Reconciling Alternative Views About the Appropriate Social Discount Rate

David F. Burgess
Department of Economics
University of Western Ontario
June 6, 2012

Abstract

This paper shows that, in an economy with an exogenous rate of return and a given capital income tax distortion, and with lump sum taxes as the marginal tax instrument, the SOC and MCF criteria both correctly identify all worthwhile projects if the criteria are properly applied. The equivalence between the SOC and MCF criteria continues to hold i) if distortionary taxes are used to balance the budget, and ii) if the rate of return to capital is endogenous. Apparent differences between the SOC and MCF criteria arise from different definitions of a project's indirect revenue effect. Neither criterion has an implementation advantage because the information requirements for each are identical

1 Introduction

This paper is an attempt to reconcile two prevalent criteria for project evaluation in a tax-distorted economy: the social opportunity cost of capital (SOC) criterion first proposed by Harberger (1969) and confirmed by Sandmo-Dreze (1971), which discounts benefits and costs at the rate of return foregone in the private sector when the government borrows to finance the project (a weighted average of the pre-tax and after-tax rates of return); and the "marginal cost of funds" (MCF) approach recently proposed by Liu (2003), which discounts benefits at the after-tax rate of return and discounts costs (including any "indirect revenue effects") at the pre-tax rate of return, but multiplies all costs and indirect revenue effects by a parameter referred to as the MCF.

The MCF criterion recognizes that using a lump sum tax to raise an additional dollar of revenue will have a social welfare cost that differs from a dollar when there is a pre-existing capital income tax distortion, and this welfare cost is compounded if the marginal tax instrument is a distortionary tax on labour income. Reasoning from the criterion for the optimum supply of a public good in a tax-distorted static (one period) economy, Liu (2003) argues that the marginal cost of funds parameter must be an integral part of any multi-period project evaluation, and (except in special circumstances) the SOC criterion is deficient because it fails to take the MCF parameter into account. He further argues that there is no general formula for the weights that are required to calculate the social opportunity cost of capital so the appropriate discount rate is "project specific", making the SOC criterion almost impossible to apply in practice. The MCF criterion supposedly avoids these difficulties because the discount rates for evaluating benefits and costs and the MCF parameter are all project independent.

However, I believe there is some misunderstanding about the SOC criterion. If a project provides benefits that the private sector regards as equivalent to income, a straightforward application of the SOC criterion is appropriate; benefits and costs should be discounted at a rate equal to the social (economic) opportunity cost of borrowed funds, which is a weighted average of the pre-tax and after-tax rates of return where the weights reflect the proportions of funding that displace private investment and consumption respectively. For any project whose benefits are not treated as income there will be indirect revenue effects, but they reflect the *compensated* effect of the project on tax revenue (i.e., the effect holding utility fixed) rather than the *uncompensated* effect (i.e., the effect holding income fixed) that enters the MCF criterion, and these effects should be incorporated by adding to (or subtracting from) the project's benefits, not by adjusting the discount rate. When properly applied, the SOC criterion is perfectly consistent with the MCF criterion; both criteria correctly identify all worthy projects.

Section 2 demonstrates the fundamental equivalence between the MCF and SOC criteria when the pre-tax rate of return is exogenous and the marginal tax instrument is a lump sum tax. Section 3 extends the analysis to situations where a distortionary tax on labour income is used to achieve intertemporal budget balance. Section 4 generalizes the analysis to situations where the pre-tax rate of return is endogenous and shows that the fundamental equivalence between the SOC criterion and a modified version of the MCF criterion continues to hold. Section 5 illustrates the main results using the example of a project that generates a perpetuity. Section 6 concludes.

2 MCF criterion versus SOC criterion with lump sum taxation

Consider the following simplified version of the infinitely lived representative agent (ILA) model used by Liu (2003). The representative agent earns an exogenous pre-tax wage w for a given amount of work effort L, and earns an exogenous pre-tax rate of return ρ on assets, but incurs a time stream of lump sum taxes $\{T^t\}$ and a time-invariant tax at proportional rate τ on capital income. There are two goods available in each period: a composite private good c^t, and a publicly provided good g^t.

Given the time streams of the publicly provided good $g = \{g^t\}$ and lump sum taxes $T = \{T^t\}$, a time stream for private consumption $c = \{c^t\}$ is chosen to maximize

$W(c, g) = \Sigma_{t=0}^{\infty} \beta^t U(c^t, g^t)$

subject to

$\Sigma_{t=0}^{\infty} c^t/(1 + r)^t = \Sigma_{t=0}^{\infty}(wL - T^t)/(1 + r)^t + A^0$

where A^0 is initial wealth and $\beta < 1$, so $(1 - \beta)/\beta$ is the pure rate of time preference. The representative agent's discount rate is the after-tax rate of return $r = (1-\tau)\rho$. From the private sector's first order conditions, consumption in each period is a function of the time stream of lump sum taxes and the time stream of the publicly provided good so $c^t(g, T)$.[1] Well-being can therefore be written as $W(c(g, T), g) = V(g, T)$.

The benefits of the publicly provided good are available to the representative agent free of charge. The government's budget constraint therefore requires that the discounted sum of tax revenue $\{R^t\}$ minus project expenditures $\{I_g^t\}$ is equal to its initial net indebtedness D^0. Government revenue in period t consists of lump sum taxes and capital income taxes, and capital income taxes depend upon assets held at the beginning of period t, which depend upon the time stream of lump sum taxes and, conceivably, the time stream of the publicly provided good. Thus $R^t = T^t + \tau\rho A^t(g, T)$.

In this formulation interest payments on government debt are taxed at the same rate as returns on private capital. Therefore government debt evolves according to $D^{t+1} = (1 + \rho)D^t + I_g^t - T^t - \tau\rho A^t$. The government's budget constraint can then be written in integrated form as

$\Sigma_{t=0}^{\infty}(T^t + \tau\rho A^t - I_g^t)/(1 + \rho)^t = D^0$.

[1] Other determinants of consumption include inital wealth A^0, the pre-tax rate of return ρ, the capital income tax rate τ, and the wage rate w. We suppress these variables because they are assumed to be unaffected by the project.

where the discount rate is the pre-tax rate of return.[2]

Now consider a small project that produces a stream of output $\{dg^t\}$ but requires an increase in government expenditures $\{dI_g^t\}$ and lump sum taxes $\{dT^t\}$ to balance the budget. The project is worthwhile if the representative agent is made better off. Assume (with no loss in generality) that the project is financed by an increase in lump sum taxes in period 0.[3] The project will make the representative agent better off if

$$\Sigma_{t=0}^{\infty}(\partial V/\partial g^t)dg^t + (\partial V/\partial T^0)dT^0 > 0$$

Dividing through by $-\partial V/\partial T^0$ and making use of the envelope theorem (which ensures that $\partial V/\partial g^t = \beta^t \partial U/\partial g^t$ and $\partial V/\partial T^0 = -\partial U/\partial c^0$), this can be re-written as

$$\Sigma_{t=0}^{\infty}\beta^t \frac{\partial U/\partial g^t}{\partial U/\partial c^0}dg^t - dT^0 > 0$$

The project's benefit in period t, denoted by B^t, is equal to $p_g^t dg^t$, where $p_g^t = (\partial U/\partial g^t)/(\partial U/\partial c^t)$ represents the marginal rate of substitution between the publicly provided good and the composite private good in period t. From the private sector's first order conditions, $\beta^t(\partial U/\partial c^t)(1+r)^t = \partial U/\partial c^0$. Therefore, the representative agent will be better off if the benefits discounted at the after-tax rate of return exceed the required lump sum tax increase in period 0, i.e. if

$$(1)\ \Sigma_{t=0}^{\infty}B^t/(1+r)^t - dT^0 > 0.$$

A project that requires a sequence of expenditures $\{dI_g^t\}$ and is financed by a lump sum tax increase dT^0 is fiscally feasible if the present value of the additional tax revenue collected is equal to the present value of the project's expenditure requirements. Thus

$$(2)\ dT^0 + \Sigma_{t=1}^{\infty}dR^t/(1+\rho)^t = \Sigma_{t=0}^{\infty}dI_g^t/(1+\rho)^t$$

The second term on the left hand side of (2) captures the combined effects of the project's output stream and the required lump sum tax increase on the present value of capital income tax revenue. Since $R^t = T^t + \tau\rho A^t$, the change in tax revenue collected in period $t = 1, ..., \infty$ is

$$dR^t = \tau\rho\left[\Sigma_{i=0}^{\infty}(\partial A^t/\partial g^i)dg^i + (\partial A^t/\partial T^0)dT^0\right]$$

The first term in this expression represents the "indirect revenue effect" of the project in period t, i.e. the effect of the project's output stream on capital income tax revenue collected in period t. Thus $IR^t = \tau\rho\Sigma_{i=0}^{\infty}(\partial A^t/\partial g^i)dg^i$. The second term represents the impact on capital income tax revenue in period t of financing the project with a lump sum tax increase in period 0.

