Chapter 8: Slutsky Equation

Elements of Decision: Lecture Notes of Intermediate Microeconomics 1

Charles Z. Zheng

Department of Economics, University of Western Ontario

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We have seen in Chapter 2 comparative statics on a firm's input-output decision. Now comes the analogous exercise on a consumer's decision, analyzing how a consumer's optimal bundle may change given an exogenous change in parameters such as prices and income. What is new here is that the price of a good not only has a direct effect on the decision through altering the relative price between the goods, but also has an indirect effect through changing the income of the consumer.

If we could solve the optimal bundle explicitly as a function of prices and income, as in Chapter 7, the function contains all the information that we need for comparative statics. In general, however, a closed-form solution of the optimal bundle may be unavailable; worse yet, often unavailable is the information of a consumer's preferences for us to calculate his optimum. This chapter introduces a technique to handle these problems.

Let us assume that the preference relation is smooth (c.f. §3, Chapter 6), monotone and exhibits diminishing MRS. Thus, given any positive prices p_1, p_2 and positive income m, the optimal bundle

$$(\tilde{x}_1(p_1, m), \tilde{x}_2(p_1, m))$$

is determined by the MRS condition coupled with the budget line equation. (The notation p_2 is suppressed here because we assume it constant to focus on the effect of p_1.) Comparative statics regarding the effect of p_1 amounts to finding the sign of $\frac{\partial}{\partial p_1}\tilde{x}_1(p_1, m)$, i.e., whether the demand curve is downward sloping or not. But how can we achieve that without a closed-form expression of $\tilde{x}_1(p_1, m)$?

1 The idea

The trick is to decompose the effect of price change into two components, one directly affecting the consumer's demand through the price per se, the other indirectly affecting his demand through altering his purchasing power. Look at Figure 1. There, the price of good 1 increases from p_1 to p_1', while that of good 2 and the money income m remain constant. This price change means that in Figure 1 the budget line pivots from l_A to the steeper l_C around the x_2-intercept $(0, m/p_2)$. Correspondingly, the optimal bundle changes from point A to point C, and the total change of the consumer's demand for good 1 is equal to $x_1^C - x_1^A$. In Figure 1, $x_1^C - x_1^A < 0$, as the indifference curves happen to be positioned there. In general, how do we know if $x_1^C - x_1^A < 0$ is true or not when the indifference curves may be positioned differently? The answer comes from a smartly chosen point B, as in Figure 1. Clearly, we can view the total change $x_1^C - x_1^A$ as the sum of two changes: from points A to B, by the horizontal distance $x_1^B - x_1^A$; and from points B to C, by the horizontal distance $x_1^C - x_1^B$. Thus, in terms of the rates of change, we have an identity

$$\frac{x_1^C - x_1^A}{p_1' - p_1} = \frac{x_1^B - x_1^A}{p_1' - p_1} + \frac{x_1^C - x_1^B}{p_1' - p_1}. \tag{1}$$

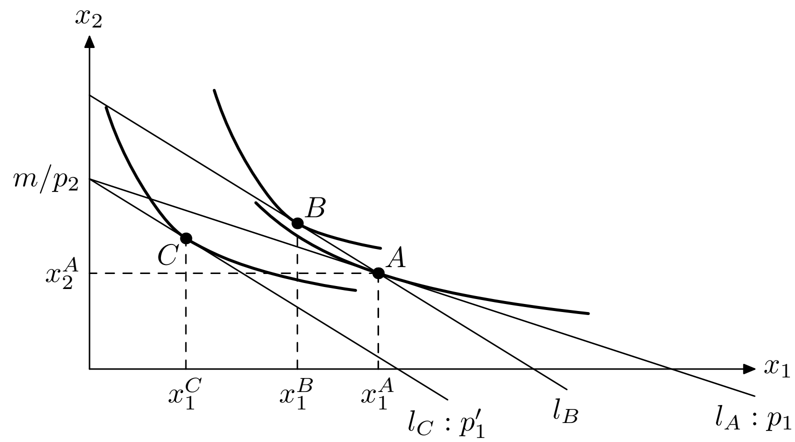

Figure 1: Substitution effect $= \frac{x_1^B - x_1^A}{p_1' - p_1}$; income effect $= \frac{x_1^C - x_1^B}{p_1' - p_1}$

