakness. Consequently, conflict would not occur if the unfavored player also accepts the proposal, and hence neither would he fear that the weakness that his acceptance might signal would hurt him in the event of conflict.

In contrast to the unequal split in the optimal proposal, symmetry between the two players is assumed in the model, so their types are drawn from the same distribution, and they weigh equally in the mediator's objective. Although such inequality could be mitigated, in theory, if the mediator randomizes which player is offered the larger share, in practice such ex ante randomizations in large stake disputes are rare. Moreover, randomization or not, the ensuing split would still remain unequal. Such inequality as a normative solution is quite different from the usual notion that symmetric contestants should be treated equally such as the symmetry axiom in the Nash bargaining solution.[1]

Our result suggests a new insight: It should not be taken for granted, even from a benevolent social planner's standpoint, that a peace proposal should offer a fair share to each contestant. Counterintuitively, a proposal favoring one contestant against the other is conducive to peacemaking. This allows one to interpret in a new light the United States announcement of its embassy relocation to Jerusalem in 2018. The announcement can be viewed as a proposal for a new status quo that recognizes Israel's full ownership of Jerusalem. Soon after the announcement, the number of Arab League countries that agreed to establish diplomatic relations with Israel jumped from two to six. Another example of an unequal peace proposal is the Vatican mediation of the Beagle Channel Dispute between Argentina and Chile. In the shadow of a war between the two countries, the Pope issued a proposal that awarded Chile all of the disputed islands, granting Argentina only the navigation rights in the area waters and a shared resource right in a part of the sea. Chile immediately accepted the proposal while Argentina was initially reluctant but eventually accepted it (cf. Garrett [14], and Greig and Diehl [15]).[2]

Our approach differs from existing models by focusing on what can be accomplished when the mediator does not condition her proposal on any signal from the contestants and

[1]The notion of equal split between equal claimants dates back to the Talmud (cf. Aumann and Maschler [1]). Yaari and Bar-Hillel [25] suggest several ways to justify the equal split between contesting claimants (including the equal treatment property in general equilibrium). Recently, Keniston et al. [18] provide a rationale for, and conduct an experimental study of, the equal split of the perceived surplus between two bargainers in a dynamic game.

[2]If we view trade unions as settlements among countries to avoid potential trade conflicts, the Maastricht Treaty for the UK to join the European Union is yet another episode of unequal proposals. The treaty offered the UK the opt-outs from the single currency mandate and the Social Chapter of employment regulations, while none of the other member nations were offered such opt-outs (cf. Baun [6] and Burton [8, Chapter 5]).

instead makes a single unconditional proposal for a resolution. This restriction would have rendered mediation useless to incentive compatible communications between the contestants should the outcome of conflict be exogenous. In those cases, communications between the contestants are relevant only before they accept or reject a proposal, after which the players have no more action to take. Thus, as in Hörner et al.'s [16] exogenous conflict model, the prediction that mediation outperforms the bilateral communication between the contestants relies on the mediator's capability to collect confidential information from the contestants.[3] In our model, by contrast, communications are relevant even after the proposal has been rejected, because the outcome of conflict thereafter depends on the contestants' bids, which are chosen after negotiation failure. Even though the mediator's proposal conveys no signal between the contestants, and the contestants cannot communicate with each other once conflict ensues, the proposal nonetheless provides an implicit channel for them to communicate: Upon seeing each other's responses to the proposal the contestants update their beliefs about each other, and such posterior beliefs determine their equilibrium bids in the conflict. Thus, the mediator in proposing an appropriate split can still indirectly influence their beliefs and hence their total welfare. Such signal-independent proposals, albeit restrictive, have a transparency appeal that makes them relevant to situations where a mediator cannot fully control the communication mechanism, say due to the likeliness of leaks (e.g., Feerick [11]) or the doubts about the mediator's commitment to truthful conveyance of communications (cf. Kydd [19], Rauchhaus [22], and Smith and Stam [23]).

The paper complements the existing literature by considering the mediator's objective as maximizing the contestants' total welfare that includes their expected payoffs both in the event of peace and in the event of conflict, instead of maximizing the probability of a peaceful resolution (e.g., Balzer and Schneider [4] and Hörner et al.'s [16]). The two objectives coincide in models where full prevention of conflict is possible (e.g., Celik and Peters [9], Zheng [27], and Balzer and Schneider [5]), where a peace proposal that is accepted by both for sure maximizes both the probability of peace and the contestants' total welfare. In a model

[3] Hörner et al.'s [16] result requires the assumption that a contestant's payoff from conflict depends on both contestants' types so that each would like to learn about the opponent's type before taking actions. When that payoff depends only on the contestant's type, Fey and Ramsay [12] show that a mediator can do no better than a one-shot cheap talk between the contestants. Our model roughly corresponds to [16], as the outcome of conflict depends endogenously on both contestants' bids during the conflict, which in turn are conditional on their types.

where full prevention of conflict is impossible such as in the current paper, the objective of maximizing the probability of peace would lead the mediator to enlarge the chance of peace resolution at the expense of the contestants' payoffs in the event of conflict. That may hurt the contestants' overall welfare as the probability of conflict cannot be eliminated.

The possibility of full prevention of conflict is considered by Bester and Wärneryd [7], Compte and Jehiel [10], Fey and Ramsay [13], Hörner et al. [16], Meirowitz et al. [21], and Spier [24], who model conflict as an exogenous lottery for the contestants, and recently by Zheng [26, 27], Celik and Peters [9] and Lu et al. [20], who model conflict as a continuation game after negotiation failure. Our model differs from this literature by precluding the possibility of full prevention of conflict. Balzer and Schneider [4] have also considered a model where full prevention of conflict is impossible. They consider communication mechanisms that maximize the probability of peaceful resolution and focus on the case where the designer is an arbitrator with full commitment power. While they also consider a mediation case, the mediator is assumed able to communicate separately and confidentially to the contestants and able to condition such communications on the negotiation outcome. In our model, by contrast, a mediator maximizes the total welfare of both parties, taking into account that conflict is unavoidable, and can only indirectly influence the posterior systems through a message-independent peace proposal.

Endogenizing the initial status quo through a mediator's decision, our study complements a literature of conflict where contestants themselves take the initiative to mitigate or escalate conflict with an implicitly exogenous status quo that defines the sequence of actions. In Baliga and Sjöström [2], the two contestants decide simultaneously whether to escalate the conflict. In Baliga and Sjöström [3], given an exogenous initial status quo, each contestant decides whether to challenge it. In Lu et al. [20], one of the two contestants has the bargaining power to make a take-it-or-leave offer to the other player for a peace settlement. The focus in this literature is the dynamic interaction between the contestants given the implicit status quo. We simplify this interaction into a static all-pay auction game and focus on the determination of the initial status quo.

The next section defines the model. Section 3 derives the players' interim and ex ante expected payoffs and describes how equilibria vary with the peace proposal. Section 5 presents the result. Section 6 concludes and suggests a couple of possible extensions. The appendix contains all omitted details.

2 The Model

Two players, named 1 and 2, contest a prize of size one. Each player's type is independently drawn from the same binary distribution, whose realization is either w ("weak") with probability θ, or s ("strong") with probability $1-\theta$, such that $0 < \theta < 1$ and $s > w > 0$.[4] Denote

$$\alpha := 1 - w/s.$$

Thus $0 < \alpha < 1$. After each player's type t_i is drawn and privately learned by the player, a neutral mediator makes a *peace proposal*, which proposes a split of the prize:

$$(x_1, x_2) \in [0,1]^2 \quad \text{such that} \quad x_1 + x_2 = 1.$$

Then each player independently and publicly announces whether to accept (A) or reject (R) the proposal. If both choose A, the game ends with player i getting a payoff equal to x_i ($\forall i = 1, 2$). If at least one player chooses R, then conflict takes place in the form of an all-pay auction: Each player i, after observing the actions (choices between A and R) of both, submits a sealed bid $b_i \in \mathbb{R}_+$; the higher bidder wins the prize, with ties broken randomly with equal probabilities; the payoff for player i of type t_i is equal to $\frac{1}{\alpha}(1 - b_i/t_i)$ if i wins, and equal to $\frac{1}{\alpha}(-b_i/t_i)$ otherwise. Then the game ends. A player's bid represents the player's total amount of warring efforts in the conflict, and the reciprocal $1/t_i$ of a player's type t_i represents the player's marginal cost of warring efforts in the conflict.[5]

Any proposed split (x_1, x_2) determines a two-stage game, for which perfect Bayesian equilibrium (PBE) is the solution concept. We call any pair of a proposed split and a PBE of such relationship *proposal-PBE pair*, or *solution* for short.

We measure the social welfare achieved by a proposal-PBE pair by the total welfare generated on path of the PBE. By *total welfare* we mean the sum of the two players' ex ante expected payoffs (before realization of types). A peace proposal of particular interest is the

[4]Our assumption of binary types is in line with much of the conflict resolution literature such as Balzer and Schneider [4, 5], Hörner et al. [16], and Meirowitz et al. [21],

[5]We scale the payoff from the conflict by the parameter $1/\alpha$ purely for notational convenience. That is because α emerges as a multiple of each player's expected payoff from any equilibrium of the conflict continuation game (Section 3.1), and our scalar $1/\alpha$ cancels out the multiple. Without the scalar $1/\alpha$ to cancel out α, α would appear in most expressions in the paper thereby complicating them, though all our results remain true.

equal split $(1/2, 1/2)$, treating the two ex ante identical players equally. Another proposal of interest is $(\theta, 1-\theta)$, splitting the prize according to the prior probabilities assigned to the weak and strong types.

Throughout the paper we maintain the following assumption, which constitutes our major point of departure from the previous conflict mediation literature:

$$\theta > 1/2. \tag{1}$$

This inequality is the necessary and sufficient condition for nonexistence of any negotiation mechanism that admits a PBE where conflict occurs with zero probability.[6] That is, due to (1), full preemption of conflict is impossible, and conflict is necessarily an on-path event. Thus, it is appropriate for a mediator to adopt an objective—such as the total welfare considered in this paper—that incorporates the players' welfare in both peace and conflict.

3 Interim Payoffs and Posterior Beliefs

3.1 The Post-Mediation Payoff in the Conflict

Let us start by considering the continuation game where conflict ensues (due to at least one player having chosen R at the proposal stage). The belief about a rival is updated conditional on the rival's response to the proposal. For each player $i \in \{1, 2\}$, denote by p_i the posterior probability of player i being type s (strong). This, together with the players' private information of their own types t_i, defines a Bayesian game.

Given any pair $(p_1, p_2) \in [0, 1]^2$ of posterior probabilities, one can show that there is a unique Bayesian Nash equilibrium (BNE) of the continuation game. Both players randomly select their bids from an interval $\left[0, \bar{b}\right]$ ($\bar{b}$ endogenous to the equilibrium). The strong type of a player selects his bid from an upper subinterval of $\left[0, \bar{b}\right]$, and the weak type of the player, from the complement of the upper subinterval. The player whose posterior probability p_i of being the strong type is lower than the other's bids zero with a positive probability when

[6] To see this, apply Zheng [27, Example 4]. Since we have scaled up the payoff in the conflict to $1/\alpha$ times the quantity assumed in [27], the peace-implementability threshold $c_* = \alpha\theta$ there becomes $(1/\alpha)c_* = \theta$. Thus the necessary and sufficient condition for peace implementability becomes $2\theta \leq 1$. If $2\theta \leq 1$, one can split the prize such that each player gets a share at least as large as θ, and it is an equilibrium for both to accept any such splits, the equal split $(1/2, 1/2)$ being one of them.

his type is weak, while the other player bids zero with zero probability and hence enjoys a positive probability of winning even by bidding zero. For each player $i \in \{1, 2\}$ and each type $t \in \{s, w\}$, let $U_i^t(p_i, p_{-i})$ denote the expected payoff for player i of type t in this BNE. One can show (Appendix A):

$$U_i^s(p_i, p_{-i}) = 1 - \min\{p_i, p_{-i}\}, \tag{2}$$

$$U_i^w(p_i, p_{-i}) = p_i - \min\{p_i, p_{-i}\}. \tag{3}$$

The functions $U_i^s(p_i, \cdot)$ and $U_i^w(p_i, \cdot)$ are graphed in Figure 1. These conflict payoffs play

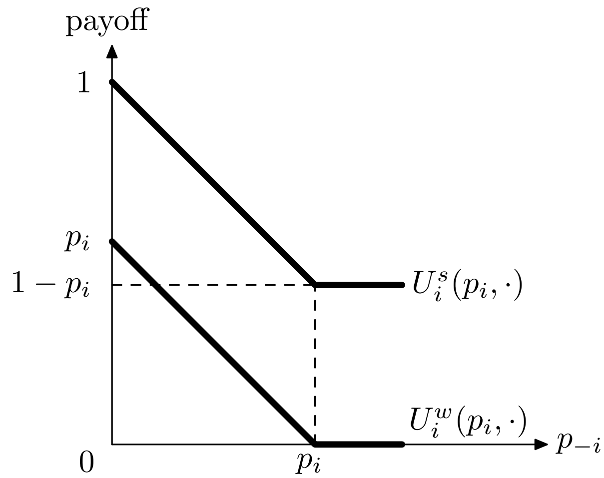

Figure 1: Payoff in the conflict as a function of the opponent's posterior

a similar role as the ex post payoff that a designer would like to concavify in the information design framework, except that in our game concavification need not bring about larger total welfare, as they are the payoffs only in the event of conflict.

Much of the tradeoff faced by the mediator grows out of the following observation.

Remark 1 An increase in p_i hurts the strong type of player i and benefits the weak type of i. In other words, a strong type would like to reduce, and a weak type would like to enlarge, the posterior probability that his rival assigns to the event that his type is strong.

Remark 1 can be observed from Figure 1, as an increase in p_i corresponds to a downward shift of the graph of $U_i^s(p_i, \cdot)$, and an upward shift of the graph of $U_i^w(p_i, \cdot)$. Intuitively speaking, to the strong type of, say, player 1, the issue is not whether he can win the prize but rather how much he has to pay to win. When the rival player 2 is complacent, believing that player 1 is unlikely to be strong, player 2's bid (which is costly, win or lose) becomes low stochastically, and so the strong type of player 1 can win at a low cost in expectation. To the weak type of player 1, by contrast, the issue is whether he can win at all, and he gets a positive expected payoff only when player 2 bids zero. The more often is player 1 believed to be weak, the less often would player 2 bid zero (as he sees little need to concede to a weak player 1), and the less expected payoff the weak type of player 1 gets.

3.2 Interim Payoffs in Mediation

Given any proposal-PBE pair, let q_i denote player i's ($\forall i \in \{1, 2\}$) ex ante probability (before realization of i's type) of choosing R, namely,

$$q_i := \theta\sigma_i(w) + (1 - \theta)\sigma_i(s), \tag{4}$$

and let p_i^A (resp. p_i^R) denote the posterior probability of player i being type s conditional on i having chosen A (resp. R) in response to the peace proposal. Given type $t \in \{w, s\}$ and anticipating the continuation payoff U_i^t in the event of conflict, player i's expected payoff from choosing A is equal to

$$V_i^A(t) := q_{-i}U_i^t\left(p_i^A, p_{-i}^R\right) + (1 - q_{-i})x_i, \tag{5}$$

and that from choosing R is equal to

$$V_i^R(t) := q_{-i}U_i^t\left(p_i^R, p_{-i}^R\right) + (1 - q_{-i})U_i^t\left(p_i^R, p_{-i}^A\right). \tag{6}$$

One can derive from Bayes's rule the next condition, called Bayesian plausibility in the information design literature.

$$q_i p_i^R + (1 - q_i)\, p_i^A = 1 - \theta. \tag{7}$$

Thus, the point $(1-\theta, V_i^R(t))$ is the convex combination between the two points on the graph of $U_i^t(p_i^R, \cdot)$ whose horizontal coordinates are p_{-i}^R and p_{-i}^A. This is illustrated by Figure 2, where p_{-i}^A, $1 - \theta$ and p_{-i}^R are positioned according to an intuitive Lemma 3 (Appendix B):

$$\forall i \in \{1, 2\} : p_i^A \leq 1 - \theta \leq p_i^R. \tag{8}$$

That is, R (rejecting the peace proposal) signals one's strength more than A does.

