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# Higher education subsidies and heterogeneity: a dynamic analysis

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**Abstract**

In this paper, we develop a simple dynamic general equilibrium framework to address the effects of increasing higher education subsidies in the US, from their already substantial levels, on inequality, welfare, and efficiency. We focus on three policies. The first is a tax and subsidy scheme that ensures that the parental decision to send a child to college is independent of income. Such a policy decreases the efficiency of the utilization of education resources, while the welfare gain is minimal. The second policy maximizes the fraction of college educated labor. This results in a large drop in the above-mentioned efficiency with little or no welfare gain. The third is the provision of merit-based aid to the poor as opposed to purely need-based aid. This policy can increase education efficiency with little decrease in welfare. Based on these experiments, we conclude that the case for further increases in higher education subsidies might have been overstated. 

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## 1. Introduction

The case for increasing the level of higher education subsidies is repeatedly made by politicians and policy makers.[1] The current level of subsidies is already quite substantial. McPherson and Schapiro (1991) report that \$7,839 of the instructional cost of \$8,760 in public universities is subsidized. In private institutions, \$7,224 of the cost of \$12,669 is subsidized. What are the effects of further increasing subsidies on inequality, welfare, and efficiency? Most of the studies that have attempted to answer these questions have either been descriptive or empirical.[2] In this paper, we impose the discipline of dynamic general equilibrium to study issues in the subsidization of higher education. Human capital accumulation is essentially a dynamic phenomenon and equilibrium conditions in the labor market heavily influence the return to this process; therefore, such an approach seems warranted. We develop and analyze a simple heterogeneous agent model of parental decision to send a child to college. We then calibrate our model to the US economy, and conduct experiments to study the effect of education policy on this decision. We view our dynamic modeling and quantitative approach as complementary to most existing work in this area.

The case for subsidization is usually based on equality of opportunity (improving the access of poor people to college education in the face of increasing college premiums) and on growth externalities (the notion that economic growth depends on the average education level of the entire workforce, thus justifying subsidization by the government to get human capital investment to efficient levels).[3] We consider the issue of economic growth in a companion paper, and focus here on the efficacy of higher education subsidies when the government is unable to fully observe the potential of students to successfully complete college.[4]

We develop a simple overlapping generation model with two-period lived agents. A liquidity constrained parent makes the decision of whether or not to incur the cost of sending a child to college, taking into account the child's academic ability. The probability that a child who is sent to college will successfully graduate depends positively on this ability. The parent is herself either a school educated or a college educated

[1] To cite a few instances of the perpetual interest in this issue—in 1998, the senate approved a "higher education bill that will cut student loan rates to the lowest level in nearly 20 yr and make grants available to low-income students". (Reuters news report, September 30, 1998.) "This legislation marks an important step forward in my effort to help more Americans enter the doors of college", then President Clinton remarked. A senator called it the "financial key to unlock the door to higher education". The same report places at \$48.5 billion, the financial aid provided by Washington for 8.5 million students. This is in addition to \$216 million spent directly for colleges.

A more recent report states "Clinton said the goal of making college accessible to many more people was within reach. 'At long last we've got the money to do it. The only question is whether we have the vision and will to do it". (Reuters news report, June 10, 2000.) Proposals for governmental support for higher education have also figured prominently in the present Bush presidency.

[2] See, for instance, McPherson and Schapiro (1991), which is a comprehensive descriptive and empirical work, and the references therein.

[3] See McPherson and Schapiro (1991) and Heckman and Klenow (1997) for an elaboration of each of these cases.

[4] See Caucutt et al. (2000) for the extension to growth.

worker. The two types of workers are imperfect substitutes in production. We study in detail the parental decision and the resulting dynamics, and prove the existence and uniqueness of equilibrium. This gives us confidence in the robustness of our setup and allows us to devise strategies to compute and calibrate the model to the US economy; we then compare outcomes under different policy regimes relative to this benchmark.

The liquidity constraint leads to persistence in higher education attainment; children whose parents are college educated are themselves more likely to be college educated. This occurs even though the academic ability of children is independently and identically distributed across both types of parents.

The model gives rise to three types of dynamic equilibria depending on the curvature parameter of the utility function (or alternately, the cost of education): 1. a unique steady state with a positive fraction of college educated workers, 2. one with a development trap, and 3. multiple steady states which feature both of these. The model thus has the potential to explain the broad diversity in educational attainment that is observed across countries, though in this paper we focus only on the case with a unique steady state.[5]

We then add a government that is equipped with a simple tax and subsidy scheme, and conduct several policy experiments. The liquidity constraint, and the ensuing persistence in attainment, provides a natural justification for subsidies. We assume that the government, unlike parents, is unable to observe student ability. An alternate interpretation of the model is that the government can observe student ability, but is forced to give need-based, rather than merit-based, aid because of political considerations. Thus, college subsidies are given only to children of school-educated parents, and the level of subsidies is common to all.

We concentrate on three policies that are typically discussed in the context of higher education. The first is equality of opportunity—the government can design a tax and subsidy scheme that guarantees this, but only at the expense of a decrease in the efficiency of utilization of education resources. Increased subsidies attract inframarginal students and cause an increase in the dropout rate. The welfare gain that comes from increasing the fraction of people with higher marginal productivity is minimal, on account of the negative effect of taxes and drop in the value to being skilled, especially when the transition is taken into account. In the presence of missing markets, transitional analysis gives a more accurate picture of welfare effects than an examination of steady states alone. The second is a policy that aims to maximize the fraction of college educated labor by "unlocking the door to higher education". Such a step results in a big drop in the above-mentioned efficiency with little or no welfare gain. The third is a merit-based policy. If the government has the political will to use any available

[5] A development trap occurs when the initial fraction of college educated people in the workforce is too low. The wages of the school educated workers are too low for them to find it profitable to send their children to college. This results in a decrease in the fraction of college educated people next period, which further decreases the wages of school educated workers, and reinforces the above-mentioned behavior.

In Caucutt and Kumar (2002), we argue that education subsidies may be a potent tool for countries that are caught in a development trap. A small but sufficient level of subsidy can cause the economy to emerge from the trap and also allow the gains to be realized in a timely manner.

signal on ability and provide merit-based aid, it can increase this efficiency with little decrease in welfare.

As mentioned earlier, there are few analytical studies of higher education subsidies in the literature, especially using the discipline of dynamic general equilibrium. Keane and Wolpin (2001) use an econometric approach that can be viewed as complementary to ours. They construct a dynamic model from the child's point of view, taking transfers made by parents as given. They estimate this model using NLSY data and study the effect of changing parental transfers and easing borrowing constraints on college enrollment. Parental decision making is central to our approach and we rely on calibration to parametrize our model. In the concluding section we compare our results to theirs.

Our model bears an analytical similarity to Andrade (1998), who uses a model with two types of agents to study issues on growth and inequality. We study a different set of issues, and assume that educational expenditure is fixed as opposed to varying. He does not model heterogeneity in ability, a feature that is central to our analysis. Aiyagari et al. (2002) also develop a model of investing in children in the presence of heterogeneity of income and ability, and compare efficient allocations to those with incomplete financial markets or missing childcare markets. Their focus is on investment in the early years of childhood, while we take that as given and concentrate on higher education.

An early influence on modeling general educational investment with liquidity constraints and heterogeneity in income is the work by Loury (1981). In his model, ability matters only for production, and even parents do not know the child's ability while investing in her education. In our basic model, ability matters only for college completion, and parents are aware of the child's academic ability while deciding whether to send her to college. This difference has important policy implications. In Loury's model it is possible to design redistributive policies that make *all* parents better off in an ex ante sense. Since parents are unaware of the child's ability while investing, redistribution can provide insurance—policies that decrease the spread of income distribution will be favored by all parents, rich or poor. Providing insurance is not the primary motive for subsidies in our setup; the aim is to induce participation through an income effect. Rich parents become worse off under redistribution, even when an aggregate measure of welfare increases. In fact, when there are opposite effects on the welfare of the rich and the poor, it is not clear what the right measure of aggregate welfare is; this is one of the reasons we also consider a positive measure of education efficiency. We feel that it is unrealistic to presume that parents have no idea of a child's working ability while making the investment decision. Working ability is likely to be strongly correlated with academic ability, and a parent has several opportunities to observe signals from her child and gauge the child's ability, especially by the time investment in college education is to be made. But it is true that our model with its perfect knowledge about a child's working ability would understate the importance of subsidies relative to a setup that features some imperfection in this knowledge.