On the right-hand side, the first term, $\frac{x_1^B - x_1^A}{p_1' - p_1}$, is called *substitution effect*, and the second term $\frac{x_1^C - x_1^B}{p_1' - p_1}$, *income effect*. We will see that *the substitution effect is always negative, and the income effect negative if good 1 is a normal good, and positive if inferior, to the consumer*. Thus, *the total effect* $\frac{x_1^C - x_1^A}{p_1' - p_1}$ *of a price change is ascribed to the sum of substitution and income effects, and is unambiguously negative when the good is normal*.

This magic is due to the smart choice of the point B. It is the optimal bundle if the consumer's budget line were l_B in the figure, with l_B defined to be the straight line parallel to the new budget line l_C and passing through the old bundle A. Parallel to l_C, l_B reflects the new price of good 1; passing through A, l_B, were it the consumer's budget line, would have given him the exact amount of money to buy bundle A at the new price. Thus, if the consumer's budget line were l_B, he would face the new price while his purchasing power, with respect to the previously optimal bundle A, remains the same. Hence the difference $x_1^B - x_1^A$ can be ascribed purely to the effect on the market rate of exchange between the two goods, with the effect on the consumer's purchasing power filtered out. Such a filtering construction, we will see soon, allows us to predict the sign of $(x_1^B - x_1^A)/(p_1' - p_1)$ unambiguously. The latter effect is captured by the other difference, $x_1^C - x_1^B$, as it is caused by a parallel shift of the budget line from l_B to l_C. The slope of the budget line unchanged in this shift, the consumer faces the same market rate of exchange between the two goods while his purchasing power is altered. Hence the difference $x_1^C - x_1^B$ is ascribed purely to the effect on the purchasing power, which is why $(x_1^C - x_1^B)/(p_1' - p_1)$ is called income effect.

In general, the new price p_1' of good 1 can be higher or lower than the original price p_1. Denote the price difference by

$$\Delta p_1 := p_1' - p_1,$$

which can be positive or negative. Correspondingly, the points B and C in Figure 1 can be to the right or to the left of point A. Denote the differences by

$$\begin{aligned} \Delta x_{\text{sub}} &:= x_1^B - x_1^A, \\ \Delta x_{\text{inc}} &:= x_1^C - x_1^B, \\ \Delta x_{\text{total}} &:= x_1^C - x_1^A. \end{aligned}$$

Then Eq. (1) is equivalent to

$$\frac{\Delta x_{\text{total}}}{\Delta p_1} = \underbrace{\frac{\Delta x_{\text{sub}}}{\Delta p_1}}_{\text{substitution effect}} + \underbrace{\frac{\Delta x_{\text{inc}}}{\Delta p_1}}_{\text{income effect}} . \tag{2}$$

2 Substitution effect

In Eq. (2), the term labeled substitution effect has an unambiguous sign:

$$\frac{\Delta x_{\text{sub}}}{\Delta p_1} < 0. \tag{3}$$

To understand why, consider the case where $\Delta p_1 > 0$, i.e., the price of good 1 increases to $p_1 + \Delta p_1$ while that of good 2 stays constant. We want to know the sign of Δx_{sub}, i.e., whether $x_1^B < x_1^A$ or $x_1^B > x_1^A$. That is, when the budget line switches from l_A to l_B in Figure 1, whether point B belong to the left or the right of A when the indifference curves may be different from those in the figure. To figure that out, recall that l_B, by definition, is obtained by rotating l_A around point A until the rotated line becomes parallel to line l_C in Figure 1. Given $\Delta p_1 > 0$ in the case that we are considering, that means line l_B is steeper than line l_A, as shown in Figure 2. Since A is optimal given the original budget line l_A, it is uniquely so within the original budget set as MRS is diminishing. Hence every bundle in the grey region of Figure 2, other than A itself, is worse than A. Thus, none of such bundles would be chosen by the consumer when his budget line becomes the new one, because the better bundle A, belonging to the new budget line, is still affordable given the new price $p_1 + \Delta p_1$. That leaves the dark area in Figure 2 as the only possible bundles that the consumer would choose given price $p_1 + \Delta p_1$. Hence the optimal bundle given l_B as the budget

