# Chapter 7: Demand Function

**Elements of Decision: Lecture Notes of Intermediate Microeconomics 1**

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Abstractly speaking, an individual can be thought of as a mathematical object, a function, which in response to the surrounding environment—the "parameters"—resorts to some actions—the "solution"—possibly including actions aimed at changing the environment. In other words, an individual behaves as if he were just a mapping from parameters to solutions, and we could write down the mathematical formula of him the function had we known fully how he makes decisions. To illustrate how that works, let us go back to our toy model of a consumer in a competitive market of two commodities trying to pick a most preferred consumption bundle among those that he can afford. Hence he is reduced to a *demand function*, which maps any configuration of prices and income to his optimal bundle. We can derive this function if we know what his preferences are.

## 1 Deriving the demand function

### 1.1 Smooth preference

Suppose that our consumer is driven by the utility function $u(x_1, x_2) := x_1^3 x_2^{1/2}$ for all nonnegative quantities $x_1$ and $x_2$ of the two goods. Given any positive prices $(p_1, p_2)$ of the goods and the total income $m$ he has, we have calculated, in Chapter 6, that the optimal bundle is $(6m/(7p_1), m/(7p_2))$. Thus, given this utility function, we obtain this consumer's demand function, denoted by $(\tilde{x}_1, \tilde{x}_2)$:

$$\begin{aligned} \tilde{x}_1(p_1, p_2, m) &= 6m/(7p_1) \\ \tilde{x}_2(p_1, p_2, m) &= m/(7p_2). \end{aligned} \tag{1}$$

Note that the right-hand sides of these equations contain only the parameters, and the left-hand sides only the solutions. That is what I mean in saying that a demand function is a mapping from prices and income—the parameters—to consumption bundles—the solution. A frequently made mistake among beginners is to stop the derivation after getting an equation from the MRS condition (c.f. Chapter 6), $x_2 = \frac{p_1}{6p_2}x_1$ in this case, and mistake this equation for the demand function, whereas the equation merely maps one part of the solution to another part of the solution.

From the demand function given by Eq. (1) we have

$$\frac{\partial}{\partial p_1}\tilde{x}_1(p_1, p_2, m) = -\frac{6m}{7p_1^2} < 0 \quad \text{and} \quad \frac{\partial}{\partial m}\tilde{x}_1(p_1, p_2, m) = \frac{6}{7p_1} > 0,$$

which implies a comparative statics observation that, given the above utility function, the optimal consumption of good 1 is decreasing in its price, and increasing in the consumer's income. The comparative statics about good 2 is similar. To graph such comparative statics observation of good 1, we have to conform to economists' seemingly odd convention of putting quantity on the horizontal axis thereby leaving price to the vertical axis:[1] rewrite Eq. (1) as

$$p_1 = \frac{6m}{7\tilde{x}_1(p_1, p_2, m)} = \frac{6m}{7\tilde{x}_1},$$

where for the second equality we suppress $(p_1, p_2, m)$ to lighten the notation; draw a plane with horizontal axis representing $\tilde{x}_1$ and vertical axis $p_1$; on this plane graph the equation $p_1 = \frac{6m}{7\tilde{x}_1}$ with $m$ treated as a constant. This curve, called *demand curve* for good 1, depicts the relationship between the optimal consumption quantity of good 1 and its price. In this example, one readily sees that the curve is downward sloping, meaning that the relationship is negative. Likewise, to graph the relationship between the optimal consumption and income, rewrite Eq. (1) as

$$m = \frac{7}{6} p_1 \tilde{x}_1;$$

draw a plane with horizontal axis standing for $\tilde{x}_1$ and vertical axis $m$; graph the equation $m = \frac{7}{6} p_1 \tilde{x}_1$ on the plane, with $p_1$ treated as a constant. This curve, called *Engel curve*, depicts the relationship between the optimal consumption quantity of good 1 and the consumer's income. In this example, it is obvious that the curve is a straight line with positive slope $\frac{7}{6} p_1$, meaning that optimal consumption of good 1 increases as income increases.

In general, however, a demand curve is not necessarily downward sloping, nor an Engel curve necessarily upward sloping (c.f. Exercise 6). A commodity is called *normal good* to a consumer if his Engel curve of the good is upward sloping, *inferior good* if his Engel curve is downward sloping, and *Giffen good* if his demand curve is not downward sloping. For example, in the example calculated above, both goods are normal, and neither of them is Giffen.