Analytically, our model differs from Loury's in its fixed educational investment and limited heterogeneity, which makes it simpler on some dimensions as well as more readily applicable to issues in higher education subsidies and the college wage premium.

Our model is also readily amenable to transitional analysis. In his model, educational investment varies with income, and there is a continuous distribution of earnings; the focus is on the stationary distribution that arises from a Markovian transition function. [6]

The rest of the paper is organized as follows. In Section 2, we present the basic model without the government. The model is characterized in Section 3. The government is introduced in Section 4, calibration is discussed in Section 5, and results from the policy experiments are presented in Section 6. Section 7 concludes the paper.

## 2. The model economy

### 2.1. Demographics

The economy is populated by a continuum of two-period lived agents in an overlapping generations setup. The size (measure) of each generation is normalized to one. Agents are children in the first period and parents in the second. Children complete schooling and some of them will go on to attend college. We do not model the schooling decision. However, the parental decision to send a schooled child to college is central to the model.

We thus take the stance that the education decision has a strong intergenerational aspect to it, a stance that seems empirically valid. This stands in contrast to a strategy in which there is no intergenerational connection and students born without endowments finance their own education in a perfect credit market. McPherson and Schapiro (1991) present convincing evidence on the intergenerational connection. [7] For a particular sample they analyzed, parents' price over gross cost ranged from 19.4% to 88.2%. More relevant to our assumption that parental financing forms the bulk of family expenditure, they find that parental contribution is not $< 65\%$ and for some income groups is as high as 95% of the total family cost. Keane and Wolpin (2001) also provide some supporting evidence for this approach to educational financing. They find parental transfers are sizeable and the maximum debt amount is quite small.

School as well as college educated children enter the labor pool in the second period of their lives. At this point each adult worker becomes a parent, has a schooled child, and the economy continues. A parent derives utility from her consumption and from her child's utility. Therefore, altruism provides the intergenerational linkage. Throughout the paper, we will use "rich", "college educated", and "skilled" interchangeably, as we will "poor", "school educated", and "unskilled".

### 2.2. Parents' education problem

Children differ in their ability to complete college. We assume that, conditional on being sent to college, a child with ability $a$ completes college with probability $\pi(a)$.

[6] However, it should be possible to use the computational techniques of Krusell and Smith (1998) to conduct transitional analysis on Loury's model.

[7] See, for instance, their Tables 5–7.

With probability $(1 - \pi(a))$, the child drops out of college and enters the schooled labor force irrespective of the number of years she has attended college. Partial college education is useless in this model. This all-or-nothing aspect to college education limits the heterogeneity in education levels, and therefore income, to two types. [8]

**Assumption 1.** $a \in [0, 1]$. $0 \leqslant \pi(a) \leqslant 1$, $\pi'(a) > 0$, $\forall a$. $\pi(0) = 0$. The ability of the child is fully observed by the parent, but not others (including the government, which we will introduce in Section 4).

The above informational assumption captures the idea that a parent, by spending more time with her child than anyone else does, is in a better position to observe the child's ability. Let $F(\cdot)$ denote the distribution function for ability on the support $[0, 1]$, and $f(\cdot)$ the corresponding density function. The distribution is identical across types and within parents of the same type. All ability draws are independent of each other. In this section, we assume that the cost of college education is the same for all children. Sending a child to college involves a real cost of $e$ units of consumption.

**Assumption 2.** A parent cannot borrow to finance her child's college education. [9]

Incompleteness in the human capital market, on account of the moral hazard inherent in the process of accumulation and problems of enforcement, is a widely accepted assumption. See, for example, the discussion in Becker (1991, pp. 247–248). Disallowing borrowing for human capital investments is a much more limited assumption than disallowing *all* borrowing, a fairly common assumption in the (macroeconomic) life-cycle literature. [10]

[8] Following the lead of Rogerson (1988), we achieve convexification by making the process of skill accumulation probabilistic. This is not merely an analytical convenience. The assumption that not all students who enter college complete it successfully is empirically justified. OECD (1997) reports that tertiary non-completion rates in OECD countries (which typically offer generous subsidies) were as high as 64% of those who were enrolled in college (p. 93). In Section 4 we calibrate the model to be broadly consistent with US dropout rates.

[9] With this assumption, the private information aspect in Assumption 1 does not have any bite in the private sector economy. It will matter when we introduce the government.

[10] However, Heckman and Klenow (1997) cite evidence from Cameron and Heckman (1998) as reason to be skeptical on the prevalence of liquidity constraints. Therefore, our reasons for continuing to assume that parents are liquidity constrained are worth noting. First, Cameron and Heckman find that the average derivative of probabilities of schooling transition with respect to family income decreases in magnitude when a proxy for ability, the AFQT score, is controlled for. Based on this evidence, they suggest that long-term family characteristics that affect the AFQT scores are more important than short-term liquidity constraints. In most of the equilibrium outcomes of our model with liquidity constraints, the ablest of the poor children attend college, just as the ablest of the rich children. In other words, conditioning on ability lowers the dependence on income of the enrollment decision even in our model, which is qualitatively consistent with the evidence in Cameron and Heckman. Second, in spite of controlling for AFQT scores and family characteristics other than income, they find positive and significant values for the above-mentioned average derivatives; so liquidity effects cannot be completely ruled out. Third, it seems highly unlikely that family characteristics are uncorrelated with income. As long as there is incomplete biological transmission of family characteristics, and there is a dearth of socio-economic models in which family characteristics and

We also implicitly assume that market to insure against the ability risk of grandchildren is missing. However, this is likely to play a minor role when compared to credit constraints in studying differential effects of policies on the rich and the poor since this risk is common to both types of parents.

The goods cost ("tuition") of $e$ is the only cost of higher education. Ignoring the opportunity cost of foregone earnings, an important component of the cost of education, is not likely to be a serious omission for purposes of studying the efficacy of higher education subsidies. [11]

Let the fraction (measure) of college educated people entering the labor force at any time be denoted by $n_c$. This is the only aggregate state variable in this economy. Let $w_c(n_c)$ denote the wage earnings of a college educated parent as a function of the aggregate state $n_c$, and let $w_s(n_c)$ denote the wage of a school educated parent. Consider a parent of type $i$, $(i = c, s)$, who has the good (ill) fortune of having a child of ability $a$. This liquidity constrained parent can decide to further educate the (already schooled) child in college by expending $e$, or not educate the child any further and incur no cost. Let $V_i(a; n_c)$ be the value of this parent who optimally decides whether or not to send her child to college. We have included the aggregate state $n_c$, in addition to the individual state $a$, as an argument of the value function, to highlight the role it plays in parental decisions. This parent's Bellman equation is

$$V_i(a; n_c) = \max_{c,s}\{u(w_i(n_c) - e) + \beta[\pi(a)\mathrm{E}_{a'} V_c(a'; n_c') + (1 - \pi(a))\mathrm{E}_{a'} V_s(a'; n_c')],$$
$$u(w_i(n_c)) + \beta \mathrm{E}_{a'} V_s(a'; n_c')\}, \quad i = c, s. \tag{1}$$

Here, $\mathrm{E}_{a'} V_i(a'; n_c') = \int_0^1 V_i(a'; n_c')\,\mathrm{d}F(a')$, $i = c, s$, is the child's expected utility, which depends on whether the child enters adulthood as a college educated or a school educated worker. Henceforth, we will denote this quantity by $\mathrm{E}V_i(n_c')$. We take $\beta$ to be an intergenerational discount (altruism) factor. The decision is between sending the child to college (c) and letting the child remain school educated (s). The aggregate state that will prevail when the child enters the labor force is denoted by $n_c'$. We assume that all parents posit that the aggregate state follows the law of motion:

$$n_c' = \Phi(n_c), \tag{2}$$

*footnote 10 (contd.)*
income are jointly determined, it seems premature to dismiss models with liquidity constraints entirely. Fourth, in spite of liquidity constraints, our model retains the policy implications of Cameron and Heckman's study that policies that raise family income attract lower ability students into college, and ignoring heterogeneity in abilities is likely to overstate the gains of such policies (see their discussion in pp. 312–313). Finally, as Heckman and Klenow note, "the absence of any convincing evidence on liquidity constraints may be a consequence of the generosity of existing programs". Modeling liquidity constraints can help one to structurally evaluate the force of these constraints in the face of subsidies, though, admittedly, assuming that they always apply (as we do) is a bit extreme.

