Chapter 5: Consumer's Decision

Elements of Decision: Lecture Notes of Intermediate Microeconomics

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1 Budget constraint

Chapter 4 has presented a foundation of quantitative representations for a decision-maker's objective. Thus we can consider a consumer's objective, in deciding which consumption bundle to buy, as a utility function. To formalize this decision, we need to specify the set of feasible alternatives for the consumer. For simplicity, assume that there are only two goods; the market price is p_1 dollars per unit of good 1, and p_2 dollars per unit of good 2; and the consumer's income to spend on these goods is a fixed amount m dollars. Thus, any consumption bundle (x_1, x_2) he chooses needs to satisfy the *budget constraint* that the total expense does not exceed his income:

$$p_1x_1 + p_2x_2 \le m.$$

This, coupled with the nonnegativity constraint that the quantities x_1 and x_2 be nonnegative, constitutes the *budget set*, the set of the consumer's feasible alternatives:

$$\left\{(x_1, x_2) \in \mathbb{R}^2_+ : p_1x_1 + p_2x_2 \le m\right\}.$$

As long as p_1, p_2 and m are positive, the budget set corresponds to, on the x_1-x_2-plane, the triangle area bounded by the two axes and the *budget line*

$$p_1x_1 + p_2x_2 = m, \tag{1}$$

a straight line of slope $-p_1/p_2$ and x_2-intercept m/p_2. The x_2-intercept m/p_2 is simply the maximum quantity of good 2 he can have (by spending all income on good 2); the x_1-intercept m/p_1 is analogous. The slope $-p_1/p_2$ corresponds to the rate of exchange between the two goods: Suppose the consumer is maxing out his income, then to consume one additional unit of good 1 he needs to cut his consumption of good 2 thereby coming up with an additional p_1 dollars; exactly what is the quantity of good 2 does he need to reduce? Since each unit of good 2 costs him p_2 dollars, the p_1 dollars would have afforded him p_1/p_2 units of good 2. Thus, to increase his good-1 consumption by one unit, the consumer needs to cut his good-2 consumption by p_1/p_2 units.

2 A consumer's optimal consumption bundle

Given preference relation represented by a utility function u, a consumer's decision is to maximize his utility by choosing a consumption bundle from his budget set:

$$\begin{aligned} \max_{(x_1,x_2)\in\mathbb{R}^2_+} \quad & u(x_1, x_2) \\ \text{subject to} \quad & p_1x_1 + p_2x_2 \le m. \end{aligned} \tag{2}$$

Recall from Chapter 4 that in many cases the utility function can be graphically represented by indifference curves, each in the form of $u(x_1, x_2) = c$ for some constant c equal to the utility level of any point on the curve. The higher is an indifference curve in terms of its utility level c, the more preferred is any bundle on the curve.[1] With the preference relation assumed monotone, none of the points in the interior of the triangle is optimal to the consumer, as he can do better by consuming slightly more of each good, which he can afford because the point is not on the boundary of the budget constraint. Thus, any solution of Problem (2) lies on the budget line. In other words, *the budget constraint, albeit an inequality, can be replaced by the equality (1) without loss (i.e., without ruling out an optimal solution), as long as preferences are monotone*. Furthermore, a consumption bundle solves Problem (2) iff the bundle corresponds to a point at which the budget line touches the highest possible indifference curve among those with which the line shares common points.[2]

For example, if $u(x_1, x_2) = 2x_1 + 3x_2$ for all nonnegative (x_1, x_2), each indifference curve is of the form $2x_1 + 3x_2 = c$ for some constant c, which is a straight line of slope $-2/3$ and x_2-intercept $c/3$. When the price is \$4 for both good, the slope of the budget line is -1, steeper than the indifference curves, so the highest indifference curve that touches the budget line at its x_2-intercept, m/p_2. Hence the consumer's optimal bundle is $(0, m/4)$.[3] By contrast, when the price changes to \$2 for good 1 and \$4 for good 2, the slope of the budget line becomes $-1/2$, flatter than the indifference curves; hence the optimal bundle becomes the x_1-intercept, $(m/2, 0)$. As a third possibility, consider the case where the price is \$4 for good 1 and \$6 for good 2. Then the slope of the budget line is $-2/3$, equal to that of the budget line. Thus, the budget line coincides with the highest indifference curve that it touches. In other words, Problem (2) has multiple solutions: every nonnegative point on the budget line constitutes an optimal bundle.