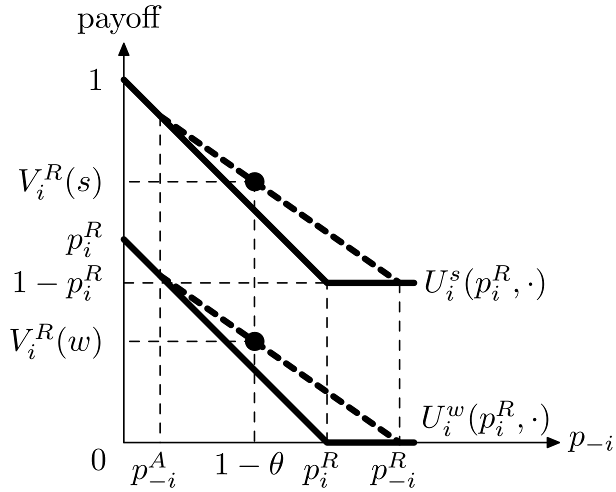

Figure 2: Interim expected payoffs as convex combinations

Remark 2 Figure 2 reveals the following: (a) The interim payoff for type w (weak) in the conflict is bounded from above by θ, and attains this upper bound when $p_i^R = 1$. (b) The interim payoff for type s (strong) in the conflict is bounded from below by θ, and attains this lower bound when $p_i^R \geq p_{-i}^R$. (c) It follows from (b) that, in any proposal-PBE pair, the strong type of each player can always secure an interim payoff no less than θ through choosing R.

4 Equilibrium and Total Welfare

4.1 The Equilibria

There is always a trivial PBE where conflict occurs for sure regardless of what the peace proposal is: each player always chooses R because he expects the same from the opponent. The other PBEs are determined by the peace proposal. For each player $i \in \{1, 2\}$ and each type $t \in \{w, s\}$, let $\sigma_i(t)$ denote the probability with which player i of type t chooses R at the proposal stage, and let p_i^A (resp. p_i^R) denote the posterior probability of player i being type s conditional on having chosen A (resp. R). The posteriors $(p_i^A, p_i^R)_{i=1}^2$ determine the players' expected payoffs $(U_i^t)_{i=1}^2)_{t=w}^s$ in the event of conflict (Section 3.1), which in turn determine their interim expected payoffs at the proposal stage (Section 3.2). Then (σ_1, σ_2) is determined by the mutual best response condition based on the interim expected payoffs.

Without loss of generality, suppose that the larger share in the proposed split (x_1, x_2)

is offered to player 1, namely, $x_1 \geq x_2$. When x_1 varies in $[1/2, 1)$ (which is the entire range of x_1 except $x_1 = 1$, where the trivial PBE prevails), the nontrivial PBEs change in the manner listed below. We assume $2/3 \leq \theta \leq 3/4$.

1. $x_1 \in [\theta, 1)$: This is the necessary and sufficient condition for any PBE of the following form to exist: $\sigma_1(s) = \sigma_1(w) = 0$, $\sigma_2(s) = 1$, and $\sigma_2(w) \in (0, 1)$. In any such a PBE, the share $x_1 \in [\theta, 1)$ offered to player 1 is so large that both types of player 1 choose A for sure, and the strong type of player 2 chooses R for sure, leaving only his weak type to mix between A and R. We call any PBE in this format *lopsided equilibrium*.

2. $x_1 \in [2(1 - \theta), \theta)$ is a necessary condition for any PBE of the following form to exist: $\sigma_1(s), \sigma_1(w), \sigma_2(w) \in (0, 1)$, $\sigma_2(s) = 1$, and $p_1^R \geq p_2^R$. Now that the share x_1 offered to player 1 falls below the threshold θ, he no longer chooses A for sure. Player 2's strategy remains similar to that in the previous case.

3. There exists a unique $\xi \in [1/2, 2(1 - \theta)]$ such that:

   a. $\xi < x_1 < 2(1 - \theta)$ is a necessary condition for any PBE of the following form to exist: $\sigma_1(s), \sigma_1(w), \sigma_2(w) \in (0, 1)$, $\sigma_2(s) = 1$, and $p_1^R < p_2^R$. With the share x_1 offered to him lower than before, player 1 is willing to reject the offer more often than he does in the previous case even if his type is weak, and hence the posterior p_1^R of his type being strong signaled by R drops below p_2^R.

   b. Given any $x_1 \in [1/2, \xi]$, a PBE of the following form may exist: $\sigma_i(t) \in (0, 1)$ for all $i \in \{1, 2\}$ and all $t \in \{w, s\}$. The proposal is so near to the equal split that the two players behave similarly, each type mixing between A and R.

4. If $\theta = 3/4$ then when $x_1 = 1/2$ $(= 2(1-\theta))$ there is also a PBE for which $\sigma_1(w), \sigma_2(w) \in (0, 1)$ and $\sigma_1(s) = \sigma_2(s) = 1$. Under the equal-split proposal, the strong type of both players chooses R for sure, and the weak type of each player mixes between A and R.

The above list covers all the possible nontrivial PBEs (Lemma 2, Appendix B). Among them and the trivial equilibria, the lopsided equilibrium associated with the proposal $x_1 = \theta$ will be shown to be optimal (Section 5). The rest of this section will prove the existence of lopsided equilibria and the necessity of $x_1 \geq \theta$ for their existence.[7]

[7] The necessity of $x_1 \in [2(1 - \theta), \theta)$ for the existence of the PBE in Case 2 is deferred to Lemma 7

Construction of Lopsided Equilibria For any $x_1 \in [\theta, 1)$ as in Case 1, define $p_2^R := 2 - \theta - x_1$. We will see at the end of the construction that this p_2^R rationalizes player 2's strategy. Define the off-path posterior $p_1^R := p_2^R$ for player 1.[8]

Being offered the share $x_1 \geq \theta$, player 1 chooses A for sure whether his type is strong or weak. The strong type chooses A because deviation leads to the off-path posterior p_1^R that is greater than or equal to its counterpart p_2 for the rival player 2, whether $p_2 = p_2^A$ when the rival chooses A ($p_1^R \geq 1 - \theta \geq p_2^A$ by (8)), or $p_2 = p_2^R$ when the rival chooses R ($p_1^R = p_2^R$ by the previous definition). That reduces the strong player 1's expected payoff $U_1^s(p_1^R, p_2)$ in the event of conflict to its minimum θ (Remark 2.b). By contrast, his expected payoff is at least θ from choosing A: If the rival chooses A, player 1 gets the share $x_1 \geq \theta$; if the rival chooses R, player 1 gets $U_1^s(p_1^A, p_2^R)$, which is equal to θ because $p_1^A = 1 - \theta$ at any lopsided equilibrium and $1 - \theta \leq p_2^R$ by (8).

To see that the weak type of player 1 chooses A for sure, note that his expected payoff is zero conditional on the rival choosing R. This follows from the fact $p_1 \leq p_2 \Rightarrow U_1^w(p_1, p_2) = 0$ (Eq. (3)). With the rival choosing R, $p_2 = p_2^R \geq 1-\theta$ by (8). If player 1 chooses A as expected on path, $p_1 = p_1^A = 1-\theta$; if he deviates to R, the off-path posterior is $p_1^R = p_2^R$. Thus, $p_1 \leq p_2$ either way and so $U_1^w(p_1, p_2) = 0$. It follows that the weak type of player 1 chooses A if he prefers so conditional on the rival choosing A. In that event, player 1 gets the offered share $x_1 \geq \theta$ from choosing A. That is better than R, which gets him into the conflict and yields at most θ (Remark 2.a).

Meanwhile, the strong type of player 2 chooses R for sure because the share $x_2 = 1-x_1$ offered to him is no more than $1 - \theta$, which is less than θ by (1), while he can secure an expected payoff at least θ in conflict (Remark 2.b). To see why the weak type of player 2 mixes between A and R, note that he gets x_2 from choosing A, and $U_2^w(p_2^R, p_1^A)$ from choosing R, since player 1 chooses A for sure. Since the on-path action of player 1 signals no news, $p_1^A = 1 - \theta$. Since the strong type of player 2 chooses R for sure, $p_2^R \geq 1 - \theta$ by Bayes's rule. Thus $U_2^w(p_2^R, p_1^A) = p_2^R - (1 - \theta)$ by (3). Consequently, the weak type of player 2 is willing to mix between A and R because $p_2^R - (1 - \theta) = x_2$ due to the definition of p_2^R at the outset

in Appendix H.1, and that of $x_1 \in (\xi, 2(1 - \theta))$ for Case 3a., deferred to Lemma 10 in Appendix H.2, where $\hat{x}_2$ is equal to $1-\xi$ for the cutoff ξ. For both Case 3b. (detailed in Appendix I) and Case 4 (detailed in Appendix J), we do not bother to show the existence of the corresponding PBEs or the set of x_1 necessary for their existence, because we will show that any PBE in either case is suboptimal (Claims 3 and 4, Section 5).

[8] Any other $p_1^R \geq p_2^R$ works as well, with slightly longer calculations.

(σ_2 is then derived from p_2^R via Bayes's rule).

Necessity of $x_1 \geq \theta$ for any Lopsided Equilibrium Suppose, to the contrary, that a lopsided equilibrium exists despite $x_1 < \theta$. Conditional on such an equilibrium, the strong type of player 1 would deviate to R, which secures for him an expected payoff at least θ (Remark 2.b), while choosing A gives him less than θ: he would get $x_1 < \theta$ if the opponent chooses A, and θ if the opponent chooses R (shown in the construction of lopsided equilibria). Consequently, the strong type of player 1 chooses R sometimes, contradiction.

The Weak Type's Incentive to Mix Let us illustrate the incentive for a weak type to mix between A and R with the player 2 in Case 2. Within that case, observe that the weak player 2's expected payoff conditional on the opponent choosing R is zero regardless of his choice: If player 2 choses A, the posterior system is (p_1^R, p_2^A) and we have $p_1^R \geq 1 - \theta \geq p_2^A$ by (8); if player 2 choses R, the posterior system (p_1^R, p_2^R) is such that $p_1^R \geq p_2^R$ as defined in Case 2. Thus, whichever he chooses, $p_1 \geq p_2$ holds and hence $U_2^w(p_2, p_1) = 0$ by (3). Thus, the decision of the weak type of player 2 is purely based on the event where the opponent chooses A. The weak player 2 therefore mixes between A and R if $x_2 = U_2^w(p_2^R, p_1^A)$, which by (3) and (8) is equivalent to $x_2 = p_2^R - p_1^A$. This indifference is valid because one can show that a solution of (p_1^A, p_2^R) for this equation exists.

4.2 The Total Welfare

By *total welfare* we mean the sum of the ex ante expected payoffs (before realization of types) across the two players. The total welfare of the lopsided equilibrium is easy to calculate (Lemma 5, Appendix F). For the equilibrium in the other cases in Section 4.1, R is chosen with positive probabilities by both types of each player, and hence the total welfare is equal to $\sum_{i=1}^2 \left(\theta V_i^R(w) + (1-\theta) V_i^R(s)\right)$. The next lemma provides a formula for this sum.

Lemma 1 *Let $(\sigma_i, p_i^A, p_i^R)_{i=1}^2$ represent any PBE that is not lopsided, and define q_i by (4) for each $i = 1, 2$. Relabel the players if necessary so that $p_1^R \geq p_2^R$. Then the total welfare at this PBE is equal to $2\theta p_1^R + (q_1 - \theta)\left(p_1^R - p_2^R\right)$.*

Proof This lemma is based on (6) and (7), or the convex combination observation about a player's interim expected payoff in Figure 2. The upper solid graph in that figure represents a strong type's expected payoff from choosing R as a function of the rival's posterior probability of being strong, and the lower solid graph represents the counterpart for the weak type. The two graphs are reproduced separately for player 1 in Figure 3 (for the strong type) and Figure 4 (for the weak type). Since the lemma labels the players so that $p_1^R \geq p_2^R$, player 1's

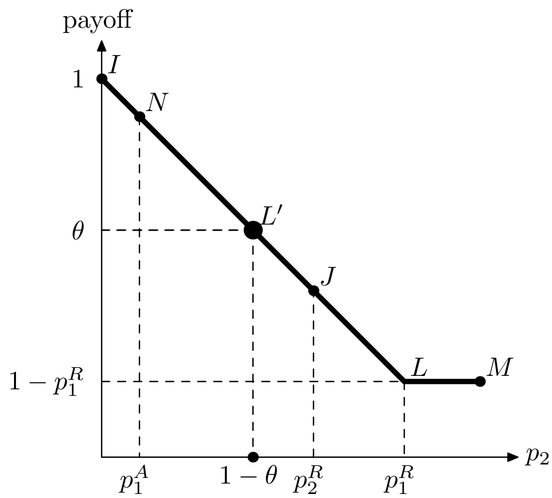

Figure 3: Strong player 1's payoff: L' as a convex combination between N and J

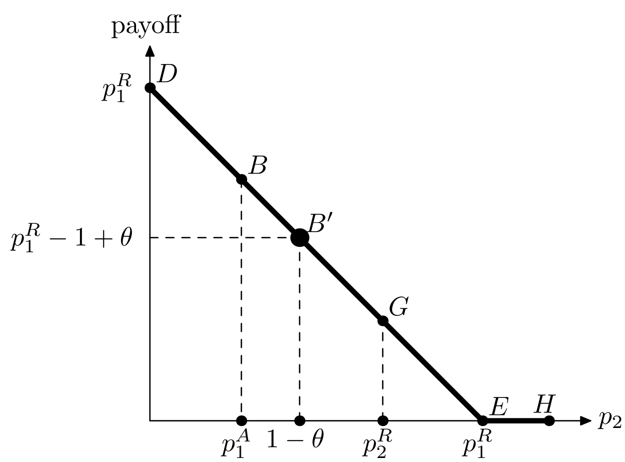

Figure 4: Weak player 1's payoff: B' as a convex combination between B and G

expected payoff $U_1^t(p_1^R, p_2^R)$ from choosing R in the event where the rival also chooses R corresponds to the point J in Figure 3 if player 1's type is strong, or the point G in Figure 4 if player 1's type is weak. It then follows from (6) and (7) that player 1's interim expected payoff from choosing R corresponds to the point L' in Figure 3 if his type is strong, and the point B' in Figure 4 if his type is weak. That is,

$$\begin{aligned} V_1^R(s) &= \theta, & (9) \\ V_1^R(w) &= p_1^R - 1 + \theta. & (10) \end{aligned}$$

Taking the weighted sum of (9) and (10) according to the prior distribution ($\Pr\{s\} = 1-\theta$), we see that the ex ante expected payoff for player 1 is equal to θp_1^R.

Comparing Figure 3 with its counterpart for the strong type of player 2, and comparing Figure 4 with its counterpart for the weak type of player 2, one can show (Appendix C):

$$\begin{aligned} V_2^R(s) - V_1^R(s) &= q_1 \left(p_1^R - p_2^R\right) & (11) \\ V_2^R(w) - V_1^R(w) &= -(1-q_1)\left(p_1^R - p_2^R\right). & (12) \end{aligned}$$

The weighted sum of (11) and (12) according to the prior $\Pr\{s\} = 1-\theta$ yields the difference in the ex ante expected payoffs between player 2 and player 1: $(q_1 - \theta)(p_1^R - p_2^R)$.