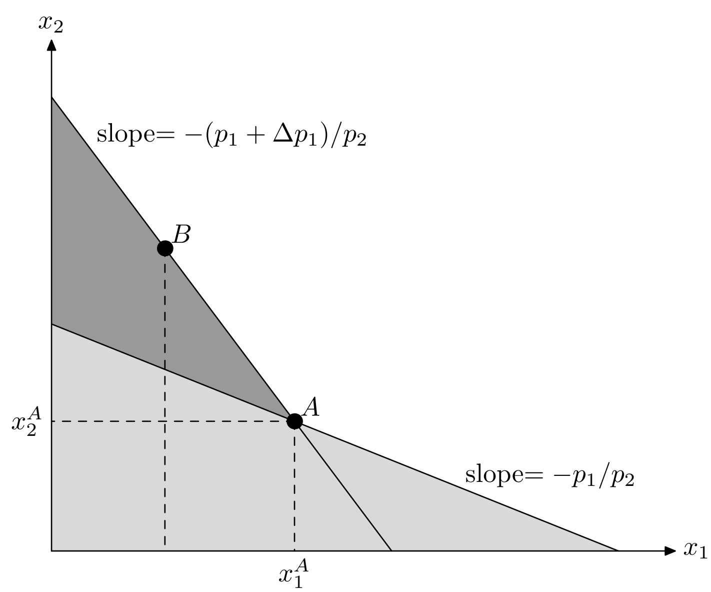

Figure 2: $A \to B$: the substitution effect of $p_1 \to p_1 + \Delta p_1$

line is located to the left of A. Consequently,

$$\Delta x_{\text{sub}} = x_1^B - x_1^A < 0, \tag{4}$$

which coupled with $\Delta p_1 > 0$ implies Ineq. (3). In the other case, $\Delta p_1 < 0$, one can show analogously that the "$<$" in Ineq. (4) switches to "$>$" (c.f. Exercise 6). Divide both sides of the inequality thereof by the negative Δp_1 and we obtain Ineq. (3) again.

3 Income effect

The term labeled income effect in Eq. (2), by contrast, can be negative in some cases and positive in others. That is because the change from points B to C, as illustrated in Figure 1, is caused by a parallel shift of the budget line from l_B to l_C; depending on the positioning of the indifference curves, C may belong to the left of B or to the right of B, as exemplified by Figure 3.