Multiplicity of optimal bundles, such as the last case in the above example, is ruled out if the preference relation exhibits diminishing MRS. With diminishing MRS, the highest indifference curve touched by the budget line is curved away from the budget line except at the point where the two meet, hence the solution of Problem (2) is unique.

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[1] In cases where the utility function is differentiable, one can calculate the gradient of the utility function u as

$$\nabla u(x_1, x_2) = \begin{bmatrix} \frac{\partial}{\partial x_1} u(x_1, x_2) \\ \frac{\partial}{\partial x_2} u(x_1, x_2) \end{bmatrix}.$$

As long as u is assumed increasing in x_1 and in x_2, both partial derivatives in this vector are positive. Hence the gradient $\nabla u(x_1, x_2)$ always points to the northeast in the x_1-x_2 plane.

[2] The above observation is analogous to the supporting-hyperplane idea in Chapter 2. For any consumption bundle (x_1, x_2), consider the set of all the consumption bundles that are at least as preferred as (x_1, x_2) and call it the *weakly preferred set* with respect to (x_1, x_2). When the preference relation is monotone and satisfies the diminishing MRS assumption, note that the weakly preferred set with respect to any bundle (x_1, x_2) is convex (cf. Exercise 11.b, Chapter 4), enveloped from below by the indifference curve to which the bundle (x_1, x_2) belongs. Then *a bundle is optimal iff the budget line is a supporting hyperplane of the weakly preferred set that this bundle belongs to*.

[3] To gain a deeper understanding of this corner solution, compare the gradient of the utility function $u(x_1, x_2) := 2x_1 + 3x_2$ and that of the constraint function $g(x_1, x_2) := m - 4x_1 - 4x_2$ (as the equality part of the budget constraint is $m - 4x_1 - 4x_2 = 0$). At any point (x_1, x_2), the former gradient is $[2, 3]$ while the latter is $[-4, -4]$. Note that the two gradients are nowhere aligned. Furthermore, the joint force of the two gradients always points to the northwest in the x_1-x_2 plane; consequently, the consumer ends with the northwest corner of his budget set.

3 The MRS condition

Call a preference relation *smooth* iff any indifference curve that passes through some non-boundary point of $\mathbb{R}^2_+$ represents higher preference than any point on the boundary of $\mathbb{R}^2_+$ and, at any point of the curve, has a unique, finite slope. Call a smooth preference relation *strongly monotone* iff its utility function has positive partial derivatives at all $(x_1, x_2) \in (0, \infty)^2$. Given a preference relation that satisfies all assumptions introduced previously, if it is also smooth and strongly monotone, then as one can easily see by drawing a graph the solution of Problem (2) is the tangent point between the budget line and the highest indifference curve supported by the line. Thus, the slopes of the two at the solution (x_1^*, x_2^*) are equal:

$$\mathrm{MRS}(x_1^*, x_2^*) = -\frac{p_1}{p_2}. \tag{3}$$

This *MRS condition*, coupled with the budget line equation (1), pins down the solution (x_1^*, x_2^*). If the utility function is differentiable, $\mathrm{MRS} = -\mathrm{MU}_1/\mathrm{MU}_2$ as shown in Chapter 4, and so Eq. (3) becomes a more explicit equation

$$\frac{\mathrm{MU}_1(x_1^*, x_2^*)}{\mathrm{MU}_2(x_1^*, x_2^*)} = \frac{p_1}{p_2}. \tag{4}$$