## 1.2 Perfect substitutes

Let the utility function be $u(x_1, x_2) := 2x_1 + 3x_2$ for all nonnegative $(x_1, x_2)$. Then the indifference curves are straight lines of the same slope, $-2/3$. Thus, given any positive prices $(p_1, p_2)$ and income $m$, the optimal bundle is $(0, m/p_2)$ if $p_1/p_2 > 2/3$, $(m/p_1, 0)$ if $p_1/p_2 < 2/3$, and any bundle on the budget line if $p_1/p_2 = 2/3$. Thus, the demand function, if we focus only on good 1, is

$$\tilde{x}_1(p_1, p_2, m) = \begin{cases} m/p_1 & \text{if } p_1 < 2p_2/3 \\ \text{any number in } [0, m/p_1] & \text{if } p_1 = 2p_2/3 \\ 0 & \text{if } p_1 > 2p_2/3. \end{cases}$$

To graph the demand curve for good 1, start with a plane with horizontal axis being $p_1$ and vertical axis $\tilde{x}_1$; then plot the curve for the above equation; finally, to respect the aforementioned convention in economics of putting the consumption quantity on the horizontal axis, rotate the plane by 90 degree to obtain Figure 1.

The shape of the Engel curve in this example depends on whether $p_1 < 2p_2/3$ or not. By the above equation, when $p_1 < 2p_2/3$, $\tilde{x}_1 = m/p_1$, i.e., $m = p_1 \tilde{x}_1$, hence the Engel curve is the straight

[1] This convention looks odd, from the perspective of other science disciplines, only within the confine of a consumer's decision, to which prices are independent variables and consumption quantity the dependent one. In the general equilibrium theory, which aggregates the solutions of these individual decisions into market behaviors, prices become dependent variables that clear the market, hence it makes sense to be on the vertical axis.

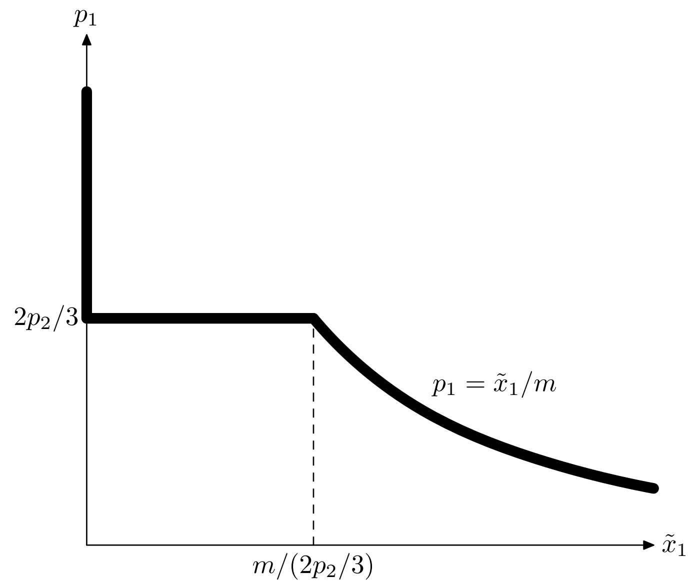


Figure 1: Demand curve given perfect substitute preferences

line starting from $(0,0)$ with slope $p_1$. When $p_1 > 2p_2/3$, the above equation says that $\tilde{x}_1 = 0$, hence the Engel curve is the nonnegative part of the vertical $(m)$ axis. When $p_1 = 2p_2/3$, we have $0 \le \tilde{x}_1 \le m/p_1$, hence the Engel "curve" is the triangular area bounded between the nonnegative vertical axis and the aforementioned straight line.

### 1.3 Quasilinear preferences

As introduced in Chapter 6, such preferences are represented by utility functions of the form

$$u(x_1, x_2) = v(x_1) + x_2, \tag{2}$$

where $v$ is an increasing, concave and differentiable function. To be specific, let $v$ be given by

$$v(x_1) = \alpha \ln(x_1 + 1) \tag{3}$$

for all $x_1 \ge 0$, where $\alpha$ is a positive parameter. Then $\frac{d}{dx_1} v(x_1) = \alpha/(x_1 + 1)$.