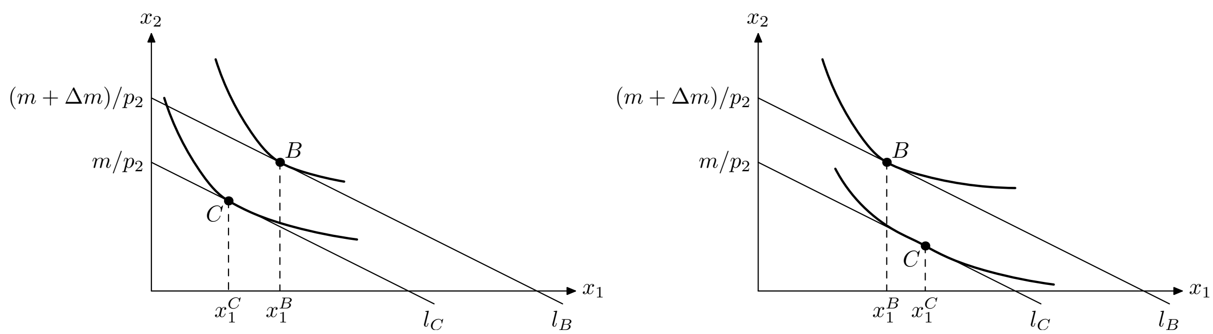

Figure 3: Left panel: good 1 is normal; right panel: good 1 is inferior

In Figure 3, replacing the consumer's budget line l_C by l_B amounts to a parallel movement of the line such that its vertical intercept rises by some distance $\Delta m/p_2$, as if the consumer were given an extra income equal to some Δm dollars. The effect of this extra income depends on whether good 1 is normal or inferior. On the left panel of Figure 3, the consumer chooses to buy more of good 1 given the higher budget line l_B than given the lower budget line l_C, i.e., when he has more income. A good of such a property is called *normal good* to the consumer. On the right panel, by contrast, the consumer buys less of good 1 when he is given the higher l_B than given l_C as the budget line; that is, he consumes less of good 1 when having more income. Goods of such property is called *inferior good* to the consumer. Specifically, if Δm denotes the increase in the consumer's income when his budget line shifts from l_C to the higher l_B, then

$$\frac{x_1^B - x_1^C}{\Delta m} > 0$$

if good 1 is normal, and the strict inequality is reversed if good 1 is inferior.

Keeping the distinction between normal and inferior goods in mind, we inspect the income effect $\Delta x_{\text{inc}}/\Delta p_1$ in Eqs. (2). There, the change $\Delta x_{\text{inc}} = x_1^C - x_1^B$ is due to the parallel shift of the budget line from l_B to l_C (Figure 1), as if the consumer's income decreased by Δm. Note that

$$\frac{\Delta x_{\text{inc}}}{\Delta p_1} = \frac{\Delta x_{\text{inc}}}{\Delta m} \cdot \frac{\Delta m}{\Delta p_1} = -\frac{x_1^B - x_1^C}{\Delta m} \cdot \frac{\Delta m}{\Delta p_1}. \tag{5}$$

Here the second factor $\frac{\Delta m}{\Delta p_1}$ we can calculate precisely: Recall that m was the consumer's income when he bought bundle A given the original prices (p_1, p_2), hence

$$m = p_1 x_1^A + p_2 x_2^A;$$

and $m + \Delta m$ is the expense of bundle A given the new prices $(p_1 + \Delta p_1, p_2)$ (as shown in Figure 3, $(m + \Delta m)/p_2$ is the vertical intercept of the budget line l_B, which by definition has slope $-(p_1 + \Delta p_1)/p_2$ and passes through point A), hence

$$m + \Delta m = (p_1 + \Delta p_1)x_1^A + p_2 x_2^A.$$

Subtract the two above-displayed equations to obtain $\Delta m = x_1^A \Delta p_1$. Thus,

$$\frac{\Delta m}{\Delta p_1} = x_1^A.$$

Plug this into Eq. (5) and we see that the income effect of a price change Δp_1 is equal to

$$\frac{\Delta x_{\text{inc}}}{\Delta p_1} = -\frac{x_1^B - x_1^C}{\Delta m} \cdot x_1^A. \tag{6}$$

Since $x_1^A > 0$, Eq. (6) implies that the income effect is negative if good 1 is normal (i.e., $\frac{x_1^B - x_1^C}{\Delta m} > 0$), and positive if inferior (i.e., $\frac{x_1^B - x_1^C}{\Delta m} < 0$). Furthermore, from Eq. (6) we learn that *the income effect is proportional to x_1^A, the quantity of good 1 consumption before the price change.*

4 Total effect

Plugging Eq. (6) into Eq. (2), we obtain the Slutsky equation

$$\frac{\Delta x_{\text{total}}}{\Delta p_1} = \underbrace{\frac{\Delta x_{\text{sub}}}{\Delta p_1}}_{\text{substitution effect}} \underbrace{-\frac{x_1^B - x_1^C}{\Delta m} \cdot x_1^A}_{\text{income effect}}, \tag{7}$$

which, even without explicit formulas of the demand function or specific information of the consumer's preferences, implies the following bifurcated prediction:

When good 1 is a normal good, $\frac{x_1^B - x_1^C}{\Delta m} > 0$, hence the income effect—the term that starts with and includes the negative sign in Eq. (7)—is negative. This coupled with Ineq. (3) implies that the total effect, the left-hand side of Eq. (7), is negative, i.e., the demand curve for good 1 is downward sloping. This corresponds to the movement of the consumption bundle from point A to point C in Figure 4.