[11] As noted by Heckman and Klenow (1997), the government subsidizes only the direct cost of college, not the opportunity cost faced by students.

which they assume to be outside their control. Since there are no aggregate shocks in this economy, this is a deterministic function; parents effectively take the current and future wages as given in solving their optimization problem. We assume a standard utility function, with $u' > 0$, and $u'' < 0$.

Before proceeding to describe production in this economy, we note two aspects of intergenerational transmission as we have modeled it. We have assumed that a child's ability is independent of her parent's ability. This is the reason we can write the ex ante expected utility as a function of the aggregate state alone. We are thus focusing on the intrinsic ability that would help children complete college and are ignoring any biological transmission of ability from parent to child. When we calibrate the model and conduct policy experiments, we use different probability functions for the children of college and school educated parents, $\pi_c(a) > \pi_s(a)$, $\forall a \in [0,1]$. This allows us to model the advantages that children of rich parents have from higher quality primary and secondary schooling ($K$-12), and captures one dimension of persistence. Second, in our setup only educational bequests are possible. We speculate on the effects of adding physical capital bequests in the final section.

## 2.3. Production and wages

In the earlier subsection we assumed that parents took current and future wages of college and school educated labor as given.

**Assumption 3.** We assume that $w_c(n_c)$ is strictly decreasing, $w_s(n_c)$ is strictly increasing, and $w_c(n_c) > w_s(n_c)$, $\forall n_c \in [0,1]$.

The results do not materially depend on $w_c > w_s$ holding uniformly in the interval $[0,1]$; the assumption is made purely for convenience. Even if this inequality does not hold in the entire interval, equilibrium considerations demand that we focus attention on the interval $[0,\overline{n_c})$, $\overline{n_c} < 1$, in which $w_c > w_s$. Since college education uses real resources, parents will send their children to college only if they anticipate college-educated wage to be higher. If at any time $w_c < w_s$ is anticipated, no one will be sent to college, which will make the next period college wage very high and induce college enrollment.

We now motivate one production structure that yields wage functions with the above properties. Suppose firms in the economy manufacture a single consumption good using school and college educated labor as distinct inputs. The production function is constant returns to scale. Therefore, we assume a single production firm that manufactures the good according to the CES specification:

$$Y = A[\theta(N_c + \gamma N_s)^v + (1-\theta)(N_s + \varepsilon N_c)^v]^{1/v}, \tag{3}$$

where $0 < \varepsilon,\ v < 1$, and $\gamma \ll \varepsilon$. The first term within the square brackets can be thought of as "brain" and the second term as "brawn".[12] $N_c$ is the number of college educated

[12] The terminology is inspired by Stokey (1996).

workers employed by the firm, while $N_s$ is the number of school educated workers employed. College educated workers are the primary suppliers of "brain" (the weight of school educated workers in this factor, $\gamma$, is close to zero, and is included mainly for the technical reason of keeping wages bounded). Both types of workers contribute toward "brawn". Since going to college can take away time that one could have spent developing one's biceps, each unit of college educated labor is assumed to contribute less to "brawn" than does a unit of school educated labor—hence the factor $\varepsilon < 1$ that multiplies $N_c$ in the second term of the production function. This production function captures the notion that college educated workers can do (almost) all that school educated workers can do, and more. Note that the mere employment of $N_c$ workers contributes to "brain" and "brawn"—there is no decision involved on the part of these workers in devoting time to one activity versus the other. Simple manipulation of the marginal products yields a sufficient condition that guarantees higher wages for college-educated workers in the entire interval $[0,1]$: $\varepsilon > ((1-\theta)/(1-\gamma))^{1/1-\upsilon}$.[13] The coefficient of skilled labor in the production of "brawn" should be high enough to guarantee that skilled wage is always higher. Since the marginal product of college educated labor is always higher than that of school educated labor, output is maximized when $n_c$ is maximized.

This completes the description of the economy.

## 3. Characterization of the model

### 3.1. Parents' decision rules

We begin by characterizing the behavior of parents. Each type of parent has to make the decision of whether or not to send her child to college, given the child's ability $a$ (the individual state) and $n_c$ (the aggregate state). Let $C_i(n_c)$ be the set of intervals such that a parent of type $i$ sends her child to college if $a$ belongs to any of the set's elements. We assume that if a parent is indifferent between the two options, she will choose the college option. In Appendix A, where all results are derived or proved, we show the following result.

**Lemma 1.** *In any equilibrium in which parents behave optimally, for any* $n_c \in [0,1]$, $\mathrm{E}V_c(n_c) > \mathrm{E}V_s(n_c)$.

The above intuitive lemma asserts that college educated parents cannot get lower ex ante utility than the school educated parents on average. From the parents' problem in (1), it is clear that the option of not sending the child to college yields a value that is independent of the child's ability, $a$. Given Assumption 1 and the above lemma, it is also clear that the value of sending the child to college is increasing in $a$. It is

[13] The earning function in Loury (1981), where earnings positively depend on educational investment and ability, can be viewed as a reduced form of the production function we consider.

therefore intuitive to expect the parent's decision to exhibit a threshold (or reservation) ability behavior. This intuition is formalized in the following lemma.

**Lemma 2.** *For any given $n_c$, there exist unique $a_i^*(n_c) \in (0,1]$, $i = c, s$, such that a parent of type $i$ sends her child to college if $a \geqslant a_i^*(n_c)$, and does not otherwise. That is, $C_i(n_c)$ is the singleton set $\{[a_i^*(n_c), 1]\}$.*

When we discuss the parents' problems, we really need to write $a_i^*(n_c;\Phi)$. Since $\Phi$ is the law of motion for $n_c$ that parents posit, their decision rule would depend on this function. However, for the sake of notational convenience we will be explicit about this only when absolutely necessary. For a parent whose child is at the threshold ability, we can examine the two options of (1) and write:

$$\beta\pi(a_i^*(n_c))(\mathrm{E}V_c(n_c') - \mathrm{E}V_s(n_c')) \leqslant u(w_i(n_c)) - u(w_i(n_c) - e), \tag{4}$$

with equality if $a_i^*(n_c) < 1$. This expression says that for the marginally able child, the discounted benefit of going to college is exactly equal to the utility cost of college education. Sending any child who is more able will result in the parent benefiting by more than the utility cost. It is not worth the parent's utility cost to send any child who is less able. If the above holds as an inequality even when $a_i^*(n_c) = 1$, even the most able child will not be sent to college. That is, *no* parent of type $i$ sends her child to college. It is convenient to introduce new notation for a few expressions that we will frequently encounter. Define:

$$\Lambda(n_c) \equiv \mathrm{E}V_c(n_c) - \mathrm{E}V_s(n_c) \tag{5}$$

$$g_i(n_c) \equiv u(w_i(n_c)) - u(w_i(n_c) - e). \tag{6}$$

Here, $\Lambda(n_c)$ is the expected value to being college educated (that depends only on the aggregate state, and hence common to both parent types), and $g_i(n_c)$, $i = c, s$, is the utility cost of a parent of type $i$ of sending a child to college. From the production function (3) and the concavity of $u$, we obtain $g_c'(n_c) > 0$ and $g_s'(n_c) < 0$. Expression (4) can now be written as $\beta\pi(a_i^*(n_c))\Lambda(\Phi(n_c)) \leqslant g_i(n_c)$.

Given Assumption 3, it seems reasonable to expect that the richer college educated parents can afford to send even children of lower ability to college while the poorer school educated parents can afford to send only higher ability children to college; the utility cost of sending a child to college is less for the rich. We therefore have the following result.

**Lemma 3.** *In any equilibrium in which parents behave optimally and some college education occurs, $a_c^*(n_c) < a_s^*(n_c)$, for $n_c \in [0,1]$. It also follows that $a_c^*(n_c) < 1$.*

This lemma can be viewed as a result on persistence for our model. That is, children whose parents are college educated are themselves more likely to be college educated, a piece of evidence that is often mentioned. We get this result even though ability is i.i.d., on account of the liquidity constraint parents face in financing college education and curvature of the utility function.[14]

Though we do not focus on this effect, an increase in tuition $e$ would increase $a^*$ (decrease enrollment), by increasing $g$.