For example, consider the utility function $u(x_1, x_2) := x_1^3 x_2^{1/2}$. It represents a smooth preference relation. To see that consider any indifference curve $x_1^3 x_2^{1/2} = c$ that passes through some non-boundary point of $\mathbb{R}^2_+$, i.e., for some $a > 0$ and $b > 0$, $a^3 b^{1/2} = c$, hence $c > 0$. Thus, no boundary point of $\mathbb{R}^2_+$, in the form of $(0, x_2)$ or $(x_1, 0)$, can belong to this curve. Hence the curve does not intersect with the boundary of $\mathbb{R}^2_+$. Furthermore, as calculated in Chapter 4, the slope of the curve, $-6x_2/x_1$, is finite everywhere on the curve. Thus we can apply Eq. (4), as well as the marginal utilities calculated in Chapter 4, to obtain

$$\frac{6x_2}{x_1} = \frac{p_1}{p_2},$$

i.e., $x_2 = p_1 x_1/(6p_2)$. Plug this into Eq. (1) to get

$$p_1 x_1 + p_2 \cdot \frac{p_1 x_1}{6p_2} = m, \quad \text{i.e.,} \quad \frac{7}{6} p_1 x_1 = m.$$

Hence $x_1 = 6m/(7p_1)$ and hence

$$x_2 = \frac{p_1 x_1}{6p_2} = \frac{p_1 \cdot 6m/(7p_1)}{6p_2} = \frac{m}{7p_2}.$$

Thus the optimal bundle is $(6m/(7p_1), m/(7p_2))$.

Alternatively, we can obtain the MRS condition through the Lagrange method with equality constraints. To apply the method, first recall from the previous section that, with preferences monotone, the inequality in Problem (2) can be replaced by Eq. (1) without loss of generality. Second, we can also replace the domain $\mathbb{R}^2_+$ by the open domain $(0, \infty)^2$. That is because, with preferences smooth and parameters p_1, p_2 and m all positive, one can show that no solution of

Problem (2) belongs to the boundary of $\mathbb{R}^2_+$.[4] Thus Problem (2) is equivalent to

$$\begin{array}{cc} \max\limits_{(x_1,x_2)\in(0,\infty)^2} & u(x_1,x_2) \\ \text{subject to} & m - p_1x_1 - p_2x_2 = 0. \end{array}$$

Hence we can apply the Lagrange method. First, form the Lagrangian as

$$L(x_1, x_2; \lambda) := u(x_1, x_2) + \lambda \left(m - p_1x_1 - p_2x_2\right).$$

Next write down the first-order conditions:

$$\begin{aligned} \frac{\partial}{\partial x_1}L &= \frac{\partial}{\partial x_1}u(x_1^*, x_2^*) - \lambda p_1 = 0, \\ \frac{\partial}{\partial x_2}L &= \frac{\partial}{\partial x_2}u(x_1^*, x_2^*) - \lambda p_2 = 0, \\ \frac{\partial}{\partial m}L &= m - p_1x_1^* - p_2x_2^* = 0. \end{aligned}$$

The last equation is simply the budget line equation (1). The first two equations are equivalent to

$$\mathrm{MU}_1(x_1^*, x_2^*) = \lambda p_1, \tag{5}$$

$$\mathrm{MU}_2(x_1^*, x_2^*) = \lambda p_2. \tag{6}$$

Note that $\lambda \neq 0$, otherwise Eq. (6) implies $\mathrm{MU}_2(x_1^*, x_2^*) = 0$, contradicting strong monotonicity of the preference. Hence we can divide equations (5) by (6), canceling out λ, to obtain Eq. (4).

4 Corner solution

Without the smooth, strongly monotone and diminishing MRS assumptions, the MRS condition need not hold and Problem (2) may have a corner solution. Such an example we have seen in Section 2. Here we illustrate a technique to handle corner solutions by considering a preference relation represented by a *quasilinear* utility function, in the form of

$$u(x_1, x_2) := v(x_1) + x_2,$$

with v an increasing and differentiable function whose derivative is a decreasing function. Hence any indifference curve is of the form $x_2 = c - v(x_1)$ for some constant c. Thus MRS, the slope of the indifference curve, can be easily obtained:

$$\mathrm{MRS}(x_1, x_2) = \frac{d}{dx_1}\left(c - v(x_1)\right) = -\frac{d}{dx_1}v(x_1). \tag{7}$$

Eq. (7) implies an important property of quasilinear utility functions: $\mathrm{MRS}(x_1, x_2)$ is independent of x_2 and depends only on x_1. In other words, *indifference curves are vertically parallel to one another*: On the x_1-x_2 plane, draw any vertical line and look at its intersection points with various indifference curves; the slopes of the indifference curves at these intersection points are all identical.