Consequently, the total welfare $\sum_{i=1}^2 \left(\theta V_i^R(w) + (1-\theta)V_i^R(s)\right)$ is equal to

$$\theta p_1^R + \theta p_1^R + (q_1 - \theta)(p_1^R - p_2^R) = 2\theta p_1^R + (q_1 - \theta)(p_1^R - p_2^R). \quad \blacksquare$$

Remark 3 The proof of Lemma 1 reveals which player gets the larger share of the total welfare in a non-lopsided equilibrium: It is the player with the larger posterior p_i^R, provided that his ex ante probability q_i of choosing R is less than θ. Since the lemma labels the players so that $p_1^R \geq p_2^R$, it is player 1 who gets the larger share of the total welfare provided that $q_1 - \theta < 0$, for then $(q_1-\theta)(p_1^R - p_2^R)$, the amount by which the rival's ex ante expected payoff "exceeds" player 1's, is nonpositive. In other words, when rejecting a peace proposal is an on-path action for both players, the player who is perceived to become stronger conditional on having rejected the proposal gets the larger share of the total welfare, provided that he does not reject the proposal too often from the ex ante viewpoint. Roughly speaking, showing off strength through aggression pays off provided that one is rarely aggressive.

5 The Optimality of a Lopsided Proposal

A lopsided equilibrium (Case 1, Section 4.1) has the advantage that one of the players chooses A independently of his own type. That is, the player accepts the peace proposal without fearing that his acceptance may betray some information that the opponent may use against him later. Among the peace proposals whose associated equilibria are lopsided, the mediator prefers those that offer more shares to the unfavored player thereby having a larger probability for him to accept the proposal as well, as long as acceptance from the favored player is still guaranteed. Since the strong type of a player can always secure an expected payoff no less than θ by choosing R (Remark 2), the share offered to the favored player cannot fall below θ and still guarantee his acceptance. The threshold θ constitutes the optimal split to offer:

Proposition *If $2/3 \leq \theta \leq 3/4$, the proposal that maximizes total welfare among all peace proposals is to offer θ to one player and $1-\theta$ to the other player.*

To appreciate the proposition, recall that the received insight in the literature (e.g., [27]) is to discourage players from rejecting a peace proposal through maximizing p_i^R to one, namely, assigning probability one to the event that a player who has rejected the proposal is of the strong type. In our model, if $p_i^R = 1$, the interim payoff from choosing R is minimized to θ for the strong type of player i, and maximized to θ for the weak type of i (Remark 2). This would have constituted an optimal solution should each player be offered a share at least as large as θ so that each is willing to accept the proposal. Given our assumption $\theta > 1/2$, however, such proposals do not exist, as any split of the prize (of size one) renders the share for some player below θ. Thus, any PBE of any proposal sees some player reject the proposal sometimes. Consequently, a player's interim payoff from choosing rejection, namely R, is part of the total welfare. This, coupled with the fact that an increase in p_i^R benefits the weak and hurts the strong (Remark 1), means that the calculus of p_i^R is more involved than that in the existing literature.

Nonetheless, there are two intuitive reasons why the previous insight of achieving optimality through maximizing p_i^R might still work. First, since a strong type incurs less marginal cost in conflict than a weak type does, one would expect that a strong type is more inclined than a weak type to reject a peace proposal. Thus, if we are to pick a type to deter it from choosing R, it would be the strong type, and so we would reduce its interim payoff from R through enlarging p_i^R. Second, from the ex ante viewpoint, any quantity of payoff to a weak type contributes more to the total welfare than the same quantity of payoff to a strong type does, due to the assumption $\theta > 1/2$. Thus, one would expect that an increase

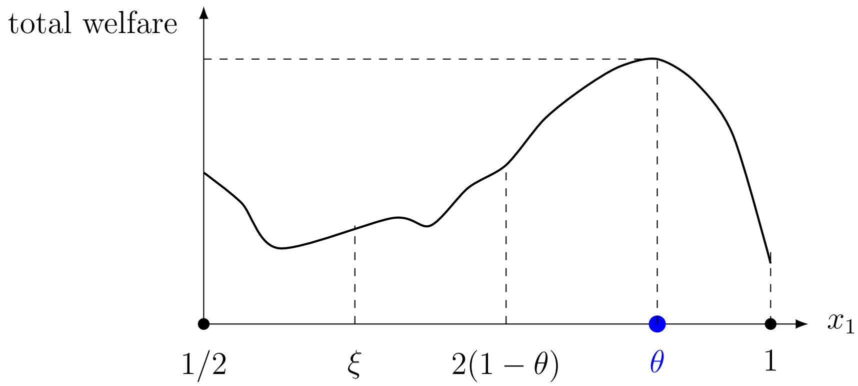

Figure 5: The lopsided proposal $(\theta, 1-\theta)$ as the global optimum

in p_i^R, benefiting the weak at the expense of the strong, enlarges the total welfare.

It is therefore conceivable that, the less constrained is p_i^R, the more can p_i^R be maxed out and hence the larger is the total welfare. That is where lopsided equilibria have an advantage over non-lopsided ones. In a non-lopsided equilibrium, both A and R being on path for each player, each component of the posterior system $(p_i^A, p_i^R)_{i=1}^2$ is constrained by Bayes's rule. In a lopsided equilibrium, by contrast, R is off path for the favored player, say player 1; hence the posterior probability p_1^R is unconstrained by Bayes's rule.

Proof of the Proposition Relabel the players if necessary so that player 1 is offered the larger share in the peace proposal, namely, $x_1 \geq x_2$. A peace proposal is then represented by x_1, whose entire range is $[1/2, 1]$. We shall prove that $x_1 = \theta$ maximizes the total welfare among all $x_1 \in [1/2, 1]$. We do that by establishing four claims, illustrated by Figure 5.

Claim 1 $x_1 = \theta$ maximizes the total welfare among all lopsided equilibria associated with any $x_1 \in [\theta, 1)$.

Let us observe that the total welfare based on the lopsided equilibrium given any $x_1 \in [\theta, 1)$ (Case 1, Section 4.1) is a strictly increasing function of p_2^R. This follows from the construction of any such equilibrium (Section 4.1): The ex ante expected payoff for player 2 is strictly increasing in p_2^R because her on-path interim expected payoff is equal to $p_2^R - 1 + \theta$ when her type is weak, and θ when her type is strong. To see the same monotonicity property for player 1, first note that player 1 prefers smaller q_2 (ex ante probability of player 2 choosing R) to larger q_2: If player 2 chooses A, player 1 (who chooses A for sure) gets $x_1 \geq \theta$; else player 1 gets θ if his type is strong, and zero if his type is weak. Thus smaller q_2 makes player 1's ex ante expected payoff strictly larger. Then apply Bayes's rule to see that $q_2 p_2^R = 1 - \theta$: smaller q_2 means bigger p_2^R. Thus, both players considered, the total welfare is maximized among all lopsided equilibria when the p_2^R in the equilibrium is maximized among all such equilibria. Since $x_1 \in [\theta, 1)$ is necessary and sufficient for any lopsided equilibrium to exist (Section 4.1), and since $p_2^R = 2 - \theta - x_1$ by the construction of such equilibria (Section 4.1), maximizing p_2^R is equivalent to minimizing x_1. Thus, $x_1 = \theta$ maximizes the total welfare among all such equilibria.

Claim 2 When x_1 increases in $[2(1-\theta), \theta)$, the total welfare of the PBE in the form of Case 2 in Section 4.1 increases; as x_1 converges to θ from below, the total welfare of the PBE converges to the total welfare of the lopsided equilibrium associated with $x_1 = \theta$.

Any PBE in the form of Case 2 in Section 4.1 is characterized by

$$\sigma_1(s), \sigma_1(w), \sigma_2(w) \in (0,1), \quad \sigma_2(s) = 1, \quad \text{and} \quad p_1^R \geq p_2^R. \tag{13}$$

Accordingly, one can calculate the PBE (Lemma 7, Appendix H.1) and obtain

$$\begin{aligned} p_1^R &= \frac{3 - 2\theta - x_2}{2}, \\ p_2^R &= 2 - 2\theta, \\ q_1 &= \frac{2(\theta - 1 + x_2)}{2\theta + x_2 - 1}. \end{aligned}$$

Since the equilibrium is non-lopsided, Lemma 1 implies that the total welfare is equal to

$$S(x_2) = 2\theta p_1^R + (q_1 - \theta)\left(p_1^R - p_2^R\right) = \theta p_1^R + q_1(p_1^R - p_2^R) + \theta p_2^R,$$

where the total welfare is denoted as a function of x_2 $(= 1 - x_1)$ because the variables on the right-hand side are each a function of x_2 according to the equations displayed above.

As stated in Case 2, Section 4.1, a PBE of this form exists only when $x_1 \in [2(1-\theta), \theta)$, namely, $x_2 \in (1-\theta, 1-2(1-\theta)]$. To prove the monotonicity claim, note from the above formula that $S(x_2)$ is determined by the values of p_1^R, p_2^R and q_1 when the PBE varies with x_2. As displayed above, p_2^R is constant. Thus the total welfare of the PBE is determined solely by the values of p_1^R and q_1. By the above-displayed formulas of p_1^R and q_1, an increase of x_2 has two opposite effects. First, it increases q_1 (player 1 choosing R more often as the share x_1 offered to him shrinks). Second, it decreases p_1^R (with the weak type of player 1 more willing to reject the shrinking x_1, R signals less about the strength of player 1). The above formula of $S(x_2)$ says that the total welfare is enlarged by the first effect, and reduced by the second effect. One can show (Lemma 9 in Appendix H.1, with the assumption $\theta \leq 3/4$) that the second effect outweighs the first,[9] and hence $S(x_2)$ is strictly decreasing when x_2 increases.

To prove the convergence part of the claim, simply use the four equations displayed above to show that $\lim_{x_2 \downarrow 1-\theta} S(x_2)$ is equal to the total welfare of the lopsided equilibrium associated with $x_1 = \theta$ (Lemma 8, Appendix H.1).

Claim 3 Any PBE in the form of Cases 3a. or 3b. in Section 4.1 generates less total welfare than the lopsided equilibrium associated with $x_1 = \theta$ does.

According to Section 4.1, Cases 3a. and 3b. correspond to

$$\text{either} \quad \sigma_1(s), \sigma_1(w), \sigma_2(w) \in (0,1), \quad \sigma_2(s) = 1, \quad \text{and} \quad p_1^R < p_2^R; \tag{14}$$

$$\text{or} \qquad \forall i \in \{1,2\}: \ \sigma_i(w), \sigma_i(s) \in (0,1). \tag{15}$$

To prove the claim, we first establish that in any such a PBE, $p_2^R \geq p_1^R$ and $q_2 < \theta$. If the PBE is in the form of (14), $p_2^R \geq p_1^R$ is part of the definition, and $q_2 < \theta$ is derived (Lemma 11, Appendix H.2) from the assumption $2/3 \leq \theta \leq 3/4$ and the fact $x_1 \geq \xi > 1/2$ (a necessary condition for the form (14), stated in Case 3a., Section 4.1). Else, the PBE is in the form of (15), which satisfies $p_2^R \geq p_1^R$ and $q_2 < \theta$ due to (48) and Lemma 16 in Appendix I. Second, apply Lemma 1 to the non-lopsided equilibrium, switching the roles between players 1 and 2 in the lemma because $p_2^R \geq p_1^R$ here. It then follows that the total welfare of the equilibrium is less than $2\theta p_2^R$. This quantity is less than the total welfare generated by the lopsided equilibrium given $x_1 = \theta$, due to the assumption $\theta \geq 2/3$ (Lemmas 12 and 17).

Claim 4 If $\theta \leq 3/4$ then any PBE of the following form generates less total welfare than the lopsided equilibrium associated with $x_1 = \theta$:

$$\sigma_1(w), \sigma_2(w) \in (0,1) \quad \text{and} \quad \sigma_1(s) = \sigma_2(s) = 1. \tag{16}$$

Among all the PBE in the form of (16), the total welfare maximum is attained by the one in Case 4 of Section 4.1, associated with the equal-split proposal, $x_1 = 1/2$ (Lemma 18, Appendix J). Then we show that the total welfare generated by this local maximum is still

[9] This is in line with the previous intuition that an increase in p_1^R could improve the total welfare.

less than the one generated by the lopsided equilibrium under the proposal $x_1 = \theta$ (last paragraph, Appendix J), where the assumption $\theta \leq 3/4$ is used).

Finally, it is easy to show that the lopsided equilibrium converges to a trivial (always conflict) equilibrium when $x_1 \to 1$. As the total welfare is decreasing when x_1 increases in $[\theta, 1)$ (Claim 1), the trivial equilibrium is suboptimal. Thus, all other trivial equilibria are suboptimal because they have the same posterior system (Lemma 2.a, Appendix B) and hence generate the same total welfare. Now that all possible equilibria when x_1 varies in its entire range $[1/2, 1]$ have been covered, the optimality of $x_1 = \theta$ is proved. ■

Remark 4 The assumption $2/3 \leq \theta \leq 3/4$ in the proposition, though partially relaxable with more calculations, reflects an intuition that the equal-split proposal ($x_1 = 1/2$) could be optimal when θ is close to 1/2 or 1. Since the equal-split proposal fully prevents conflict when $\theta \leq 1/2$ (cf. Footnote 6), it might remain optimal when θ is just slightly above 1/2. When $\theta \approx 1$, the total welfare puts a heavy weight on the weak type, and one can show that the total expected payoff for the weak type of both players under the equal-split proposal is almost equal to the full size of the prize.[10]

Remark 5 While a lopsided equilibrium involves an off-path posterior, the equilibrium under the optimal (lopsided) proposal $x_1 = \theta$ satisfies both the Intuitive and D1 criteria of refinement (Appendix D).

Remark 6 Although the player who is offered the larger share of the good is "favored" at face value by the peace proposal, he need not end with a larger share of the total welfare at equilibrium. In any non-lopsided equilibrium, for instance, it is the player i for whom $p_i^R > p_{-i}^R$ and $q_i < \theta$ that has a larger share of the total welfare (Remark 3). As shown by Claim 3 in the above proof, when player 1 is offered the larger share of the good and the equilibrium takes the form of (14) or (15), $p_1^R \leq p_2^R$ and $q_2 < \theta$. That is, the player who is offered less by the proposed split ends with a larger share of the total welfare. Nonetheless, in the lopsided equilibrium given the optimal proposal $x_1 = \theta$, the two senses of favoritism coincide. Here player 1 is offered a larger share in the proposed split; meanwhile, as shown in Appendix F, player 1's equilibrium ex ante expected payoff $1 - \theta/2$ is no less than its

[10] In a PBE under the equal-split proposal, $p_1^R = p_2^R = 1/2$ (Lemma 18, Appendix J) and hence each player's weak type gets $p_1^R - (1 - \theta) = \theta - 1/2$ (Figure 8). Thus the total expected payoff for them, $2\theta - 1$, converges to one as $\theta \to 1$.

counterpart $2 - 2\theta$ for player 2 due to the assumption $\theta \geq 2/3$. The alternative of giving player 2 a larger share of the total welfare turns out to be suboptimal as shown in Claim 3. Intuitively speaking, player 2 is offered a smaller share of the good and hence his rejecting the proposal may be attributed to the smaller offer rather than his strength. Thus it is inefficient to raise p_2^R thereby to enlarge his ex ante payoff advantage $|q_2 - \theta|(p_2^R - p_1^R)$ over player 1. In other words, it is inefficient to enlarge the total welfare through transferring welfare from the player favored by the proposed split to his opponent.

6 Conclusion

Humanity is often trapped in conflict situations where full preemption of conflict is impossible. In such situations, it is inadequate for a benevolent social planner to aim merely at minimizing the likelihood of conflict, as the social welfare in both the event of peace and the event of conflict should be taken into account. This paper contributes to the conflict mediation literature by incorporating both conflict and peace into maximization of total welfare and presenting an explicit solution for the maximization problem. In our model, a mediator is restricted in instruments so that she cannot effect any information structure deemed desirable with tailor-made communication mechanisms, but rather can only indirectly influence the outcome through simple mechanisms whose integrity is easy to trust. Thus, techniques in the information-design literature are not readily available, and this paper contributes an explicit analysis on how a mediator can nonetheless achieve a constrained optimal posterior information structure given simple, message-independent mechanisms.