### 3.2. Definition of equilibrium

We are now in a position to define an equilibrium.

**Definition 1.** A *competitive equilibrium* is a collection of functions $w_i(n_c)$, $a_i^*(n_c)$, $i = c, s$, $\Lambda(n_c)$, and $\Phi(n_c)$, on $[0, 1]$, such that

- Given $w_i(n_c)$, $\Lambda(n_c)$, and $\Phi(n_c)$, $a_i^*(n_c)$ solves the college decision problem of parent $i$. That is, (4) is satisfied.
- Given wages, the firm maximizes profits. That is, both types of workers are paid their marginal products.
- The labor market clears. That is, $N_c = n_c$, and $N_s = 1 - n_c$.
- $\Lambda(n_c)$ is consistent with parental decisions. That is, $\mathrm{E}V_c(n_c)$ and $\mathrm{E}V_s(n_c)$ satisfy the equation system:

$$\begin{aligned} \mathrm{E}V_c(n_c) = &\int_0^{a_c^*(n_c)} \{u(w_c(n_c)) + \beta \mathrm{E}V_s(n_c')\}\,\mathrm{d}F(a) \\ &+ \int_{a_c^*(n_c)}^1 \{u(w_c(n_c) - e) \\ &+ \beta\pi(a)\mathrm{E}V_c(n_c') + \beta(1 - \pi(a))\mathrm{E}V_s(n_c')\}\,\mathrm{d}F(a), \\ \mathrm{E}V_s(n_c) = &\int_0^{a_s^*(n_c)} \{u(w_s(n_c)) + \beta \mathrm{E}V_s(n_c')\}\,\mathrm{d}F(a) \\ &+ \int_{a_s^*(n_c)}^1 \{u(w_s(n_c) - e) \\ &+ \beta\pi(a)\mathrm{E}V_c(n_c') + \beta(1 - \pi(a))\mathrm{E}V_s(n_c')\}\,\mathrm{d}F(a), \end{aligned} \tag{7}$$

  where, $n_c'$ is given by $\Phi(n_c)$.
- $\Phi(n_c)$ is consistent with parental decisions. That is, the law of motion for $n_c$ (the transition function) satisfies:

$$\Phi(n_c) = n_c \int_{a_c^*(n_c;\Phi)}^1 \pi(a)\,\mathrm{d}F(a) + (1 - n_c) \int_{a_s^*(n_c;\Phi)}^1 \pi(a)\,\mathrm{d}F(a). \tag{8}$$

**Definition 2.** A *steady state* is a competitive equilibrium with $n_c = n_c^* \in [0, 1]$, which satisfies $\Phi(n_c^*) = n_c^*$.

[14] Loury's (1981) assumption that education is a normal good implies persistence in income in his model.

On a steady state, the wages, reservation abilities, expected utilities, and the fraction (measure) of college educated parents are all constant over time. Analytically, a steady state is fixed point of the transition function $\Phi$.[15]

### 3.3. Dynamics

In Appendix A, we derive the following equation by manipulating the equation system (7):

$$\Lambda(n_c) = x(n_c) + \left[\beta \int_{a_c^*(n_c)}^{a_s^*(n_c)} \pi(a)\,\mathrm{d}F(a)\right] \Lambda(\Phi(n_c)), \tag{9}$$

where

$$\begin{aligned} x(n_c) \equiv & [F(a_c^*(n_c))u(w_c(n_c)) + (1 - F(a_c^*(n_c)))u(w_c(n_c) - e)] \\ & - [F(a_s^*(n_c))u(w_s(n_c)) + (1 - F(a_s^*(n_c)))u(w_s(n_c) - e)]. \end{aligned} \tag{10}$$

Eq. (9) is the central relationship characterizing the dynamics of the model. Here, $x$ is extra contemporaneous (ex ante, expected) utility a college educated parent gets, taking into account the endogenous response of the higher wage earning college parent having a higher probability of sending a child to college (recall Lemma 3). The value of being college educated has two components—a contemporaneous utility gain and a discounted future value. If utility is linear, the utility cost for both types of parents is the same. Eq. (4) then implies that $a_s^* = a_c^*$, and the second term on the right-hand side of (9) vanishes. That is, the only advantage of being college educated is the greater utility resulting from higher $w_c$. But for a concave utility function, the second term is positive—there is an additional gain that results from the parent being able to send more of her children to college, more than a school educated parent will be able to. The discounted value from these additional college educated children feeds into today's value. This is what makes the parent's problem truly dynamic.

We repeat the equations resulting from (4):

$$\beta\pi(a_c^*(n_c))\Lambda(\Phi(n_c)) = g_c(n_c) \tag{11}$$

$$\beta\pi(a_s^*(n_c))\Lambda(\Phi(n_c)) \leqslant g_s(n_c), \quad w.e.i\ a_s^*(n_c) < 1. \tag{12}$$

Eqs. (8)–(12) are four *functional* equations in the four functions $\Lambda$, $\Phi$, $a_c^*$, and $a_s^*$. We can characterize the dynamics of the economy by characterizing these four functions. When $n_c$ is replaced by $n_c^*$, we get four equations in four steady-state *quantities*.

Eq. (9) is an equilibrium expression, with the ability thresholds and the transition function being endogenous quantities. There is no obvious reason why the functional

[15] The law of motion (8), as well as the existence of threshold abilities, and the interaction between ability and human capital, are features in our model that are similar to those in Galor and Tsiddon (1997), who use linearity instead of limited types to simplify the problem of heterogeneity. Their focus, however, is very different from ours.

equation should have a solution. Since we cannot relate this system of functional equations to any existing setup, we prove (in Appendix A), by providing reasonable conditions, that a unique solution exists to this system. To simplify the algebra involved, we specialize to the case of a linear probability function ($\pi(a)=a$) and uniform distribution of ability ($F(a)=a$). These simplifying assumptions are also consistent with agnosticism regarding these directly unobservable functions.[16]

Since $w_c(n_c)$ is a decreasing function, it seems reasonable to expect the extra value obtained by being college educated, $\Lambda$, to also be a decreasing function (provided $\Phi(n_c)$ is increasing and the endogenous response of $a_c^*$ and $a_s^*$ do not cause perverse effects in the contemporaneous extra utility $x$). We, therefore, work in the space of weakly decreasing functions and study the mapping $T$ defined by

$$T\Lambda(n_c)=x(n_c)+\left[\beta\int_{a_c^*(n_c)}^{a_s^*(n_c)}\pi(a)\,\mathrm{d}F(a)\right]\Lambda(\Phi(n_c)),$$

in conjunction with (8), (11), and (12).

When $n_c$ increases, it can be seen from (11) that $a_c^*(n_c)$ increases—the utility cost of sending a child to college (the right-hand side) increases with $n_c$, while the benefit $\Lambda$ decreases (provided the anticipated state tomorrow, $\Phi(n_c)$ is increasing.) However, for the school educated parent, the two effects work in opposite directions. When $n_c$ increases, the utility cost decreases, exerting a downward pressure on $a_s^*(n_c)$ (an income effect); $\Lambda$ decreases, which causes the parent to shift away from the child's college education into personal consumption, exerting an upward pressure on $a_s^*(n_c)$ (a substitution effect). In the appendix, we provide sufficient conditions to ensure that the income effect is strong enough relative to the substitution effect, the transition function increases, and the above-mentioned perverse effect in $x$ is avoided. These conditions allow us to prove the following theorem.

**Theorem 1.** *The equilibrium mapping*, $T$, *is a contraction. A unique solution to the functional equations* (8) *through* (12), *and hence a unique competitive equilibrium exists.*

To provide a sharp characterization of the dynamics, we specialize to the isoelastic (CRRA) utility function:

$$u(c)=\frac{c^{1-\sigma}}{1-\sigma},\quad \sigma\geqslant 0,$$

with the $\sigma=1$ case interpreted as $\log(c)$. We want to argue that the curvature parameter, $\sigma$, plays an important role in the types of equilibria that we can obtain. It is, therefore, useful to consider the linear utility case, $u(c)=c$, first. As mentioned above, in this case $a_s^*=a_c^*$, and there is no real dynamics.