[4] To see that, note from the definition of smooth preferences that any boundary point of $\mathbb{R}^2_+$ is worse than any interior point.

Eq. (7) also implies that a quasilinear preference relation exhibits diminishing MRS: The equation means $|\text{MRS}(x_1, x_2)|$ equals $\frac{d}{dx_1}v(x_1)$, which is decreasing in x_1 by assumption.

However, the preference is not smooth, because an indifference curve $x_2 = c - v(x_1)$ intersects with the x_1 axis at the point $(\bar{x}_1, 0)$ such that $v(\bar{x}_1) = c$. Thus, the MRS condition need not hold at an optimal bundle.

Nevertheless, the MRS condition fails only when the optimal bundle is a corner of the budget set. When the optimal bundle is not on the boundary of $\mathbb{R}^2_+$, it is necessarily the tangent point between the budget line and an indifference curve, as the preference is monotone and MRS diminishing. Furthermore, diminishing MRS also implies that the tangent point between the budget line and an indifference curve is unique and that the tangent point is the optimal bundle if it has nonnegative quantities for both goods. In addition, since the budget line is downward sloping (with prices positive), the tangent point cannot be negative in both coordinates. Thus comes a trick to solve Problem (2):

1. use the MRS condition and the budget line equation to solve for the tangent point between the budget line and an indifference curve;

2. if the solution is nonnegative in both coordinates, it is the optimal bundle; else—

   a. if $x_1 < 0$ then the optimal bundle is $(0, m/p_2)$, the x_2-intercept of the budget line;

   b. if $x_2 < 0$ then the optimal bundle is $(m/p_1, 0)$, the x_1-intercept of the budget line.

To understand the second step, look at Figure 1, where point A is the tangent point between the budget line and an indifference curve. Its vertical coordinate negative, A is outside the budget set, and Case (2b.) of the above procedure claims that the corner $(m/p_1, 0)$ is the optimal bundle. To understand the claim, consider any indifference curve that shares some common point with the budget set, say the dashed curve labeled α in Figure 1. Let us compare the slope of the budget line with the slope of curve α at the point B where the curve crosses the budget line. The slope of the budget line, tangent to the higher indifference curve labeled β in the figure, is equal to the slope of β at the tangent point A. By diminishing MRS, curve β gets steeper at point C than at A. Since indifference curves of quasilinear preferences are vertically parallel to one another, curve β has the same slope at C as curve α at B. It follows that curve α is steeper than the budget line at B. That being true for any indifference curve that has a common point with the budget set, we see that the only optimal bundle is the rightmost corner of the budget set, $(m/p_1, 0)$.

For example, let $u(x_1, x_2) = 2\sqrt{x_1} + x_2$ for all bundles (x_1, x_2) and the market price be \$2 for good 1 and \$6 for good 2. Hence $v(x_1) = 2\sqrt{x_1}$ and Eq. (7) implies