#### 1.3.1 Demand curve

Following the technique in Chapter 6, we first calculate the tangent point between the budget line and an indifference curve by solving the equations

$$\frac{d}{dx_1} v(x_1) = \frac{p_1}{p_2}, \tag{4}$$
$$p_1 x_1 + p_2 x_2 = m. \tag{5}$$

Eq. (4), by $\frac{d}{dx_1} v(x_1) = \alpha/(x_1 + 1)$, means $\alpha/(x_1 + 1) = p_1/p_2$, i.e., $x_1 = \alpha p_2/p_1 - 1$. Plug this into Eq. (5) to get

$$p_2 x_2 = m - p_1 (\alpha p_2/p_1 - 1) = m - (\alpha p_2 - p_1).$$

Thus, the tangent point is $(\alpha p_2/p_1 - 1, (m - \alpha p_2 + p_1)/p_2)$. Second, find out the condition for both coordinates of the tangent point to be nonnegative: the first coordinate is nonnegative iff $\alpha p_2/p_1 - 1 \geq 0$ iff $p_1 \leq \alpha p_2$, and the second coordinate nonnegative iff $m - \alpha p_2 + p_1 \geq 0$ iff $p_1 \geq \alpha p_2 - m$. Thus, if $\alpha p_2 - m \leq p_1 \leq \alpha p_2$, both coordinates are nonnegative, and the tangent point is the optimal bundle. Else the optimal bundle is the rightmost corner $(m/p_1, 0)$ if $p_1 < \alpha p_2 - m$ (negative vertical coordinate), and the uppermost corner $(0, m/p_2)$ of the budget set if $p_1 > \alpha p_2$ (negative horizontal coordinate). In other words, the demand function for good 1 is

$$\tilde{x}_1(p_1, p_2, m) = \begin{cases} m/p_1 & \text{if } p_1 < \alpha p_2 - m \\ \alpha p_2/p_1 - 1 & \text{if } \alpha p_2 - m \leq p_1 \leq \alpha p_2 \\ 0 & \text{if } p_1 > \alpha p_2. \end{cases} \tag{6}$$

For simplicity and to focus on the demand for good 1, assume

$$\alpha p_2 \leq m \quad \text{and} \quad p_2 = 1. \tag{7}$$

(See Exercise 5 for a case without (7).) Then the upper branch of Eq. (6) is vacuous because $\alpha_2 p_2 - m \leq 0 \leq p_1$ for any positive price $p_1$. Thus the demand function for good 1 becomes

$$\tilde{x}_1(p_1, m) = \begin{cases} \alpha/p_1 - 1 & \text{if } p_1 \leq \alpha \\ 0 & \text{if } p_1 > \alpha. \end{cases} \tag{8}$$

To graph the demand curve, rewrite the first branch of Eq. (8), $\tilde{x}_1 = \alpha/p_1 - 1$, into $p_1 = \alpha/(\tilde{x}_1 + 1)$. From this, coupled with the second branch of Eq. (8), we obtain the demand curve for good 1, as in Figure 2. Given any price of good 1, draw a horizontal line, with vertical coordinate equal to

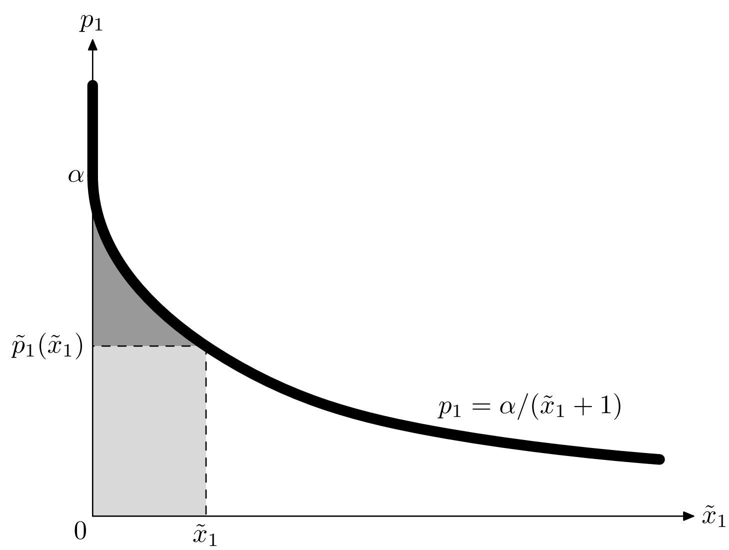


Figure 2: The thick curve: the demand curve given $u(x_1, x_2) = \alpha \ln(x_1 + 1)$

the price level, until it intersects with the demand curve, labeled as $(\tilde{x}_1, \tilde{p}_1(\tilde{x}_1))$ in the figure; the horizontal coordinate of the intersection is the quantity demanded for good 1 by this consumer.