If we substitute the expression for dR^t into equation (2), the fiscal feasibility constraint can be re-written as

[2] If interest payments on government bonds were tax exempt, tax revenue in period t would be $R^t = T^t + \tau\rho K^t$, where K^t is private capital invested at the beginning of period t. Since $A^t = K^t + D^t$ government debt would evolve according to $D^{t+1} = (1+r)D^t + I_g^t - T^t - \tau\rho K^t$, which in integrated form becomes $\Sigma_{t=0}^{\infty}(R^t - I_g^t)/(1+r)^t = D^0$. The decision to tax or not to tax interest payments on government debt will affect the *financial* cost of borrowing, but not the *economic opportunity cost* of borrowing because a dollar of government borrowing will displace a dollar of private capital whose marginal rate of productivity is ρ.

[3] Ricardian equivalence holds in the ILA model, so the *timing* of any lump sum tax increase is irrelevant.

(3) $dT^0 = \left[1 + \tau\rho\Sigma_{t=1}^{\infty}(\partial A^t/\partial T^0)/(1+\rho)^t\right]^{-1}\left[\Sigma_{t=0}^{\infty} dI_g^t/(1+\rho)^t - \Sigma_{t=1}^{\infty} IR^t/(1+\rho)^t\right]$

The first term in square brackets with the inverse sign is the ratio of the increase in lump sum tax revenue to the increase in the present value of total tax revenue. Since the increase in lump sum tax revenue is equal to the reduction in private sector welfare (all measured in terms of period 0 consumption), this term represents the welfare cost per dollar increase in government revenue resulting from a lump sum tax increase. Liu refers to this as the "marginal cost of funds" (MCF) for a lump sum tax.[4] Thus

(4) $MCF = \left[\Sigma_{t=0}^{\infty}(\partial R^t/\partial T^0)/(1+\rho)^t\right]^{-1} = \left[1 + \Sigma_{t=1}^{\infty}\tau\rho(\partial A^t/\partial T^0)/(1+\rho)^t\right]^{-1}$

Now use (3) to eliminate dT^0 from (1) and we find that the representative agent will be better off with the project if

(5) $\Sigma_{t=0}^{\infty} B^t/(1+r)^t - MCF\left[\Sigma_{t=0}^{\infty} dI_g^t/(1+\rho)^t - \Sigma_{t=1}^{\infty} IR^t/(1+\rho)^t\right] > 0$

In words, the present value of the project's benefits discounted at the after-tax rate of return r must exceed the present value of the project's expenditure requirements minus its indirect revenue effects all discounted at the pre-tax rate of return ρ and mulitiplied by the MCF parameter. This is the MCF criterion proposed by Liu (2003).[5]

Liu maintains that the SOC criterion for the project to be worthwhile takes the form

$\Sigma_{t=0}^{\infty}(B^t - dI_g^t)/(1+\omega)^t > 0$

where ω is a weighted average of ρ and r, the weights being determined by the proportions of resources that are drawn from investment and consumption. He claims that the appropriate weighted average discount rate will be "project specific", making the SOC criterion almost impossible to apply in practice. This is because (in his view) the SOC criterion looks only at a project's *direct* benefits and costs, ignoring indirect revenue effects. Thus a project with positive indirect revenue effects (i.e. a project whose output stimulates private investment and thereby generates additional capital income tax revenue) will warrant a lower weighted average discount rate than a project with no indirect revenue effects.

These statements reveal a misunderstanding about the SOC criterion and how to apply it.[6] The appropriate social discount rate reflects what Harberger

[4] Jones (2005) refers to this term as the "shadow value of government revenue" when a lump sum tax is used to transfer a dollar of revenue to the government. He takes the conventional (Harberger) view that the marginal cost of funds is *by definition* one plus the marginal excess burden, so the marginal cost of funds for a lump sum tax is always unity.

[5] Liu's MCF parameter is project independent. The "spending effect" of the project, i.e. how the project's output affects tax revenue, is captured by the indirect revenue effect. A "project specific" MCF would incorporate the indirect revenue effect in the MCF parameter. It measures the welfare cost of raising the revenue necessary to finance the project, taking into account how the project itself affects tax revenue. Measures of the MCF that incorporate the spending effect are discussed by Wildasin (1984) and Ballard and Fullerton (1992), among others. Dahlby (2008) provides a good overview of the literature on the conceptual foundations of the MCF parameter.

[6] The misunderstanding is not unique to Liu (2003). It is reflected, for example, in the views of Diamond (1968), Feldstein (1972) and Stiglitz (1982), who follow Sandmo/Dreze (1971) in defining the SOC rate as the marginal rate of return on worthwhile public investment at the second best optimum under the pre-existing capital income tax.

(1969) calls the "social opportunity cost of borrowed funds". This is the rate of return foregone when the government, at pre-existing tax rates, induces the private sector voluntarily to relinquish funds that would otherwise finance private investment and consumption, and it is independent of the project being assessed. If the pre-tax rate of return ρ is endogenous the SOC rate will be a weighted average of the pre-tax and post-tax rates of return, where the weights reflect the proportions of funds that are drawn away from financing private investment and consumption respectively.[7] If the pre-tax rate of return ρ is exogenous, as in Liu's model, the SOC rate will equal the pre-tax rate of return because each dollar of borrowing will displace a dollar of private investment.

With respect to indirect revenue effects, the SOC criterion *does* recognize the possibility of indirect revenue effects but, as this paper makes clear, the appropriate measure of the indirect revenue effect is the **compensated** effect of the project on capital income tax revenue, not the **uncompensated** (Marshallian) effect that appears in the MCF criterion.[8] This is because the SOC criterion looks at the effect of the project on government revenue when the private sector is kept at pre-project utility. The benchmark for measuring indirect revenue effects using the SOC criterion is therefore a project whose compensated effect on capital income tax revenue is zero, i.e. a project whose benefits are "just like income".

Finally, the marginal cost of funds parameter as Liu defines it *does* exceed unity for a lump sum tax (when labour supply is exogenous) but it is not the same as the conventional (Harberger) measure of the MCF, which is the welfare cost of using a particular tax instrument to raise a dollar of revenue that is (lump sum) rebated to balance the budget. The conventional measure of the MCF parameter for a lump sum tax is therefore equal to one. Rather, Liu's MCF parameter represents the "shadow value of government revenue", which is the welfare cost of using a lump sum tax to transfer a dollar of revenue to the government budget. The shadow value of government revenue therefore contains an income effect that is not present in the conventional measure of the MCF. The shadow value of government revenue is indeed taken into account when

[7] Sjaastad and Wisecarver (1977, p 517-18 and p 533) emphasize that Harberger's SOC rate refers only to the raising of funds, while acknowledging that how the funds are spent can affect private sector decisions. Sandmo-Dreze (1971) derive Harberger's SOC rate as the marginal rate of return on public investment at the second best optimum when public investment produces a perfect substitute for the output produced by private investment. This ensures that the marginal rate of return at the second best optimum coincides with the economic opportunity cost of borrowed funds. Burgess (1988) shows that Harberger's SOC rate applies to projects that are complements or substitutes for private investment provided that the project's effect on private investment, and therefore capital income tax revenue, is included along with the benefits.

[8] The uncompensated effect is the effect of the project on capital income tax revenue holding private sector income fixed. It is the **Marshallian** uncompensated effect. The compensated effect holds private sector utility fixed. Hatta (1977) shows that the compensated effect is equal to the uncompensated **Bailey** effect divided by the Hatta coefficient, the inverse of which coincides with the shadow value of government revenue. The uncompensated Bailey effect is the effect of the project on capital income tax revenue with private sector income adjusted to balance the government's budget. Thus the uncompensated Bailey effect is a scalar multiple of the compensated effect. For further details see Jones (2005).

applying the SOC criterion; it is incorporated in the project's compensated indirect revenue effect. Benefits, costs and (compensated) indirect revenue effects are all discounted at the social opportunity cost of borrowed funds, which is independent of the project.