$$\text{MRS}(x_1, x_2) = -2(1/2)x_1^{-1/2} = -\frac{1}{\sqrt{x_1}}.$$

Thus the MRS condition, Eq. (3) is

$$-\frac{1}{\sqrt{x_1}} = -\frac{2}{6} = -\frac{1}{3},$$

i.e., $x_1 = 9$. Plug this into the budget line equation (1) to obtain

$$2 \times 9 + 6x_2 = m,$$

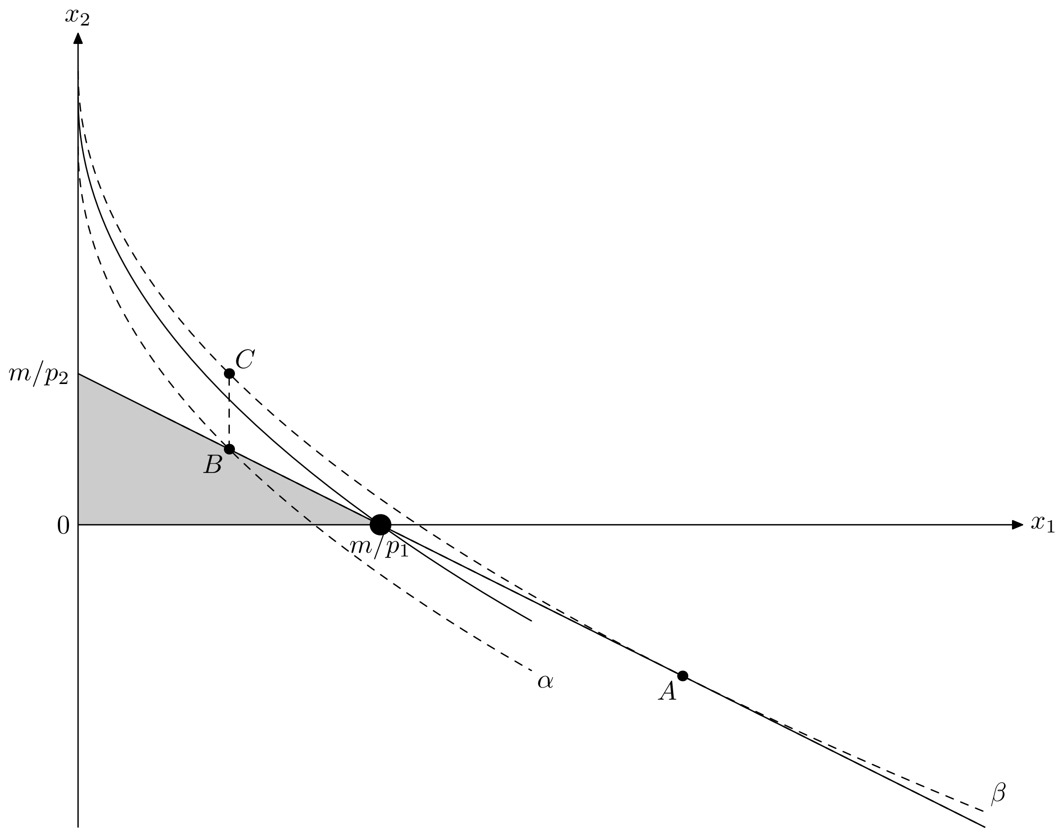

Figure 1: Grey area: the budget set; point A: the tangent point

i.e.,

$$x_2 = m/6 - 3.$$

Thus, when the consumer's given income m is above \$18, $x_2 > 18/6 - 3 = 0$, so the tangent point $(9, m/6 - 3)$ is an interior solution, and as explained previously is the optimal bundle. When $m \leq 18$, whereas, the tangent is not interior to $\mathbb{R}^2_+$, so the rightmost corner of the budget set $(m/2, 0)$ is the optimal bundle.

5 Endowment and trade

So far we have assumed that the consumer's income m is a fixed lump sum of money. Alternatively, instead of a lump sum of money as the income, the consumer is given a consumption bundle at the outset, called *endowment* and denoted by (ω_1, ω_2). Then given market prices p_1 and p_2 of the two goods, the sum of money for the consumer to spend is now determined by

$$m = p_1\omega_1 + p_2\omega_2.$$

Everything in previous sections remains the same except that m is replaced by $p_1\omega_1 + p_2\omega_2$. Following the same method, we obtain the optimal bundle say (x_1^*, x_2^*). Then $x_1^* - \omega_1^*$ is called *excess demand* for good 1, and $x_2^* - \omega_2^*$ excess demand for good 2. When $x_1^* - \omega_1^* > 0$, the consumer is a buyer of good 1, as he is consuming more of good 1 than he is endowed with. Symmetrically, when $x_1^* - \omega_1^* < 0$, the consumer is a seller of good 1, consuming less than his endowment.

6 Exercises

1. Suppose that a consumer's preferences are represented by a utility function $u(x_1, x_2) := x_1^{\alpha} x_2^{\beta}$ for all nonnegative quantities x_1 of good 1 and x_2 of good 2, with α and β being positive parameters. Suppose also that the consumer's income is a given dollar amount m, and the market prices are given to be p_1 for good 1 and p_2 for good 2, with parameters m, p_1 and p_2 all positive.

   a. Is the consumer's preference relation monotone? smooth? exhibiting diminishing MRS?

   b. Why is the Lagrange method with equality constraints applicable to this consumer's optimization problem?

   c. Use the Lagrange method to find the consumer's optimal bundle.

   d. Calculate the gradients of the utility function and the constraint function $m - px_1 - p_2 x_2$ at the optimal bundle. Are the two gradients aligned?

