Application of Mechanism Design to Electric Power Markets

Carlos Silva, *Member, IEEE*, Bruce F. Wollenberg, *Fellow, IEEE*, and Charles Z. Zheng

***Abstract*—As competition is introduced across the electric power industry around the world, market design for the industry is urgently needed to shape its future structure and performance. When generator companies compete with one another in a deregulated market, they may not be willing to share the information needed to perform an economic dispatch of the generation. Using game theory, this paper designs a new mechanism that achieves efficiency (economic dispatch) in spite of this information problem. In this mechanism, when each company acts in the best of its own interest, the outcome is efficient. The paper demonstrates the merits of the mechanism by simulations including the IEEE 14-bus case.**

***Index Terms*—Economics, game theory, power generation dispatch.**

I. INTRODUCTION

THE APPLICATION of competitive markets has been successful in several sectors of the U.S. economy. However, the application of competition in electric power markets has not been completed. What makes the electric power market different is the presence of a transmission system, where the flow of power cannot be easily controlled and the scarcity of transmission capacity leads to congestion or potential overloading. In spite of the efforts made to create an open, free market, we still have to cope with the transmission congestion problem.

In this paper we propose solving the congestion problem through a technique in economics, called mechanism design. Using this technique and taking the transmission network constraints seriously, we have designed a mechanism such that, when each participant acts in its own best interest, the outcome in the daily operation of the electric power market is efficient (achieving economic dispatch).

Our mechanism differs from the existing proposals for electricity deregulation. These proposals, such as "nodal pricing" by Schweppe [7] and Hogan [8] and "tradable physical rights" by Chao and Peck [9], focus on the concrete pricing methods of transmission rights.

Our work, in contrast, asks a more fundamental question: in a deregulated power market, how should any desirable mechanism (nodal pricing or tradable physical rights, etc.) look like? It turns out that any such mechanism, in an abstract level, looks like a bidding process, which can be summarized as:

Each generator company submits a production cost; based on these costs, an agent (coordination entity, pool operator, power exchange, etc.) allocates electricity production and payments.

This paper provides an explicit formula for such allocations.

If one views an electric power market as consisting of two layers, infrastructure planning and daily operation, then our mechanism concerns the latter, and it takes the companies' investment decision as given. Nevertheless, any two-layer solution needs to deal with the daily operation, so our mechanism provides an efficient daily-operation layer for that solution.

This paper is organized as follows. Section II describes economic dispatch, the standard procedure to deal with the transmission congestion problem in power system engineering. Section III points out that deregulation creates an information problem in economic dispatch. Section IV introduces the basic concepts of mechanism design, new to power system engineers. Section V describes our proposed market mechanism that achieves economic dispatch in spite of the information problem. Through simulations, Section VI demonstrates that the mechanism works. Section VII concludes.

II. PHYSICAL ENVIRONMENT

This section describes the physical environment and the corresponding economic dispatch. Throughout this paper, we use the DC power flow model [4].

A. Economic Dispatch

An economic dispatch can be described as a minimization of the total production cost subject to i) power flow constraints, ii) line thermal limits, and iii) generation limits, i.e.,

$$\min \sum_{i=1}^{N_g} C_i(q_i), \tag{1}$$

subject to:

i) power flow equation for each bus i,

$$\sum_{j\in\Phi_i} \frac{1}{x_{ij}} (\theta_i - \theta_j) = q_i - q_{iL}, \tag{2}$$

ii) line thermal limit constraint for all pairs i–j corresponding to existing lines,

$$Cap_{i,\,\min} \le \frac{1}{x_{ij}} (\theta_i - \theta_j) \le Cap_{i,\,\max}, \tag{3}$$

iii) generation constraints for all generators i,

$$q_{i,\,\min} \le q_i \le q_{i,\,\max}, \tag{4}$$

Manuscript received August 4, 1999; revised August 14, 2000.

C. Silva and B. F. Wollenberg are with the Department of Electrical and Computer Engineering, University of Minnesota, Minneapolis, MN 55455.

C. Z. Zheng is with the Department of Economics, Northwestern University, Evanston, IL 60208-2600.

Publisher Item Identifier S 0885-8950(01)02300-8.

where:

q_i	—the quantity of power produced by the generator at bus i (in economics it is conventional to use the letter p for price and q for quantity).
$C_i(q_i)$	—the cost of generator i as a function of.
N_g	—the number of generators.
θ_i	—bus i's phase angle.
q_{iL}	—bus i's load.
Φ_i	—the set of lines connected to bus i.
$Cap_{i,\min}$, $Cap_{i,\max}$	—line i's minimum and maximum thermal capacities, respectively.
$q_{i,\min}$, $q_{i,\max}$	—the minimum and maximum quantities that generator i can produce, respectively.

Solving this problem requires that the system coordination entity know the true cost functions of all generators.