Our solution produces a surprising implication: Even though the adversaries are ex ante identical, and are assigned equal welfare weights, the socially optimal peace proposal favors one adversary against the other so much that the favored party always accepts the proposal. Thus it should not be taken for granted that a peace proposal should offer a fair share to each contestant even from the viewpoint of a benevolent mediator. The insight conveyed by our result is that a peace proposal biased towards one side may, counterintuitively, achieve better social welfare than an unbiased one because the favored side is willing to accept the peace deal without fearing being viewed to be weak and taken advantage of later, so that the mediator can devote more resources to compensate the unfavored side.

While the design objective we consider is to maximize the total welfare, the optimality of a lopsided peace proposal demonstrated by our result is extendable to models where the design objective is to minimize the probability of conflict. Given the same intermediate range of the weak-type probability θ for which the lopsided proposal maximizes total welfare, one can show that the lopsided proposal also minimizes the probability of conflict. In addition, the equal-split proposal minimizes the probability of conflict when the probability of being weak is very high or when it is low enough to be near to the region where peace can be guaranteed. This is similar to the pattern with respect to the objective that we consider.

An open question is what happens if a contestant can renege on its acceptance of a peace deal. After Iran accepted the nuclear deal in 2015, the United States withdrew from the agreement in 2018 thereby resuming the hostile relationship. It is conceivable that Iran, in retrospect, would attribute the US withdrawal to Iran's acceptance of the deal in 2015, which might have revealed Iran's weak position in the conflict. That taken into account, Iran will be more reluctant to accept any nuclear deal in the future than before, for fear of its weakness being further revealed and exploited. Thus we conjecture that the inscrutability of a contestant's response to a peace proposal can only become more important when contestants may renege. In the sense that a lopsided solution guarantees acceptance from the favored side thereby making its private information inscrutable from its acceptance, the optimality of lopsided solutions may be robust to such limited commitment situations. See Kamranzadeh [17, Chapter 4] for details.

For tractability, and for a clear contrast with the lopsided solution, we assume that the two contestants are ex ante identical with a common value of the contested prize. A natural question is to what extent a lopsided solution may remain optimal when ex ante asymmetry or private values are considered. While we conjecture that the inscrutability advantage that a lopsided solution provides for the favored party remains crucial, the ex ante asymmetry between the two sides is likely to bring about new questions.

A Derivation of (2) and (3) for the All-Pay Auction

Consider any Bayesian Nash equilibrium (BNE) of the all-pay auction where p_i denotes the probability with which player i's type is s (strong) for each $i = 1, 2$. If player i's type is t_i ($t_i \in \{w, s\}$) and if G_{-i} is the c.d.f. of the bid from the rival $-i$ at equilibrium, then i's expected payoff from bidding b is equal to

$$\frac{1}{\alpha}\left(G_{-i}(b) - \frac{b}{t_i}\right)$$

unless b is an atom of G_{-i}. According to the all-pay auction literature, there exists a unique equilibrium and (G_1, G_2) is characterized by the first-order condition

$$G_i'(b) = \begin{cases} 1/s & \text{if } G_{-i}(b) > 1 - p_{-i} \\ 1/w & \text{if } G_{-i}(b) < 1 - p_{-i} \end{cases}$$

for each $i \in \{1, 2\}$. Assume for now that $p_1 \geq p_2$. Coupled with the equilibrium boundary condition that $G_i(0) = 0$ for at least one player, this differential equation system admits a unique solution.[11] One way to solve it is to start with the endogenous maximum bid $\bar{b}$, common to both players, and trace the graphs of G_1 and G_2 according to the differential equation system when the bid decreases from $\bar{b}$ to zero. As in Figure 6, both graphs start by decreasing at the rate equal to $1/s$. Then the graph of G_1 changes to the steeper slope $1/w$ at the bid b for which $G_2(b) = 1 - p_2$, while G_2 remains decreasing at the rate $1/s$ until $G_1(b) = 1 - p_1$ (because $p_1 \geq p_2$). Thus, when the bid decreases down to zero, $G_2(0) \geq G_1(0)$. Since the zero bid cannot be an atom for both bidders (or an equilibrium condition is violated), $G_1(0) = 0$. That pins down $\bar{b}$ and $G_2(0)$:

$$\begin{aligned} \bar{b}/s &= 1 - (1 - w/s)(1 - p_2) = 1 - \alpha(1 - p_2), \\ G_2(0) &= (1 - w/s)(p_1 - p_2) = \alpha(p_1 - p_2), \end{aligned}$$

where we have used the notation $\alpha := 1 - w/s$. Thus, for each player i, the expected payoff for the strong type in the equilibrium is equal to

$$U_i^s(p_1, p_2) = \frac{1}{\alpha}\left(1 - \bar{b}/s\right) = 1 - p_2 = 1 - \min\{p_1, p_2\}.$$

[11] Since G_i and G_{-i} need not be differentiable, the differential equation system holds only for almost all b in their common support. However, one can prove that G_i and G_{-i} are each absolutely continuous and hence the system coupled with a boundary condition admits a unique solution. See Zheng [27] for details.

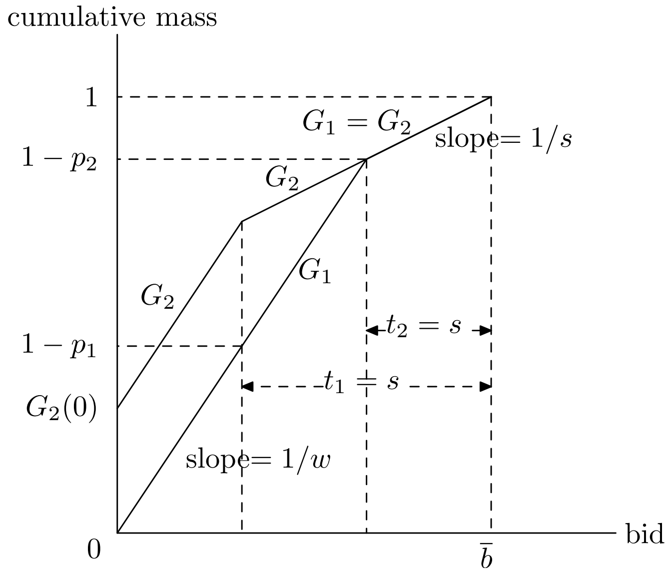

Figure 6: The equilibrium in the all-pay auction

The expected payoffs for the weak type of the two players are:

$$\begin{aligned} U_1^w(p_1, p_2) &= \frac{1}{\alpha}\left(G_2(0) - 0/w\right) = p_1 - p_2 = p_1 - \min\{p_1, p_2\}, \\ U_2^w(p_2, p_1) &= 0 = p_2 - \min\{p_2, p_1\}. \end{aligned}$$

Then remove the assumption $p_1 \geq p_2$ to generalize the above to (2) and (3).

B Categorization of All Equilibria

The next lemma classifies all the possible cases of proposal-PBE pairs, called solutions for short. Case (a) corresponds to the trivial (always conflict) equilibria, Case (b) corresponds to lopsided equilibria, Case (c) the PBEs that satisfy (16), Case (d) those satisfying (15), and Case (e) those satisfying (13) or (14).

Lemma 2 *For any solution $(x_i, \sigma_i, p_i^A, p_i^R, q_i)_{i=1}^2$, exactly one of the following is true:*

a. $q_i = 1$ for some player i, and the on-path posterior is equal to the prior for both players;

b. for some $i \in \{1, 2\}$, $\sigma_i(w) = \sigma_i(s) = 0$ and $0 < \sigma_{-i}(w) < 1 = \sigma_{-i}(s)$;

c. for each $i \in \{1, 2\}$, $0 < \sigma_i(w) < 1 = \sigma_i(s)$;

d. for each $i \in \{1, 2\}$, $\sigma_i(w), \sigma_i(s) \in (0, 1)$;

e. for some $i \in \{1,2\}$, $\sigma_i(w), \sigma_i(s), \sigma_{-i}(w) \in (0,1)$, and $\sigma_{-i}(s) = 1$.

Proof First, we observe that the lemma follows from the following claims:

1. If $q_i = 1$ for some player i, the on-path posterior is equal to the prior for each player.

2. If $q_i < 1$ for each player i, then there does not exist any $i \in \{1,2\}$ for whom:

   i. $\sigma_i(w) = 0 < \sigma_i(s) \leq 1$; or

   ii. $\sigma_i(s) = 0 < \sigma_i(w) \leq 1$; or

   iii. $0 < \sigma_i(s) < 1 = \sigma_i(w)$; or

   iv. $\sigma_i(w) = \sigma_i(s) = 0$ and $\sigma_{-i}(w), \sigma_{-i}(s) \in (0,1)$.

To see why the claims suffice, note that Claims 2.i and 2.ii together imply $\sigma_i(w) = 0 \Leftrightarrow \sigma_i(s) = 0$, and Claim 2.iii implies $0 < \sigma_i(s) < 1 \Rightarrow \sigma_i(w) < 1$. This coupled with Claim 2.i implies $0 < \sigma_i(s) < 1 \Rightarrow 0 < \sigma_i(w) < 1$. In sum, for each player $i \in \{1,2\}$, if $q_i < 1$ then there are only three possibilities: either $\sigma_i(w) = \sigma_i(s) = 0$, or "$0 < \sigma_i(w) < 1$ and $0 < \sigma_i(s) < 1$," or "$0 < \sigma_i(w) < 1$ and $\sigma_i(s) = 1$" (where $\sigma_i(s) \neq 0$ because of the first implication). Thus, in any equilibrium where $q_i < 1$ for both players i (i.e., outside Case (a) in the lemma), there are only nine combinations for (σ_1, σ_2), as in the following table:

	$\sigma_2(w) = \sigma_2(s) = 0$	$\sigma_2(w), \sigma_2(s) \in (0,1)$	$0 < \sigma_2(w) < 1 = \sigma_2(s)$
$\sigma_1(w) = \sigma_1(s) = 0$	impossible	impossible	case (b)
$\sigma_1(w), \sigma_1(s) \in (0,1)$	impossible	case (d)	case (e)
$0 < \sigma_1(w) < 1 = \sigma_1(s)$	case (b)	case (e)	case (c)

In this table, the cell $(1,1)$ ($\sigma_1(w) = \sigma_1(s) = 0 = \sigma_2(w) = \sigma_2(s)$) is impossible because our assumption $\theta > 1/2$ implies that it is impossible to have $\sigma_i(s) = \sigma_i(w) = 0$ for both players i (Footnote 6). Claim 2.iv says that the cells $(1,2)$ and $(2,1)$ (one player's strategy is totally mixed and the other chooses A for sure) are each impossible. The other cells are the possible ones, where we fill in the corresponding cases in the lemma.

The rest of the proof establishes the claims listed above.

Claim 1 Let $q_i = 1$ for some player i. Then the on-path posterior about i is $p_i^R = 1-\theta$. For player $-i$, suppose that the action A is on path and p_{-i}^A is not equal to the prior $1-\theta$. Then Bayes's rule requires that the other action R be on path as well such that $p_{-i}^R \neq 1-\theta$ and (7) be satisfied. Thus, one of p_{-i}^A and p_{-i}^R is above $1-\theta$, and the other below $1-\theta$. If $p_{-i}^A > 1-\theta > p_{-i}^R$, then by (2) and (3) (or simply Figure 1),

$$
\begin{aligned}
U_{-i}^s(p_{-i}^A, 1-\theta) = \theta \quad &< \quad 1 - p_{-i}^R = U_{-i}^s(p_{-i}^R, 1-\theta), \\
U_{-i}^w(p_{-i}^R, 1-\theta) = 0 \quad &< \quad p_{-i}^A - \theta + 1 = U_{-i}^w(p_{-i}^A, 1-\theta);
\end{aligned}
$$

thus player $-i$ of type s would choose R for sure, and $-i$ of type w, A for sure. That implies $p_{-i}^R = 1$ and $p_{-i}^A = 0$, contradicting $p_{-i}^A > 1-\theta > p_{-i}^R$. The other case, where $p_{-i}^A < 1-\theta < p_{-i}^R$, is self-contradicting analogously. This proves Claim 1.

Claim 2.i Suppose, to the contrary, that $\sigma_i(w) = 0 < \sigma_i(s) \le 1$ for some player i. By Bayes's rule, $\sigma_i(w) = 0$ implies $p_i^R = 1$. Then the two graphs in Figure 1 coincide, with p_i there equal to $p_i^R = 1$, and hence $V_i^R(s) = V_i^R(w) = 1 - (1-\theta) = \theta$ by (6)—simply put, the dashed segment in Figure 2 coincides with the solid thick line because any p_{-i}^A and p_{-i}^R are less than or equal to $1 = p_i^R$. Recall from (5) that $V_i^A(t)$ denotes i's expected payoff from choosing A given type $t \in \{s, w\}$. By the best response condition,

$$
\begin{aligned}
\sigma_i(w) = 0 \quad &\Rightarrow \quad V_i^A(w) \ge V_i^R(w) = \theta, \\
\sigma_i(s) > 0 \quad &\Rightarrow \quad V_i^A(s) \le V_i^R(s) = \theta.
\end{aligned}
$$

Thus $V_i^A(w) \ge V_i^A(s)$. Meanwhile, (5) implies that $V_i^A(w) \le V_i^A(s)$, as $U_i^w(p_i^A, \cdot) \le U_i^s(p_i^A, \cdot)$ for any $p_i^A \in [0,1]$. Consequently, $V_i^A(w) = V_i^A(s)$. Then (5) coupled with $q_{-i} > 0$ implies that $U_i^w(p_i^A, p_{-i}^R) = U_i^s(p_i^A, p_{-i}^R)$. Compare (2) with (3)—or simply inspect Figure 1—to see that the equation is possible only if $p_i^A = 1$. But that violates Bayes's rule given that $\sigma_i(w) < 1$. Thus Claim 2.i follows.

Claim 2.ii Suppose, to the contrary, that $q_i < 1$ for both players i, and $\sigma_i(s) = 0 < \sigma_i(w) \le 1$ for some player i. By Bayes's rule, $\sigma_i(s) = 0$ implies $p_i^R = 0$. By (2) and (3), $U_i^s(p_i^R, \cdot) = 1$ and $U_i^w(p_i^R, \cdot) = 0$. It follows from (6) that $V_i^R(s) = 1$ and $V_i^R(w) = 0$. By the best response condition for $\sigma_i(w) > 0$,

$$
0 = V_i^R(w) \ge V_i^A(w) \overset{(5)}{=} q_{-i} U_i^w(p_i^A, p_{-i}^R) + (1 - q_{-i}) x_i \ge (1 - q_{-i}) x_i
$$

and hence $x_i = 0$ (since $1 - q_{-i} > 0$). This coupled with the best response condition for $\sigma_i(s) = 0$ implies

$$1 = V_i^R(s) \leq V_i^A(s) = 0 + q_{-i} U_i^s(p_i^A, p_{-i}^R) \overset{(2)}{=} q_{-i}\left(1 - \min\{p_i^A, p_{-i}^R\}\right).$$

Thus, $q_{-i} = 1$, contradiction.

Claim 2.iii Suppose, to the contrary, that $q_i < 1$ for both players i, and $0 < \sigma_i(s) < 1 = \sigma_i(w)$ for some player i. By Bayes's rule, $\sigma_i(w) = 1$ implies $p_i^A = 1$. It then follows from (2) and (3) that $U_i^s(p_i^A, \cdot) = U_i^w(p_i^A, \cdot)$ and hence, by (5), $V_i^A(s) = V_i^A(w)$. By the best response condition, $0 < \sigma_i(s) < 1$ implies $V_i^R(s) = V_i^A(s)$, and $\sigma_i(w) > 0$ implies $V_i^R(w) \geq V_i^A(w)$. Thus, $V_i^R(w) \geq V_i^R(s)$. This, by inspection of Figure 2—or (6)—is possible only if $p_i^R = 1$. But $p_i^R = 1$ violates Bayes's rule since $\sigma_i(w) > 0$, contradiction.