2. Suppose a consumer's utility function is $u(x_1, x_2) := \min\{3x_1, 2x_2\}$, an example of "perfect complements" preferences. Suppose also that the consumer's income is \$100, the price for good 1 is \$5 per unit, and the price for good 2 \$4 per unit.

   a. Is the Lagrange method with equality constraints applicable to this consumer's decision?

   b. Graph the budget line and a couple of indifference curves; based on the graph, calculate the optimal bundle.

3. Suppose that a consumer's preferences are lexicographic, with good 1 always ranked above good 2, as defined in Chapter 4, and he chooses quantities of the two goods that can be in any nonnegative real numbers. The market prices p_1 and p_2 of goods 1 and good 2 are given to be positive parameters; the consumer's income is a given positive constant m.

   a. Can this consumer's decision problem be expressed in the form of Problem (2) for some utility function u?

   b. Calculate the consumer's optimal bundle as a function of the parameters p_1, p_2 and m, via the following step:

      i. Draw an x_1-x_2-plane. Pick any point (bundle) (x_1, x_2) such that both coordinates are positive. Draw the weakly preferred set with respect to (x_1, x_2) (cf. Footnote 2). Note that this set is weakly convex in the sense that if it contains any two points then it contains the entire straight segment between the two points.

      ii. Analogously, draw the weakly preferred set with respect to a bundle $(0, x_2)$ such that $x_2 > 0$; and the weakly preferred set with respect to bundle $(x_1, 0)$ such that $x_1 > 0$.

      iii. Pick any parameters p_1, p_2, m that are each positive. Draw the budget line. Find the consumption bundle for whose weakly preferred set the budget line is a supporting hyperplane. What is the optimal bundle?

      iv. Now suppose that $p_1 = 0$ while p_2 and m are each positive. Draw the budget line. Is there any consumption bundle whose weakly preferred set has the budget line as a supporting hyperplane? Does an optimal bundle exist?

4. A consumer decides how much money to spend now and how much to save for the future. The income available to him now is \$1000, and that available to him in the future is \$500. However, he does not need to stick to spending \$1000 now and \$500 in the future, as he can add more to the future by spending less now, or add more to the current consumption by borrowing money now (and paying back the debt and its interest in the future). Denote x_1 for the amount of money he spends now, and x_2 for the amount of money available to him in the future. His preference relation is represented by the utility function $u(x_1, x_2) := 2 \ln x_1 + 3 \ln x_2$. The interest rate is 20%; that is, each dollar he saves now will become 1.2 dollars in the future.

   a. In the x_1-x_2-plane, draw the consumer's endowment point, $(1000, 500)$.

   b. Denote p_1 for the price of money available to now, and p_2 for the price of money available in the future; recall the "rate of exchange" interpretation of the slope $-p_1/p_2$ of a budget line (c.f. the end of Section 1). What is this slope equal to in our case?

   c. Combine the previous steps to draw the budget line for this consumer. Express the budget line as an equation.

   d. Combine the MRS condition with the budget line equation to find the optimal bundle.

   e. What is the optimal amount of saving for this consumer?

5. Joe the millionaire is concerned about the uncertain outcome of a US tax cut legislation. If tax cut gets enacted, Joe's endowment becomes 5 million dollars; whereas, if it is not enacted, with a higher tax Joe's endowment is reduced to 4 million dollars. He believes that the tax cut bill will get enacted with probability 2/3 and fail to be enacted with probability 1/3. There is a market of contracts contingent on the outcome of the tax bill. For each contract he buys, Joe needs to pay 75 cents, and with that contract he gets paid one dollar if the tax bill is enacted, and else gets paid nothing. For each contract he sells, Joe receives 75 cents, and with that contract sold he needs to pay one dollar if the tax bill is enacted, and else pays nothing. Joe is free to buy or sell any quantity of such contracts given the above-described prices. Denote x_1 for the money quantity, measured in millions, that Joe ends with when the tax bill is enacted, and x_2 for that when the tax bill is not enacted. Joe's preferences among such (x_1, x_2) are represented by the expected utility based on the vNM utility function u defined by $u(x) := \sqrt{x}$ for all $x \geq 0$, with x measured in the unit of millions.