B. *Physical Environment Assumptions*

1) Linear Cost Functions: In order to simplify the application of mechanism design, this paper assumes that a generator's cost is a linear function of its output (power). Thus, a generator's cost is determined by a constant marginal cost.

2) Static Load: This paper assumes that loads are constant. That is, the quantity of demand for electric power at bus i is a given constant, for all i.

3) Non-Empty Feasible Set: We assume that the feasible set defined by constraints (2)–(4) is nonempty. Without such an assumption, the economic dispatch problem would have no solution anyway.

C. *Pareto Efficiency: An Aside*

Interestingly, economic dispatch is consistent with Pareto efficiency, the standard efficiency criterion in economics. An allocation of resources is said to be Pareto efficient if there is no other feasible allocation that makes some participant better-off and no others worse-off.

For example, consider a one-bus system with two generators and a load. Let us temporarily assume in this paragraph that marginal costs increase with output levels. On one hand, Pareto efficiency requires that the generators produce at the same marginal cost. Otherwise, we could have the generator with higher marginal cost produce less (thereby lowering its marginal cost) and the other produce more (thereby raising its marginal cost). One readily sees that such a re-allocation reduces the total cost. On the other hand, operating at identical marginal cost is also a requirement of economic dispatch, as is well-known in power system engineering.

III. THE INFORMATION PROBLEM DUE TO DEREGULATION

The electric power market deregulation creates an information problem to the economic dispatch. When firms compete with one another in the deregulated market, a generator generally will not be willing to provide its production costs. Consequently, the marginal cost of generator i becomes its private information. Therefore, if one still dispatches electric power production according to economic dispatch, then the outcomes are likely to be inefficient, because the marginal costs may be misrepresented.

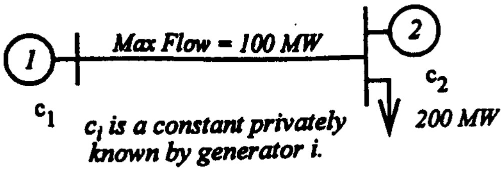

Fig. 1. 2-bus example.

A. *Inefficiency in Economic Dispatch*

The example in Fig. 1 shows how the above information problem makes the economic dispatch allocation inefficient.

Notice that generator 2 will always be assigned a certain amount of production. Therefore, it has an incentive to exaggerate its marginal cost, because it is guaranteed to produce at least 100 MW. Then the outcome of economic dispatch is inefficient. To see this, imagine the case when the true marginal cost of generator 1 is higher than that of generator 2. Given the true marginal costs, efficiency would require generator 2 to produce the entire 200 MW. However, since true marginal costs are private information, and generator 2 exaggerates its cost, economic dispatch would assign only 100 MW for generator 2, which is an inefficient allocation.

B. *Information Assumptions*

This subsection formulates the above information problem into the following assumptions.

1) Private and Public Information: We assume that the marginal cost of a generator is its private information. The reason is that a generator knows its own inputs (e.g., fuel, boiler efficiency, etc.) better than anyone else. With generators competing in a deregulated environment, they are not willing to provide their cost unless given sufficient incentive to do so.

We also assume that a generator j regards the marginal cost of another generator i ($i \neq j$) as a random variable from a commonly known distribution. This common knowledge may be obtained from the publicly available information about a generator's technology.

The other parameters of an electric system, including transmission line capacities, line admittances, loads and the technical limits of the generators, are assumed to be common knowledge among all market participants.

2) The Distribution of Marginal Costs: We further assume that the distribution of a generator's (true) marginal cost has the following properties:

- The domain of possible marginal cost is bounded. In other words, others know that generator i's marginal cost lies between ceiling and floor values.
- The probability density function of the distribution is continuous.

Many distribution functions satisfy the above properties. Fig. 2 shows two of such examples.

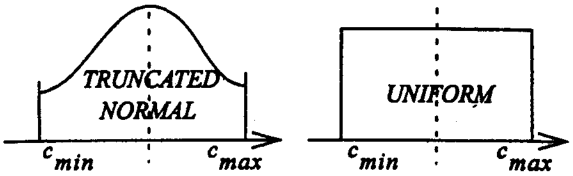

Fig. 2. Distribution functions of the true marginal cost.

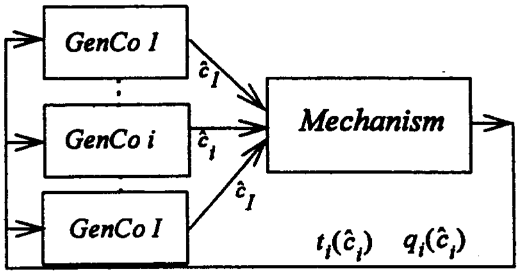

Fig. 3. Mechanism.