Claim 2.iv Suppose, to the contrary, that for each player i we have $q_i < 1$ and $\sigma_i(w) = \sigma_i(s) = 0$, $0 < \sigma_{-i}(w) < 1$ and $0 < \sigma_{-i}(s) < 1$. With $\sigma_i(w) = \sigma_i(s) = 0$, we have $q_i = 0$ and $p_i^A = 1 - \theta$. Plug them into (6)—or simply noting that the convex combination in Figure 2 degenerates to the point $1-\theta$—to see that $V_{-i}^R(w) = p_{-i}^R - (1-\theta)$ and $V_{-i}^R(s) = 1-(1-\theta) = \theta$. Since $\sigma_{-i}(w) > 0$, $p_{-i}^R < 1$ and hence $p_{-i}^R - (1-\theta) < \theta$. Consequently, $V_{-i}^R(w) < V_{-i}^R(s)$. Meanwhile, by the best response condition and $q_i = 0$,

$$0 < \sigma_{-i}(w) < 1 \quad \Rightarrow \quad x_{-i} = V_{-i}^A(w) = V_{-i}^R(w),$$
$$0 < \sigma_{-i}(s) < 1 \quad \Rightarrow \quad x_{-i} = V_{-i}^A(s) = V_{-i}^R(s).$$

Thus $V_{-i}^R(w) = V_{-i}^R(s)$, contradiction. ■

An implication of Lemma 2 is that the condition $p_i^R \geq 1 - \theta \geq p_i^A$ in Figures 2–7 is indeed satisfied.

Lemma 3 *For any solution $(x_i, \sigma_i, p_i^A, p_i^R, q_i)_{i=1}^2$, either $q_i = 1$ for some player i and the on-path posterior is equal to the prior for both players, or $q_i < 1$ for both players i and, for each player i, $q_i > 0 \Rightarrow p_i^R > 1 - \theta > p_i^A$.*

Proof By Lemma 2, either Case (a) is true, which means the on-path posterior is equal to the prior for both players, or (a) is not true and hence $q_i < 1$ for both players i. In the latter alternative, if $q_i > 0$ then we have either (I) $\sigma_i(w), \sigma_i(s) \in (0,1)$—which is true for both players in case (d), and player i in case (e), in Lemma 2—or (II) $\sigma_i(s) > \sigma_i(w)$ (which is true for player $-i$ in case (b), both players in case (c), and player $-i$ in case (e)). In (I), the best response condition implies

$$V_i^R(s) - V_i^A(s) = 0 = V_i^R(w) - V_i^A(w),$$

which, by (17), simplifies to $1 - p_i^R = q_{-i}(1 - p_i^A)$. This coupled with $q_{-i} < 1$ implies $1 - p_i^R < 1 - p_i^A$, i.e., $p_i^R > p_i^A$. In (II), by Bayes's rule $\sigma_i(s) = q_i p_i^R/(1-\theta)$ and $\sigma_i(w) = q_i(1-p_i^R)/\theta$, and by $q_i > 0$, we have $p_i^R/(1-\theta) > (1-p_i^R)/\theta$, namely, $p_i^R > 1-\theta$. Both cases considered, we have shown that $q_i > 0$ implies $p_i^R > p_i^A$ or $p_i^R > 1-\theta$. In either case, the Bayesian plausibility condition (7) implies $p_i^R > 1-\theta > p_i^A$. ■

C Proof of (11) and (12) for Lemma 1

Figure 7 depicts the expected payoff from choosing R for the strong type of each player, with curve ILM for player 1, and curve IJK for player 2. Curve ILM lies below curve IJK because the lemma labels the players so that $p_1^R \geq p_2^R$. Similarly, Figure 8 depicts the expected payoff from choosing R for the weak type of each player, with curve DEH for player 1, and curve FGH for player 2. Curve DEH lies above curve FGH again by $p_1^R \geq p_2^R$.

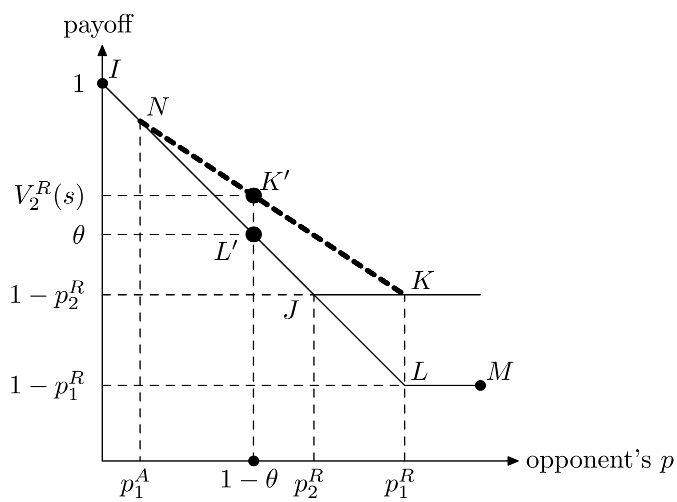

Figure 7: Rejection payoffs for the strong type

To prove (11), note in Figure 7 that $\Delta NK'L'$ and ΔNKL are similar triangles. Thus,

$$\frac{|K'L'|}{|KL|} = \frac{1-\theta-p_1^A}{p_1^R - p_1^A}.$$

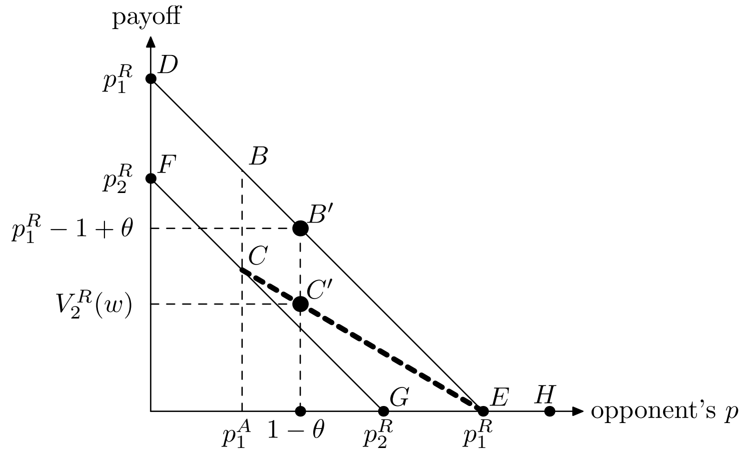

Figure 8: Rejection payoffs for the weak type

Consequently, since $|K'L'| = V_2^R(s) - V_1^R(s)$ and $|KL| = p_1^R - p_2^R$, we have

$$V_2^R(s) - V_1^R(s) = \frac{1-\theta-p_1^A}{p_1^R - p_1^A}(p_1^R - p_2^R) = q_1(p_1^R - p_2^R),$$

with the second equality due to the Bayesian plausibility condition (7). Thus (11) follows.

Analogously, in Figure 8, $\Delta EB'C'$ and ΔEBC are similar triangles. Thus,

$$\frac{|B'C'|}{|BC|} = \frac{p_1^R - (1-\theta)}{p_1^R - p_1^A} = 1 - q_1,$$

with the second equality again due to (7). Consequently, since $V_2^R(w) - V_1^R(w) = -|B'C'|$ and $|BC| = p_1^R - p_2^R$, Eq. (12) follows.

D Verification of the Intuitive and D1 Criteria

It is easy to derive from the construction of lopsided equilibria (Section 4.1) that $q_2 = 1/2$ and $p_2^R = 2(1-\theta)$ in the lopsided equilibrium when the proposal is $x_1 = \theta$. Note that the only observable deviation from the equilibrium is player 1 choosing R. Also note that player 1's expected payoff from this equilibrium is equal to $V_1^A(s) = \theta$ when his type is s, and $V_1^A(w) = \theta/2$ when the type is w. For each $t \in \{s, w\}$ and any $p_1^R \in [0,1]$, let $\tilde{V}_1^R(t, p_1^R)$ denote type-t player 1's expected payoff from the deviation provided that the posterior probability of him being strong is p_1^R (together with the on-path posterior probability $p_2^R = 2(1-\theta)$ of player 2 being strong).

Intuitive Criterion Denote J for the set of player 1's types whose equilibrium payoff is higher than any payoff it could get by playing R, as long as player 2's action is rationalizable. That is,

$$J := \left\{ t \in \{s, w\} \;\middle|\; V_1^A(t) > \max_{p_1^R \in [0,1]} \tilde{V}_1^R(t, p_1^R) \right\}.$$

Observe that $J = \varnothing$: $s \notin J$ because the equilibrium payoff θ is the minimum payoff that a strong type s can achieve from playing R (Remark 2); $w \notin J$ because the equilibrium payoff $\theta/2$ is less than θ, which is equal to $\tilde{V}_1^R(w, 1)$ because $p_1^R = 1 > 2(1-\theta) = p_2^R$ implies via (10) that $\tilde{V}_1^R(w, 1) = 1 - (1-\theta) = \theta$. Now that $J = \varnothing$, the set of distributions of player 1's type whose supports exclude J (the empty set) contains the posterior distribution that supports the lopsided equilibrium. Thus, the equilibrium satisfies the Intuitive Criterion.

D1 Criterion It suffices to falsify the following inequality for each $t \in \{s, w\}$ (and $\{t'\} := \{s, w\} \setminus \{t\}$):

$$\left\{ p_1^R \in [0,1] \;\middle|\; V_1^A(t) \le \tilde{V}_1^R(t, p_1^R) \right\} \subsetneq \left\{ p_1^R \in [0,1] \;\middle|\; V_1^A(t') < \tilde{V}_1^R(t', p_1^R) \right\}.$$

To that end, consider first $t = s$ (so $t' = w$). Since $V_1^A(s) = \theta$ is the minimum payoff that a strong type s can achieve from playing R (Remark 2), the left-hand side is equal to $[0,1]$ and hence the (strict) inequality cannot hold. Next consider $t = w$ (and so $t' = s$). Note that $p_1^R = 1$ belongs to the left-hand side, as $V_1^A(w) = \theta/2 < \theta = \tilde{V}_1^R(w, 1)$, shown in the previous paragraph. However, $p_1^R = 1$ does not belong to the right-hand side, because $V_1^A(s) = \theta$ and $\tilde{V}_1^R(s, 1) = \theta$ by (9). Thus again the inequality displayed above does not hold. Both cases considered, the D1 Criterion is satisfied.

E Three Useful Equations

Lemma 4 *In any solution* $(x_i, \sigma_i, p_i^A, p_i^R, q_i)_{i=1}^2$,

$$V_i^R(s) - V_i^A(s) - \left(V_i^R(w) - V_i^A(w)\right) = 1 - p_i^R - q_{-i}(1 - p_i^A) \tag{17}$$

for each player i*, and if* $p_i^R \ge p_{-i}^R \ge 1 - \theta$*, then*

$$\begin{aligned} V_i^R(w) - V_i^A(w) &= p_i^R - (1 - q_{-i})x_i - 1 + \theta, & (18) \\ V_{-i}^R(w) - V_{-i}^A(w) &= (1 - q_i)\left(p_{-i}^R - p_i^A - x_{-i}\right). & (19) \end{aligned}$$

Proof To prove (17), note from (5) and (6) that the left-hand side is equal to

$$\begin{aligned}
& q_{-i}\left(U_i^s(p_i^R,p_{-i}^R)-U_i^s(p_i^A,p_{-i}^R)-U_i^w(p_i^R,p_{-i}^R)+U_i^w(p_i^A,p_{-i}^R)\right)\\
& +(1-q_{-i})\left(U_i^s(p_i^R,p_{-i}^A)-U_i^w(p_i^R,p_{-i}^A)\right)\\
\overset{(2),(3)}{=}\ & q_{-i}\left(1-\min\{p_i^R,p_{-i}^R\}-1+\min\{p_i^A,p_{-i}^R\}-p_i^R+\min\{p_i^R,p_{-i}^R\}+p_i^A-\min\{p_i^A,p_{-i}^R\}\right)\\
& +(1-q_{-i})\left(1-\min\{p_i^R,p_{-i}^A\}-p_i^R+\min\{p_i^R,p_{-i}^A\}\right)\\
=\ & q_{-i}\left(-p_i^R+p_i^A\right)+(1-q_{-i})\left(1-p_i^R\right),
\end{aligned}$$

which is equal to the right-hand side. To prove (18), assume without loss that $p_1^R \geq p_2^R$. Thus for each player i, $p_i^R \geq 1-\theta$ and hence, by the Bayesian plausibility condition (7), $p_i^A \leq 1-\theta$. Use (5) and (6) to obtain

$$\begin{aligned}
V_1^R(w)-V_1^A(w) &= q_2\left(U_1^w(p_1^R,p_2^R)-U_1^w(p_1^A,p_2^R)\right)+(1-q_2)\left(U_1^w(p_1^R,p_2^A)-x_1\right)\\
&\overset{(3)}{=} q_2\left(p_1^R-\min\{p_1^R,p_2^R\}-p_1^A+\min\{p_1^A,p_2^R\}\right)+(1-q_2)\left(p_1^R-\min\{p_1^R,p_2^A\}-x_1\right)\\
&= q_2\left(p_1^R-p_2^R-p_1^A+p_1^A\right)+(1-q_2)\left(p_1^R-p_2^A-x_1\right)\\
&= p_1^R-q_2p_2^R-(1-q_2)p_2^A-(1-q_2)x_1\\
&= p_1^R-(1-\theta)-(1-q_2)x_1,
\end{aligned}$$

with the third line due to $p_1^R \geq p_2^R \geq 1-\theta \geq p_j^A$ for each player j, and the last line due to the Bayesian plausibility condition (7). Thus (18) is true. Analogously, (19) follows from

$$\begin{aligned}
V_2^R(w)-V_2^A(w) &= q_1\left(p_2^R-p_2^R-p_2^A+p_2^A\right)+(1-q_1)\left(p_2^R-p_1^A-x_2\right)\\
&= (1-q_1)\left(p_2^R-p_1^A-x_2\right). \quad \blacksquare
\end{aligned}$$

F The Total Welfare of the Optimal Lopsided Solution

Lemma 5 *The total welfare generated by the lopsided equilibrium associated with the proposal $(\theta,1-\theta)$ is equal to $\theta(3-5\theta/2)$.*

Proof By definition of any lopsided equilibrium, $q_1=0$ and $0<\sigma_2(w)<1=\sigma_1(s)$. Thus the total welfare from $(\theta,1-\theta)$ is equal to

$$\underbrace{(1-q_2)\theta+q_2\left[\theta U_1^w(p_1^A,p_2^R)+(1-\theta)U_1^s(p_1^A,p_2^R)\right]}_{\text{player 1}}+\underbrace{\theta U_2^w(p_2^R,p_1^A)+(1-\theta)U_2^s(p_2^R,p_1^A)}_{\text{player 2}}.$$

By Bayes's rule, $p_1^A = 1-\theta$, $p_2^A = 0$ and $q_2 = (1-\theta)/p_2^R$. As explained in the construction of lopsided equilibria (Section 4.1), $p_2^R = 1-\theta+x_2 = 2(1-\theta)$. Combine them with (2) and (3) to calculate the above-displayed sum:

$$\begin{aligned}
& (1-q_2)\theta + q_2\left(\theta\cdot 0 + (1-\theta)(1-1+\theta)\right) + \theta(p_2^R - 1+\theta) + (1-\theta)(1-1+\theta) \\
= & \left(1-\frac{1-\theta}{p_2^R}\right)\theta + \frac{1-\theta}{p_2^R}(1-\theta)\theta + \theta(p_2^R-1+\theta) + (1-\theta)\theta \\
= & \left(1-\frac{1-\theta}{2(1-\theta)}\right)\theta + \frac{1-\theta}{2(1-\theta)}(1-\theta)\theta + \theta(2(1-\theta)-1+\theta) + (1-\theta)\theta \\
= & \underbrace{(2-\theta)\theta/2}_{\text{player 1}} + \underbrace{(2-2\theta)\theta}_{\text{player 2}} \\
= & \theta(3-5\theta/2). \quad \blacksquare
\end{aligned}$$

G Suboptimality of Any Trivial Equilibrium

By Claim 1 in the proof of Lemma 2, any trivial PBE, namely, any Case-(a) solution, has the on-path posterior equal to the prior for each player. Since $q_i = 1$ for some player i, conflict takes place for sure and hence each player's ex ante payoff from the PBE is equal to

$$\theta U_i^w(1-\theta, 1-\theta) + (1-\theta)U_i^s(1-\theta,1-\theta) = 0 + (1-\theta)(1-(1-\theta)) = \theta(1-\theta).$$

Thus, the total welfare generated by the PBE is equal to $2\theta(1-\theta)$, which is less than $\theta(3-5\theta/2)$, the total welfare generated by the lopsided proposal $(\theta, 1-\theta)$ (Lemma 5). Thus, any PBE that belongs to Case (a) is suboptimal.