The compensated indirect revenue effect that enters into the SOC criterion is the effect on capital income tax revenue of the project's output stream combined with a stream of lump sum tax increases equal to the private sector's willingness to pay for the project's benefits. Thus $IR_c^t = \tau\rho\Sigma_{i=0}^{\infty}[(\partial A^t/\partial g^i)dg^i + (\partial A^t/\partial T^i)dT^i]$, where $dT^i = B^i = p_g^i dg^i$. Henceforth we will express $IR_c^t = \tau\rho\Sigma_{i=0}^{\infty}(\partial A^t/\partial g^i)_U dg^i$, where $(\partial A^t/\partial g^i)_U = (\partial A^t/\partial g^i)dg^i + p_g^i(\partial A^t/\partial T^i)$. A project with benefits $\{B^t\}$, costs $\{dI_g^t\}$, and (compensated) indirect revenue effects $\{IR_c^t\}$ is worthwhile according to the SOC criterion if:

(6) $\Sigma_{t=0}^{\infty}(B^t - dI_g^t + IR_c^t)/(1+\rho)^t > 0$

The compensated indirect revenue effect can be derived from the uncompensated effect using Liu's MCF parameter for a lump sum tax. If the project's benefits are worth $\Sigma_{t=0}^{\infty}B^t/(1+r)^t$ and the MCF parameter represents the welfare cost of raising the present value of government revenue (discounted at the SOC rate) by a dollar using a lump sum tax, then the increase in the present value of government revenue from raising lump sum taxes by an amount equal to the private sector's willingness to pay for the project's benefits is $(1/MCF)\Sigma_{t=0}^{\infty}B^t/(1+r)^t$. But the present value (discounted at the SOC rate) of the increase in lump sum tax revenue from the sequence of lump sum tax increases is $\Sigma_{t=0}^{\infty}B^t/(1+\rho)^t$. The present value of the increase in capital income tax revenue is then the difference between the increase in total tax revenue and the increase in lump sum tax revenue. Thus $\Delta CITR = (1/MCF)\Sigma_{t=0}^{\infty}B^t/(1+r)^t - \Sigma_{t=0}^{\infty}B^t/(1+\rho)^t$.[9] The compensated indirect revenue effect can then be expressed as:

(7) $\Sigma_{t=0}^{\infty}IR_c^t/(1+\rho)^t = \Sigma_{t=0}^{\infty}IR^t/(1+\rho)^t + (1/MCF)\Sigma_{t=0}^{\infty}B^t/(1+r)^t - \Sigma_{t=0}^{\infty}B^t/(1+\rho)^t$.

By substituting (7) into (6) it is easy to see that the SOC criterion in (6), with indirect revenue effects properly incorporated, is perfectly consistent with Liu's MCF criterion in (5). If the private sector treats the project's benefits as equivalent to income, e.g. if the publicly provided good is a perfect substitute for a private good, the compensated indirect revenue effect will be zero and the SOC criterion simplifies to the standard SOC formula. However, in this situation Liu's MCF criterion will include a non-zero indirect revenue effect which is still easily measured given data on the project's benefits and the MCF parameter. On the other hand, if the project has no effect on private sector behaviour because its benefits are separable from other private goods the

[9] Ricardian equivalence ensures that a welfare preserving change in the timing of a lump sum tax increase leaves the present value of government revenue unchanged. Thus $dPVR/dT^i = (dPVR/dT^0)(1+r)^{-i}$. But $\partial PVR/\partial T^i = (1+\rho)^{-i} + \tau\rho\Sigma_{t=1}^{\infty}(\partial A^t/\partial T^i)/(1+\rho)^t$ and $(\partial PVR/\partial T^0)/(1+r)^i = (1+r)^{-i}[1+\tau\rho\Sigma_{t=1}^{\infty}(\partial A^t/\partial T^0)/(1+\rho)^t]$, where the expression in square brackets is MCF^{-1}. Therefore $\Sigma_{i=0}^{\infty}B^i(\partial PVR/\partial T^i) = \Sigma_{i=0}^{\infty}B^i(\partial PVR/\partial T^0)/(1+r)^i$ from which it follows that $\tau\rho\Sigma_{i=0}^{\infty}B^i\Sigma_{t=1}^{\infty}(\partial A^t/\partial T^i)/(1+\rho)^t = MCF^{-1}\Sigma_{i=0}^{\infty}B^i/(1+r)^i - \Sigma_{i=0}^{\infty}B^i/(1+\rho)^i$.

uncompensated indirect revenue effect will be zero. This simplifies Liu's MCF criterion (by eliminating indirect revenue effects), but even though the SOC criterion now contains a non-zero (compensated) indirect revenue effect it is still easily measured given data on the project's benefits and the MCF parameter.[10]

3 MCF criterion versus SOC criterion with distortionary taxation

So far it has been assumed that lump sum taxes are feasible, but what if they are not? In this section the model of section 2 is modified to incorporate labour-leisure choice, with l^t representing leisure time in period t and the marginal tax instrument being a distortionary tax on labour income. This is the model specified by Liu (2003), which we summarize briefly as follows.

The representative agent chooses $\{c^t\}$ and $\{l^t\}$, given $\{g^t\}$, τ, τ_L, and A^0, to maximize

$$W(c,l,g) = \Sigma_{t=0}^{\infty}\beta^t U(c^t, l^t, g^t)$$

subject to

$$\Sigma_{t=0}^{\infty}c^t/(1+r)^t = \Sigma_{t=0}^{\infty}w(1-\tau_L)L^t/(1+r)^t + A^0 \text{ and } l^t + L^t = H$$

where L^t represents working time in period t and H is the total time endowment in each period.

From the private sector's first order conditions consumption and leisure in each period can be expressed as functions of initial wealth A^0, the proportional tax rates on capital and labour income, τ and τ_L, and the time stream of the publicly provided good g, so $c^t(A^0, \tau, \tau_L, g)$ and $l^t(A^0, \tau, \tau_L, g)$. Well-being is then given by $V(A^0, \tau, \tau_L, g)$.

Since the project's benefits are inappropriable, government revenue is the sum of capital income tax revenue and labour income tax revenue so $R^t = \tau\rho A^t + \tau_L w L^t$. The government's budget constraint can then be written in integrated form as

$$\Sigma_{t=0}^{\infty}(\tau\rho A^t + \tau_L w L^t)/(1+\rho)^t - \Sigma_{t=0}^{\infty}I_g^t/(1+\rho)^t = D^0$$

where ρ is the government's borrowing rate (because interest on government bonds is taxed at the same rate as returns on private capital). More importantly, ρ represents the opportunity cost of government revenue because a dollar of government revenue not used for program spending at any point in time could

[10] According to equation (7) one needs information on the MCF parameter plus information on a project's *uncompensated* indirect revenue effect to determine a project's *compensated* indirect revenue effect. However, if the private sector's preferences are specified by an expenditure function rather than a utility function, duality theory yields an estimate of the project's compensated indirect revenue effect without recourse to an MCF parameter. The compensated demand function for good i is derived from the expenditure function $E(p,g,u)$ by differentiating with respect to p^i. Thus $\partial E/\partial p^i = x^i(p,g,u)$. If good i is taxed at rate τ^i generating revenue $R = T + \tau^i x^i$ the compensated indirect revenue effect of a marginal increase in the publicly provided good g is $\partial R(p,g,u)/\partial g = \tau^i \partial x^i(p,g,u)/\partial g$. With a dual formulation, the MCF parameter is required to derive an estimate of a project's *uncompensated* indirect revenue effect from knowledge of its *compensated* indirect revenue effect. Thus neither measure of a project's indirect revenue effect has an implementation advantage over the other.

be used to redeem a dollar of government debt, which reduces debt service payments by r and increases capital income tax revenue by $\tau\rho$ in all subsequent periods. A dollar diverted from current program spending therefore increases the revenue available for future program spending by $r + \tau\rho = \rho$ dollars in each subsequent period.

Now consider a small project that requires an increase in program spending of$\{dI_g^t\}$ and results in an increase in the publicly provided good of$\{dg^t\}$. The government balances its budget by a permanent increase in the labour income tax rate holding the capital income tax rate fixed. The project will be worthwhile if $\Sigma_{t=0}^{\infty}(\partial V/\partial g^t)dg^t + (\partial V/\partial \tau_L)d\tau_L > 0$.

Since $\partial U/\partial c^0 = \lambda$, $\beta^t \partial U/\partial c^t = \lambda/(1+r)^t$ and $\beta^t \partial U/\partial l^t = \lambda w(1-\tau_L)/(1+r)^t$ by the private sector's first order conditions, and $\partial V/\partial A^0 = \lambda$, $\partial V/\partial \tau_L = -\lambda \Sigma_{t=0}^{\infty} wL^t/(1+r)^t$ and $\partial V/\partial g^t = \beta^t \partial U/\partial g^t$ by the envelope theorem, the project is worthwhile if

(8) $\Sigma_{t=0}^{\infty} B^t/(1+r)^t - [\Sigma_{t=0}^{\infty} wL^t/(1+r)^t]d\tau_L > 0.$

where $B^t = p_g^t dg^t$ and $p_g^t = (\partial U/\partial g^t)/(\partial U/\partial c^t)$ is the marginal willingness to pay for a unit of the publicly provided good in period t.

