**Microeconomics Core Exam Question**

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**Question** Consider a pure-exchange economy with two kinds of goods and two consumers. Consumer 1 is endowed with one unit of good 1 and none of good 2; consumer 2 is endowed with one unit of good 1 and one unit of good 2. Their consumption sets are each $\mathbb{R}^2_+$, and their utility functions $u_1$ and $u_2$ are defined by, for each consumption bundle $(x, y) \in \mathbb{R}^2_+$,

$$\begin{aligned} u_1(x,y) &:= \max\{x,y\}, \\ u_2(x,y) &:= \min\{x,y\}. \end{aligned}$$

a. On a clearly labeled Edgeworth box, first locate the coordinate of the endowment point and label it by $E$, then graph the indifference curves corresponding to the following equations (and be precise about the coordinate position of each graph).

   i. $u_1(x_1, y_1) = 1/4$ for Consumer 1

   ii. $u_2(x_2, y_2) = 1/4$ for Consumer 2

b. For each of the following items, write down the explicit solution for this economy, in set-theoretic notations, and label the solution in a diagram with clearly marked axes, origins, coordinates and indifference maps:

   i. The set of all Pareto optimal allocations

   ii. The set of all Walras equilibrium allocations (be sure to specify the supporting price vectors)

   iii. The set of all price equilibria with transfers (be sure to specify the supporting price vectors)

   iv. The core

c. For each of the following theorems, point out whether the theorem is applicable to this economy and explain your answer concisely, in no more than 20 words (i.e., for each theorem, check whether all its conditions are satisfied or not):

   i. The *first* fundamental theorem of welfare economics

   ii. The *second* fundamental theorem of welfare economics

**Solution** Part (a): Figure 1. (Note that the graph of $u_1 = 1/4$ does not go beyond the box, whereas that of $u_2 = 1/4$ does.)

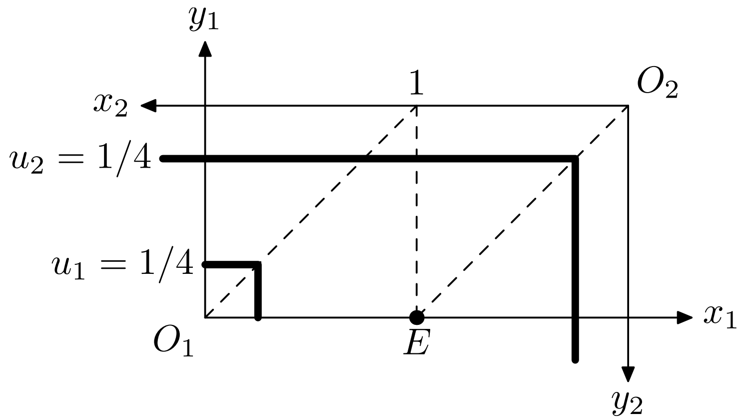


Figure 1: Solution for Part (a)

Part (b):

i. The set of Pareto optima corresponds to the grey area $O_2EF$ in Figure 2. Any point

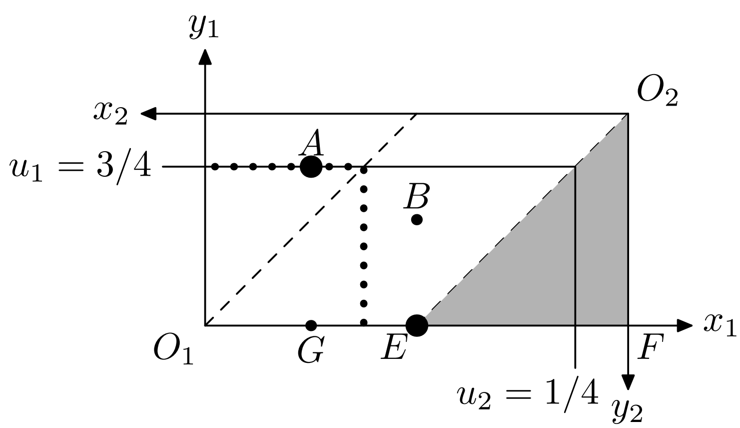


Figure 2: Solution for Part (b.i)

in the Edgeworth box outside the grey area and above line $O_1E$ is Pareto dominated by some point to its southeast (e.g., point $A$ is dominated by $B$, with the dotted line being Consumer 1's indifference curve passing through $A$, and the upside-down L-shape labeled "$u_2 = 1/4$" Consumer 2's indifference curve passing through $A$). Any point on segment $O_1E$, exclusive $E$, is Pareto dominated by points to its east (e.g., $G$ is dominated by $E$).

ii. Walras equilibria: There is only one WE: the endowment point $E$, supported by any

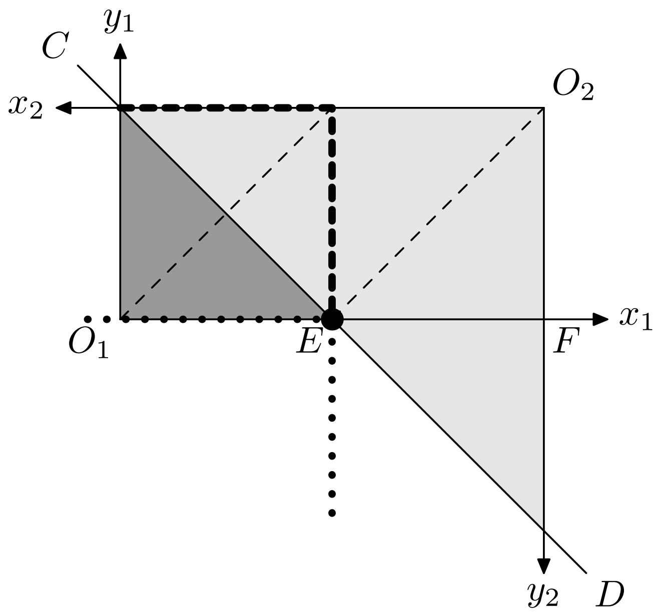


Figure 3: Solution for Part (b.ii)

price vector $(p_1, p_2) \in \mathbb{R}_{++}$ such that $p_1/p_2 \in (0, 1]$, e.g., line $CD$ in Figure 3. The thick dashed line is Consumer 1's indifference curve passing through $E$, and the thick dotted line is Consumer 2's. Given price line $CD$, Consumer 1's budget set is the dark area in Figure 3, and Consumer 2's is the light grey area in that figure. (See part (iii) for explanation of unicity.)

iii. It is the singleton consisting of the endowment point $E$, with supporting price vectors same as (ii). By the first welfare theorem (valid because preferences here are locally nonsatiable), we need only to consider the Pareto optima, the points in the triangle area $EFO_2$. Any price equilibrium, with or without transfers, is an optimal choice for each consumers. Thus, from Consumer 1's perspective, the equilibrium point belongs to segment $EF$. For any point on $EF$ to be optimal to Consumer 1, the price line needs to have a slope between zero and negative one. With such price lines, $E$ is the only point that can be supported as an optimal choice for Consumer 2.

iv. Core: The singleton consisting of the endowment point $E$ in Figure 2.

Part (c): (i) Yes because preferences are locally nonsatiable. (ii) No because preferences are not all convex.