H Suboptimality of Any Equilibrium in Eqs. (13) or (14)

Equilibria in the form of Eqs. (13) or (14) correspond to Case (e) in Lemma 2: exactly one of the two players is totally mixing A and R for each type. Relabeling the players if necessary, assume without loss that in any Case-(e) PBEs it is player 1 who is totally mixing, i.e.,

$$0 < \sigma_1(w) < 1, \quad 0 < \sigma_1(s) < 1, \quad 0 < \sigma_2(w) < 1, \quad \sigma_2(s) = 1. \tag{20}$$

Call a Case-(e) solution *Case (e)-i* if $p_2^R \le p_1^R$, and *Case (e)-ii* if $p_1^R < p_2^R$. This labeling of the players implies $x_1 \ge x_2$ (and hence is consisting with the labeling in Section 4.1), because $x_2 \le 1/2$ according to Lemma 7 for Subcase-(e)-i, and Lemma 10 for Subcase-(e)-ii.

Lemma 6 *A tuple* $(x_i, \sigma_i, p_i^A, p_i^R, q_i)_{i=1}^2$ *constitutes a Case-(e)-i solution if and only if it satisfies (20) and all the following:*

$$\begin{aligned}
1 - p_1^R &= q_2(1 - p_1^A), && (21)\\
1 - p_2^R &\geq q_1, && (22)\\
p_2^R &\leq p_1^R, && (23)\\
p_1^R + \theta - 1 &= (1 - q_2)x_1, && (24)\\
p_2^R &= p_1^A + x_2. && (25)
\end{aligned}$$

A tuple $(x_i, \sigma_i, p_i^A, p_i^R, q_i)_{i=1}^2$ *constitutes a Case-(e)-ii solution if and only if it satisfies (20), (21), (22) and all the following:*

$$\begin{aligned}
p_1^R &< p_2^R, && (26)\\
p_1^R &= x_1, && (27)\\
p_2^R + \theta - 1 &= (1 - q_1)x_2. && (28)
\end{aligned}$$

Proof The best response condition for (20) to constitute a PBE is that $V_1^R(w) - V_1^A(w) = V_1^R(s) - V_1^A(s) = 0$ for player 1 and $V_2^R(w) - V_2^A(w) = 0 \leq V_2^R(s) - V_2^A(s)$ for player 2. By (17), that is equivalent to simultaneous satisfaction of $V_1^R(w) - V_1^A(w) = V_2^R(w) - V_2^A(w) = 0$, $\left(1 - p_1^R\right) = q_2\left(1 - p_1^A\right)$ and $1 - p_2^R \geq q_1$ (i.e., Ineq. (22), the derivation of which also uses the fact $p_2^A = 0$ implied by Bayes's rule with respect to $\sigma_2(s) = 1$). To write the condition $V_1^R(w) - V_1^A(w) = V_2^R(w) - V_2^A(w) = 0$ explicitly, note for each player i that $q_i < 1$ in this PBE and hence $p_i^A < 1 - \theta < p_i^R$ by Lemmas 3. If the solution belongs to Subcsae (i) of Case (e), $p_1^R \geq p_2^R$, then (18) and (19) apply to the case $i = 1$ and hence

$$\begin{aligned}
V_1^R(w) - V_1^A(w) &= p_1^R - (1 - \theta) - (1 - q_2)x_1,\\
V_2^R(w) - V_2^A(w) &= (1 - q_1)\left(p_2^R - p_1^A - x_2\right).
\end{aligned}$$

Thus the condition $V_1^R(w) - V_1^A(w) = 0$ becomes (24), and the condition $V_2^R(w) - V_2^A(w) = 0$ becomes (25). Analogously, if it is Subcase (ii) of Case (e), $p_1^R \leq p_2^R$, then (18) and (19) apply to the case $i = 2$ and hence

$$\begin{aligned}
V_2^R(w) - V_2^A(w) &= p_2^R - (1 - \theta) - (1 - q_1)x_2,\\
V_1^R(w) - V_1^A(w) &= (1 - q_2)\left(p_1^R - p_2^A - x_1\right) = (1 - q_2)\left(p_1^R - x_1\right),
\end{aligned}$$

with the last "=" due to $p_2^A = 0$ (since $\sigma_2(s) = 1$). Thus, the condition $V_i^R(w) - V_i^A(w) = 0$ for both players i becomes (27) and (28). ■

H.1 Subcase (i): $p_1^R \geq p_2^R$ (Eq. (13))

Lemma 7 *For any $x_2 \in [0,1]$ there is at most one tuple $\left(\sigma_i, p_i^A, p_i^R, q_i\right)_{i=1}^2$ that constitutes a Case-(e)-i solution, and for any such solution, $1-\theta < x_2 \leq 2\theta - 1$, where $2\theta - 1 \leq 1/2$ if $\theta \leq 3/4$.*

Proof Let $x_2 \in [0,1]$ and $\left(x_i, \sigma_i, p_i^A, p_i^R, q_i\right)_{i=1}^2$ be a Case-(e)-i solution. By Lemma 6, the tuple satisfies Eqs. (21), (24) and (25). Combine (21), (24) and (25) with $q_2 = \theta\sigma_2(w) + 1 - \theta$ (definition of q_i), $p_2^R = (1-\theta)/q_2$ (Bayes's rule with respect to $\sigma_2(s) = 1$) and $x_1 + x_2 = 1$ (definition of peace proposal) to obtain

$$\sigma_2(w) = 1 - \frac{1}{2\theta}. \tag{29}$$

Plug (29) into the system consisting of (20), (21), (24) and (25) to obtain a unique solution for all components of the tuple:

$$\begin{aligned}
q_2 &= \theta\left(1 - \frac{1}{2\theta}\right) + 1 - \theta = \frac{1}{2}, \\
p_2^R &= \frac{1-\theta}{q_2} = 2 - 2\theta, && (30)\\
p_1^R &= 1 - \theta + (1 - 1/2)(1 - x_2) = \frac{3 - 2\theta - x_2}{2}, && (31)\\
p_1^A &= p_2^R - x_2 = 2(1-\theta) - x_2, \\
q_1 &= \frac{1 - \theta - p_1^A}{p_1^R - p_1^A} = \frac{2(\theta - 1 + x_2)}{2\theta + x_2 - 1}, && (32)\\
\sigma_1(w) &= \frac{\theta - 1 + x_2}{\theta}. && (33)
\end{aligned}$$

In particular, (33) follows from

$$\begin{aligned}
\theta\sigma_1(w) &= q_1 - (1-\theta)\sigma_1(s) = q_1 - p_1^R q_1 \\
&= \frac{2(\theta - 1 + x_2)}{2\theta + x_2 - 1}\left(1 - \frac{3 - 2\theta - x_2}{2}\right) \\
&= \theta - 1 + x_2.
\end{aligned}$$

Since $\sigma_1(w) > 0$ by definition of any Case-(e) solution, (33) implies $x_2 > 1 - \theta$.

To prove $x_2 \leq 2\theta - 1$, plug (30) and (31) into the condition $p_1^R \geq p_2^R$ that defines Subcase (e)-i to obtain

$$p_1^R \geq p_2^R \iff \frac{3 - 2\theta - x_2}{2} \geq 2 - 2\theta \iff x_2 \leq 2\theta - 1. \quad \blacksquare$$

Lemma 8 *When x_1 converges to θ from above, the total welfare generated by any Case-(e)-i solution with proposal (x_1, x_2) converges to the total welfare generated by the lopsided equilibrium associated with the proposal $(\theta, 1-\theta)$.*

Proof By Lemma 7, any Case-(e)-i solution is uniquely determined by the x_2 in the tuple, with 2 being the label for the player for whom $p_2^R \leq p_1^R$. Thus, the total welfare generated by the solution is uniquely determined by x_2. Hence denote $S_e(x_2)$ for the total welfare generated by a Case-(e)-i solution that offers x_2 to the player $-i$ for whom $p_{-i}^R \leq p_i^R$. Since Reject is a best reply for each type of each player in any Case-(e) solution, Lemma 1 implies

$$S_e(x_2) = 2\theta p_1^R + (q_1 - \theta)(p_1^R - p_2^R). \tag{34}$$

By Lemma 7, $x_2 > 1 - \theta$. Taking the limit of (31) and (32) as x_2 converges to $1 - \theta$ from above, we have

$$\begin{aligned} \lim_{x_2 \downarrow 1-\theta} p_1^R &= \frac{2-\theta}{2}, \\ \lim_{x_2 \downarrow 1-\theta} q_1 &= 0. \end{aligned}$$

Combine them with the above formula of $S_e(x_2)$ and (30) to obtain

$$\begin{aligned} \lim_{x_2 \downarrow 1-\theta} S_e(x_2) &= 2\theta p_1^R - \theta(p_1^R - p_2^R) = \theta(p_1^R + p_2^R) \\ &= \theta\left(\frac{2-\theta}{2} + 2 - 2\theta\right) \\ &= \theta\left(3 - \frac{5}{2}\theta\right), \end{aligned}$$

which by Lemma 5 is equal to the total welfare generated by the lopsided equilibrium given proposal $(\theta, 1-\theta)$. ■

Lemma 9 *If $\theta \leq 3/4$, the lopsided equilibrium given proposal $(\theta, 1-\theta)$ generates larger total welfare than any Case-(e)-i solution.*

Proof By Lemma 8, it suffices to prove that $\frac{d}{dx_2} S_e(x_2) < 0$ for all $x_2 > 1 - \theta$. To prove that, use (34) and $dp_2^R / dx_2 = 0$ (Eq. (30)) to obtain

$$\begin{aligned} \frac{d}{dx_2} S_e(x_2) &= \frac{\partial S_e}{\partial p_1^R} \frac{dp_1^R}{dx_2} + \frac{\partial S_e}{\partial q_1} \frac{dq_1}{dx_2} = (q_1 + \theta) \frac{dp_1^R}{dx_2} + \left(p_1^R - p_2^R\right) \frac{dq_1}{dx_2} \\ &= -\frac{q_1 + \theta}{2} + \left(p_1^R - p_2^R\right) \frac{2\theta}{(2\theta + x_2 - 1)^2}, \end{aligned} \tag{35}$$

with the last equality due to (31) and (32). Note that the expression (35) is strictly decreasing in x_2: By (30) and (31), $p_1^R - p_2^R = (2\theta - 1 - x_2)/2$, which is strictly decreasing in x_2; as can be seen above (due to (32)),

$$\frac{dq_1}{dx_2} = \frac{2\theta}{(2\theta + x_2 - 1)^2} > 0$$

and so $-\frac{q_1+\theta}{2}$ is strictly decreasing in x_2 as well. Thus, $\frac{d}{dx_2}S_e(x_2)$ is strictly decreasing in x_2.

Now that $\frac{d}{dx_2}S_e(x_2)$ is strictly decreasing in x_2 for all $x_2 > 1 - \theta$, and $x_2 > 1 - \theta$ for any Case-(e)-i solution, to show that $S_e(x_2)$ is strictly decreasing in x_2, we need only

$$\lim_{x_2 \downarrow 1-\theta} \frac{d}{dx_2} S_e(x_2) < 0.$$

To show that, take the limit of (35) as x_2 converges to $1 - \theta$ from above and use (30), (31), and (32) (so $\lim_{x_2 \downarrow 1-\theta} q_1 = 0$ and $\lim_{x_2 \downarrow 1-\theta} \left(p_1^R - p_2^R\right) = \frac{3\theta-2}{2}$) to obtain

$$\lim_{x_2 \downarrow 1-\theta} \frac{d}{dx_2} S_e(x_2) = -\frac{\theta}{2} + \frac{(3\theta - 2)}{2}\frac{2}{\theta} = \frac{-\theta^2 + 6\theta - 4}{2\theta} = -\frac{1}{2\theta}\left((\theta - 3)^2 - 5\right),$$

which is negative because the condition $\theta \leq 3/4$ in the lemma implies $\theta < 3 - \sqrt{5}$. Thus, the supremum of $\frac{d}{dx_2}S_e(x_2)$ is negative among all $x_2 > 1 - \theta$, so $\lim_{x_2 \downarrow 1-\theta} S_e(x_2)$ is the supremum total welfare among all Case-(e)-i solutions. By Lemma 8, the supremum is equal to the total welfare generated by the lopsided equilibrium given proposal $[\theta, 1 - \theta]$. ■

H.2 Subcase (ii): $p_1^R < p_2^R$ (Eq. (14))

Lemma 10 *For any $x_2 \in [0, 1]$ there is at most one tuple $\left(\sigma_i, p_i^A, p_i^R, q_i\right)_{i=1}^2$ that constitutes a Case-(e)-ii solution; if $\theta \leq 3/4$ in addition, then $2\theta - 1 < x_2 < \hat{x}_2$ for any such solution, where $\hat{x}_2$ is uniquely determined by θ and belongs to $[2\theta - 1, 1/2]$.*

Proof By Lemma 6, the tuple satisfies Eqs. (21), (27) and (28). Plug Bayes's rule $1 - p_1^R = \theta\sigma_1(w)/q_1$ into Eq. (27) to obtain

$$\sigma_1(w) = \frac{(1 - \theta)(1 - x_1)}{\theta x_1}\sigma_1(s). \tag{36}$$

Eq. (27), combined with $1 - p_1^R = \theta\sigma_1(w)/q_1$ and $x_1 + x_2 = 1$, also implies

$$q_1 = \frac{\theta\sigma_1(w)}{x_2}. \tag{37}$$

Thus, from Bayes's rule we have

$$1 - p_1^A = \frac{\theta(1 - \sigma_1(w))}{1 - q_1} = \frac{\theta - q_1 x_2}{1 - q_1}.$$

Plug this into (21), replace p_1^R via $p_1^R = x_1$ (Eq. (27)) and replace q_2 through $q_2 = (1-\theta)/p_2^R$ (due to (7) and $p_2^A = 0$, the latter due to $\sigma_2(s) = 1$), and eliminate p_2^R by (28). Then

$$x_2 = \frac{(1-\theta)}{1-\theta+(1-q_1)x_2}\frac{\theta - q_1 x_2}{1-q_1},$$

which is simplified to a quadratic equation

$$(q_1)^2(x_2)^2 - 2q_1(x_2)^2 + x^2 + (1-\theta)(x-\theta) = 0,$$

namely,

$$x_2^2(q_1-1)^2 = (1-\theta)(\theta - x_2).$$

We claim $\theta - x_2 > 0$. To see that, note $p_1^A < p_1^R$ due to Lemma 3 and $\sigma_1(w) < 1$ and hence $q_1 < 1$ in any Case-(e) PBE. Then the Bayesian plausibility condition (7) implies $p_1^R > 1-\theta$. This, combined with Bayes's rule $p_1^R = (1-\theta)\sigma_1/q_1$ and $1 - p_1^R = \theta\sigma_1(w)/q_1$, implies $\sigma_1(w) < \sigma_1(s)$. Then (36 implies $1 - x_1 < \theta$, namely,

$$\theta - x_2 > 0. \tag{38}$$

Thus, the above quadratic equation implies $x_2(q_1-1) = -\sqrt{(1-\theta)(\theta-x_2)}$, namely,

$$q_1 = 1 - \frac{1}{x_2}\sqrt{(1-\theta)(\theta - x_2)}. \tag{39}$$

Thus, the Case-(e) solution is uniquely determined by x_2. In particular,

$$p_1^R \overset{(27)}{=} 1 - x_2, \tag{40}$$

$$p_2^R \overset{(28)}{=} 1-\theta+\sqrt{(1-\theta)(\theta-x_2)}, \tag{41}$$

$$\sigma_1(w) \overset{(37)}{=} \frac{x_2 - \sqrt{(1-\theta)(\theta-x_2)}}{\theta}, \tag{42}$$

$$q_2 = \frac{1-\theta}{p_2^R}, \tag{43}$$

with (43) due to Bayes's rule with respect to $\sigma_2(s) = 1$.