### 1.3.2 Consumer's surplus

Having traveled thus far, we are rewarded with a formal foundation of the notion *consumer's surplus* in Introductory Economics. Given the quasilinear preferences described above, if our individual consumes a quantity $x_1$ of good 1 at price $p_1$ per unit, then his surplus is obviously equal to $v(x_1) - p_1 x_1$. But suppose that we do not know exactly what his function $v$ is and observe only his demand curve. Is there a way to find out his surplus? The answer is Yes if his demand curve is generated by quasilinear preferences. Here is how. Note that the non-vertical portion of the demand curve in Figure 2 says that for the consumer to demand any positive quantity $\tilde{x}_1$ of good 1 its price $p_1$ needs to be equal to $\alpha/(\tilde{x}_1 + 1)$. I.e., the non-vertical portion of the demand curve is the graph of a function

$$\tilde{p}_1(x_1) := \alpha/(x_1 + 1)$$

that specifies the value of $p_1$ given which the consumer's demand for good 1 equals $x_1$. Note from Eq. (3) that $\alpha/(x_1 + 1) = \frac{d}{dx_1} v(\tilde{x}_1)$,[2] Thus, for all $\tilde{x}_1 > 0$,

$$\tilde{p}_1(x_1) = \frac{d}{dx_1} v(\tilde{x}_1).$$

It then follows from the second fundamental theorem of calculus that, for any $\tilde{x}_1 > 0$,

$$v(\tilde{x}_1) = v(0) + \int_0^{\tilde{x}_1} \frac{d}{dx_1} v(x_1) dx_1 = v(0) + \int_0^{\tilde{x}_1} \tilde{p}(x_1) dx_1 = \int_0^{\tilde{x}_1} \tilde{p}(x_1) dx_1,$$

with the last equality due to $v(0) = \alpha \ln(0 + 1) = 0$. Put intuitively, the equation $v(\tilde{x}_1) = \int_0^{\tilde{x}_1} \tilde{p}(x_1) dx_1$ says that the gross surplus $v(\tilde{x}_1)$ that the consumer obtains from consuming a quantity $\tilde{x}_1$ of good 1 is equal to the shaded area, dark and grey combined, in Figure 2. For the consumer to demand exactly $\tilde{x}_1$ units of good 1, its price needs to be equal to $\tilde{p}_1(\tilde{x}_1)$, given which the consumer's expense on good 1 is equal to $\tilde{p}_1(\tilde{x}_1)\tilde{x}_1$, the grey area in Figure 2. That leaves $v(\tilde{x}_1) - \tilde{p}_1(\tilde{x}_1)\tilde{x}_1$, the dark area in Figure 2, as the consumer's net surplus, i.e., *consumer's surplus*.

# 2 Market demand

## 2.1 Market demand curve

With the foundation of a consumer's demand curve established, we define the market demand curve, which beginners see in Introductory Economics, to be the horizontal sum of the demand curves of the individual consumers, horizontal because the horizontal axis represents the quantity of the good. If all individual demand curves are downward sloping, so is the market demand curve.

Specifically, suppose that there are $n$ consumers in the market and each consumer's preference relation is represented by the quasilinear utility function defined by Eqs. (2) and (3), with the parameter $\alpha$ for consumer $i$ labeled by $\alpha_i$, and his income by $m_i$. As in (7), assume $p_2 = 1$ and $\alpha_i p_2 \le m_i$ for any consumer $i$. Then, as in (8), consumer $i$'s demand function for good 1 is

$$\tilde{x}_1^i(p_1, m_i) = \begin{cases} \alpha_i/p_1 - 1 & \text{if } p_1 \le \alpha_i \\ 0 & \text{if } p_1 > \alpha_i. \end{cases}$$

[2] This equation is no coincidence, as the equation $p_1 = \alpha/(\tilde{x}_1 + 1)$ is derived from the MRS condition, Eq. (4), which combined with $p_2 = 1$ and Eq. (3) says that $p_1 = \frac{d}{dx_1} v(\tilde{x}_1)$.