When good 1 is an inferior good, $\frac{x_1^B - x_1^C}{\Delta m} < 0$, so the income effect is positive. Thus the right-hand side of Eq. (7) is the sum of two terms of opposite signs: the negative substitution effect and the positive income effect. Hence its left-hand side, the total effect, may be negative, such as the movement from A to C' in Figure 4, or positive, such as the movement from A to C'' in Figure 4. Even in such a case, however, we are not clueless, for the income effect, by Eq. (6), is in the order of x_1^A, the consumption quantity of good 1. Thus, when good 1 is inferior, if the consumption quantity of good 1 is sufficiently large, the positive income effect becomes large enough to outweigh the negative substitution effect, thereby rendering the total effect, and the slope of the demand curve, positive. Should that happen, good 1 is called *Giffen good*.

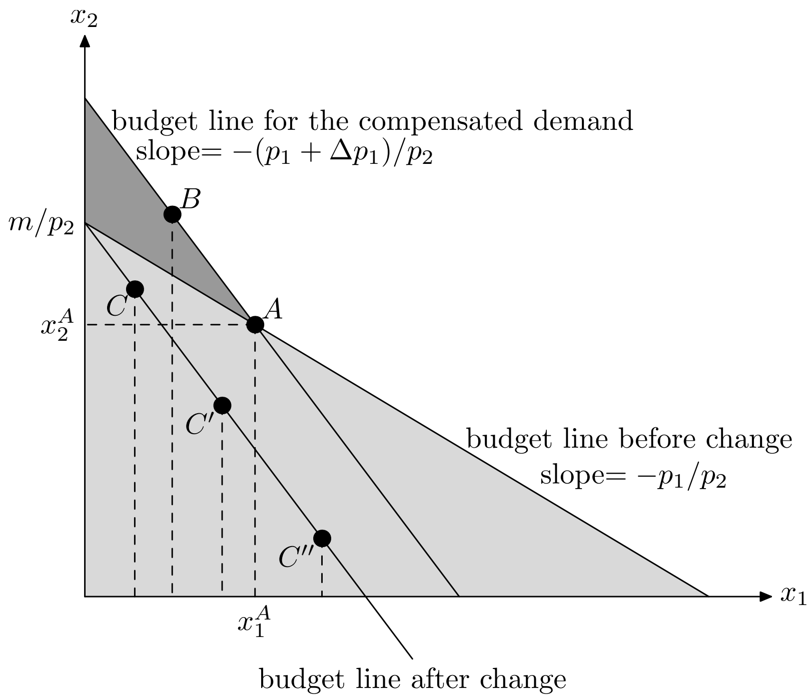

Figure 4: Good 1 is normal if $B \to C$, inferior (but not Giffen) if $B \to C'$, and Giffen if $B \to C''$

5 With proper notations

The notations x_1^A, x_1^B and x_1^C we have been using thus far, albeit easy to recognize, are ad hoc because we need to refer to a graph for their meanings. Let us replace them with proper ones derived from the notation $\tilde{x}_1(p_1, m)$ of the demand function for good 1, with the constant p_2 suppressed. To that end, note, by inspection of Figure 1, that

$$\begin{aligned}
x_1^A &= \tilde{x}_1(p_1, m) = \tilde{x}_1(p_1, p_1 x_1^A + p_2 x_2^A), \\
x_1^B &= \tilde{x}_1(p_1', p_1' x_1^A + p_2 x_2^A) \;=\; \tilde{x}_1(p_1', m + \Delta m), \\
x_1^C &= \tilde{x}_1(p_1', m),
\end{aligned}$$

where the second equality in the first line is due to A's belonging to line l_A, and the second equality in the second line due to the fact $\Delta m = x_1^A \Delta p_1$ obtained in Section 3. Since (x_1^A, x_2^B) and p_2 are held constant, let us shorten the notations by introducing the *Slutsky compensated demand*

$$x_1^S(p_1) := \tilde{x}_1(p_1, p_1 x_1^A + p_2 x_2^A) \tag{8}$$

given price p_1 for good 1, and likewise $x_1^S(p_1') := \tilde{x}_1(p_1', p_1' x_1^A + p_2 x_2^A)$ given p_1'. Then

$$\begin{aligned}
\Delta x_{\text{sub}} &= x_1^B - x_1^A \;=\; x_1^S(p_1') - x_1^S(p_1), \\
\Delta x_{\text{inc}} &= x_1^C - x_1^B \;=\; \tilde{x}_1(p_1', m) - \tilde{x}_1(p_1', m + \Delta m).
\end{aligned}$$