But the project is fiscally feasible if the present value of the increase in tax revenue equals the present value of the increase in project expenditure, i.e. $\Sigma_{t=0}^{\infty}[dR^t - dI_g^t]/(1+\rho)^t = 0$. The increase in tax revenue will be the result of the increase in the labour income tax rate $d\tau_L$ plus any indirect revenue effect of the project's output $\{dg^i\}$. Thus $dR^t = [\tau\rho(\partial A^t/\partial \tau_L) + \tau_L w(\partial L^t/\partial \tau_L) + wL^t]d\tau_L + IR^t$, where

(9) $IR^t = \tau\rho[\Sigma_{i=0}^{\infty}(\partial A^t/\partial g^i)dg^i] + \tau_L w[\Sigma_{i=0}^{\infty}(\partial L^t/\partial g^i)dg^i]$

Substituting for dR^t in the government's budget constraint, the required change in the labour income tax rate $d\tau_L$ must satisfy:

(10) $[\Sigma_{t=0}^{\infty}(wL^t + \tau_L w\partial L^t/\partial \tau_L + \tau\rho\partial A^t/\partial \tau_L)/(1+\rho)^t]d\tau_L = \Sigma_{t=0}^{\infty}[dI_g^t - IR^t]/(1+\rho)^t$

The term in square brackets on the left hand side of (10) is the present value of the increase in tax revenue resulting from an increase in the labour income tax rate. If we now substitute the expression for $d\tau_L$ derived from (10) into (8) we arrive at Liu's criterion for a worthwhile project, namely:.

(11) $\Sigma_{t=0}^{\infty} B^t/(1+r)^t - MCF_{\tau L}\{\Sigma_{t=0}^{\infty}[dI_g^t - IR^t]/(1+\rho)^t\} > 0$

where the parameter $MCF_{\tau L}$ in (11) is given by

(12) $MCF_{\tau L} = [\Sigma_{t=0}^{\infty} wL^t/(1+r)^t]/[\Sigma_{t=0}^{\infty}(wL^t + \tau_L w\partial L^t/\partial \tau_L + \tau\rho\partial A^t/\partial \tau_L)/(1+\rho)^t]$

It represents the welfare cost of raising an additional dollar of revenue using the (distortionary) labour income tax.[11] The numerator in (12) is the cost to

[11] In defining $MCF_{\tau L}$ it is assumed that the government retains the revenue to balance its budget, which has been put in deficit by the project. The conventional Harberger measure assumes that the government lump sum rebates the revenue to balance its budget, which is in balance when the tax is imposed. Jones (2005) shows that the "modified" MCF parameter $MCF_{\tau L}$ that appears in (12) is equal to the conventional measure $MCF_{\tau L}^c$ multiplied by the "shadow value of government revenue" for a lump sum tax S_R. The shadow value of government revenue using a lump sum tax is Liu's measure of the MCF parameter for a lump sum tax that appears in equation (4).

private surplus of an increase in the labour income tax rate, and the denominator is the corresponding increase in government revenue. Thus the project is worthwhile if the benefits discounted at the after-tax rate r exceed the costs minus the indirect revenue effects all discounted at the pre-tax rate ρ and multiplied by the parameter $MCF_{\tau L}$.

Comparing equation (11) with equation (5) it is clear that introducing labour-leisure choice and replacing a lump sum tax with a distortionary tax on labour income changes the value of the MCF parameter as well as the value of the project's indirect revenue effect (whenever the project affects labour supply). The project's benefits are still discounted at the after-tax rate, and the costs plus indirect revenue effects discounted at the pre-tax rate. However, if in the presence of labour-leisure choice a lump sum tax were used to balance the budget instead of a labour income tax the MCF criterion would be (11) with $MCF_{\tau L}$ replaced by MCF. The value of the MCF parameter for a lump sum tax will differ from the value in Section 2, but the value of the uncompensated indirect revenue effect given by equation (9) is independent of the choice of marginal tax instrument.

The standard SOC criterion assumes that the marginal tax instrument is a lump sum tax. The justification for this is to avoid conflating project evaluation with issues of tax reform. However, contrary to claims made by Liu, the SOC criterion can be adapted to situations where a distortionary tax is used to balance the budget. To see this, first recognize that the compensated indirect revenue effect of the project is now the uncompensated indirect revenue effect plus the effect on **capital plus labour** income tax revenue of a sequence of lump sum tax increases equal to the private sector's willingness to pay for the project's benefits. In other words, equation (7) remains valid, with the uncompensated indirect revenue effect given by equation (9). Now substitute (7) into (11) in order to replace the project's uncompensated indirect revenue effect $\Sigma_{t=0}^{\infty} IR^t/(1+\rho)^t$ by its compensated indirect revenue effect $\Sigma_{t=0}^{\infty} IR_c^t/(1+\rho)^t$, and re-arrange terms to get the following:

(13) $\Sigma_{t=0}^{\infty}(B^t - dI_g^t + IR_c^t)/(1+\rho)^t > [(MCF)^{-1} - (MCF_{\tau L})^{-1}]\Sigma_{t=0}^{\infty} B^t/(1+r)^t$

According to this expression the project is worthwhile if the benefits minus the costs plus the (compensated) indirect revenue effects, all discounted at the SOC rate, exceed a term that represents the excess budgetary cost of appropriating the project's benefits using a distortionary labour income tax rather than a lump sum tax. In other words, it is the revenue that is lost if a lump sum tax is replaced by a distortionary labour income tax while keeping the private sector at pre-project utility.[12] Importantly, the compensated indirect revenue effect that enters the SOC formula is unaffected by the choice of marginal tax instrument, so the SOC criterion is just as easy to implement as Liu's MCF criterion. Indeed, the information requirements for the two criteria are identical.

If the project's benefits are a perfect substitute for income its compensated

[12] Since $MCF = S_R$, and $MCF_{\tau L} = MCF_{\tau L}^c.S_R$, then $(MCF)^{-1} - (MCF_{\tau L})^{-1} = [meb_\tau/(1+meb_\tau)](S_R)^{-1}$ where $meb_\tau = MCF_{\tau L}^c - 1$ is the marginal excess burden of the tax τ_L.

indirect revenue effect will be zero, i.e. $\Sigma_{t=0}^{\infty} IR_c^t/(1+\rho)^t = 0$ in (13). The SOC criterion then requires that the benefits minus the costs discounted at the SOC rate exceed the excess budgetary cost of appropriating project benefits with a distortionary tax. On the other hand, if the project's benefits are separable from private consumption and labour supply (i.e. the project leaves no behavioural trace) then the uncompensated indirect revenue effect will be zero, i.e. $\Sigma_{t=0}^{\infty} IR^t/(1+\rho)^t = 0$. This further simplifies Liu's MCF criterion in (11) by eliminating indirect revenue effects, but the SOC criterion is still just as easy to implement because the information requirements for the two criteria are identical. To see this, set $\Sigma_{t=0}^{\infty} IR^t/(1+\rho)^t = 0$. in (11) and substitute the resulting expression for $\Sigma_{t=0}^{\infty} IR_c^t/(1+\rho)^t$ into (13) to get:

(14) $\Sigma_{t=0}^{\infty}(B^t - dI_g^t)/(1+\rho)^t > -\{(MCF)^{-1}\Sigma_{t=0}^{\infty} B^t/(1+r)^t - \Sigma_{t=0}^{\infty} B^t/(1+\rho)^t\} + [(MCF)^{-1} - (MCF_{\tau L})^{-1}]\Sigma_{t=0}^{\infty} B^t/(1+r)^t$

The right hand side of this expression consists of two components: the first component in braces is the change in capital plus labour income tax revenue that results if a lump sum tax is used to appropriate project benefits, and the second component is the loss in tax revenue if a lump sum tax is replaced by a labour income tax while keeping the private sector at pre-project utility. A project whose benefits are separable from private consumption will be worthwhile according to the SOC criterion if the benefits minus the costs discounted at the SOC rate exceed the sum of these two components.