Finally we verify that $2\theta - 1 < x_2 < 1/2$ in any Case-(e)-ii solution. Recall from the definition of Case-(e)-ii solutions that $p_2^R > p_1^R$. By (40) and (41),

$$\begin{aligned} p_2^R > p_1^R &\iff 1-\theta+\sqrt{(1-\theta)(\theta-x_2)} > 1 - x_2 \\ &\iff \sqrt{(1-\theta)(\theta-x_2)} > \theta - x_2. \end{aligned} \tag{44}$$

By (38), the above inequality is equivalent to

$$\left(\sqrt{(1-\theta)(\theta-x_2)}\right)^2 > (\theta-x_2)^2,$$

namely, $1-\theta > \theta - x_2$. Thus

$$x_2 > 2\theta - 1. \tag{45}$$

To prove $x_2 < \hat{x}_2$, the claim about $\hat{x}_2$ in the lemma, recall that (22) holds for any Case-(e)-ii solution (Lemma 6), namely, $q_1 \leq 1 - p_2^R$. Plug (39) and (41) into this inequality to obtain

$$\left(\frac{1}{x_2} - 1\right)\sqrt{(1-\theta)(\theta-x_2)} \geq 1-\theta,$$

namely,

$$(x_2)^2(1-\theta) - (\theta-x_2)(1-x_2)^2 \leq 0. \tag{46}$$

Note: the left-hand side of (46) is strictly increasing in x_2. By the assumption $\theta \leq 3/4$, the left-hand side of (46) is equal to $(1-\theta)(4\theta-3) \leq 0$ when $x_2 = 2\theta - 1$, and equal to $3/8 - \theta/2 \geq 0$ when $x_2 = 1/2$. Thus, there exists a unique $\hat{x}_2 \in [2\theta-1, 1/2]$ for which (46) holds at equality when $x_2 = \hat{x}_2$, and holds strictly for all $x_2 < \hat{x}_2$, as asserted. ■

Lemma 11 *If $2/3 \leq \theta \leq 3/4$, then $q_2 < \theta$ in any Case-(e)-ii PBE.*

Proof By (41) and (43).

$$\begin{aligned} q_2 < \theta &\iff \frac{1-\theta}{1-\theta+\sqrt{(1-\theta)(\theta-x_2)}} < \theta \\ &\iff (1-\theta)^2 \leq \theta\sqrt{(1-\theta)(\theta-x_2)} \\ &\iff x_2 \leq \theta - \frac{(1-\theta)^3}{\theta^2}. \end{aligned}$$

Thus, since $x_2 < 1/2$ by Lemma 10, it suffices to show $1/2 \leq \theta - (1-\theta)^3/\theta^2$, namely,

$$\frac{4\theta^3 - 7\theta^2 + 6\theta - 2}{2\theta^2} \geq 0.$$

Thus we are done if $4\theta^3 - 7\theta^2 + 6\theta - 2 \geq 0$. To show that, note

$$\frac{d}{d\theta}\left[4\theta^3 - 7\theta^2 + 6\theta - 2\right] = 12\theta^2 - 14\theta + 6 = 6\theta(2\theta-1) + 2(3-4\theta) > 0$$

because $2\theta > 1$ by (1) and $\theta \leq 3/4$ by assumption. Thus, the term $4\theta^3 - 7\theta^2 + 6\theta - 2$ is strictly increasing in θ. Since it is equal to $2/27$ at $\theta = 2/3$, it follows that $4\theta^3 - 7\theta^2 + 6\theta - 2 > 0$ for all $\theta \in [2/3, 3/4]$. This proves $q_2 < \theta$, as desired. ■

Lemma 12 *If $2/3 \le \theta \le 3/4$, then the lopsided equilibrium associated with proposal $(\theta, 1-\theta)$ generates larger total welfare than any Case-(e)-ii solution.*

Proof Since any Case-(e)-ii solution corresponds to a non-lopsided equilibrium, Lemma 1 applies with the roles of players 1 and 2 switched due to $p_2^R \ge p_1^R$ in Case-(e)-ii. Thus the total welfare is equal to

$$S'_e := 2\theta p_2^R + (q_2 - \theta)(p_2^R - p_1^R).$$

To prove that S'_e is less than the total welfare generated by the lopsided equilibrium given proposal $(\theta, 1-\theta)$, which is equal to $\theta(3-5\theta/2)$ by Lemma 5, it suffices to prove $p_2^R < 2-2\theta$ for any Case-(e)-ii solution: Since $q_2 < \theta$ by Lemma 11, we have $S'_e < 2\theta p_2^R$ because $p_2^R - p_1^R > 0$ in any Case-(e)-ii solution; if, in addition, $p_2^R < 2 - 2\theta$, then

$$S'_e < 2\theta p_2^R < 2\theta(2-2\theta) \le \theta(3 - 5\theta/2),$$

with the last inequality due to the condition $\theta \ge 2/3$ in the lemma.

Thus, we verify that $p_2^R < 2 - 2\theta$. Note from (41) that $p_2^R < 2 - 2\theta$ is equivalent to

$$\begin{aligned} 1 - \theta + \sqrt{(1-\theta)(\theta - x_2)} < 2 - 2\theta \iff& \sqrt{(1-\theta)(\theta - x_2)} < 1 - \theta \\ \iff& 1 - \theta < \theta - x_2 \iff 2\theta - 1 < x_2, \end{aligned}$$

where $2\theta - 1 < x_2$ is true by Lemma 10. Thus, $p_2^R < 2 - 2\theta$, as desired. ■

I Suboptimality of Any Equilibrium in Eq. (15)

Equilibria in the form of Eq. (15) correspond to Case (d) in Lemma 2. In any such PBE, each type of each player is totally mixing A and R:

$$0 < \sigma_i(w) < 1, \quad 0 < \sigma_i(s) < 1, \quad \forall i \in \{1, 2\}. \tag{47}$$

This being symmetric between the two players, let us assume without loss that

$$p_2^R \ge p_1^R. \tag{48}$$

This labeling of the players will be shown to imply $x_1 \ge x_2$ (Lemma 15) and hence consistent with the labeling in Section 4.1.

Lemma 13 *A tuple* $(x_i, \sigma_i, p_i^A, p_i^R, q_i)_{i=1}^2$ *that satisfies (48) constitutes a Case-(d) solution if and only if it satisfies (47) and all the following:*

$$\begin{aligned} 1 - p_1^R &= q_2(1 - p_1^A), && (49)\\ 1 - p_2^R &= q_1(1 - p_2^A), && (50)\\ p_1^R &= p_2^A + x_1, && (51)\\ p_2^R + \theta - 1 &= (1 - q_1)x_2. && (52) \end{aligned}$$

Proof The best response condition for (47) to constitute a PBE is that $V_i^R(w) - V_i^A(w) = V_i^R(s) - V_i^A(s) = 0$ for each player i. By (17), that is equivalent to simultaneous satisfaction of $V_1^R(w) - V_1^A(w) = V_2^R(w) - V_2^A(w) = 0$, $\left(1 - p_1^R\right) = q_2\left(1 - p_1^A\right)$, and $1 - p_2^R = q_1(1 - p_2^A)$. To write the condition $V_1^R(w) - V_1^A(w) = V_2^R(w) - V_2^A(w) = 0$ explicitly, note for each player i that $q_i < 1$ in this PBE and hence $p_i^A < 1 - \theta < p_i^R$ by Lemmas 3. This combined with (48) implies that (18) and (19) apply to the case $i = 2$ and hence

$$\begin{aligned} V_2^R(w) - V_2^A(w) &= p_2^R - (1 - \theta) - (1 - q_1)x_2,\\ V_1^R(w) - V_1^A(w) &= (1 - q_2)\left(p_1^R - p_2^A - x_1\right). \end{aligned}$$

Consequently, with $q_2 < 1$,

$$\begin{aligned} V_1^R(w) - V_1^A(w) = 0 &\iff p_1^R = p_2^A + x_1,\\ V_2^R(w) - V_2^A(w) = 0 &\iff p_2^R + \theta - 1 = (1 - q_1)x_2. \quad \blacksquare \end{aligned}$$

Lemma 14 *If* $(x_i, \sigma_i, p_i^A, p_i^R, q_i)_{i=1}^2$ *is a Case-(d) solution such that* $p_2^R \geq p_1^R$*, then*

$$\begin{aligned} \sigma_1(w) &= \frac{\theta + x_1 - 1 + q_1(1 - 2x_1)}{\theta}, && (53)\\ \sigma_1(s) &= \frac{q_1 - \theta\sigma_1(w)}{1 - \theta}, && (54)\\ \sigma_2(w) &= 1 - \frac{x_2}{\theta}, && (55)\\ \sigma_2(s) &= \frac{\theta - x_2}{1 - \theta} \cdot \frac{1 - \theta + x_2(1 - q_1)}{\theta + x_2(q_1 - 1)}, && (56)\\ x_2 &< \theta, \quad \textit{and} && (57) \end{aligned}$$

$$(q_1)^3 x_2 (1 - 2x_2) + (q_1)^2 x_2 (3x_2 - 1 - \theta) + q_1 (3x_2 - 1 - \theta)(\theta - x_2) + (\theta - x_2)^2 = 0. \quad (58)$$

Proof Eq. (54) follows trivially from $q_1 = \theta\sigma_1(w) + (1-\theta)\sigma_1(s)$. To prove the rest, first apply Bayes's rule to $1 - p_2^A$ and then to $1 - p_2^R$ to obtain

$$1 - p_2^A = \frac{\theta(1-\sigma_2(w))}{1-q_2} = \frac{(1-p_2^R)\theta(1-\sigma_2(w))}{1-p_2^R-(1-p_2^R)q_2} = \frac{(1-p_2^R)\theta(1-\sigma_2(w))}{1-p_2^R-\theta\sigma_2(w)}.$$

Then

$$p_2^R - p_2^A = (1-p_2^A) - (1-p_2^R) = \frac{(1-p_2^R)\theta(1-\sigma_2(w))}{1-p_2^R-\theta\sigma_2(w)} - (1-p_2^R) = \frac{(1-p_2^R)(\theta+p_2^R-1)}{1-p_2^R-\theta\sigma_2(w)}.$$

By (50) we have $q_1 = (1-p_2^R)/(1-p_2^A)$. Plug this into (52) to obtain

$$(\theta + p_2^R - 1)(1-p_2^A) = (p_2^R - p_2^A)x_2.$$

Plugging into this equation the formulas of $1-p_2^A$ and $p_2^R - p_2^A$ obtained above, we have

$$(\theta+p_2^R-1)\frac{(1-p_2^R)\theta(1-\sigma_2(w))}{1-p_2^R-\theta\sigma_2(w)} = \frac{(1-p_2^R)(\theta+p_2^R-1)}{1-p_2^R-\theta\sigma_2(w)}x_2,$$

namely,

$$\theta(1-\sigma_2(w)) = x_2.$$

Thus (55) is true. Then Eq. (55) coupled with $\sigma_2(w) > 0$ implies (57).

Second, plug Eqs. (50) and (51) into Eq. (52) to obtain

$$1 - q_1(1-p_1^R+x_1) = 1-\theta+(1-q_1)x_2.$$

eliminate $1-\theta$ therein by Eq. (7) and cancel and combine terms to obtain

$$(1-q_1)(1-p_1^A) = x_2 - q_1\left(x_2 - x_1\right),$$

which, by Bayes's rule, is equivalent to

$$\theta(1-\sigma_1(w)) = x_2 - q_1\left(x_2-x_1\right), \tag{59}$$

which in turn is equivalent to Eq. (53).

Third, rewrite (50) as $q_1 = (1-p_2^R)/(1-p_2^A)$ and then rewrite the right-hand side by Bayes's rule to obtain

$$q_1 = \frac{\theta\sigma_2(w)}{\theta(1-\sigma_2(w))}\cdot\frac{\theta(1-\sigma_2(w))+(1-\theta)(1-\sigma_2(s))}{\theta\sigma_2(w)+(1-\theta)\sigma_2(s)} \overset{(55)}{=} \frac{\theta-x_2}{x_2}\cdot\frac{x_2+(1-\theta)(1-\sigma_2(s))}{\theta-x_2+(1-\theta)\sigma_2(s)},$$

which implies Eq. (56).

Finally, we prove Eq. (58). Use Bayes's rule on player 2 and then use (55) to obtain

$$(1-q_2)(1-p_2^A) = \theta(1-\sigma_2(w)) = x_2.$$

Eliminate the q_2 in this equation by (49), and p_2^A by (51), to rewrite the above equation as

$$\left(1 - \frac{1-p_1^R}{1-p_1^A}\right)(1-p_1^R+x_1) = x_2,$$

namely,

$$(1-p_1^A)x_2 = \left(p_1^R - p_1^A\right)\left(1-p_1^R+x_1\right). \tag{60}$$

Meanwhile, use Bayes's rule on player 1 and then use (59) to obtain

$$1-p_1^A = \frac{\theta(1-\sigma_1(w))}{1-q_1} = \frac{x_2 - q_1\left(x_2-x_1\right)}{1-q_1}.$$

Analogously, use Bayes's rule on player 1 and then use Eq. (53) to obtain

$$1-p_1^R = \frac{\theta\sigma_1(w)}{q_1} = \frac{\theta+x_1-1+q_1(1-2x_1)}{q_1}.$$

From the two formulas we get

$$\begin{aligned} p_1^R - p_1^A &= \frac{x_2 - q_1\left(x_2-x_1\right)}{1-q_1} - \frac{\theta+x_1-1+q_1(x_2-x_1)}{q_1} \\ &= \frac{-\theta - x_1 + 1 + q_1\theta + 2q_1x_1 - q}{q_1(1-q_1)} \\ &= \frac{x_2-\theta-q_1(x_2-x_1-\theta)}{q_1(1-q_1)} \quad \text{(by } x_1+x_2=1\text{).} \end{aligned}$$

Replace the $1-p_1^A$, $1-p_1^R$ and $p_1^R-p_1^A$ in (60) with the above formulas to rewrite (60) as

$$\begin{aligned} \frac{x_2-q_1\left(x_2-x_1\right)}{1-q_1}x_2 &= \left(\frac{x_2-\theta-q_1\left(x_2-x_1-\theta\right)}{(1-q_1)q_1}\right)\left(\frac{\theta+x_1-1+q_1(1-2x_1)}{q_1}+x_1\right) \\ &= \left(\frac{x_2-\theta-q_1\left(x_2-x_1-\theta\right)}{(1-q_1)q_1}\right)\left(\frac{q_1x_2+\theta-x_2}{q_1}\right), \end{aligned}$$

with the second line due to $x_1+x_2=1$. Simplify the above equation into

$$x_2(x_2-q_1(x_2-x_1)) = \frac{x_2-\theta-q_1(x_2-x_1-\theta)}{q_1}\cdot\frac{q_1x_2+\theta-x_2}{q_1},$$

namely,

$$(q_1)^2\,x_2\left(q_1x_1+(1-q_1)x_2\right) = \left(q_1x_1-(1-q_1)(\theta-x_2)\right)\left(q_1x_2+\theta-x_2\right).$$