Thus the market demand for good 1 is equal to

$$X_1(p_1) := \sum_{i=1}^{n} \tilde{x}_1^i(p_1, m_i). \tag{9}$$

Since $\frac{\partial}{\partial p_1}\tilde{x}_1^i(p_1, m_i) < 0$ for all consumers $i$, $\frac{d}{dp_1}X_1(p_1) < 0$, i.e., the market demand curve is downward sloping, as is assumed in Introductory Economics. Furthermore, if the market price for good 1 is $p_1^*$, each consumer $i$'s surplus is equal to the area bounded between the price line $p_1 = p_1^*$ and his demand curve, as illustrated in Figure 2 and explained in Section 1.3.2. Thus, the aggregate of these surpluses across all consumers, called *consumers' surplus*, is simply the area bounded between the market demand curve and the price line.

## 2.2 Price elasticity

While the sign of the slope of the demand curve tells us the qualitative information regarding the relationship between price and quantity, the magnitude of the slope, being a ratio of changes, depends on the arbitrary choice of the units for the currency and the good. To have a unit-free measurement, economists consider the ratio of the *percentage* changes

$$\frac{\left(X_1(p_1 + \Delta p_1) - X_1(p_1)\right)/X_1(p_1)}{\Delta p_1/p_1}.$$

When $\Delta p_1 \to 0$, this ratio converges to

$$\varepsilon(p_1) := \frac{p_1}{X_1(p_1)} \cdot \frac{d}{dp_1}X_1(p_1), \tag{10}$$

called *price elasticity* of the market demand for good 1 at price $p_1$. With the demand curve downward sloping, $\frac{d}{dp_1}X_1(p_1) < 0$, hence $\varepsilon(p) < 0$. The market demand is said *elastic* when $|\varepsilon(p_1)| > 1$, as $|\varepsilon(p_1)| > 1$ means, by Eq, (10), that the quantity demand for the good would shrink by more than one percent if its price rises by one percent. Correspondingly, call the market demand *inelastic* when $|\varepsilon(p_1)| < 1$.

Price elasticity is a useful concept to figure out monopoly behaviors. Consider a firm that corners an entire market for good 1, with market demand function $X_1$, so the firm's revenue is equal to $p_1X_1(p_1)$ if it charges price $p_1$ for the good. For simplicity assume that the firm's marginal cost is zero—not a farfetched assumption to some internet giants—then the firm simply maximizes its revenue $p_1X_1(p_1)$ by choosing a price $p_1$. Note

$$\frac{d}{dp_1}\left(p_1X_1(p_1)\right) = X_1(p) + p_1\frac{d}{dp_1}X_1(p_1) = X_1(p)\left(1 + \frac{p_1}{X_1(p_1)} \cdot \frac{d}{dp_1}X_1(p_1)\right) \overset{(10)}{=} X_1(p)\left(1 + \varepsilon(p_1)\right).$$

Thus, assuming $X_1(p) > 0$,

$$\frac{d}{dp_1}\left(p_1X_1(p_1)\right) \begin{cases} > 0 & \text{if } \varepsilon(p_1) > -1 \quad \text{i.e., demand is inelastic} \\ = 0 & \text{if } \varepsilon(p_1) = -1 \quad \text{i.e., demand is unit-elastic} \\ < 0 & \text{if } \varepsilon(p_1) < -1 \quad \text{i.e., demand is elastic.} \end{cases}$$

Hence the firm would raise the price if and only if demand is inelastic (so we know what Netflix or Apple will eventually do to us).

## 3 Exercises

1. Consider a utility function defined by $u(x_1, x_2) := x_1^{1/3} x_2^{2/3}$ for all nonnegative $x_1$ and $x_2$.

   a. Is the preference relation represented by this $u$ smooth?

   b. Calculate the demand function. Specifically, write down the quantity demand $\tilde{x}_1(p_1, p_2, m)$ for good 1 as a function of the prices $p_1$, $p_2$ and the consumer's entire income $m$.

   c. Graph the demand curve for good 1.

   d. Graph the Engel curve for good 1.

   e. Using the result obtained in Step 1b. and holding $p_2$ constant:

      i. Calculate $\frac{\partial}{\partial p_1} \tilde{x}_1(p_1, p_2, m)$; is good 1 Giffen?

      ii. Calculate $\frac{\partial}{\partial m} \tilde{x}_1(p_1, p_2, m)$; is good 1 normal or inferior?