To conclude this section, it should be noted that while the criteria we have derived apply to situations where the project's output is not appropriable by the government via ordinary market transactions, it is straightforward to extend the analysis to situations where the publicly provided good is in the nature of a private good. If the project's benefits can be appropriated using normal market transactions there will be no need to raise distortionary taxes to balance the budget, and therefore there will be no "excess budgetary cost". The right hand side of equation (13) will therefore equal zero, and so will the last term on the right hand side of equation (14). The appropriate MCF criterion for this situation is given in equation (11), with the MCF$_{\tau L}$ parameter being replaced by MCF. This is because a dollar of benefits transferred to the government's budget by charging the private sector its marginal willingness to pay for the project's output has the same effect on government revenue as a dollar increase in lump sum taxes.

4 MCF criterion versus SOC criterion when the rate of return is endogenous

Liu's MCF criterion, as he presents it, is valid only in situations where the pretax rate of return is exogenous. This "partial equilibrium" assumption would seem to limit its usefulness, but in fact a modified version of the MCF criterion applies when ρ is a decreasing function of the capital stock making the demand for investible funds less than perfectly elastic.[13] The key insight from the analysis so far is that the SOC criterion and the MCF criterion are conducting project evaluation from two different perspectives, and thus they use different numeraires. The MCF criterion looks at the impact of the project on the present value of private surplus (consumption discounted at the after-tax rate of return) by converting the project's present value cost to government revenue (discounted at the pre-tax rate representing the opportunity cost of government revenue when ρ is exogenous) into its cost to private surplus by multiplying by the appropriate MCF parameter. On the other hand, the SOC criterion looks at the impact of the project on the present value of government revenue holding private surplus at its pre-project level. Any project that increases private surplus while keeping the present value of government revenue unchanged can increase the present value of government revenue keeping private surplus unchanged, and vice-versa. There is no reason why this basic proposition should only hold when ρ is exogenous, i.e. when the economic opportunity cost of borrowed funds (what Liu (2003, p.1715) refers to the opportunity cost of government revenue) is equal to the pre-tax rate of return.

In this section, when the government enters the capital market in period t to finance project spending a proportion $\alpha^t < 1$ of funds displaces private investment and a proportion $1-\alpha^t$ displaces consumption. In a well functioning, but tax-distorted capital market this happens because, with the demand for investible funds being less than perfectly elastic and a limited supply of funds, the government's additional demand for funds reduces the funding available for private sector projects thereby driving up the pre-tax rate of return on the marginal private project and the after-tax rate of return on an increment of saving. The act of borrowing leaves private sector *utility* unchanged, just as in Liu's model, but now not all of the funding displaces private investment; some will cause a postponement of consumption.[14] The economic opportunity cost of a dollar of funds borrowed in period t and repaid in period $t+1$ is therefore $\omega^t = (1-\alpha^t)r^t + \alpha^t\rho^t$, where ρ^t and r^t are the pre-tax and after-tax rates of return on funds invested at the beginning of period t.

Even if the capital income tax rate is constant over time, the economic opportunity cost of borrowed funds could vary from one period to the next because of the endogeneity of ρ, and also because the proportions of funding that displace investment and consumption depend upon the relative amounts of investment, saving and government borrowing occuring in each period. However, if the economy is in a steady-state (or a state of balanced growth) before the project is introduced, the pre-project values for ρ, r, α, and ω will all be time-independent; and if the project is small the pre-project values for ρ and r will accurately measure, respectively, the (time-independent) marginal rate of productivity of investment displaced and the marginal rate of time preference

[13] Liu's exogenous rate of return assumption follows from a technology in which capital and labour are perfect substitutes, whereas a neoclassical technology exhibits a diminishing marginal rate of substitution between capital and labour.

[14] Government debt that finances public investment is perceived as net wealth by the private sector.

of consumption postponed.

In this section we ignore labour-leisure choice and assume that lump sum taxes are feasible. Output in each period, y^t, depends upon a neoclassical production function with a diminishing marginal product of capital so $y^t = F(K^t)$, where $\partial F/\partial K^t > 0$, and $\partial^2 F/(\partial K^t)^2 < 0$. The representative agent with initial wealth A^0 chooses time streams for consumption $\{c^t\}$ and capital investment $\{K^t\}$ conditional on the capital income tax rate τ, the time stream for the after-tax rate of return $\{r^t\}$, the time stream for the publicly provided good $\{g^t\}$, and the time stream of lump sum taxes $\{T^t\}$, to maximize: $W(c,g) = \Sigma_{t=0}^{\infty}\beta^t U(c^t, g^t)$, subject to: $c^0 + I^0 + T^0 + \Sigma_{t=1}^{\infty}(c^t - F(K^t) - \tau\rho^t K^t + I^t + T^t)/\Pi_{i=1}^{t}(1+r^i) = A^0$, where $I^t = K^{t+1}$(because capital depreciates fully each period) and $A^0 = F(K^0) - \tau\rho^0 K^0 + D^0$. The first order conditions for $\{c^t\}$ and $\{K^t\}$, namely $\beta(\partial U/\partial c^t)/(\partial U/\partial c^{t-1}) = 1/(1+r^t)$ and $\partial F/\partial K^t = 1 + r^t/(1-\tau)$ reflect a constant wedge between the marginal rate of time preference and the marginal rate of productivity of capital. These conditions, together with the condition that the present value of consumption equals initial wealth plus the present value of after-tax earnings (the transversality condition), determine unique time paths for $\{c^t\}, \{K^t\}$,and $\{r^t\}$ that converge to steady state values in an economy with constant levels of g and T and an initial level of outstanding government debt D^0.[15] The private sector's choice of consumption in period t can then be specified as $c^t(A^0, g, T)$, and the present value of lifetime utility can be expressed as $V^* = \Sigma_{t=0}^{\infty}\beta^t U(c^t(A^0, g, T), g) = V^*(A^0, g, T)$.

As in section 2, interest payments on government debt are taxed at the same rate as returns on private capital so the government's borrowing rate equals the pre-tax rate of return. Government revenue in period t is equal to lump sum taxes plus capital income taxes so $R^t = T^t + \tau\rho^t A^t$. The government's budget constraint can be written in integrated form as: $R^0 - I_g^0 + \Sigma_{t=1}^{\infty}(R^t - I_g^t)/\Pi_{i=1}^{t}(1+\rho^i) = D^0$. However, this is merely an accounting statement; it does not mean that the economic opportunity cost of borrowed funds in period t is the pre-tax rate of return ρ^t. The government budget constraint follows from the private sector's budget constraint and the market clearing condition, that each period's output must be either consumed, invested privately or purchased by government.[16]

For a small project, the evaluation criteria that we derive depend on pre-project values of the after-tax and before-tax rates of return. These rates of return will not be constant over time unless the economy is in a steady state

[15] For an exhaustive treatment see Arrow and Kurz (1970).

[16] Since $A^t = K^t + D^t$, the tax on debt interest can be deducted from the cost of debt service so the evolution of government debt can alternatively be expressed as $D^{t+1} = (1+r^t)D^t + I_g^t - T^t - \tau\rho^t K^t$. The government budget constraint can then be written as $T^0 - I_g^0 + \Sigma_{t=1}^{\infty}[T^t + \tau\rho^t K^t - I_g^t]/\Pi_{i=1}^{t}(1+r^i) = D^0$ Combine this with the private sector's budget constraint $c^0 - w^0 L - T^0 + \Sigma_{t=1}^{\infty}[c^t - w^t L - T^t]/\Pi_{i=1}^{t}(1+r^i) = A^0$ and note that $y^t = (1+\rho^t)K^t + w^t L$. Therefore $y^0 - (1+r^0)K^0 + \Sigma_{t=1}^{\infty}[y^t - (1+r^t)K^t]/\Pi_{i=1}^{t}(1+r^i) = c^0 + I_g^0 + \Sigma_{t=1}^{\infty}[c^t + I_g^t]/\Pi_{i=1}^{t}(1+r^i)$

But $K^t = I^{t-1}, t = 1, 2, ..\infty$, and $A^0 = (1+r^0)K^0 + D^0$ so $y^0 + \Sigma_{t=1}^{\infty}y^t/\Pi_{i=1}^{t}(1+r^i) = c^0 + I_g^0 + I^0 + \Sigma_{t=1}^{\infty}[c^t + I_g^t + I^t]/\Pi_{i=1}^{t}(1+r^i)$ from which it follows that $y^t = c^t + I_g^t + I^t$ for all t.

prior to the project. If the pre-project equilibrium is a steady-state the private sector's first order conditions become $\beta(\partial U/\partial c^{t+1})/(\partial U/\partial c^t) = (1+r)^{-1}$ and $\partial F/\partial K^t = 1+\rho$ for $t = 0, 1, ...\infty$, where $\rho(1-\tau) = r$. The steady-state after-tax rate of return r equals $(1-\beta)/\beta$ and the pre-tax rate of return ρ equals $(1-\beta)/\beta(1-\tau)$. Steady-state consumption is then $c^t = c = rA^0 + w - T$, and steady-state project expenditure is $I_g = T + \tau\rho K - rD^0$.[17]