With such notations, Eq. (7) becomes

$$\frac{\tilde{x}_1(p_1', m) - \tilde{x}_1(p_1, m)}{\Delta p_1} = \frac{x_1^S(p_1') - x_1^S(p_1)}{\Delta p_1} - \frac{\tilde{x}_1(p_1', m + \Delta m) - \tilde{x}_1(p_1', m)}{\Delta m} \cdot \tilde{x}_1(p_1, m).$$

In deriving the above equation, we did not restrict the magnitude of the price change Δp_1. Thus we can take the limit of the equation when Δp_1 converges to zero (i.e., $p_1' \to p_1$), thereby obtaining (by the definition of derivatives in calculus) the Slutsky equation in derivative format:

$$\frac{\partial}{\partial p_1}\tilde{x}_1(p_1, m) = \frac{d}{dp_1} x_1^S(p_1)\Big|_{\left[\begin{smallmatrix} x_1 \\ x_2 \end{smallmatrix}\right] = \left[\begin{smallmatrix} \tilde{x}_1(p_1, m) \\ \tilde{x}_2(p_1, m) \end{smallmatrix}\right]} - \frac{\partial}{\partial m}\tilde{x}_1(p_1, m) \cdot \tilde{x}_1(p_1, m). \tag{9}$$

6 Exercises

1. Suppose that a consumer's preference relation is monotone, smooth (c.f. §3, Chapter 6) and exhibiting diminishing MRS, and that the given market prices and income are each positive.

   a. Explain why the consumer's optimum is unique and belongs to the budget line.

   b. Explain why the bundle B in Figure 2 cannot be the same as A in that figure.

2. Suppose that the price for good 1 increases from \$2 to \$5 while the price of good 2 remains to be \$4, and the consumer's money income unchanged. And suppose that before this price change the consumer's optimal consumption bundle was $(7, 3)$. Suppose further that the consumer's preference relation is monotone, smooth and exhibiting diminishing MRS.

   a. Write down the equation corresponding to the consumer's budget line before the price change; what is the x_2-intercept of this budget line?

   b. Write down the equation corresponding to the straight line passing through the bundle $(7, 3)$ and having slope $-\$5/\$4$ (given the new price); what is its x_2-intercept?

   c. To make the line obtained in Step (b) the budget line for the consumer, how much money is needed to give to the consumer (in addition to her fixed income)?

   d. Suppose that the price for good 1 *decreases*, rather than increase, from \$2 to \$0.5, with every other parameter same before. In order to ensure that the consumer has the *exact* (i.e., no more and no less) amount of income to afford the original bundle $(7, 3)$ given the new price, how much money needs to be taken away from him?

3. Nathan's preference relation is monotone, smooth and exhibiting diminishing MRS. He is given a fixed quantity of income (and nothing else). When the price is \$2 for good 1 and \$6 for good 2, Nathan chooses the bundle $(3, 3)$. Now the price of good 1 has increased to \$4 while the price of good 2 and Nathan's income remain unchanged, and Nathan ends up choosing a new bundle such that his consumption of good 1 is 2.5 units.

   a. Write down the equation for Nathan's budget line before the price change, and that after the price change.

   b. Suppose that, after the price change, Nathan's income is adjusted so that he has the exact amount of income to afford the previous bundle $(3, 3)$ under the new price. Then, given such adjustment and the new price:

      i. write down the equation for Nathan's budget line;

ii. if the quantity of good 1 optimally chosen by Nathan in this circumstance is one of the following: 2 units, 3 units, 4 units, or 9 units, then which one should it be? (hint: Figure 2)

c. Based on the answer in Step 3(b.)ii and the fact that Nathan's consumption of good 1 becomes 2.5 units when p_1 becomes \$4 while p_2 and original income m are unchanged, is good 1 a normal or inferior good? If it is the latter, is the good also Giffen? (Hint: deduce the sign of the income effect based on the Slutsky equation.)