[16] These assumptions also imply that the fraction of college enrollees who are successful is bounded by 1/2. (This follows from the fact, $n_c\int_0^1 a\,\mathrm{d}a+(1-n_c)\int_0^1 a\,\mathrm{d}a=1/2$.) This is clearly unrealistic—for the calibration we use more general probability functions and also assume $\pi_c(a)>\pi_s(a)\ \forall a\in[0,1]$.

**Proposition 1** (Linear utility). *When $\sigma = 0$, there is a unique steady state which is reached in a single step.*

Once we prove the existence and uniqueness of equilibrium, we rely on computations to characterize the types of dynamic equilibria that obtain—that is, the shape of the transition function that results. The effect of the assumptions made on the income and substitution effects is such that $\Lambda(\Phi(n_c))$ decreases at a slower rate than does $g_s(n_c)$. If for any $\widehat{n_c}$, $a_s^*(\widehat{n_c}) = 1$, that is, $\beta\Lambda(\Phi(\widehat{n_c})) < g_s(\widehat{n_c})$, then it follows that $a_s^*(n_c) = 1$, $\forall n_c < \widehat{n_c}$. In the interval where no school educated parent sends a child to college, the transition function is

$$\Phi(n_c) = n_c \int_{a_c^*(n_c;\Phi)}^{1} \pi(a)\,\mathrm{d}F(a) < n_c.$$

Graphically, the transition function is a straight line that lies below the 45° line. Any economy that starts out in this region will asymptotically reach the origin. This fact is useful in discussing the behavior of the economy in the neighborhood of $n_c = 0$.

It is helpful to distinguish among three cases for the curvature parameter $\sigma$, "high", "low", and "medium" while discussing the types of dynamic equilibria.

- "**Low**" $\sigma$: For a given set of production parameters, when $\sigma$ is low the utility cost of school educated parents is low even when $n_c = 0$ (note that the wages of school educated parents are highest when $n_c = 1$ and lowest when $n_c = 0$). School educated parents will always send their children to college, $\Phi(n_c)$ is concave and intersects the 45° line once. A unique steady state with a positive $n_c^*$ is obtained.
- "**Medium**" $\sigma$: For intermediate values of $\sigma$, $a_s^*(n_c) = 1$, for low values of $n_c$. Once the school educated parent's wage becomes high enough (that is, $n_c$ becomes large enough), $a_s^*(n_c) < 1$, $\Phi(n_c)$ increases at a rapid rate to cross the 45° line, and then becomes concave again to re-intersect the 45° line. There are three steady states, zero and two positive steady states, with the lower of the two positive steady states being unstable.
- "**High**" $\sigma$: When $\sigma$ is high, the utility cost of school educated parents is very high. School educated parents will either never send their children to college or start sending them to college only for high wages. The increase in the fraction of college educated labor is low enough to cause $\Phi(n_c) < n_c$ in the entire interval in this case. The only possible steady state then is $n_c^* = 0$. That is, a *development trap* obtains.

The dynamic behavior of the economy around the origin is governed mainly by the utility cost of school educated rather than college educated parents. Rich parents always send a positive fraction of their children to college and especially so when their wages are very high ($n_c \to 0$). But given that they are a very small fraction of the labor force when $n_c$ is in the vicinity of 0, their behavior matters little to the dynamics of the economy. Whether the fraction of college educated workers continues to grow in the vicinity of $n_c = 0$, and if so whether it grows at a rate that can sustain a long run equilibrium with a positive fraction of such workers, depend on the behavior of the poor parents.[17]

### 3.4. Computation

The equations that describe the dynamics of the system are highly non-linear, which limits the extent of analytical characterization we can provide. We use numerical algorithms to compute and more completely characterize the model. A detailed outline of the algorithm used for computation is provided in the Appendix A. In this subsection we provide only a synopsis.

As mentioned earlier, Eqs. (8)–(12) are four functional equations in the four functions $\Lambda$, $\Phi$, $a_c^*$, and $a_s^*$. The computation involves an iteration scheme to find the fixed points of the transition function $\Phi$, and the costate $\Lambda$. Suppose parents take as given these functions. Then for any given state $n_c$, they can solve for $a_s^*(n_c)$ and $a_c^*(n_c)$. (When (12) holds with inequality, $a_s^*(n_c)$ is set to 1.) These reservation abilities can be used in (8) to update $\Phi$, and in (9) to update $\Lambda$ to be used in the next iteration. In practice, we use Chebyshev approximation and interpolation and compute the functions $\Phi$ and $\Lambda$ at a discrete set of grid points in $[0,1]$.[18] See Judd (1998) for details on this method.

### 3.5. Numerical examples

We postpone the calibration until the next section (where we discuss the subsidies that are necessary to match US data), and present numerical examples here with the sole purpose of illustrating the three types of equilibrium mentioned above. We keep the production and probability functions fixed and vary only $\sigma$, the curvature of the utility function.[19]

In Fig. 1, we show the transition functions for the three cases. When $\sigma = 2$ ("low" $\sigma$), we get a unique positive steady state, with about 25% of the labor force being college educated, the rich send around 95% of their children to college and the poor send close to 25% of their children to college. When $\sigma = 2.65$ ("medium" $\sigma$), we get the second type of equilibrium, with a low ($n_c^* = 0$) as well as a high ($n_c^* = 12.6\%$) steady state. The middle ($\widetilde{n_c} = 4.7\%$) steady state is an unstable one. If an economy starts with an $n_c < \widetilde{n_c}$, the low steady state is reached. For any other starting value for $n_c$, the higher steady state is reached. Finally, when $\sigma = 4$ (high $\sigma$), we get the

[17] These three types of equilibria can also be illustrated with constant $\sigma$ but varying cost $e$. Increasing either would have the effect of increasing the utility cost $g_s(n_c)$, especially in the vicinity of $n_c = 0$.

[18] There are cases where $a_s^* = 1$ for an interval of $n_c$. This causes $\Phi$ and $\Lambda$ to be kinked at the point where $a_s^*$ becomes $< 1$. The Chebyshev method has a tendency to smooth out functions and miss out on such kinks. So, we used a lot more grid points than is normally used with this method. FORTRAN programs used to compute numerical solutions are available from the authors upon request.

[19] The parameters used in the computation are same as those used in the calibration section, except for taxes and subsidies which are set to zero here.

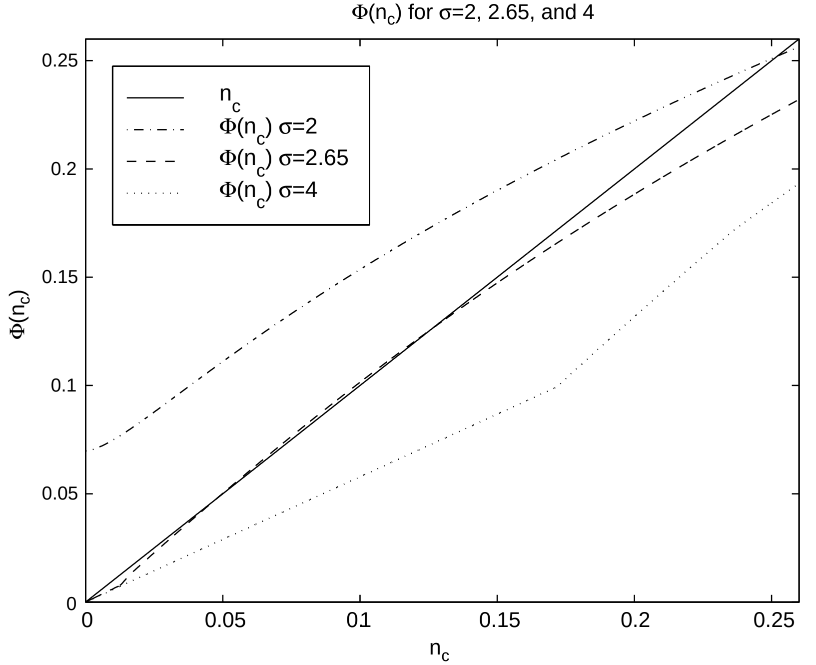


Fig. 1.

