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Corrigendum

Corrigendum to "Core equivalence theorem with production" [J. Econ. Theory 137 (1) (2007) 246–270]

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Abstract

One of the proofs in Xiong and Zheng (2007) has a subtle mistake. This note provides a simple correction without changing the structure of the proof or altering any assumption in the said paper.

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The Core Equivalence Theorem has been established for private ownership productive economies by Xiong and Zheng (2007, Theorem 11). A step in its proof is to show core equivalence for cores defined by a preliminary notion of blocking, wishful blocking (Proposition 6). However, this step contains a subtle mistake. Starting with any type-symmetric allocation $((x_{i_1})_{i_1 \in I_1}, (y_{j_1})_{j_1 \in J_1})$ that is not wishfully blocked in any replica economy, the step (Appendix C, p. 265) defines

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$$\Gamma_{i_1} := \left\{ z_{i_1} \in \mathbb{R}^l : x_{i_1} \prec_{i_1} z_{i_1} + e_{i_1} + \sum_{j_1 \in J_1} \theta_{i_1 j_1} y'_{j_1} \text{ for some } \left(y'_{j_1}\right)_{j_1 \in J_1} \in \prod_{j_1 \in J_1} Y_{j_1} \right\}$$

for any type $i_1 \in I_1$ and claims that the zero vector $\mathbf{0}$ does not belong to the convex hull Γ of $\bigcup_{i_1 \in I_1} \Gamma_{i_1}$, so that a hyperplane separating $\mathbf{0}$ from Γ and passing through $\mathbf{0}$ exists and the rest of the proof follows.

Xiong and Zheng's (2007) argument for this claim is based on an assertion that, given any $z_{i_1} \in \Gamma_{i_1}$ and a $\left(y'_{j_1}\right)_{j_1 \in J_1} \in \prod_{j_1 \in J_1} Y_{j_1}$ that justifies $z_{i_1} \in \Gamma_{i_1}$ (i.e., $x_{i_1} \prec_{i_1} z_{i_1} + e_{i_1} + \sum_{j_1 \in J_1} \theta_{i_1 j_1} y'_{j_1}$), if $(z^k_{i_1})^\infty_{k=1}$ is a sequence converging to z_{i_1} as $k \to \infty$, then $x_{i_1} \prec_{i_1} z^k_{i_1} + e_{i_1} + \sum_{j_1 \in J_1} \theta_{i_1 j_1} y'_{j_1}$ for all sufficiently large k. However, the assertion is not necessarily true. That is because $z_{i_1} + e_{i_1} + \sum_{j_1 \in J_1} \theta_{i_1 j_1} y'_{j_1}$ can be on the boundary of the consumption set $\mathbb{R}^l_+$ so that $z^k_{i_1} + e_{i_1} + \sum_{j_1 \in J_1} \theta_{i_1 j_1} y'_{j_1}$ is not necessarily an element of $\mathbb{R}^l_+$, and hence not necessarily comparable to the bona fide consumption bundle x_{i_1} via the preference relation $\succ_{i_1}$, however large is k.

Note that the boundary possibility pointed out above cannot be ruled out by adding to Prop. 6 the assumption of boundary aversion (Assumption 9, Xiong and Zheng, 2007). That is because preferences with boundary aversion still allow for the case where both x_{i_1} and $z_{i_1} + e_{i_1} + \sum_{j_1 \in J_1} \theta_{i_1 j_1} y'_{j_1}$ have zero coordinates, thereby making it possible for $z^k_{i_1} + e_{i_1} + \sum_{j_1 \in J_1} \theta_{i_1 j_1} y'_{j_1}$ to have negative coordinates.[2]

Nonetheless, the above mistake can be corrected without changing the main structure of the proof of Prop. 6, so that both the proposition and Xiong and Zheng's (2007) core equivalence theorem remain intact. We need only to amend the definition of Γ_{i_1} as follows:

$$\Gamma_{i_1} := \left\{ z_{i_1} \in \mathbb{R}^l : x_{i_1} \prec_{i_1} z_{i_1} + e_{i_1} + \sum_{j_1 \in J_1} \theta_{i_1 j_1} y'_{j_1} \in \mathbb{R}^l_{++} \text{ for some } \left(y'_{j_1}\right)_{j_1 \in J_1} \in \prod_{j_1 \in J_1} Y_{j_1} \right\}.$$