4. Consider a utility function defined by $u(x_1, x_2) := x_1^{1/3} x_2^{2/3}$ for all nonnegative x_1 and x_2. Recall the demand function $\tilde{x}_1(p_1, p_2, m)$ for good 1 you calculated in the corresponding exercise in Chapter 7. Assume p_2 to be constant.

   a. Calculate $\frac{\partial}{\partial p_1} \tilde{x}_1(p_1, p_2, m)$.

   b. Calculate $\frac{\partial}{\partial m} \tilde{x}_1(p_1, p_2, m)$.

   c. Calculate $\tilde{x}_1(p_1, p_2, p_1 x_1 + p_2 x_2)$ for any consumption bundle (x_1, x_2); note from Eq. (8) that $\tilde{x}_1(p_1, p_2, p_1 x_1 + p_2 x_2)$ is equal to the Slutsky compensated demand at bundle (x_1, x_2).

   d. Calculate the right-hand side of Eq. (9); verify that the result obtained thereof is equal to the one in Step 4a.

5. A consumer's utility function is given by

$$u(x_1, x_2) := \sqrt{x_1} + x_2$$

   for any nonnegative x_1 and x_2, representing the consumption quantities of goods 1 and 2, respectively. Suppose that the price of good 2 is constantly \$1, and that the consumer is given income m dollars (and nothing else). Denote p_1 for the price of good 1. Based on the quantity demand $\tilde{x}_1(p_1, m)$ for good 1 derived in the corresponding exercise in Chapter 7:

   a. Given that $p_1 > 1/(4m)$, calculate $\frac{\partial}{\partial p_1} \tilde{x}_1(p_1, m)$ and $\frac{\partial}{\partial m} \tilde{x}_1(p_1, p_2, m)$, then use the Slutsky equation to deduce what the substitution effect $\frac{d}{dp_1} x_1^S(p_1, x_2)$ is equal to.

   b. Given that $p_1 < 1/(4m)$, calculate $\frac{\partial}{\partial p_1} \tilde{x}_1(p_1, m)$ and $\frac{\partial}{\partial m} \tilde{x}_1(p_1, p_2, m)$, then use the Slutsky equation to deduce what the substitution effect $\frac{d}{dp_1} x_1^S(p_1, x_2)$ is equal to.

   c. Is it true that the income effect is zero whenever the consumer's preferences are quasi-linear? If not, what additional condition is needed to make that true?

6. Suppose that $\Delta p_1 < 0$. Draw a graph analogous to Figure 2 to describe the case in Section 2, and a graph analogous to Figure 4 to describe the case in Section 3.

7. Consider a utility function defined by

$$u(x_1, x_2) := \min \left\{ x_1 + x_2, \frac{9}{10} x_2 + 1 \right\}$$

   for all nonnegative $x_1, x_2 \geq 0$. Let $m > 0$ be the consumer's entire income m. Denote p_1 for the price of good 1, and p_2 that of good 2. Let $\tilde{x}_1(p_1, p_2, m)$ denote the demand function for good 1 obtained in the corresponding exercise in Chapter 7. Given that $0 < p_1 < p_2$ and $m/p_2 < 10$:

a. Calculate $\frac{\partial}{\partial p_1}\tilde{x}_1(p_1, p_2, m)$.

b. Calculate $\frac{\partial}{\partial m}\tilde{x}_1(p_1, p_2, m)$.

c. Draw a graph to illustrate that the substitution effect is equal to zero.

d. Plug the results obtained in Steps 7b. and 7c. into the right-hand side of Eq. (9); verify that the result obtained thereof is equal to the one in Step 7a.