Now consider a small project that requires a stream of expenditures $\{dI_g^t\}$ and produces a stream of output $\{dg^t\}$ for which the private sector is willing to pay $\{B^t\}$. Assuming that the project's benefits are available free of charge, the private sector will be indifferent to the project if $\{dT^t\} = \{B^t\}$.[18] To evaluate the project's impact on government revenue the appropriate discount rate is the economic opportunity cost of borrowed funds, i.e. the rate of return the economy foregoes when revenue is spent on the project rather than used to redeem government debt. In previous sections this rate was ρ, but it is now equal to $\omega^t = (1-\alpha^t)r^t + \alpha^t\rho^t$ where ρ^t and r^t are the pre-project values of the pre-tax and after-tax rates of return in period t. To understand why, note first that government debt evolves according to $D^{t+1} = (1+\rho^t)D^t + I_g^t - T^t - \tau\rho^t A^t$, so a small project will alter the time stream of government debt as follows when lump sum taxes are raised in each period by an amount equal to the private sector's willingness to pay for the project's benefits in that period, i.e. if $\{dT^t\} = \{B^t\}$.

(15) $dD^1 = dI_g^0$; $dD^{t+1} = (1+\rho^t)dD^t + dI_g^t - B^t - \tau\rho^t dA^t$ for $t = 1, 2, ...\infty$

To determine the project's effect on asset holdings at the beginning of period t note that capital market equilibrium at the beginning of period t requires that:

$K^t(r^t/(1-\tau)) = A^t(V^0, r^t, g) - D^t$

where $A^t(.)$ represents assets held at the beginning of period t when the private sector is kept at pre-project utility.[19] The effect of the project on A^t

[17] Out of steady-state equilibrium the private sector's consumption plan follows a saddle path, with consumption increasing over time and the pre and post-tax rates of return decreasing. The economy will converge to a steady-state for a given capital income tax rate τ, a constant level of project expenditure I_g that produces a constant stream of public goods g, a constant lump sum tax T, and a given initial level of government debt D^0.

[18] Pre-project indirect utility is $V^*(A^0, g, T)$. From the envelope theorem $\partial V^*/\partial g^t = \beta^t \partial U/\partial g^t$ and $\partial V^*/\partial T^t = -\lambda/\Pi_{i=1}^t(1+r^i)$ where λ is the marginal utility of consumption in period 0, i.e. $\lambda = \partial U/\partial c^0$. For a project with output stream $\{dg^t\}$ where $B^t = p_g^t dg^t$, $p_g^t = \partial U/\partial g^t/\partial U/\partial c^t$, and $\partial U/\partial c^t = \lambda/\Pi_{i=1}^t(1+r^i)$ that is accompanied by a sequence of lump sum tax increases $\{dT^t\}$, present value consumption changes by: $dV^*/\lambda = \Sigma_{t=0}^{\infty}[B^t - dT^t]/\Pi_{i=1}^t(1+r^i)$ Therefore if $dT^t = B^t$ the private sector remains at pre-project utility.

[19] A^t is expressed as a function of the rate of return in period t to incorporate the private sector's willingness to alter consumption in period $t-1$ in response to a change in r^t. The change in period $t-1$ consumption is a compensated response holding utility fixed. Thus, any deviation of A^t from its pre-project value reflects the effect of only two factors: the (compensated) effect of the project's output stream, dg; and the effect of a change in the rate of return on assets in period, dr^t. How the change in A^t is allocated between consumption in period t and assets at the beginning of period $t+1$ depends on the rate of return that clears the capital market in period $t+1$. Thus the incremental amount of consumption postponed in period $t-1$ is governed solely by the increase in the rate of return in period t. This is consistent with the economic opportunity cost of borrowed funds reflecting the cost of *raising* the funds, and not on how the funds are spent.

can then be expressed as $dA^t = \Sigma_{i=1}^{\infty}(\partial A^t/\partial g^i)_U dg^i + (\partial A^t/\partial r^t)_U dr^t$, where $dr^t = \{\Sigma_{i=1}^{\infty}(\partial A^t/\partial g^i)_U dg^i - dD^t\}/((\partial A^t/\partial r^t)_U - \partial K^t/\partial r^t)$ from the capital market equilibrium condition. Substituting for dr^t in the expression for dA^t, the effect of the project on A^t can be written as follows:

(16) $\quad dA^t = \alpha^t \Sigma_{i=1}^{\infty}(\partial A^t/\partial g^i)_U dg^i + (1-\alpha^t)dD^t$.

where $\alpha^t = -(\partial K^t/\partial r^t)/((\partial A^t/\partial r^t)_U - \partial K^t/\partial r^t)$ is the proportion of an increase in government borrowing that displaces investment, and therefore $1-\alpha^t$ is the proportion that displaces consumption.[20] If we now substitute the expression for dA^t in (16) into the expression for dD^{t+1} in (15) and note that $\omega^t = \rho^t - \tau\rho^t(1-\alpha^t)$, the effect of the project on the time stream of government debt can be expressed as follows:

(17) $\quad dD^1 = dI_g^0;\; dD^{t+1} = (1+\omega^t)dD^t + dI_g^t - B^t - \tau\rho^t\alpha^t\Sigma_{i=1}^{\infty}(\partial A^t/\partial g^i)_U dg^i$ for $t = 1, 2, ...\infty$

A project will be worthwhile if it increases the present value of government revenue (discounted at the SOC rate) when the private sector is kept at pre-project utility. The present value of the change in government revenue will be positive if $\lim_{t\to\infty} dD^{t+1}\Omega^t < 0$ where $\Omega^t = 1/\Pi_{i=1}^t(1+\omega^i)$ is the discount factor applied to revenue changes occurring in period $t = 1, 2, ...$, and $\Omega^0 = 1$.[21] Applying this "no Ponzi scheme" condition to equation (17), the project will be worthwhile if:

(18) $\quad \Sigma_{t=1}^{\infty}\{B^t + \tau\rho^t\alpha^t[\Sigma_{i=1}^{\infty}(\partial A^t/\partial g^i)_U dg^i] - dI_g^t\}/\Pi_{i=1}^t(1+\omega^i) - dI_g^0 > 0$

The expression in square brackets in (18) multiplied by $\tau\rho^t\alpha^t$ is the compensated effect of the project on capital income tax revenue in period t. Therefore, the project is worthwhile if the present value of its benefits plus its (compensated) indirect revenue effects minus its costs, all multiplied by the discount factor $\Omega^t = 1/\Pi_{i=1}^t(1+\omega^i)$, is positive. This is the multi-period version of the SOC criterion proposed by Harberger (1969), but extended to situations where the (compensated) indirect revenue effects are not equal to zero. If the pre-project equilibrium is a steady-state then $\rho^t = \rho$, $\omega^t = \omega$, and $\alpha^t = \alpha$ for all t and the SOC criterion simplifies to:

(19) $\quad \Sigma_{t=0}^{\infty}\{B^t + \tau\rho\alpha[\Sigma_{i=1}^{\infty}(\partial A^t/\partial g^i)_U dg^i] - dI_g^t\}/(1+\omega)^t > 0$

In contrast, the MCF criterion looks at the impact of the project on welfare (present value of consumption discounted at the after-tax rate) if the government raises lump sum taxes to restore inter-temporal budget balance. The modified MCF criterion that is applicable when the pre-tax rate of return is endogenous can now be derived from the SOC criterion given by equation (19). First, recall that the compensated effect of the project on capital income tax revenue

[20] Note that the weights α^t and $1-\alpha^t$ depend upon the *compensated* effect of consumption in period $t-1$ (and assets at the beginning of period t) to the after-tax rate of return in period t, which is in accordance with the analysis of Sandmo/Dreze (1971).

[21] $dD^{t+1}\Omega^t = dD^t\Omega^{t-1} + \Omega^t\{dI_g^t - B^t - \tau\rho^t\alpha^t\Sigma_{i=1}^{\infty}(\partial A^t/\partial g^i)_U dg^i\}$. By successive substitution we get $dD^{T+1}\Omega^T = \Sigma_{t=0}^{\infty}\Omega^t\{dI_g^t - B^t - \tau\rho^t\alpha^t\Sigma_{i=1}^{\infty}(\partial A^t/\partial g^i)_U dg^i\}$.