IV. SOME ECONOMICS

To overcome the information problem we described, economists have developed a technique called mechanism design. This section introduces the basic concepts of this technique.

A. What is a Mechanism?

A mechanism is a set of rules defining a game played by a group of participants. The purpose of these rules is to achieve a certain outcome by providing appropriate incentives to the participants. A mechanism may be decentralized (free markets, open auctions, etc.), centralized (the military, etc.), or any form in between (government-regulated markets, etc.)

In our specific application, we focus on the following type of mechanisms: Each participant (generator) submits a marginal cost (which does not need to be the true one) to an agent (coordination entity, pool operator, power exchange, etc.). Given the submitted costs, the agent allocates production tasks and payments to the participants. This is illustrated by Fig. 3, where $\hat{c}_i$ is the marginal cost submitted by i, t_i is the payment to i, and q_i is the production assigned to i.[1]

In general, a mechanism is required to satisfy several properties in order for it to serve the designer's objective. We introduce these properties below.

B. Incentive Compatibility

A mechanism is said to be incentive compatible if it is optimal for each participant to submit its true private information (true marginal cost in our application) provided that all other participants submit their true information also. A succinct way of saying this in mechanism design is "truth-telling is an equilibrium."

To make a mechanism incentive compatible in our case, we need to provide sufficient incentives to the generator companies, so that it is profit-maximizing for each to submit its true marginal cost. We should point out a subtle point here. Incentive compatibility does not mean that a participant would always tell the truth; instead, it means, "I would tell the truth if others tell the truth also."

C. Individual Rationality

A mechanism is said to be individually rational if no participant would lose profit at the truth-telling equilibrium. This condition is needed because participants usually have an outside option of quitting. In our application, for example, a generator company can turn off its plant and get zero profit.

D. Feasibility

A mechanism is said to be feasible if its allocations satisfy all physical constraints. In our application, this means that the electricity production dispatched by the mechanism needs to satisfy all the feasibility constraints given by the power flow (2), line thermal limits (3) and generation limits (4).

V. THE MECHANISM

In our application, we need to design a mechanism that is incentive compatible, individually rational and feasible, and dispatches electricity production efficiently in spite of the information problem. Such a desirable mechanism is presented below.

A. Description of the Mechanism

As we discussed in Section IV, a mechanism in our application needs to specify the production assignments and the payments to the generators as functions of the marginal costs submitted by the generators. To assure that the mechanism is feasible and achieves efficiency, each production assignment needs to be a solution of the economic dispatch problem based on the true marginal costs. Consequently, if we temporarily assume that participants tell the truth due to a suitable payment function that we will specify later, the production assignment function is easy to pin down: it is a solution of the economic dispatch problem based on the generators' submitted marginal costs.

Therefore, what we need is to design a payment scheme that makes the mechanism incentive compatible, i.e., that induces each participant to submit its true marginal cost. To do that, we look at each participant's optimization on what to submit. If a mechanism is indeed incentive compatible, then each participant would think it is optimal to submit its true marginal cost. Solving this optimization problem, we narrow the possible candidates for such a payment scheme into a family of payment functions that differ from one another by a constant. We then choose the payment scheme that satisfies the individual rationality condition. (For details of how to get this function see the Appendix.)

The following mechanism was first proposed and proven correct for a 3-bus network in [5], and in Section VI is shown by simulation to meets all the desirable properties.

The mechanism goes as follows:

- Ask generator i (from $i = 1$ to Ng) to submit its marginal cost, $\hat{c}_i$.

---

[1] There is no loss of generality in restricting this type of mechanisms, because a well known principle in mechanism design. Roughly speaking, this principle says that any conceivable mechanism in our application is equivalent to a mechanism of the above type [2].

- Assign to generator i the production resulting from the calculation described in Section II-A, which uses the marginal costs the generators have submitted. Let $q_i(\hat{c}_1, \ldots, \hat{c}_{N_g})$ denote the quantity generator i is assigned to produce.
- Pay each generator i a "cost compensation,"

$$\hat{c}_i \cdot q_i(\hat{c}_1, \ldots, \hat{c}_{N_g}), \tag{5}$$

and an "information compensation,"

$$\tau_i(\hat{c}_1, \ldots, \hat{c}_{N_g}) = \frac{1}{\Psi_i(\hat{c}_i)} \int_{\hat{c}_i}^{\overline{c}_i} \overline{q}_i(x)\, dx, \tag{6}$$

where

$\hat{c}_i$	is the marginal cost provided by i,
$\overline{q}_i(\hat{c}_i)$	is i's amount of production generator i will expect before the others submit their costs, and
$\Psi_i(\hat{c}_i)$	is the probability assessed by generator i at that time for the event that he will get to produce some quantity of electric power.