8. In addition to the Slutsky compensated demand, another way to quantify the substitution effect of price change is *Hicksian compensated demand*: Let A denote the optimal bundle given prices (p_1^A, p_2^A) and income m, represented by the budget line l_A in Figure 5, and let α denote the indifference curve that A belongs to; suppose that the prices change to (p_1^B, p_2^B); rotate the original budget line l_A along the indifference curve α until the budget line has the new slope $-p_1^B/p_2^B$, as line l_B in Figure 5; the common point between l_B and α, labeled B in Figure 5, is the Hicksian compensated demand given the original bundle A. And the *Hicksian substitution effect* is determined by the distance between A and B, defined by $(x_1^B - x_1^A)/(p_1^B - p_1^A)$. In Figure 5, it is clear that such substitution effect is negative, i.e., $(x_1^B - x_1^A)/(p_1^B - p_1^A) < 0$. But the figure implicitly rules out the possibility of corner solutions (c.f. §4, Chapter 6). The following sketches a proof for negativity of the Hicksian substitution effect without ruling out the possibility of corner solutions.

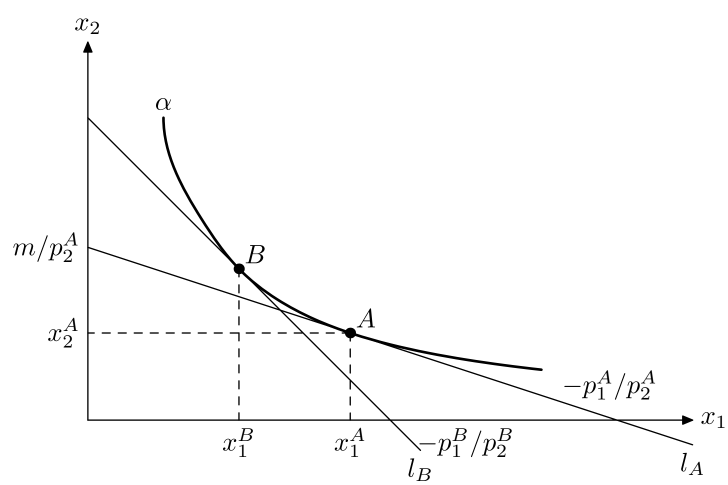

Figure 5: $(x_1^B - x_1^A)/(p_1^B - p_1^A)$: Hicksian substitution effect

a. Why is it impossible that $p_1^B x_1^A + p_2^B x_2^A \le p_1^B x_1^B + p_2^B x_2^B$ even when situations may be different from Figure 5?

b. Obtain an inequality as a logical consequence of the previous step.

c. Analogously to the previous steps, figure out whether $p_1^A x_1^B + p_2^A x_2^B \le p_1^A x_1^A + p_2^A x_2^A$ or $p_1^A x_1^B + p_2^A x_2^B > p_1^A x_1^A + p_2^A x_2^A$ is true.

d. Based on the inequalities obtained in Steps 8b. and 8c., prove that

$$(p_1^B - p_1^A)(x_1^B - x_1^A) + (p_2^B - p_2^A)(x_2^B - x_2^A) < 0. \tag{10}$$

e. Use Ineq. (10) to prove that if $p_2^B = p_2^A$ then $(x_1^B - x_1^A)/(p_1^B - p_1^A) < 0$.

9. Imagine a different setup where the consumer were given unlimited income but were bound by the restriction that he stay on the indifference curve α in Figure 5, so his decision, given prices (p_1, p_2), is to minimize the expense $p_1x_1 + p_2x_2$ by choosing a bundle $(x_1, x_2) \in \mathbb{R}^2_+$ subject to the constraint that (x_1, x_2) belongs to the indifference curve α.

   a. Does this problem sound similar to something in an earlier chapter?

   b. Let A be the optimal bundle in the above-described decision problem, given prices (p_1^A, p_2^A), and B the optimal bundle in the same problem given prices (p_1^B, p_2^B).

      i. Derive an inequality from the fact that A is the unique optimal bundle given prices (p_1^A, p_2^A), while B is an alternative, feasible bundle.

      ii. Derive an inequality from the fact that B is the unique optimal bundle given prices (p_1^B, p_2^B), while A is an alternative, feasible bundle.

      iii. Prove Ineq. (10) from the inequalities obtained in the previous two steps.