Thus the project's effect on the present value of the government debt outstanding at the beginning of period $T+1$ is the present value of its benefits minus its costs plus its indirect revenue effects in periods 0 to T, where the discount rate for period t is $\Pi_{i=1}^t(1+\omega^i)^{-1}$. The no Ponzi game condition requires that the project must not cause the government debt to increase faster than the SOC rate, i.e. that $\lim_{T\to\infty} dD^{T+1}\Omega^T \leq 0$.

in period t is the uncompensated effect plus the effect of a sequence of lump sum tax increases equal to the projects benefits so $\tau\rho\alpha\ \Sigma_{i=1}^{\infty}(\partial A^t/\partial g^i)_U dg^i = \tau\rho\alpha\Sigma_{i=1}^{\infty}(\partial A^t/\partial g^i)dg^i + \Sigma_{i=1}^{\infty}B^i(\partial A^t/\partial T^i)$. Therefore the compensated effect of the project on the present value of capital income tax revenue (discounted at the SOC rate) is related to the uncompensated effect by:

(20) $\tau\rho\alpha\ \Sigma_{t=0}^{\infty}\Sigma_{i=1}^{\infty}(\partial A^t/\partial g^i)_U dg^i/(1+\omega)^t = \tau\rho\alpha\Sigma_{t=0}^{\infty}\Sigma_{i=1}^{\infty}(\partial A^t/\partial g^i)dg^i/(1+\omega)^t + \Sigma_{t=0}^{\infty}\Sigma_{i=1}^{\infty}B^i(\partial A^t/\partial T^i)/(1+\omega)^t$

Next, note that the second term on the right hand side of equation (20), namely $\Sigma_{t=0}^{\infty}\Sigma_{i=1}^{\infty}B^i(\partial A^t/\partial T^i)/(1+\omega)^t$, represents the effect on the present value of capital income tax revenue of a lump sum tax increase equal to the private sector's willingness to pay for the project's benefits, previously referred to as $\Delta CITR$. Recall that $\Delta CITR$ is the difference between the present value of the increase total tax revenue and the increase in lump sum tax revenue. Thus if the private sector's willingness to pay for the project's benefits is $\Sigma_{t=0}^{\infty}B^t/(1+r)^t$ and the marginal cost of transferring a dollar of revenue to the government's budget using a lump sum tax is MCF, then $\Delta CITR$ is given by:

(21) $\Delta CITR = (MCF)^{-1}\ \Sigma_{t=0}^{\infty}B^t/(1+r)^t - \Sigma_{t=0}^{\infty}B^t/(1+\omega)^t$

Finally, making use of equations (20) and (21), the SOC criterion in equation (19) can be re-arranged to give the following modified MCF criterion:

(22) $\Sigma_{t=0}^{\infty}B^t/(1+r)^t - MCF\left[\Sigma_{t=0}^{\infty}dI_g^t/(1+\omega)^t - \Sigma_{t=1}^{\infty}\tau\rho\alpha\Sigma_{i=1}^{\infty}(\partial A^t/\partial g^i)dg^i/(1+\omega)^t\right] > 0$

Equation (22) measures the effect of the project on private surplus, with the first term representing the value of the project's direct benefits, the term in square brackets representing the project's impact on the government's budget, and the MCF parameter converting the project's budgetary cost into its cost to private surplus. The project is worthwhile if its benefits discounted at the after-tax rate r exceed its budgetary costs (direct project expenditures minus any indirect revenue effects) discounted at the SOC rate ω and multiplied by the MCF parameter.[22]

5 An Illustration

This section illustrates the main results of the paper using the example of a perpetuity: a project that requires an initial expenditure of dI_g^0 and generates a constant stream of output in all subsequent periods so $dg^t = dg$ for $t = 1, ..., \infty$. Assume that the pre-tax wage and rate of return are exogenous and the representative agent supplies a constant amount of work effort $L = 1$ each period. Also assume i) that the pure rate of time preference $(1-\beta)/\beta$ is equal to the after tax rate of return r, and ii) that lump sum taxes are constant over time in the initial equilibrium. Private consumption will then equal permanent (disposable) income so $c^t = c = y = w - T + rA$. Thus the representative agent

[22] The MCF parameter that appears in equation (22) assumes that the government retains the revenue rather than lump sum rebating it to balance its budget. Thus it represents what Jones (2005) calls the "shadow value of government revenue", i.e. the increase in private surplus that results from the lump sum transfer of a dollar of revenue to the private sector.

consumes the annuity value of wealth. Since $dg^t = dg$ the benefit of the project in each period as measured by the private sector's willingness to pay is $p_g dg = B$. Absent labour-leisure choice, a lump sum tax increase in period 0 of dT^0 will reduce consumption in period 0 and thereafter by $dc^t = -rdT^0/(1+r)$, assets in period 1 and thereafter will decrease by $dA^t = -dT^0/(1+r)$, and capital income tax revenue in period 1 and thereafter will decrease by $dR^t = -\tau\rho dT^0/(1+r)$. The marginal cost of funds parameter for a lump sum tax as defined by Liu is equal to $MCF = \rho(1+r)/r(1+\rho)$.[23]

If the project's benefits are separable from private consumption the project leaves no behavioural trace so there is no uncompensated indirect revenue effect. Liu's MCF criterion for the project to be worthwhile is then

(23) $B/r - MCF.dI_g^0 > 0$.

According to Liu, the SOC criterion is $B/\omega - dI_g^0 > 0$, where $\omega = r.MCF = \rho(1+r)/(1+\rho)$ is the appropriate discount rate- a weighted average of ρ and r. Following this reasoning the SOC criterion will result in project specific discount rates.[24] However, the SOC criterion requires that benefits, costs and indirect revenue effects be discounted at a rate equal to the economic opportunity cost of borrowed funds, with the indirect revenue effect representing the compensated effect of the project on capital income tax revenue. For a project whose stream of benefits B in periods $t = 1, 2, ..., \infty$ leave no behavioural trace, the compensated indirect revenue effect is the effect on capital income tax revenue of an increase in lump sum taxes of $dT^t = B$ in periods $t = 1, 2, ...\infty$. Consumption will fall by $dc = -B/(1+r)$ in all periods and assets will increase by $B/(1+r)$ in periods $t = 1, 2, ...\infty$, so capital income tax revenue will increase by $\tau\rho B/(1+r)$ in periods $t = 1, 2, ...\infty$. Therefore the compensated indirect revenue effect of the project in period $t = 1, 2, ...\infty$ is $IR_c^t = \tau\rho B/(1+r)$.

Adding the compensated indirect revenue effect to the benefits and discounting at the SOC rate ρ, the project is worthwhile according to the SOC criterion if

(24) $B/\rho + IR_c/\rho - dI_g^0 > 0$

The SOC criterion in (24) is seen to be equivalent to the MCF criterion in (23) by substituting for $IR_c = \tau\rho B/(1+r)$ and noting that the MCF parameter equals $\rho(1+r)/r(1+\rho)$.

If the project's benefits are "just like income", the compensated indirect

[23] The MCF parameter for a lump sum tax increase dT^0 is equal to $-(dPVC/dT^0)/(dPVR/dT^0)$. A lump sum tax increase of dT^0 reduces the present value of consumption (PVC) by dT^0 and increases the present value of government revenue (discounted at the SOC rate) (PVR) by $dT^0 - [\tau\rho dT^0/(1+r)]\Sigma_{t=1}^{\infty}(1+\rho)^{-t}$, which simplifies to $dT^0 r(1+\rho)/\rho(1+r)$. The MCF parameter is therefore equal to $\rho(1+r)/r(1+\rho)$. If the representative agent's pure rate of time preference differs from the after tax rate of return then $r = \delta + g\eta$, where g is the growth rate of consumpiton and η is the (constant) elasticity of the marginal utility of consumption. In this case the MCF parameter for a lump sum tax becomes $(\rho - g)(1+r)/[(r-g)(1+\rho)]$.

[24] For example, a project requiring an initial expenditure of dI^0 and generating separable benefits worth B beginning in period 2 is worthwhile according to the MCF criterion if $B/r(1+r) - MCF.dI^0 > 0$. Since $MCF = \rho(1+r)/r(1+\rho)$ it is easy to verify that the discount rate that makes this project just worthwhile is the value of ω such that $B/\omega(1+\omega) - dI^0 = 0$. This is a different value for ω than for a project whose benefits begin in period 1.

revenue effect will be zero and the uncompensated effect will be the effect on capital income tax revenue of a sequence of lump sum *transfers* equal to the project's benefits. Consumption will increase by $dc = B/(1+r)$ in all periods and assets will decrease by $B/(1+r)$ in periods $t = 1, 2, ...\infty$, so capital income tax revenue will decrease by $\tau\rho B/(1+r)$ in periods $t = 1, 2, ...\infty$. The project's uncompensated indirect revenue effect in periods $t = 1, 2, ...\infty$ is therefore $IR^t = -\tau\rho B/(1+r)$

The MCF criterion requires that indirect revenue effects be subtracted from the project's costs, discounted at the pre-tax rate ρ and multiplied by the MCF parameter. Therefore the project is worthwhile if

(25) $B/r - MCF\left\{dI_g^0 - IR/\rho\right\} > 0$

where $IR = -\tau\rho B/(1+r)$ for $t = 1, 2, ...\infty$.