The above mechanism achieves all of the following:

- It induces every generator to provide its true marginal cost (incentive compatibility).
- It dispatches electricity production efficiently (feasibility and efficiency).
- It guarantees that no generator would lose profit if it submits its true marginal cost (individual rationality).

B. Interpretation of the Payment Scheme

The payment consists of two parts. The first one is a cost compensation according to the marginal costs claimed by the generators. The second one, which is indispensable due to the information structure, is a compensation for the information advantage of the generator. This part is precisely tuned so that each generator is willing to provide the true marginal cost.

VI. SIMULATIONS

This section demonstrates the merits of our mechanism by simulation on a 2-bus network and the IEEE 14-bus network. Our simulation first approximates the probability distribution of a generator's marginal cost by a finite grid, defining a set of possible costs, each one with an associated probability. For each combination of such marginal costs across the generators, we calculate the corresponding optimal power flow. From these results, we compute both the cost and the information compensation (6), for each point of the grid. These give us an approximation of a generator's expected profit as a function of its submitted cost. The simulation results show that our mechanism is efficient whether the transmission network suffers a capacity shortage or not.

A. 2-Bus Case

This is the simplest case to consider the burden of congestion in a transmission system.

1) Description of the System: The data for this case is shown in Fig. 4 and Table I.

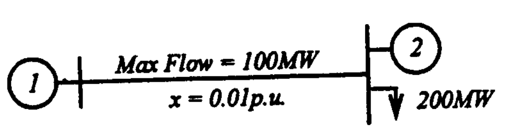

Fig. 4. 2-bus case.

TABLE I
GENERATOR DATA FOR THE 2-BUS CASE

Generator (by location)	True Cost ($/MWh)	Maximum Production (MW)	Minimum Production (MW)
1	28	150	0
2	18	300	0

TABLE II
PROBABILITY DISTRIBUTION DATA FOR THE 2-BUS CASE

Generator (by location)	C_{min} ($/MWh)	C_{max} ($/MWh)	Mean ($/MWh)	Standard Deviation
1	20	38	30	3
2	15	30	20	4

In this case, every generator company, while knowing its true cost, assumes the costs of its competitors are drawn from a truncated normal distribution (between $C_{\min}$ and $C_{\max}$).

In Table II we can see the generator cost probability distributions.

Since the transmission line between 1 and 2 allows only 100 MW we have a potential problem of congestion. Thus, left alone, generator company 2 would exaggerate its marginal cost in order to raise its expected revenue.

Let us analyze from generator 2's perspective. This company is going to claim the cost that maximizes its expected profit when asked to provide its marginal cost.

Generator 2's revenue depends on the payment it gets for its power. Below we contrast a traditional payment policy and ours.

2) A Traditional Mechanism: This mechanism simply pays the total cost (i.e., power times cost) claimed by a generator. In Fig. 5 we can see generator 2's profit as a function of its marginal cost provided in this traditional mechanism.

As seen in Fig. 5, due to its location, generator 2's optimal strategy is to claim its upper bound, 30 ($/MWh). Since generator 2 is distorting its real cost, the outcome of the economic dispatch is not likely to be efficient.

3) Our Proposed Mechanism: On the other hand, the mechanism described in Section V includes an extra payment that would correct generator 2's expected profit function. This payment eliminates any gain from overstating one's marginal cost [in this case 18 ($/MWh)].

In Fig. 6 we can see both this extra payment and the resulting profit function for generator 2.

The new profit function has a maximum at the generator 2's true cost, therefore, we can expect efficiency. In this case there is a wide range of marginal cost that maximize generator 2's profit, but it still has no incentive to distort its true cost.

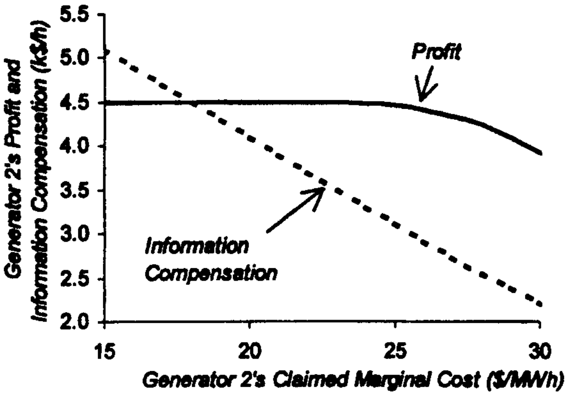

Fig. 5. Generator 2's profit using a *traditional* mechanism.