2. Consider a utility function $u(x_1, x_2) := \min\{2x_1, 3x_2\}$ for all nonnegative $x_1$ and $x_2$.

   a. Is the preference relation represented by this $u$ smooth? Is the Lagrange method applicable to the consumer's decision problem?

   b. Locate the optimal bundle in a diagram with a couple of indifference curves and the budget line, given positive parameters prices $p_1$, $p_2$ and positive income $m$.

   c. Write down the demand function.

   d. Graph the demand curve for good 1.

   e. Graph the Engel curve for good 1.

3. A consumer's utility function is given by

$$u(x_1, x_2) := \sqrt{x_1} + x_2$$

   for any nonnegative $x_1$ and $x_2$, representing the consumption quantities of goods 1 and 2, respectively. Suppose that the price of good 2 is constantly \$1, and that the consumer is given income $m$ dollars (and nothing else). Denote $p_1$ for the price of good 1.

   a. Follow the steps in Section 1.3.1 to calculate the consumer's quantity demand for good 1. (Hint: note that at any tangent point $(x_1, x_2)$ between the budget line and the highest indifference curve the former can reach, $x_2 \geq 0$ iff $m \geq 1/(4p_1)$, and that $x_1 \geq 0$ always.)

   b. Suppose within this step that $m = 1$ (dollar). Graph the demand curve for good 1. (Note that the equation for the demand curve when $p_1 < 1/4$ is different from the one when $p_1 > 1/4$.)

   c. Suppose within this step that $p_1 = 1/2$ (dollar). Graph the Engel curve for good 1.

4. A consumer's utility function is given by Eqs. (2) and (3) with $\alpha = 100 \leq m$. Suppose the price of good 2 is constantly equal to one.

   a. Follow the steps in Section 1.3.1 to graph the demand curve for good 1 in a coordinate system where the horizontal axis represents $\tilde{x}_1$, and vertical axis $p_1$; write down the equation that represents the demand curve; label on the diagram the lowest level among those prices of good 1 at which the consumer's quantity demand for good 1 is zero.

b. Suppose that the market price is \$20 per unit of good 1. Then—

   i. Use the equation of the demand curve to calculate the consumer's quantity demand $\tilde{x}_1$ for good 1 at this price.

   ii. For the diagram obtained in Step 4a., calculate the area bounded between the demand curve and the horizontal axis, when $\tilde{x}_1$ ranges between zero and the quantity demand obtained in Step 4(b.)i. (Hint: $\frac{d}{dx_1}\ln(x_1+1) = 1/(x_1+1)$.)

   iii. Subtract from the area obtained in the previous step by the area of the rectangle that corresponds to the consumer's payment for the quantity of good 1 he demands.

   iv. Plug the quantity demand $\tilde{x}_1$ obtained in Step 4(b.)i into $v(\tilde{x}_1) - p_1\tilde{x}_1$ to calculate the consumer's surplus. Check whether this number is equal to the result obtained in Step 4(b.)iii.

5. Ineq. (7) assumes $\alpha p_2 \leq m$ to simplify the quasilinear utility demand function (6) into two branches. Now replace (7) with the assumption that $p_2 = 1$ and $\alpha > m$.

   a. Plug $p_2 = 1$ into (6) and graph this three-branch demand function on the $p_1$-$\tilde{x}_1$ plane.

   b. Is this demand curve continuous at the point where the top branch of (6) switches to the middle branch (i.e., when $p_1 = \alpha - m$)?

   c. When $p_1$ drops from slightly above $\alpha - m$ to slightly below $\alpha - m$, does the slope of the demand curve jump or drop?

   d. If $p_1 < \alpha - m$, is the consumer's surplus still equal to the area under the demand curve as in Figure 2?