Plug $x_2 = 1 - x_1$ into the above displayed equation to obtain

$$
\begin{aligned}
&(q_1)^2 x_2 (q_1(1-x_2) + (1-q_1)x_2) = (q_1(1-x_2) - (1-q_1)(\theta - x_2)) \cdot (q_1 x_2 + \theta - x_2) \\
\iff\ & (q_1)^2 x_2 (q_1(1-2x_2) + x_2) = (q_1(1+\theta-2x_2) + x_2 - \theta) \cdot (q_1 x_2 + \theta - x_2)\,, \\
\iff\ & (q_1)^3 x_2(1-2x_2) + (q_1)^2 (x_2)^2 = (q_1)^2 (1+\theta-2x_2)x_2 + (\theta-x_2)q_1(1+\theta-3x_2) - (\theta-x_2)^2 \\
\iff\ & (q_1)^3 x_2(1-2x_2) + (q_1)^2 x_2(3x_2-1-\theta) + q_1(\theta-x_2)(3x_2-1-\theta) + (\theta-x_2)^2 = 0.
\end{aligned}
$$

Thus, Eq. (58) is true. ■

Lemma 15 *If $(x_i, \sigma_i, p_i^A, p_i^R, q_i)_{i=1}^2$ is a Case-(d) solution such that $p_2^R \geq p_1^R$, $x_1 \geq 1/2 \geq x_2$.*

Proof By Bayes's rule,

$$1 - p_1^R = \frac{\theta\sigma_1(w)}{q_1} \overset{(53)}{=} \frac{\theta - x_2 + q_1(2x_2-1)}{q_1},$$

with the second "=" also using $x_1 + x_2 = 1$. Meanwhile, write (52) into

$$1 - p_2^R = \theta + x_2(q_1 - 1).$$

Thus,

$$
\begin{aligned}
p_2^R \geq p_1^R \iff\ & \frac{\theta - x_2 + q_1(2x_2-1)}{q_1} \geq \theta + x_2(q_1-1) \\
\iff\ & (3x_2 - 1 - \theta)\, q_1 + (\theta - x_2) \geq (q_1)^2 x_2 \\
\iff\ & (3x_2 - 1 - \theta)(\theta - x_2)\, q_1 + (\theta - x_2)^2 \geq (q_1)^2 x_2(\theta - x_2), \qquad (61)
\end{aligned}
$$

with the last line due to $\theta - x_2 > 0$ (Ineq. (57)). Subtract Ineq. (61) by Eq. (58) and cancel some terms to see that Ineq. (61) is equivalent to

$$0 \;\geq\; (q_1)^3 x_2 (1-2x_2) + (q_1)^2 x_2 (3x_2 - 1 - \theta) + (q_1)^2 x_2(\theta - x_2),$$

namely,

$$0 \geq (q_1)^2 x_2 (1-q_1)(2x_2-1)\,.$$

Thus,

$$p_2^R \geq p_1^R \iff 0 \geq (q_1)^2 x_2 (1-q_1)(2x_2-1) \iff 0 \geq 2x_2 - 1,$$

with the second " $\iff$ " due to the fact $q_1 < 1$ in all Case-(d) PBEs. Thus we have $2x_2 \leq 1$, which by $x_1 + x_2 = 1$ means $x_1 \geq 1/2 \geq x_2$, as claimed. ■

Lemma 16 *In any Case-(d) solution, $p_2^R < 2 - 2\theta$ and, if $\theta \geq 2/3$ in addition, then $x_1 < \theta$ and $q_2 < \theta$.*

Proof First, observe a necessary condition for any Case-(d) proposal-PBE pair:

$$q_1 > \frac{\theta - x_2}{1 - x_2}. \tag{62}$$

This follows from plugging (56) into the Case-(d) condition $\sigma_2(s) < 1$, which gives

$$\frac{\theta - x_2}{1 - \theta} \cdot \frac{1 - \theta + x_2(1 - q_1)}{\theta - x_2 + x_2 q_1} < 1.$$

Since $\theta - x_2 > 0$ by (57), the above-displayed inequality simplifies to (62).

Next, we prove $p_2^R < 2 - 2\theta$. It suffices to show $(1 - q_1)x_2 < 1 - \theta$, as the two inequalities are equivalent by (52). Since $(1 - q_1)x_2 < 1 - \theta \iff q_1 > (x_2 + \theta - 1)/x_2$, the desired inequality follows from (62) if

$$\frac{\theta - x_2}{1 - x_2} \geq \frac{x_2 + \theta - 1}{x_2},$$

which is equivalent to

$$(2x_2 - 1)(1 - \theta) \leq 0.$$

The last inequality is true because $x_2 \leq 1/2$ (Lemma 15).

Now assume $\theta \geq 2/3$ to prove $x_1 < \theta$ and $q_2 < \theta$. By (53) and (54), the Case-(d) condition $\sigma_1(s) < 1$ becomes

$$\frac{q_1 - \theta\left(\theta + x_1 - 1 + q_1(1 - 2x_1)\right)/\theta}{1 - \theta} < 1,$$

which simplifies to $q_1 < 1/2$. This, coupled with (62), implies $(\theta - x_2)/(1 - x_2) < 1/2$, namely, $x_2 > 2\theta - 1$. Thus, since $2\theta - 1 \geq 1 - \theta$ (assumption $\theta \geq 2/3$) and $x_2 = 1 - x_1$, $x_1 < \theta$ follows.

To prove $q_2 < \theta$, combine the proved fact $x_1 < \theta$ (i.e., $x_2 > 1 - \theta$) with $\sigma_2(s) < 1$ (part of the definition of Case (d)) to obtain $\sigma_2(s) < x_2/(1 - \theta)$. Plug this and (55) into the definition $q_2 = \theta\sigma_2(w) + (1 - \theta)\sigma_2(s)$ to obtain $q_2 < \theta(1 - x_2/\theta) + (1 - \theta)(x_2/(1 - \theta)) = \theta$. ■

Lemma 17 *If $\theta \geq 2/3$, the lopsided equilibrium given proposal $(\theta, 1 - \theta)$ generates strictly larger total welfare than any Case-(d) solution does.*

Proof Consider any Case-(d) solution $(x_i, \sigma_i, p_i^A, p_i^R, q_i)_{i=1}^2$. As in (48), we have $p_2^R \geq p_1^R$. Then Lemma 1, with the roles between players 1 and 2 switched due to $p_2^R \geq p_1^R$, implies that the total welfare generated by this solution is equal to $2\theta p_2^R + (q_2 - \theta)\left(p_2^R - p_1^R\right)$, which by the fact $q_2 < \theta$ (Lemma 16) is less than $2\theta p_2^R$. Since the total welfare generated by the lopsided equilibrium under proposal $(\theta, 1-\theta)$ is equal to $\theta(3 - 5\theta/2)$ (Lemma 5), the proof is complete if $\theta(3 - 5\theta/2) \geq 2\theta p_2^R$. As in the proof of Lemma 12, this inequality follows from $\theta \geq 2/3$ (assumption) and $p_2^R < 2 - 2\theta$ (Lemma 16). ■

J Suboptimality of Any Equilibrium in Eq. (16)

Equilibria in the form of (16) correspond to Case (c) in Lemma 2. First we show that, within the Case-(c) PBEs, the one admitted by the equal-split proposal maximizes the total welfare.

Lemma 18 *(i) The Case-(c) PBE given the equal-split proposal maximizes the total welfare among all Case-(c) solutions. (ii) At this Case-(c) optimal solution, $p_1^R = p_2^R = 1/2$, $q_2 = 2(1-\theta)$, and the total welfare is equal to θ.*

Proof As defined in Lemma 2, a PBE belongs to Case (c) if and only if its strategy profile satisfies

$$\forall i \in \{1, 2\} : 0 < \sigma_i(w) < 1 = \sigma_i(s). \tag{63}$$

Then Bayes's rule implies $p_i^A = 0$ and hence (by (7)) $q_i p_i^R = 1 - \theta$ for each player i. The best response condition for (63) to constitute a PBE is that $V_i^R(w) - V_i^A(w) = 0$ and $V_i^R(s) - V_i^A(s) \geq 0$ for each player i. Since (63) is symmetric between the two players, let us assume without loss that

$$p_1^R \geq p_2^R. \tag{64}$$

We will see that p_1^R implies $x_2 \leq 1/2$ (Eq. (68)). Thus this assumption is consistent with the labeling of the players in Section 4.1. Apply (18) and (19) to the case $i = 1$ to obtain

$$\begin{aligned} V_1^R(w) - V_1^A(w) &= p_1^R - (1-\theta) - (1-q_2)x_1, \\ V_2^R(w) - V_2^A(w) &= (1-q_1)\left(p_2^R - p_1^A - x_2\right) = (1-q_1)(p_2^R - x_2), \end{aligned}$$

with the last "=" due to $p_i^A = 0$. Thus, the condition $V_i^R(w) - V_i^A(w) = 0$ for both i becomes

$$\begin{aligned} p_1^R &= 1 - \theta + (1-q_2)x_1, \qquad (65) \\ p_2^R &= x_2. \qquad (66) \end{aligned}$$

Plug $q_2 = (1-\theta)/p_2^R$, $x_1 = 1 - x_2$ and (66) into (65) to have

$$p_1^R = 1 - \theta + \left(1 - \frac{1-\theta}{x_2}\right)(1 - x_2) = \frac{\theta + x_2(1-2\theta) - (1-x_2)^2}{x_2}. \tag{67}$$

Thus, Ineq. (64), $p_1^R \geq p_2^R$, is equivalent to

$$\begin{aligned}
\theta + x_2(1-2\theta) - (1-x_2)^2 \geq x_2^2 \quad &\iff \quad \theta(1-2x_2) + x_2(1-x_2) - (1-x_2)^2 \geq 0 \\
&\iff \quad \theta(1-2x_2) + (1-x_2)(x_2 - 1 + x_2) \geq 0 \\
&\iff \quad (1-2x_2)(\theta - 1 + x_2) \geq 0.
\end{aligned}$$

The last inequality in the multiline displayed above is equivalent to either (i) $1 - 2x_2 \geq 0$ and $\theta - 1 + x_2 \geq 0$, namely $1 - \theta \leq x_2 \leq 1/2$, or (ii) $1 - 2x_2 \leq 0$ and $\theta - 1 + x_2 \leq 0$, namely $1/2 \leq x_2 \leq 1 - \theta$, which is impossible due to (1). Thus,

$$p_1^R \geq p_2^R \iff 1 - \theta \leq x_2 \leq 1/2. \tag{68}$$

Denote S for the total welfare generated by the PBE. By Lemma 1 (which applies directly because $p_1^R \geq p_2^R$ here) and the fact $q_1 p_1^R = 1 - \theta$,

$$S = 2\theta p_1^R + (q_1 - \theta)(p_1^R - p_2^R) = 2\theta p_1^R + \left(\frac{1-\theta}{p_1^R} - \theta\right)(p_1^R - p_2^R).$$

Since p_1^R and p_2^R are each a function of x_2 via (66) and (67), it follows that S is a function of x_2. We claim that S is strictly increasing in x_2. To prove that, first calculate:

$$\begin{aligned}
\frac{\partial S}{\partial p_1^R} &= 2\theta + \frac{1-\theta}{p_1^R} - \theta - \frac{1-\theta}{\left(p_1^R\right)^2}\left(p_1^R - p_2^R\right) = \theta + \frac{(1-\theta)p_2^R}{\left(p_1^R\right)^2}, \\
\frac{\partial S}{\partial p_2^R} &= \theta - \frac{1-\theta}{p_1^R}.
\end{aligned}$$

Second, by (66) and (67), we have $dp_2^R/dx_2 = 1$ and

$$\frac{dp_1^R}{dx_2} = \frac{1}{x_2^2}\left(((1-2\theta) + 2(1-x_2))x_2 - \theta - x_2(1-2\theta) + (1-x_2)^2\right) = \frac{1}{x_2^2}(1 - \theta - x_2^2).$$

Then plug them into

$$\frac{d}{dx_2}S = \frac{\partial S}{\partial p_1^R}\frac{dp_1^R}{dx_2} + \frac{\partial S}{\partial p_2^R}\frac{dp_2^R}{dx_2} = \frac{\partial S}{\partial p_1^R} \cdot \frac{1 - \theta - (x_2)^2}{(x_2)^2} + \frac{\partial S}{\partial p_2^R}$$

to obtain

$$
\begin{aligned}
\frac{d}{dx_2}S &= \left(\theta + \frac{(1-\theta)p_2^R}{\left(p_1^R\right)^2}\right)\left(\frac{1-\theta}{(x_2)^2} - 1\right) + \theta - \frac{1-\theta}{p_1^R} \\
&= \left(\theta + \frac{(1-\theta)p_2^R}{\left(p_1^R\right)^2}\right)\left(\frac{1-\theta}{\left(p_2^R\right)^2} - 1\right) + \theta - \frac{1-\theta}{p_1^R} \quad (\text{since } p_2^R = x_2,\ (66)) \\
&= \frac{\theta(1-\theta)}{\left(p_2^R\right)^2} - \theta + \frac{(1-\theta)^2}{\left(p_1^R\right)^2 p_2^R} - \frac{(1-\theta)p_2^R}{(p_1^R)^2} + \theta - \frac{(1-\theta)}{p_1^R} \\
&= \frac{(1-\theta)}{\left(p_1^R\right)^2\left(p_2^R\right)^2}\left[\theta\left(p_1^R\right)^2 + (1-\theta)p_2^R - p_2^R\left(p_2^R\right)^2 - p_1^R\left(p_2^R\right)^2\right] \\
&= \frac{(1-\theta)}{\left(p_1^R\right)^2\left(p_2^R\right)^2}\left[\theta\left(p_1^R\right)^2 + q_2\left(p_2^R\right)^2 - p_2^R\left(p_2^R\right)^2 - p_1^R\left(p_2^R\right)^2\right] \quad (\text{since } q_2p_2^R = 1-\theta) \\
&\geq \frac{(1-\theta)}{\left(p_1^R\right)^2\left(p_2^R\right)^2}\left[\theta\left(p_2^R\right)^2 + q_2\left(p_2^R\right)^2 - p_2^R\left(p_2^R\right)^2 - p_1^R\left(p_2^R\right)^2\right] \quad (\text{since } p_1^R \geq p_2^R) \\
&= \frac{(1-\theta)}{\left(p_1^R\right)^2}\left[\theta + q_2 - p_2^R - p_1^R\right] \\
&= \frac{(1-\theta)}{\left(p_1^R\right)^2}\left[\theta - x_2 + q_2 - p_1^R\right] \quad (\text{since } p_2^R = x_2) \\
&> 0.
\end{aligned}
$$

The inequality at the end holds because, by the fact $q_2 = (1-\theta)/p_2^R = (1-\theta)/x_2$ and (67),

$$
\begin{aligned}
\theta - x_2 + q_2 - p_1^R &= \theta - x_2 + \frac{1-\theta}{x_2} - \frac{\theta + x_2(1-2\theta) - (1-x_2)^2}{x_2} \\
&= \frac{3\theta x_2 + 2 - 2\theta - 3x_2}{x_2} = \frac{(1-\theta)(2-3x_2)}{x_2}
\end{aligned}
$$

is strictly positive because $x_2 \leq 1/2 < 2/3$ due to (64) and (68).

Now that S is strictly increasing in x_2 and $x_2 \leq 1/2$, S is maximized at $x_2 = 1/2$ among all the solutions $(x_i, \sigma_i, p_i^A, p_i^R, q_i)_{i=1}^2$ that belong to Case (c). It follows that the equal-split proposal, $x_1 = x_2 = 1/2$, attains the maximum of S among these solutions. Since it is easy to verify that the Case-(c) solution under this proposal does constitute a PBE, Claim (i) of the lemma is proved.

To prove Claim (ii) of the lemma, plug $x_1 = x_2 = 1/2$ into (65)–(67) to obtain $p_2^R = 1/2$, $q_2 = (1-\theta)/p_2^R = 2(1-\theta)$, and $p_1^R = 1/2$. By $p_1^R = p_2^R = 1/2$ and Lemma 1, the total welfare is equal to θ. Thus Claim (ii) follows. ■

By Lemma 18, the largest total welfare that any Case-(c) solution can achieve is equal to θ. By contrast, the total welfare generated by the lopsided solution $(\theta, 1-\theta)$ is equal to

$\theta(3 - 5\theta/2)$ by Lemma 5. Our assumption $\theta \leq 3/4$ in the proposition implies the desired conclusion $\theta < \theta(3 - 5\theta/2)$.

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