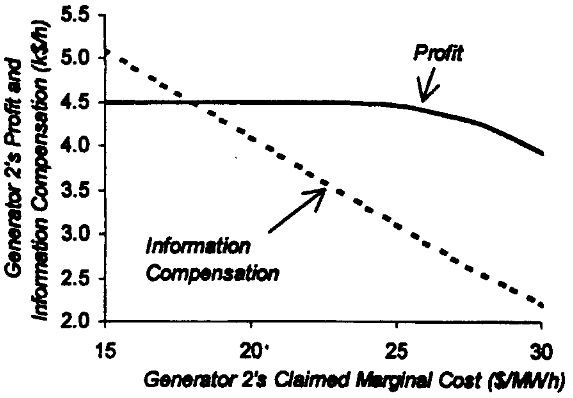

Fig. 6. Generator 2's profit and information compensation using our mechanism.

B. 14-Bus Case

As a second example we used a slightly modified version of the IEEE 14-bus test case. This system includes 21 transmission lines and 5 generators.

1) Description of the System: The system was divided into two areas, as shown in Fig. 7. The capacity of each interface between the two areas (A and B) is 100 MW.

The generator data is included in Fig. 7 and Table III.

Again in this case, every generator company, while knowing its true cost, assumes the costs of its competitors are drawn from a truncated normal distribution (between $C_{\min}$ and $C_{\max}$). In Table IV we can see the generator cost probability distributions.

Our simulations focus on the behavior of generator 6 in two scenarios: i) interfaces A and B operating, so there is enough transfer capacity between area 1 and area 2, so that generator 6 must compete in the market without market power, ii) only interface A is operating, giving generator 6 the opportunity to be a local monopoly with market power.

The expected production vs. generator 6's claimed marginal cost can be seen in Fig. 8 for both scenarios. Notice that in scenario ii), generator 6 is dispatched "out-of-merit" for 70 MW due to congestion in interface A.

As always, this company is going to claim the cost that maximizes its expected profit when asked to provide its marginal cost. As we know, generator 6's revenue depends on the payment it gets for its production. Below we contrast the

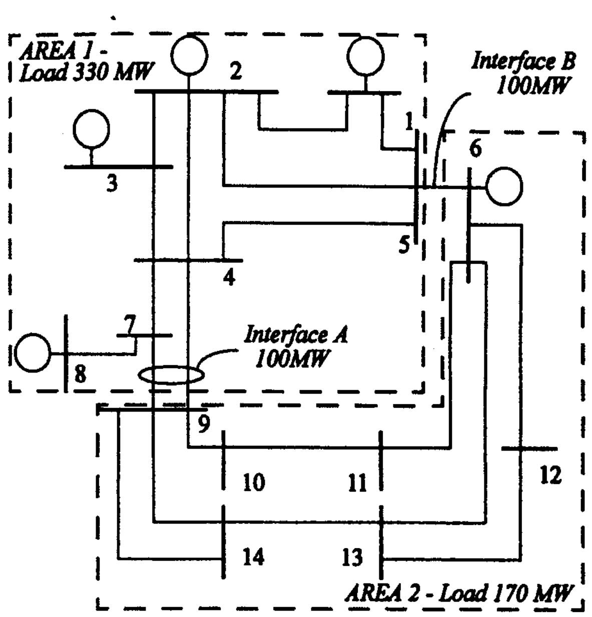

Fig. 7. IEEE 14-bus case.

TABLE III
GENERATOR DATA FOR THE IEEE 14-BUS CASE

Generator (by location)	True Cost ($/MWh)	Minimum Production (MW)	Maximum Production (MW)
1	4.5	0	600
2	8.0	0	600
3	4.0	0	600
6	6.0	0	600
8	8.0	0	600

TABLE IV
PROBABILITY DISTRIBUTION DATA FOR THE IEEE 14-BUS CASE

Generator (by location)	$C_{\min}$ ($/MWh)	$C_{\max}$ ($/MWh)	Mean ($/MWh)	Standard Deviation
1	3.0	8.0	5.0	2.0
2	6.0	10.0	9.0	2.0
3	3.0	6.0	5.0	1.0
6	2.0	18.0	5.5	3.0
8	5.0	9.0	8.0	1.0

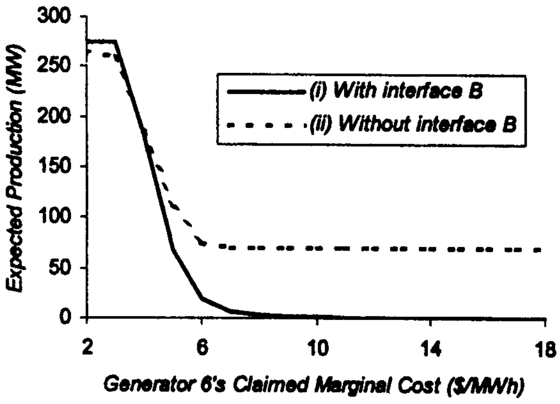

Fig. 8. Generator 6's expected production vs. its claimed marginal cost.