Substituting for $MCF = \rho(1+r)/r(1+\rho)$ and $IR = -\tau\rho B/(1+r)$ in equation (25) we see that this is equivalent to the standard SOC criterion. The project is worthwhile if the benefits discounted at the SOC rate exceed the costs:

(26) $B/\rho - dI_g^0 > 0$.

5.1 Endogenous Labour Supply

Now introduce labour-leisure choice, and let $\theta < 0$ be the income elasticity of labour supply and η_L be the uncompensated elasticity of labour supply with respect to the wage rate. If there is a proportional tax on labour income of τ_L, the static MCF parameter for a lump sum tax is $[1-\tau_L\theta/(1-\tau_L)]^{-1} < 1$, and the static $MCF\tau$ parameter for a labour income tax is $[1-\tau_L\eta_L/(1-\tau_L)]^{-1} \lesseqgtr 1$.[25] However, in a dynamic setting with a capital income tax distortion and the representative agent consuming the annuity value of wealth a permanent lump sum tax increase will reduce consumption and increase labour supply in all periods leaving assets and capital income tax revenue unchanged. The dynamic MCF parameter for a lump sum tax is then $[\rho(1+r)/r(1+\rho)][1 - \tau_L\theta/(1-\tau_L)]^{-1}$. The dynamic $MCF\tau$ parameter for a tax on labour income is $[\rho(1+r)/r(1+\rho)][1-\tau_L\eta_L/(1-\tau_L)]^{-1}$. Note that the values of the dynamic MCF parameters are magnifications of their static counterparts. Thus the dynamic MCF for a lump sum tax may be greater than or less than one even though the static MCF for a lump sum tax is less than one. Also, because the compensated elasticity of labour supply is non-negative, and $\eta_L^c = \eta_L - \theta$, the dynamic $MCF\tau$ is always greater than the dynamic MCF.

A project that requires an initial investment of dI_g^0 and produces a perpetual stream of benefits worth B will be worthwhile according to the MCF criterion if

(27) $B/r - MCF_\tau[dI_g^0 - IR/\rho] > 0$

[25] Dahlby (2008) provides a good treatment of the theory and measurement of the MCF parameter. He defines the MCF parameter as the welfare cost of transferring a dollar of revenue from the private to the public sector using the particular tax instrument. Therefore, he is actually measuring the shadow value of government revenue and not the conventional (Harberger) MCF parameter.

This same project will be worthwhile according to the SOC criterion if

(28) $[B + IR_c]/\rho - dI_g^0 > [(MCF)^{-1} - (MCF_\tau)^{-1}]B/r$

But $IR_c/\rho = IR/\rho + (MCF)^{-1}B/r - B/\rho$

Therefore the two criteria are equivalent.

The term in square brackets on the right hand side of (28) represents the excess budgetary cost of appropriating project benefits using the labour income tax. Given the formulae for MCF and MCF_τ it can be expressed as $[r(1+\rho)/\rho(1+r)]\tau_L\eta_L^c/(1-\tau_L)$. Plausible parameter values for τ_L and η_L^c are 0.3 and 0.2 respectively, and plausible values for ρ and r are 0.1 and 0.04 respectively.[26] These parameter values imply that the excess budgetary cost is approximately 4 percent of the private sector's willingness to pay for the project's benefits. If benefits, costs and (compensated) indirect revenue effects are all discounted at the SOC rate, the project will be worthwhile if the net present value exceeds approximately 4 percent of the private sector's valuation of the project's benefits.

5.2 Endogenous Rate of Return

Finally, assume that labour supply is exogenous and lump sum taxes are feasible but the economic opportunity cost of borrowed funds equals $\omega < \rho$ because a dollar of government borrowing in one period to be repaid in the next period displaces α dollars of private investment and $1-\alpha$ dollars of consumption with $\omega = \alpha\rho + (1-\alpha)r$. If lump sum taxes are feasible, the project will be worthwhile according to the modified MCF criterion provided that:

(29) $B/r - MCF[dI_g^0 - IR/\omega] > 0$

This same project will be worthwhile according to the SOC criterion if

(30) $[B + IR_c]/\omega - dI_g^0 > 0$

But $IR_c/\omega = IR/\omega + (MCF)^{-1}B/r - B/\omega$, and $MCF = \omega(1+r)/r(1+\omega)$.[27]

Therefore the two criteria are equivalent.

6 Concluding Remarks

Liu (2003) claims that the SOC criterion suffers from severe implementation problems because there is no general formula for the proportions of resources that a project draws from consumption and investment; the proportions depend upon the project, making the discount rate "project specific".[28] However, if we

[26] For estimates of τ_L,η_L and θ see Dahlby (2008), or Jones (2005). A wedge of 6% between the pre-tax and after tax rates of return is consistent with a combined corporate plus property tax rate of 40% and a personal income tax rate of 33%.

[27] A permanent increase in lump sum taxes of dT will reduce the present value of private consumption by $dPVC = -dT(1+r)/r$ if the private sector consumes the annuity value of wealth. The present value of the increase in tax revenue (discounted at the SOC rate) is $dPVR = dT(1+\omega)/\omega$. The MCF parameter is therefore equal to $\omega(1+r)/r(1+\omega)$.

[28] Liu's position is that the SOC criterion may be valid in principle, but difficult to apply in practice. This differs from the view of proponents of the "shadow price algorithm" of Marglin (1963), Feldstein (1972), Bradford (1975) and Lind (1982). They maintain (incorrectly) that the SOC criterion commits an "aggregation error" by attempting to combine two distinct

follow Harberger (1969) and define the SOC rate as the social opportunity cost of borrowed funds, the SOC rate will be unique and common to all projects. If there are indirect revenue effects they reflect the compensated effect of the project on tax revenue rather than the uncompensated effect, and these effects should be added to (or subtracted from) the project's benefits, not incorporated by adjusting the discount rate.

We have found that, when the pre-tax rate of return is exogenous and the marginal tax instrument is a lump sum tax, the SOC criterion and the MCF criterion both correctly identify all worthwhile projects. However, each may have an implementation advantage in particular circumstances. If the private sector regards the project's benefits as equivalent to income the SOC criterion has an implementation advantage because no indirect revenue effects need to be taken into account. If the private sector regards the project's benefits as separable from private consumption, Liu's MCF criterion has an implementation advantage for the same reason. If the project's benefits are neither equivalent to income nor separable from private consumption there will be indirect revenue effects to take into account using either criterion, but because there is a well defined and measurable relationship between the indirect revenue effects that apply to each criterion they will be equally easy (or difficult) to implement in practice.

The standard SOC criterion assumes that the marginal tax instrument is a lump sum tax, whereas Liu's MCF criterion is valid whether the marginal tax instrument is a lump sum tax or a distortionary tax. This would seem to confer an implementation advantage upon the MCF criterion whenever lump sum taxes are not available, but we have found that the SOC criterion can be readily adapted to such situations and the required adjustment is just as easy to apply as it is for the MCF criterion. The fundamental equivalence between the MCF criterion and the SOC criterion continues to hold.

The key insight is that the MCF criterion is evaluating the project's impact on private surplus (present value of consumption discounted at the consumption rate of interest) by converting the project's budgetary cost into its cost to private surplus by multiplying by the appropriate MCF parameter, whereas the SOC criterion is evaluating the project's impact on the government's budget holding private surplus at its pre-project level. A project that satisfies one criterion will satisfy the other.

While Liu's MCF criterion is only valid when the pre-tax rate of return is exogenous, we have found that a modified version of the MCF criterion applies when the pre-tax rate of return is endogenous. In this more realistic setting a project is worthwhile if its benefits discounted at the after-tax rate exceed its costs plus indirect revenue effects all discounted at the "weighted average" SOC rate, but multiplied by the MCF parameter. The modified MCF criterion is equivalent to the SOC criterion, which discounts benefits minus costs plus (compensated) indirect revenue effects at the SOC rate.

prices (the price of future consumption in terms of current consumption, and the price of investment in terms of contemporaneous consumption) into one discount rate. Stiglitz (1982) makes the same claim.

7 References

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