   a. Write down Joe's expected utility as a function of (x_1, x_2). Is Joe risk averse? Calculate Joe's MRS as a function of (x_1, x_2). Is Joe's preference smooth?

   b. Let z denote the quantity of the contingent contracts, measured in millions, that Joe purchases; if $z < 0$ then z means he sells $|z|$ millions of such contracts.

      i. Given z, explain why $x_1 = 5 + z - 0.75z$ (million dollars); likewise note that $x_2 = 4 - 0.75z$ (million dollars).

      ii. Solve one of the above equations for z and then plug the solution for z into the other equation. Then you obtain an equation with only two variables, x_1 and x_2, and it is the equation for Joe's budget line.

      iii. On the x_1-x_2 plane, draw Joe's endowment and his budget line; note that the latter passes through the former.

c. Calculate Joe's optimal bundle (x_1^*, x_2^*).

d. Plug the solution obtained in the previous step into one of the equations in Step 5(b.)i to find the optimal z for Joe. Is this optimal z positive or negative? According to this calculation, should Joe bet against the success or failure of the tax bill?

6. Chen's preference relation is represented by a utility function $u(x_1, x_2) := \frac{2}{3} \ln x_1 + x_2$ for any nonnegative consumption bundle (x_1, x_2). The price of good 1 is \$3 and the price of good 2 is \$9. Denote m for the amount of income given to Chen.

   a. Is Chen's preference relation monotone? exhibiting diminishing MRS? smooth? having vertically parallel indifference curves?

   b. Write down the equation corresponding to Chen's MRS condition.

   c. Write down the equation corresponding to Chen's budget line.

   d. Calculate Chen's optimal consumption bundle when Chen's income is \$30.

   e. Calculate Chen's optimal consumption bundle when Chen's income is \$5.

7. Draw a graph analogous to Figure 1 in the case where the horizontal coordinate of the tangent point is negative.

8. Adam spends all his awoken hours, 18 hours per day, on either snapchats or working in Starbucks. The work in Starbucks is the best job he can find, and his salary there is \$15 per hour, prorated if his working hours are not integers. Hence the opportunity cost, or price, for him to snapchat is \$15 per hour, as snapchatting has nothing to do with his productivity, now or future. Adam's parents, yet to succumb to the fad of helicopter parenting, cut him loose by giving him only an allowance of \$50 per day. Adam desires more money, which requires work, but he is addicted to snapchat. His preference relation is represented by the utility function $u(x_1, x_2) := x_1^3 x_2^{1/2}$, where x_1 denotes the hours per day he spends on snapchat, and x_2 amount of money (measured in dollars) he has per day.

   a. On the x_1-x_2-plane, draw the point that corresponds to Adam's endowment (per day); that is, without working at all, Adam has 18 hours to snapchat and \$50 to spend.

   b. Draw Adam's budget line by taking the price of money to be \$1 per dollar and the price of snapchat to be \$15 per hour, and noting that the budget line passes through the endowment point. Write down the equation for the budget line.

   c. To check if you got the budget line equation correctly, do the following accounting of the money Adam has per day: He gets \$50 for free; in addition, each hour of works earns him \$15, and he works for $18 - x_1$ hours. The two parts sum up to be equal to x_2. The equation obtained in this manner should be equivalent to the one in the previous step.

   d. However, not every point on the budget line is feasible to Adam, because he has only 18 awoken hours. Hence draw the vertical line $x_1 = 18$, points to the right of which are not physically feasible.

   e. Combine the MRS condition with the budget line equation to find the tangent point between the budget line and the highest indifference that it touches.

f. On the x_1-x_2-plane draw this tangent point; does it correspond to a feasible consumption bundle to Adam?

g. If your answer to the previous question is No, find the optimal bundle for Adam.