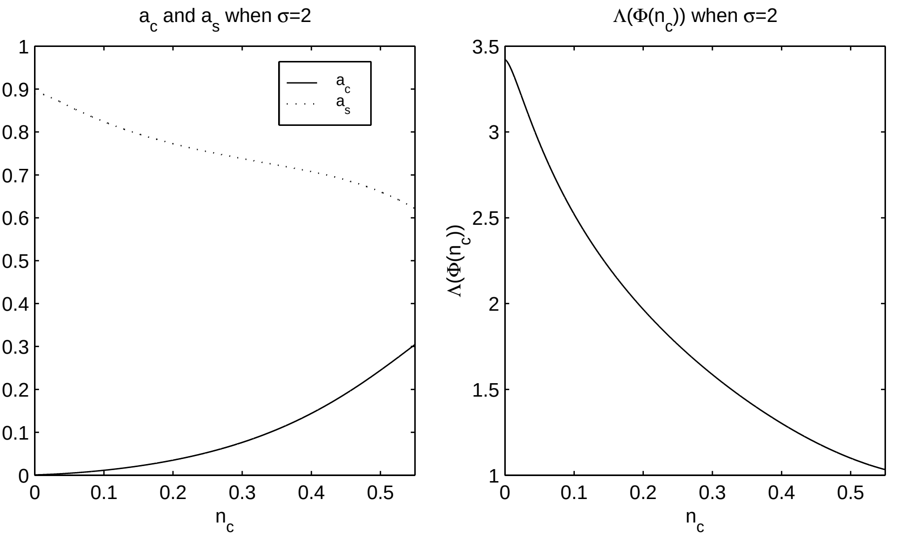


Fig. 2.

third type of equilibrium. The only steady state is a “trap”, with $n_c^* = 0$, to which the economy converges, no matter where it starts.

In Fig. 2, we show the two threshold abilities, $a_c^*$ and $a_s^*$, and the value to being skilled, $\Lambda$, when $\sigma = 2$, all as functions of the state, $n_c$. As the fraction of skilled labor increases, the skilled respond to a drop in their wages by sending only the more able of their children to college ($a_c^*$ increases), while the unskilled respond to the rise in their wages by sending more of their children ($a_s^*$ decreases). In terms of the earlier discussion, the income effect of an increase in $n_c$ and thus unskilled wages dominates the substitution effect of a decrease in $\Lambda$. But given Assumption 3, it is always the case that $a_c^* < a_s^*$. As discussed in the theoretical section, the value to being skilled decreases in $n_c$. For the higher $\sigma$ values, the main difference is that $a_s^*$ stays at 1 for a brief interval near the origin; at the low unskilled wages that prevail in this region no poor child is sent to college. For the remainder of this paper, we concentrate on the case with a unique positive steady state since it seems most relevant to the US situation.

## 4. The government and higher education policy

Thus far, we have studied the behavior of an economy with only private agents. This characterization is essential to understand the policy analysis that will be conducted. As mentioned earlier, the assumption that a child's ability is known only to the parent has no bearing on the private economy since the liquidity constrained parent fully finances the child's education. No trade or exchange is affected by the private information. But it will matter when the government is introduced.

In this model, the rich parents are more likely to send the child to college than the poor one (see Lemma 3). Therefore, in a liquidity constrained economy, less able rich children are sent to college, while more able poor children do not get the opportunity to go to college. If aggregate educational efficiency is measured as the amount of resources invested per student who successfully completes college, the fully private economy might not provide an efficient outcome. That is, by redirecting resources from educating the marginally able rich children to the more able poor children, it would seem that educational efficiency (though not necessarily, aggregate welfare) would be improved. This measure is also likely to improve equality of opportunity. Thus, a natural policy that the government could consider in this setup is the subsidization of the poor children.

This is where the informational assumption has an effect. Since the government cannot observe the ability of poor children, it will have to subsidize *all* poor children to the same extent—it cannot index the subsidy to the child's ability. That is, all poor parents face a lower cost of college, which might cause them to send more as well as less able children to college. This is a countervailing force to the one mentioned above, and can actually *decrease* educational efficiency. In our setup, given the higher response of the school educated parents to policy, the countervailing effect dominates and education efficiency decreases in most cases. The monetary rate of return would likewise decrease. Similar considerations apply if the ability were observable, but the government is under political or other constraints to provide need-based, rather than merit-based aid. In practice, lack of observability and political constraints are both likely to contribute to the primarily need-based system we observe in the US.

In addition to the educational efficiency, we also study the effect of subsidies on a weighted sum of welfare. If the government values all citizens equally, the welfare function at steady state will be given by

$$n_c^* \mathrm{E}V_c(n_c^*) + (1 - n_c^*)\mathrm{E}V_s(n_c^*) = \mathrm{E}V_s(n_c^*) + n_c^* \Lambda(n_c^*).$$

Any subsidization policy that increases the steady-state fraction of skilled people will cause a higher fraction of the population to be in the higher utility state, but will lower the value to being skilled in equilibrium; that is, the weight $n_c^*$ increases, but $\Lambda(n_c^*)$ decreases. We will shortly describe a system in which all parents are taxed but only the poor college attendees get a subsidy; the tax burden inherent in this scheme will also have a negative effect on the utility of both types of agents.[20] Steady-state welfare can therefore increase or decrease with any policy that increases the level of skill attainment.

We study transitional as well as steady-state effects on welfare while comparing different policy regimes. The sacrifices made in getting to the steady state that is better in an aggregate sense might not be worth the ultimate benefit of staying there in the long run, especially for the currently rich who will have to make most of the sacrifices.

One reasonable question at this stage is, “Why do we care about the effect of subsidies on the positive efficiency measure, especially in those cases where the aggregate welfare increases?” There are at least three reasons why one might want to study the positive measure. First, there can be little controversy on the definition of such a measure. While we chose a symmetrically weighted welfare measure because it seemed least ad hoc, there is no reason to expect it to be the universally accepted measure. This is especially true when there are opposite effects on the welfare of the rich and the poor that arise from the redistributive policies considered. Second, it is useful to know if the effect on the positive measure differs from the one on the normative measure, which it does in our case. If we envision a “Secretary of Education” who makes education budgeting decisions, this person has a stronger incentive to be concerned about the efficiency of the education sector than about aggregate welfare.[21] Finally, we have not explicitly modeled the moral hazard involved in the process of human capital accumulation that causes the incompleteness of markets in the first place. The process of completion of the market by the government can increase the moral hazard and decrease welfare; the education efficiency measure can thus be a useful complement to the welfare measure.

[20] Labor is inelastically supplied in this model. Therefore, we do not have the loss in productive efficiency that is usually found in models where taxes distort labor supply decisions. Since the portion of income taxes that go toward higher education financing is likely to be very small, it seems that we are not missing out on first-order effects by assuming inelastic labor supply.

[21] Such a Secretary is usually nominated rather than elected and is thus more remote from the political process.

### 4.1. A simple tax and subsidy scheme

Throughout this section, we will study only one kind of policy. All workers will have their income taxed at the rate $\tau$. All school educated workers will get a college subsidy of $s$, *if* they send their children to college. That is, they will face an effective college tuition of $e - s$. Since the poor receive a much higher level of subsidy in practice, such a policy assumption is empirically relevant.[22] From a calibration point of view, this would mean that we assign all subsidies seen in practice to the poor children.

The structure of the problem remains the same as in (1), but the current-period utility terms differ. College educated parents get $u((1-\tau)w_c(n_c))$ if they do not send their children to college, and $u((1-\tau)w_c(n_c)-e)$ otherwise. The corresponding utilities for the school educated parents are $u((1-\tau)w_s(n_c))$ and $u((1-\tau)w_s(n_c)-(e-s))$. In transitions, to impose some discipline on the experiments, we will consider a subsidy that is a constant percentage of education costs, but will vary $\tau$ over time to balance the government's budget. Therefore, $\tau \equiv \tau(n_c)$.

Since, the structure of the parent's problem is the same, we will get the same reservation ability behavior, but reservation abilities will now be functions of the policy parameters as well. They can be written as $a_c^*(n_c, \tau(n_c), s)$, $a_s^*(n_c, \tau(n_c), s)$. We require the government's budget to be balanced every period, which is not too restrictive given that each model period corresponds to several calendar years. The government's budget constraint is

$$(1-n_c)(1-F(a_s^*(n_c,\tau(n_c),s)))\,s = (n_c w_c(n_c) + (1-n_c)w_s(n_c))\tau(n_c). \tag{13}$$

Here, $(1-n_c)(1-F(a_s^*(n_c,\tau,s)))$ is the number of children of school educated parents sent to college, and hence the left-hand side is the total amount of subsidies paid by the government in equilibrium. These expenses have to be borne even for children who drop out, and hence $\pi$ does not appear anywhere in the expression. The right-hand side is the total amount of taxes collected from all parents.