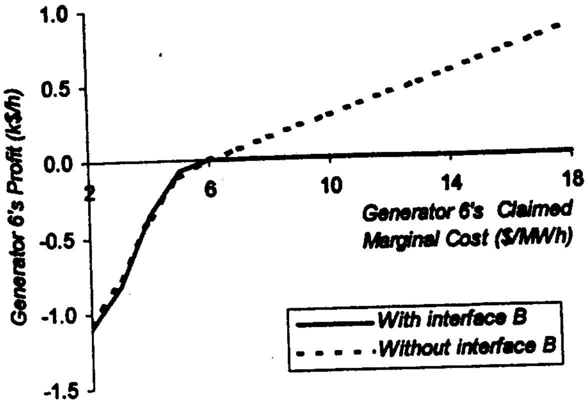

Fig. 9. Generator 6's profit vs. its claimed marginal cost using a traditional mechanism.

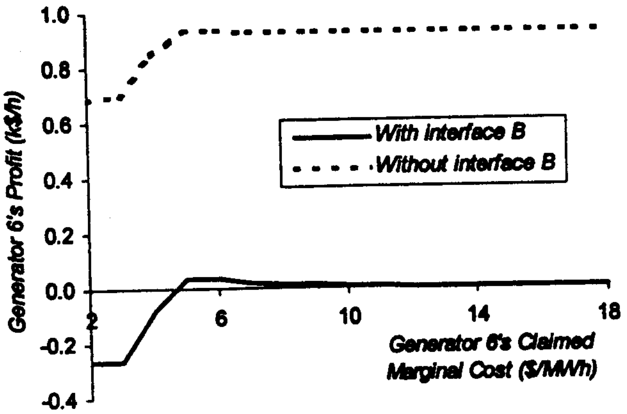

Fig. 10. Generator 6's profit vs. its claimed marginal cost using our mechanism.

previous traditional mechanism with ours in this setting for the two scenarios.

2) A Traditional Mechanism: As previously described, this mechanism simply pays the total cost claimed by a generator. Fig. 9 shows generator 6's profit as a function of its claimed marginal cost.

As seen in Fig. 9, in the first scenario generator 6's optimal strategy is to slightly exaggerate its marginal cost to 7 ($/MWh). The problem is that, claiming 7 ($/MWh), generator 6 is going to be assigned only one third of its corresponding economic dispatch production. In the second scenario generator 6's optimal strategy is to exaggerate its marginal cost to the highest level, 18 ($/MWh). Again causing its assignment to shift 5 MW from optimality (economic dispatch).

In both scenarios, since generator 6 is distorting its real cost, the outcome of the economic dispatch is not efficient. Notice that without interface B generator 6 is in a better position due to its monopoly status.

3) Our Proposed Mechanism: The mechanism described in Section V includes an extra payment that would correct generator 6's expected profit function. This payment eliminates any gain from overstating one's marginal cost, i.e., claiming a cost other than 6 ($/MWh).

In Fig. 10 we can see the resulting profit curves for both proposed scenarios. Fig. 10 shows that generator 6's best choice is to claim its true cost, since its profit is maximum at that cost.

The same argument applies to other generators, so we can expect efficiency.

Thus, whether interface B is operative or not, our mechanism is efficient and the traditional mechanism is inefficient. Furthermore, the inefficiency of the traditional mechanism is more severe when interface B is not operative, since a shortage in transmission capacity increases the monopoly power of some generator companies in the traditional mechanism.

VII. CONCLUSION

This paper points out the information obstacle to efficiently allocate electricity production in a deregulated environment.

Using game theory, we have designed a new mechanism that resolves this information problem. Simulations have demonstrated the desirable properties of this mechanism.

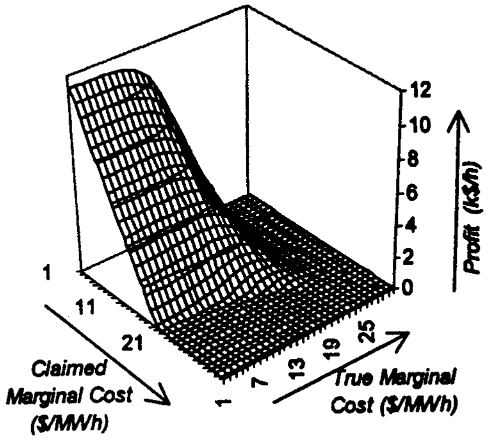

Fig. 11. Profit function.

The next step of this work will be to extend the model to include consumer companies and test the model on large systems.

APPENDIX
PAYMENT FUNCTION

In this appendix we show an informal derivation of the payment function for the mechanism described in Section V.

