Elements of Decision

Lecture Notes of Intermediate Microeconomics

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Preface

Here are my lecture notes, exercises included, for an Intermediate Microeconomics course that I have been teaching at the University of Western Ontario for more than a decade, and at Carnegie Mellon University during my visit there. The course is for the honors level sophomores majoring in economics. My goal of writing the lecture notes is to compactify the various topics into just several chapters under a single theme, to introduce the formal analysis of decision making in the pure competition context. Such compactification helps me to strike a balance between the objective of engaging the students in nontrivial economic reasoning—mathematics an integral part thereof—and the constraints due to the large number of students and their vastly diverse levels of preparedness in critical reasoning. With only a few chapters to read, students are relatively free of the distractive burden of memorizing jargons in various topics, so they have time to exercise critical reading.

I sequence the chapters in a somewhat unconventional way, starting with producer theory instead of consumer theory. I prefer such sequencing because producer theory is simpler in substance, with explicit objective functions and without the income effect complication. Furthermore, scientific reasoning on human activities requires a kind of "suspension of disbelief" (Wordswoth), as an observer can be easily biased by his own parochial experience when the subjects he studies are close to him. In contrast to a firm's decision, a consumer's decision can sound too close to students' daily experience for them to suspend their specific intuition thereby to make room for abstract reasoning.

While most of the chapters are confined to classic decision theory, as it is the main component of this course, I include some game theory materials to convey a sense of continuance from classic decision theory to game theory. First comes a a chapter on zero-sum games that are solved as if they were a consumer's optimization problems, thanks to the minimax theorem. Then is a chapter on dominant strategies, which inherits much of the decision-theoretic convenience that the opponents' responses can be set aside to a large extent. Finally comes an introduction to Nash equilibrium, where decisions are truly interactive across players.

To Students Please read these chapters *slowly, critically*. Except the exercises, almost every sentence (which can be in the form of equations) is meant to be either a definition, an assumption, or an assertion. For every sentence that appears, figure out which category it belongs to. If it is an assertion, figure out its reasoning, namely, the path from the definitions and assumptions to the assertion. And please be mindful of typos, which most likely exist, as these are notes that I update often, and any update may necessitate further changes yet to be incorporated.

Acknowledgement I got the idea of presenting producer theory before consumer theory from the lecture notes of my late advisor Marcel Ket Richter in an inspiring first-year PhD class he taught at the University of Minnesota. My chapters on revealed preference and Slutsky Equation are likely to have been influenced by Hal Varian's undergraduate textbook *Intermediate Microeconomics*, which I used as the textbook at the early stage of my teaching career.

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Part I

Decisions

Chapter 1

Firm's Supply

1.1 Introduction

Economics is a science on relations among individuals. Such relations include trades, contracts, conflicts, households, markets, networks, societies, etc. Any such relation can be modeled into a game, where one tries to pick a move as a best response to the move he expects his counterpart may pick, knowing that the counterpart's move also results from her attempt to best respond to his move that she expects him to pick. Calculations of such *interactive reasoning* constitute *game theory*, the mathematical foundation of modern economics. The fundamental building block of such calculations is the mathematical method for an individual to decide on his best move. To introduce such *decision-theoretic* method in the simplest possible setting, we start by pretending that the counterpart of the individual is not a strategic player trying to game him but rather a nonstrategic entity such as a purely competitive market, which the individual does not need to haggle with. Given this assumption of pure competition, learning the basic decision-theoretic techniques will constitute two thirds of this course. Then, for the remaining third, we will extend such techniques to basic concepts in game theory.

1.2 A firm's output decision problem

To illustrate a decision problem in its simplest form, suppose that a firm acts as an individual deciding how much of its output to sell.[1] Suppose that every unit of the output is sold for p dollars in the market (the aforementioned pure competition assumpiton), and that the firm incurs a cost $C(q)$ dollars if it sells q units of the output to the market. The question is How many units of the output should the firm supply to maximize its profit? As *profit* means revenue minus cost, the firm's profit from supplying q units is equal to $pq - C(q)$ dollars. Thus the firm's decision

[1] For a real-world example, think of a small dairy farm that sells a single kind of milk to a competitive market.

problem becomes choosing a nonnegative quantity q to maximize $pq - C(q)$, i.e.,

$$\max_{q \in \mathbb{R}_+} pq - C(q). \tag{1.1}$$

Problem (1.1) exemplifies an optimization problem: (i) There is an *objective*, the expression $pq - C(q)$ following the operator max (shorthand for maximization), which the decision maker is to maximize (or minimize if max is replaced by min). (ii) There is a *choice variable* and its *domain*, written underneath max to indicate that the decision maker is to choose an element from the domain to maximize his objective; here the choice variable is denoted q, its domain $\mathbb{R}_+$ (the set of nonnegative real numbers),[2] with the symbol $\in$ meaning "belongs to" or "is an element of." (iii) There is a *parameter*, which, p in this example, is assumed constant by the decision maker; it is important to keep in mind the distinction between a choice variable and a parameter: the former is up to the decision maker to choose, while the latter is not.

1.3 Cost function

To make Problem (1.1) more tractable, let us add two usual assumptions, and figure out their implications, of the above cost function C. It is usually *assumed* that C is of the form

$$C(q) = C_v(q) + c_0 \tag{1.2}$$

for any $q \geq 0$, where $C_v(q)$ varies with q with $C_v(0) = 0$, hence called *variable cost*, and c_0 is positive and constant to q, hence called *fixed cost*. Correspondingly, the *average cost*, defined to be $C(q)/q$ and denoted by $\mathrm{AC}(q)$, is decomposed by

$$\mathrm{AC}(q) = \frac{C(q)}{q} = \frac{C_v(q)}{q} + \frac{c_0}{q}, \tag{1.3}$$

with $C_v(q)/q$ called *average variable cost* (AVC), and c_0/q *average fixed cost*.

It is also *assumed* that C is differentiable and that its derivative $\frac{d}{dq}C$, called *marginal cost* and denoted by $\mathrm{MC}(q)$, is continuously decreasing in q when q rises from zero and, once q rises to a threshold level, becomes continuously increasing in q thereafter without upper bound. By Eq. (1.2),

$$\mathrm{MC}(q) = \frac{d}{dq}C(q) = \frac{d}{dq}C_v(q). \tag{1.4}$$

Draw a coordinate system whose horizontal axis stands for q, and vertical axis for the marginal cost and the various kinds of average costs. In this coordinate system the marginal cost function corresponds to a curve, U-shape because of the assumption stated before Eq. (1.4). Note the

[2] Interchangeably $\mathbb{R}_+$ is also denoted by $[0, \infty)$, meaning the set of real numbers between zero and infinity ∞, with the [on the left signifying that the boundary point zero is included, and the) on the right that ∞ is excluded. The notation $(0, \infty)$, with the left [replaced by (, excludes zero and denotes the set of positive real numbers.

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