### 4.2. Efficiency of the education sector

We provide a simple measure of efficiency for the education sector—a ratio of the number (measure) of students graduating from college to the total resources expended in educating them, *irrespective* of who spends this, the parent or the government. We define:

$$\mathit{eff} = \frac{n_c'}{[n_c(1-F(a_c^*)) + (1-n_c)(1-F(a_s^*))]e}.$$

As in (13), $\pi$ does not appear anywhere in the expression, since expenses have to be borne even for children who drop out. The success rate (conditional probability of

[22] In 1992–93, the students in the lowest income bracket received aid that was 48% of total costs, while those in the highest income bracket received 12% of total costs as aid. (The Condition of Education, 1996, Supplemental Table 13-1.) If one adjusts for the progressivity of taxes, the effective difference is likely to be even higher.

success), for the rich children is $S_c \equiv \int_{a_c^*}^1 \pi_c(a)\,dF(a)/(1-F(a_c^*))$, and for the poor children is $S_s \equiv \int_{a_s^*}^1 \pi_c(a)\,dF(a)/(1-F(a_s^*))$. Using (8) for $n_c'$, and the above success rates, the formula for efficiency can be written more revealingly (with the state variable argument suppressed for simplicity) as

$$eff = \left\{ \frac{n_c(1-F(a_c^*))}{n_c(1-F(a_c^*)) + (1-n_c)(1-F(a_s^*))} S_c + \frac{(1-n_c)(1-F(a_s^*))}{n_c(1-F(a_c^*)) + (1-n_c)(1-F(a_s^*))} S_s \right\} \frac{1}{e}. \tag{14}$$

The efficiency depends on a weighted sum of the success rates of the two types, where the weight is the share of each type of enrollees. It is easy to show that each of the success rates is increasing in the relevant $a^*$, since the quality of the enrolling pool is higher. So any policy that redistributes funds from the rich to the poor would have the effect of increasing $S_c$ and decreasing $S_s$. As mentioned in the introduction to this section, since the poor are located in the more responsive part of the utility function the increase in their enrollment causes the negative effect on the efficiency to dominate. [23]

An alternate quantity to use would be the monetary rate of return. At an individual level this rate of return is $\pi(a)(w_c(n_c') - w_s(n_c'))/e$. It is easy to show that at the aggregate level, this rate of return is just $eff \cdot (w_c(n_c') - w_s(n_c'))$. Since the wage gap is decreasing in $n_c'$, the return will drop even more steeply when a subsidization policy decreases efficiency as defined above; so, we present details only on our efficiency variable in all policy experiments.

### 4.3. A welfare measure

In addition to the above-mentioned efficiency measure, we also consider a symmetric welfare measure by summing up welfare over all parents in the economy. Such an equal-weighted measure seems to be the least ad hoc one. Thus, the welfare in each period is the measure-weighted sum of utilities over college educated parents who send their children to college and those who do not, and school educated parents who do and do not:

$$W_t(n_c^*)/(1+\beta) = n_c[(1-F(a_c^*))u(w_c(n_c)-e_c) + F(a_c^*)u(w_c(n_c))] + (1-n_c)[(1-F(a_s^*))u(w_s(n_c)-e_s) + F(a_s^*)u(w_s(n_c))],$$

where $e_c$ and $e_s$ are the tax or subsidy adjusted education expenditures. The welfare at a steady state is the discounted sum of the welfare in each period: $W_{ss}(n_c^*) = 1/1-\beta W_t(n_c^*)$. If we want to calculate the welfare of going from one steady state to another, we calculate the discounted sum of period welfare along the transition and then add the

[23] The weights will also be altered by such redistributive policies. But if the elasticity of $a_c^*$ is not too high, the weights will move in the same direction as the success rates.

appropriately discounted steady-state welfare, $W_{\text{transition}} = \sum_{i=0}^{T} \beta^i W_i(n_c) + \beta^{T+1} W_{ss}(n_c^*)$, where $T$ is the number of periods after which we are "very close" to the steady state.

We can translate these welfare values into an equivalent consumption measure; we calculate, $\omega$, the percentage by which consumption in *each* period needs to be increased (decreased) in order to be as well off under the benchmark as under the policy change. In other words, we calculate $\omega$ such that the $W_{\text{transition}}$ given above is related to the benchmark welfare according to the expression:

$$W_{\text{transition}} = \frac{1}{1-\beta} \{ n_{c,b}^* [(1 - F(a_{c,b}^*)) u(\omega(w_c(n_{c,b}^*) - e_{c,b})) + F(a_{c,b}^*) \omega(w_c(n_{c,b}^*))] + (1 - n_{c,b}^*)[(1 - F(a_{s,b}^*)) u(\omega(w_s(n_{c,b}^*) - e_{s,b})) + F(a_{s,b}^*) \omega(w_s(n_{c,b}^*))] \},$$

where, the subscript b on the right-hand side steady-state quantities refer to the benchmark case. Given the homogeneity of the utility function, it is easy to see that, $\omega = (W_{\text{transition}}/W_{\text{benchmark}})^{1/(1-\sigma)}$. Even though tax rates have been dropped for notational simplicity, they are applied to the wages to get the welfare measure.

## 5. Calibration

In order to conduct policy experiments, we first calibrate the above two-period overlapping generation model, where parents spend resources on their children's college education, to US data. The two types of labor are interpreted as school educated and college educated labor and the skill premium as the college premium.[24] Our calibration strategy is guided by a desire to broadly match model outcomes that are unique to our setup—enrollment and dropout rates by income group, college attainment and the skill premium—to US data.

We start by assuming values for parameters for which precedence or independent evidence exists in the literature. The generational discount factor is set at $\beta = 0.55$, which corresponds to a yearly discount factor of 0.98 compounded over 30 yr. We set $\nu = 0.35$, which corresponds to an elasticity of substitution between college and high school equivalents of 1.54. Autor et al. (1998) report that the emerging consensus on this elasticity is approximately 1.4–1.5. From the *Condition of Education 1996* which has data for the 1992–93 academic year, we approximate total college yearly expenditure of \$15,000. From the Digest of Education Statistics 1998, we get the median family

[24] The difficulties of calibrating a two-period model to the actual economy are obvious. However, there are some mitigating factors in this setup. First, even if we were to expand our model to a several-period OLG model, for any given family there will be little happening during most of the periods. This is not a model of continuous asset trading or portfolio choice—parents have a single, discrete decision to make. Second, the steady-state predictions are interesting in their own right, and it is not clear that a several-period OLG model will lead to significantly different results in this regard. Finally, the various effects in the model are a lot more transparent in this simple setup, and we can therefore provide some analytical characterization. So, we set aside the task of computing a more realistic large-scale OLG model for future research, and proceed with the calibration in the simpler setup. The aim is to get a set of benchmark parameters with which it is sensible to conduct policy analysis.