In our setting, a participant tries to maximize its expected profit, which is defined as the expected revenues minus the expected cost. The expected revenues are the sum of a cost payment, $\hat{c}_i \cdot \overline{q}_i(\hat{c}_i)$ and an information payment, $\overline{r}_i(\hat{c}_i)$. On the other hand, the total costs are given by $c_i \cdot \overline{q}_i(\hat{c}_i)$. The expression for the expected profit function, depending on the true cost, c_i, and the claimed cost, $\hat{c}_i$ would be:

$$\pi_i(\hat{c}_i, c_i) = (\hat{c}_i - c_i) \cdot \overline{q}_i(\hat{c}_i) + \overline{r}_i(\hat{c}_i). \tag{A.1}$$

Fig. 11 is an example of the function $\pi_i(\hat{c}_i, c_i)$ in a two-dimensional domain.

To achieve incentive compatibility, the profit function should not give any incentive to generator i to deviate its claim from its true cost. For example, if we fix the profit function at any feasible true cost [e.g., 16 ($/MWh)] the resulting curve is maximizes at a marginal cost provided equal to true cost level [16 ($/MWh) as shown in Fig. 12].

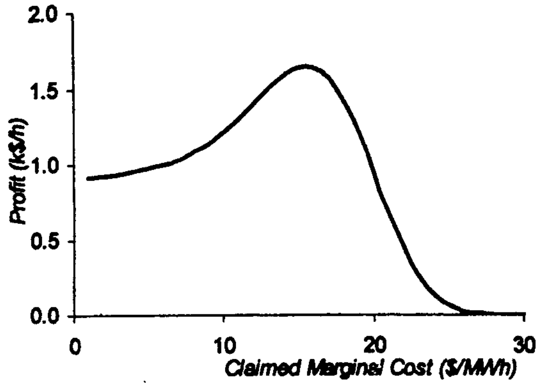

Fig. 12. Profit vs. claimed marginal cost.

Let us define the function $s(.)$ as the optimal marginal cost to claim given a true cost:

$$\hat{c}_i = s(c_i). \tag{A.2}$$

So equation (A.1) can be rewritten as,

$$\pi_i(s(c_i), c_i) = (s(c_i) - c_i) \cdot \overline{q}_i(s(c_i)) + \overline{\tau}_i(s(c_i)). \tag{A.3}$$

If we take the first derivative of $\pi_i(s(c_i), c_i)$ with respect to c_i we get,

$$\frac{d\pi_i(s(c_i), c_i)}{c_i} = D_1\pi_i(s(c_i), c_i) \cdot \frac{ds(c_i)}{dc_i} + D_2\pi_i(s(c_i), c_i). \tag{A.4}$$

By the first order necessary condition of the maximization in $\hat{c}_i$, the first derivative of $\pi_i(s(c_i), c_i)$ with respect to $\hat{c}_i = s(c_i)$ is zero. Therefore the above expression can be reduced to:

$$\frac{d\pi_i(s(c_i), c_i)}{c_i} = D_2\pi_i(s(c_i), c_i) = -\overline{q}_i(s(c_i)). \tag{A.5}$$

The incentive compatibility property implies that for any true cost, c_i, the optimal claimed cost equals the true value,

$$\hat{c}_i^* = s(c_i) = c_i. \tag{A.6}$$

so (A.5) would be,

$$\frac{d\pi_i(c_i, c_i)}{c_i} = -\overline{q}_i(c_i). \tag{A.7}$$

Applying the integral operator between the upper bound of the probability distribution, $\overline{c}_i$, and a generic c_i, we get,

$$\int_{\overline{c}_i}^{c_i} \frac{d\pi_i(c_i', c_i')}{dc_i'} \cdot dc_i' = \pi_i(c_i, c_i) - \pi_i(\overline{c}_i, \overline{c}_i) = \int_{c_i}^{\overline{c}_i} \overline{q}_i(c_i') \cdot dc_i'. \tag{A.8}$$

Replacing the value of π_i from equation (A.1) we get,

$$(c_i - c_i)\overline{q}_i(c_i) + \tau_i(c_i) - (\overline{c}_i - \overline{c}_i)\overline{q}_i(\overline{c}_i) - \tau_i(\overline{c}_i) = \int_{c_i}^{\overline{c}_i} \overline{q}_i(c_i')\, dc_i'. \tag{A.9}$$

canceling out the zero terms,

$$\tau_i(c_i) - \tau_i(\overline{c}_i) = \int_{c_i}^{\overline{c}_i} \overline{q}_i(c_i') \cdot dc_i'. \tag{A.10}$$

Finally, the generic function $\tau_i(c_i)$ that solves the above is,

$$\tau_i(c_i) = \int_{c_i}^{\overline{c}_i} \overline{q}_i(c_i') \cdot dc_i' + K. \tag{A.11}$$

In this application we set K to zero, but any real number can be used if it does allow the individual rationality property to hold (some negative numbers will not do it).