6. Consider a utility function defined by

$$u(x_1, x_2) := \min\left\{x_1 + x_2, \frac{9}{10}x_2 + 1\right\}$$

   for all nonnegative $x_1, x_2 \geq 0$.[3] Let $m > 0$ be the consumer's entire income $m$. Denote $p_1$ for the price of good 1, and $p_2$ that of good 2.

   a. Recall the indifference map of this utility function from an exercise in Chapter 5.

   b. Suppose that the market prices are such that $0 < p_1 < p_2$. Draw a budget line whose $x_2$-intercept is less than 10. Point out in the graph the optimal consumption bundle given this budget line. Find out the coordinates of this bundle.

   c. Suppose $0 < p_1 < p_2$ and $m/p_2 < 10$. Calculate the consumer's demand function for good 1, with $p_1$, $p_2$ and $m$ the parameters.

   d. Suppose $0 < p_1 < p_2$ and $m/p_2 \geq 10$. Calculate the consumer's demand function for good 1, with $p_1$, $p_2$ and $m$ the parameters.

   e. Suppose $0 < p_1 < p_2$. Graph the Engel curve for good 1. When $m/p_2 < 10$, is good 1 a normal or inferior good? Giffen good?

[3] I thank John Quah for suggesting a general direction for construction of such utility functions.

7. Suppose that, for any nonnegative bundles $(x_1, x_2)$ and $(x_1', x_2')$, our consumer strictly prefers $(x_1, x_2)$ to $(x_1', x_2')$ if and only if

$$2x_1 + 3x_2 > 2x_1' + 3x_2' \quad \text{or} \quad \left[2x_1 + 3x_2 = 2x_1' + 3x_2' \text{ and } x_1 > x_1'\right].$$

   a. Are there two consumption bundles that are indifferent to each other to the consumer? How does an indifference "curve" of his look like?
   b. Is this preference relation continuous (c.f. Chapter 5)?
   c. Find out the quantity demand $\tilde{x}_1(p_1, p_2, m)$ for good 1 as a function of the prices $p_1$, $p_2$ and the consumer's income $m$, with all three parameters positive.
   d. Graph the demand curve for good 1; compare it with the demand curve for good 1 if the consumer's preferences are represented by the perfect substitutes utility function in Section 1.2.

8. In a two-commodity world, the price for good 2 is $1, and the market for good 1 consists of two consumers, named 1 and 2. Each consumer is given an income and nothing else. The income is $4 for consumer 1, and $7 for consumer 2. Consumer 1's utility from consuming any bundle $(x_1, x_2) \in \mathbb{R}^2_+$ is equal to

$$u_1(x_1, x_2) := 4\ln(x_1 + 1) + x_2,$$

   and consumer 2's utility equal to

$$u_1(x_1, x_2) := 6\ln(x_1 + 1) + x_2.$$

   a. Compare the above information with Eqs. (2) and (3). What are the parameters $\alpha$, $m$ and $p_2$ equal to for consumer 1, and what are they equal to for consumer 2? Is the condition (7) satisfied for each consumer?
   b. Mimicking the steps in Section 1.3.1, calculate consumer 1's demand function, and consumer 2's demand function, for good 1.
   c. On a coordinate system where the horizontal axis denotes a consumer's optimal consumption quantity $\tilde{x}_1$ of good 1, and the vertical axis the price $p_1$ of good 1, graph the demand curve of consumer 1, and that of consumer 2, based on the results in Step 8b. For each demand curve, calculate the horizontal coordinates of the points of the curve whose vertical coordinates are equal to 6, 4, 3, 2, and 1, respectively.
   d. On the same diagram thereof, graph the market demand curve by summing the two individual demand curves obtained Step 8c. Calculate the horizontal coordinates of the points of the market demand curve whose vertical coordinates are equal to 6, 4, 3, 2, and 1, respectively.
   e. Use Eq. (9) to calculate the market demand function. Check that it is consistent with the curve obtained in Step 8d.
   f. Explain why the market demand curve obeys the following equation:

$$p_1 = \begin{cases} \frac{6}{\tilde{x}_1+1} & \text{if } 0 \le \tilde{x}_1 \le 1/2 \\ \frac{10}{\tilde{x}_1+2} & \text{if } \tilde{x}_1 \ge 1/2 \end{cases}$$

g. When the price $p_1$ of good 1 is \$2:

   i. What is the quantity demand for good 1 by consumer 1 and that by consumer 2?

   ii. What is the quantity of market demand for good 1?

   iii. Calculate the consumers' surplus.

   iv. Calculate the slope of the market demand curve for good 1.

   v. Calculate the price elasticity of the market demand for good 1.