The interior condition "$\in \mathbb{R}^l_{++}$" in the amended definition rules out the boundary issue pointed out above. Since Γ_{i_1} by the amended definition is nonempty ($x_{i_1} + \mathbf{1} \in \Gamma_{i_1}$, with $\mathbf{1} := (1, \ldots, 1) \in \mathbb{R}^l$, due to the strong monotonicity of preferences and the possibility of inaction on the part of the firms) and convex (due to the convexity of preferences and production sets), the convex hull of $\bigcup_{i_1 \in I_1} \Gamma_{i_1}$ is given by Γ described exactly as in the proof in Xiong and Zheng (2007), and the argument for $\mathbf{0} \notin \Gamma$ therein follows. Thus, the Separating Hyperplane Theorem applies and guarantees existence of a nonnull l-dimensional vector p such that $p \cdot z \geq 0$ for all $z \in \Gamma$.

The only other adjustment needed is the derivation of (C.2) on p. 265, since the x'_{i_1} there (immediately below the definition $\pi^*_{j_1} := \sup\left\{p \cdot y'_{j_1} : y'_{j_1} \in Y_{j_1}\right\}$) needs to be any arbitrary—not necessarily strictly positive—commodity bundle preferred by i_1 to x_{i_1}. The adjustment is to note, for all $\alpha > 0$, that $x'_{i_1} \succ_{i_1} x_{i_1}$ implies $x_{i_1} \prec_{i_1} x'_{i_1} + \alpha \mathbf{1} \in \mathbb{R}^l_{++}$ (due to the monotonicity of preferences). This implies that, for all $\left(y'_{j_1}\right)_{j_1 \in J_1} \in \prod_{j_1 \in J_1} Y_{j_1}$, the vector $x'_{i_1} + \alpha \mathbf{1} - e_{i_1} - \sum_{j_1 \in J_1} \theta_{i_1 j_1} y'_{j_1}$ belongs to $\Gamma_{i_1} \subseteq \Gamma$. The separating hyperplane property of p then gives

[2] Assuming $e_{i_1} \in \mathbb{R}^l_{++}$ for all $i_1 \in I_1$ (as in Debreu and Scarf, 1963) in addition to Assumption 9 would nevertheless be a valid alternative, since individual rationality of $((x_{i_1})_{i_1 \in I_1}, (y_{j_1})_{j_1 \in J_1})$ and the possibility of inaction on the part of the firms would then imply $x_{i_1} \in \mathbb{R}^l_{++}$, so that $z_{i_1} + e_{i_1} + \sum_{j_1 \in J_1} \theta_{i_1 j_1} y'_{j_1} \in \mathbb{R}^l_{++}$ as well. However, this additional assumption not only would lead to a weaker result, but would also be inconsistent with other parts of Xiong and Zheng (2007) (see Example 12 and the proof of Proposition 19).

$$\forall \left(y'_{j_1}\right)_{j_1 \in J_1} \in \prod_{j_1 \in J_1} Y_{j_1}: \quad p \cdot \left(x'_{i_1} + \alpha \mathbf{1}\right) \geq p \cdot \left(e_{i_1} + \sum_{j_1 \in J_1} \theta_{i_1 j_1} y'_{j_1}\right),$$

so that

$$p \cdot x'_{i_1} + \alpha\,(p \cdot \mathbf{1}) \geq p \cdot e_{i_1} + \sum_{j_1 \in J_1} \theta_{i_1 j_1} \pi^*_{j_1}.$$

Since this inequality holds for all $\alpha > 0$, (C.2) is true indeed, and the published proof can then be carried out to completion.

We thus conclude that, with the adjustment herein set forth, Proposition 6 in Xiong and Zheng (2007) remains valid in both its statement and the main structure of its proof.

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