In order to prove incentive compatibility, we subtract the profit function π_i for the real cost from the profit for any given cost. If the result is nonpositive, generator i will never get a positive gain if it deviates from its true marginal cost.

$$\begin{aligned}\pi_i(\hat{c}_i, c_i) - \pi_i(c_i, c_i)\\ = (\hat{c}_i - c_i)\overline{q}_i(\hat{c}_i) + \int_{\hat{c}_i}^{\overline{c}_i} \overline{q}_i(c_i')\, dc_i' - \int_{c_i}^{\overline{c}_i} \overline{q}_i(c_i')\, dc_i'.\end{aligned} \tag{A.12}$$

Regrouping,

$$= (\hat{c}_i - c_i) \cdot \overline{q}_i(\hat{c}_i) + \int_{\hat{c}_i}^{c_i} \overline{q}_i(c_i') \cdot dc_i'. \tag{A.13}$$

Then,

$$\pi_i(\hat{c}_i, c_i) - \pi_i(c_i, c_i) = (\hat{c}_i - c_i) \cdot (\overline{q}_i(\hat{c}_i) - \overline{q}_i(\xi)). \tag{A.14}$$

for some value ξ strictly between c_i and $\hat{c}_i$ (mean-value theorem of integration). Considering that $\overline{q}_i(\hat{c}_i)$ is a decreasing function we need to analyze two cases in order to figure out the sign of the equation above.

— If c_i is greater than $\hat{c}_i$ then $(\hat{c}_i - c_i)$ is negative and $\overline{q}_i(\hat{c}_i) - \overline{q}_i(\xi)$ is positive because ξ is greater than $\hat{c}_i$ [and $\overline{q}_i(\hat{c}_i)$ is decreasing]. Therefore the whole expression is negative.

— If $\hat{c}_i$ is greater than c_i then $(\hat{c}_i - c_i)$ is positive and $\overline{q}_i(\hat{c}_i) - \overline{q}_i(\xi)$ is negative because ξ is greater than $\hat{c}_i$ [and $\overline{q}_i(\hat{c}_i)$ is decreasing]. Therefore the whole expression is negative. Therefore $\pi_i(c_i, c_i)$ is always greater or equal than $\pi_i(\hat{c}_i, c_i)$, and the incentive compatibility property holds.

□

REFERENCES

[1] F. S. Hillier and G. J. Lieberman, *Introduction to Operations Research*, 6th ed: McGraw-Hill Series, 1995.
[2] A. Mas-Colell, M. Whinston, and J. Green, *Microeconomic Theory*: Oxford University Press, 1995.
[3] H. R. Varian, *Microeconomics Analysis*, 3rd ed: W. W. Norton & Company, Inc., 1992.
[4] A. J. Wood and B. F. Wollenberg, *Power Generation, Operation, and Control*, ser. Prentice-Hall Series in Electrical and Computer Engineering: John Wiley & Sons, 1996.
[5] C. Z. Zheng, "Efficient electricity dispatch among deregulated firms," Department of Economics, University of Minnesota, Minneapolis, MN, Working paper, 1999.
[6] F. Wu, P. Varaiya, P. Spiller, and S. Oren, "Folk theorems on transmission access: Proofs and counterexamples," *Journal of Regulatory Economics*, vol. 10, pp. 5–23, 1996.
[7] F. Schweppe, M. Caramanis, R. Tabors, and R. Bohn, *Spot Pricing of Electricity*. Boston, MA: Kluwer Academic, 1988.
[8] W. W. Hogan, "Contract networks for electric power transmission," *Journal of Regulatory Economics*, vol. 4, pp. 211–242, 1992.
[9] H. Chao and S. Peck, "A market mechanism for electric power transmission," *Journal of Regulatory Economics*, vol. 10, pp. 25–59, 1996.

Carlos Silva received both the B.Sc. and M.Sc. degrees in electrical engineering from Catholic University of Chile in 1994 and 1996, respectively. He is currently a Ph.D. candidate and a Research Assistant in the Department of Electrical and Computer Engineering at the University of Minnesota-Twin Cities.

Bruce F. Wollenberg graduated from Rensselaer Polytechnic Institute with a B.S.E.E. (1964) and M.Eng. (1966) in electric power engineering, he then attended the University of Pennsylvania, graduating with a Ph.D. (1974) in systems engineering. He worked for Leeds and Northrup Co. North Wales, PA from 1966 to 1974, Power Technologies Inc. Schenectady, NY from 1974 to 1984, and Control Data Corporation Energy Management System Division Plymouth, MN from 1984 to 1989. He took a position as a Professor of Electrical Engineering in the Electrical and Computer Engineering Department at the University of Minnesota in September 1989.

Charles Z. Zheng received his Bachelor's degree in liberal arts from Lawrence University at Wisconsin in 1993 and his Ph.D. in economics from the University of Minnesota in 1999. He is currently an Assistant Professor of Economics at Northwestern University. His research interests include mechanism design, auction theory, electricity deregulation, and bankruptcy.