Table 1
Comparison of model outcomes with US data

| Empirical quantity | Model variable | US data | Year | Source | Model |
|---|---|---|---|---|---|
| % labor force college educated | $n_c^*$ | 38.6 (full-time equiv.) | 1990 | Autor et al. (AKK) (1998) | 35.7 |
| Enrollment rate for rich (%) | $1-F(a_c^*)$ | 76.6 ("high" income) | 1990 | The Condition of Education (1998) | 80.0 |
| Enrollment rate for poor (%) | $1-F(a_s^*)$ | 46.7 ("low" income) | 1990 | The Condition of Education (1998) | 50.2 |
| Dropout rate for rich (%) | $\frac{\int_{a_c^*}^1 (1-\pi_c(a))\,dF(a)}{1-F(a_c^*)}$ | 30.2–43.0 (2 yr) 15.6–19.6 (4 yr) | 1989–90 (starting year) | Digest of Education Statistics (1998) | 32.5 |
| Dropout rate for poor (%) | $\frac{\int_{a_s^*}^1 (1-\pi_c(a))\,dF(a)}{1-F(a_s^*)}$ | 43.4–45.1 (2 yr) 25.5–29.3 (4 yr) | 1989–90 (starting year) | Digest of Education Statistics (1998) | 49.2 |
| Skill premium | $w_c^*/w_s^*$ | 1.58<br>1.66–1.73 | 1989 (avg.)<br>1990 | Murphy and Welch (1992)<br>AKK (1998) | 1.54 |
| Govt. exp. as % of GDP | $\frac{(1-F(a_s^*))s}{Y}$ | 0.55 | 1989 | McPherson and Schapiro (1991) | 1.9 |

income in 1992 to be \$36,573. We calculate the present value of educational expenses over 4 yr and yearly income over 30 yr, at a discount rate of 8% to get $e/Y$ to be around 0.12. Using a $Y=0.5$ from preliminary computations of the model, we estimate the expenditure parameter $e$ to be 0.06.[25] From the *Condition of Education 1996*, we also get a subsidy of 48% for the low-income group (enrollment-weighted total aid divided by total cost at private and public institutions). Therefore, given the $e$ of 0.06, we set $s$ to be 0.03. We use a value for $\sigma$ of 2, which is fairly standard in the business cycle and growth literature. We assume a uniform ability distribution in $[0,1]$; that is, $F(a)=a$. We then choose the remaining production parameters ($A$, $\theta$, $\varepsilon$, and $\gamma$) and the probability functions ($\pi_c(a)$, $\pi_s(a)$) to match the above-mentioned quantities.

The benchmark parameters used are summarized below:

$$\text{Production}: A=1,\ \theta=0.5,\ v=0.35,\ \varepsilon=0.1,\ \gamma=0.02,$$

$$\text{Preference}: \beta=0.55,\ \sigma=2,$$

$$\text{Education}: e=0.06,\ s=0.03,\ F(a)=a,\ \pi_c(a)=a^{0.74},\ \pi_s(a)=0.66a^{0.9}.$$

Table 1 compares the steady-state outcome of the model to US data. The model's outcomes are broadly consistent with the corresponding quantities seen in US data. The

[25] Note that this is the discount rate that is used *within* a period of the model to map the data into the model's framework, as opposed to the intergenerational rate implicit in the $\beta$ that is built into the model. Discount rates in the range of 5–8% all yielded $e/Y$ measures in the range of 0.10–0.12. Since the $Y$ used is from preliminary computations, we verify that the $e/Y$ that results in the benchmark equilibrium (0.118) is close to that assumed.

percentage of population that is college educated is very close to the 38.6% full-time college equivalents reported by Autor et al. (1998). It is also in between the range of 27.3% who have completed higher education and 45.2% who have some higher education as reported by Barro and Lee (1996). The enrollment rates are consistent with the enrollment rate at income brackets at either end of the distribution as reported by the *Condition of Education*, *1998*.[26] The enrollment gap of about 30% is almost the same as the one seen in data. The model dropout rates are closer to the 2 yr college rates rather than the 4 yr rates. Data is reported by income quartiles, and the lower ranges given in the table correspond to the higher income quartiles.[27]

The skill premium is very close to the Murphy and Welch (1992) figure and not too far from the estimates of Autor et al. (1998). The government subsidy expenditure as a fraction of GDP is higher than but in the same order of magnitude as the one calculated for US data.[28]

## 6. Policy experiments

In this section we consider the effects of the tax and subsidy policy on the transitional and long-run outcome of the economy. As mentioned in the introduction, we concentrate on three government policies that are typically proposed—guaranteeing equality of opportunity, maximizing steady-state college attainment, and replacing pure need-based aid by merit-based aid for the poor. We also discuss the optimal subsidy level and, as a check of robustness, consider the case where ability matters for production in addition to academic achievement. The first two policies entail solving a social planner's problem; the planner's problems that we solve are of the "Ramsey" type. That is, the planner takes as given the actions of individual agents in response to policy parameters, and chooses the policy vector to guarantee equality of opportunity or maximize skill attainment. The firm's actions are taken as given too, since the wages are given by the MPLs. This is in contrast with a scheme in which the planner collects all resources and allocates them as consumption and education expenditures of both types of parents, and organizes production as well. In other words, the planner's (government's) policy is purely an educational one and *not* used for general purpose redistribution. The latter is clearly outside the scope of this paper.

[26] Reallocation of the 54.4% enrollment rate for the "middle" income group reported for the US to the "low" and the "high" group is likely to increase the rate for the former and decrease it for the latter. The model's outcome would then be closer to the data for the enrollment of the poor, but higher than what is found in the data for the rich.

[27] The data shown corresponds to the "no degree, not enrolled" column of the *Digest of Education Statistics* table. There is a separate column titled, "no degree, still enrolled", which corresponds to students who have not graduated on time. Given the high likelihood that some of these students will also eventually drop out, the model outcomes are likely to be closer to the data if a portion of these students are also counted as dropouts.

[28] The US figure is computed using the nearly $32 Billion reported by McPherson and Schapiro (1991) as total federal, state, and institutional aid for higher education in 1988–89 divided by the GDP figure of about $5,200 Billion in 1989. In making this comparison we are assuming that the ratio holds within any sub-period of the model period.

### 6.1. Equality of opportunity

Equality of educational opportunity in this model is achieved in an equilibrium in which $a_c^* = a_s^*$. A child of ability $a$ will either attend or not attend college based on a common threshold ability—her parent's income will not matter. This seems the least ad hoc way of specifying equality of opportunity for the young, and is closely aligned to the concept espoused in Atkinson (1983) that, "the ex ante distribution of earnings is the same for all people with innate abilities". Equality of opportunity is sometimes viewed as equating opportunity sets. There is a sense in which that is relevant here. Enrollment in our model depends on the utility cost of sending a child to college and equating utility costs across different types of parents is what guarantees the above equilibrium outcome; however, the absolute consumption levels of the rich and the poor parents will continue to differ. The fact that we need to solve a planner's problem to find the right levels of tax and subsidy rates to equate the parents' utility costs does not mean there is any guaranteed educational *outcome* for the children; given the stochastic human capital production function some of them will not become skilled.

We can use the analogue of Eqs. (8)–(12) for the case where $\pi_c \neq \pi_s$ to derive conditions that will have to be satisfied in a steady state with equality of opportunity. We use the following notation to make the resulting expressions more succinct: $u_c \equiv u((1-\tau)w_c(n_c^*))$, $u_{c-e} \equiv u((1-\tau)w_c(n_c^*) - e)$, $u_s \equiv u((1-\tau)w_s(n_c^*))$, and $u_{s-e} \equiv u((1-\tau)w_s(n_c^*) - e + s)$. The conditions required to have $a_c^* = a_s^* = a^*$ can then be derived to be

$$\pi_s(a^*)(u_c - u_{c-e}) = \pi_c(a^*)(u_s - u_{s-e}),$$

$$u_c - u_{c-e} = \beta\pi_c(a^*)\,\frac{[F(a^*)(u_c - u_s) + (1 - F(a^*))(u_{c-e} - u_{s-e})]}{[1 - \beta\int_{a^*}^1 (\pi_c(a) - \pi_s(a))\,\mathrm{d}F(a)]},$$

$$n_c^* = n_c^* \int_{a^*}^1 \pi_c(a)\,\mathrm{d}F(a) + (1 - n_c^*)\int_{a^*}^1 \pi_s(a)\,\mathrm{d}F(a),$$

$$(1 - n_c^*)(1 - F(a^*))\,s = \tau\,(n_c^* w_c(n_c^*) + (1 - n_c^*)w_s(n_c^*)).$$

These are four equations in the four unknowns $n_c^*$, $a^*$, $s$, and $\tau$, which we solve numerically.

Table 2 compares this policy with the benchmark. This policy increases the measure of people with college attainment in the long run, relative to the benchmark case. College educated workers get taxed, but do not get any subsidies—the decrease in their net income as well as the anticipated decrease in skill premium and the value to being skilled due to an increase in the college educated workforce will cause them to send fewer of their children to college. The school educated parents are taxed but also get education subsidies, and as long as the net income effect is positive, they will send more of their children to college. The effect of school educated parents, who are larger in measure, sending their children to college swamps out the effect of college educated parents sending fewer children. A decrease in college premium (a measure of "inequality" in this setup) is an immediate consequence—a premium of 1.45 in the subsidized economy, compared to the 1.54 in the benchmark case.

Table 2
Equality of opportunity

| Variable | Interpretation | Benchmark | Eq. opp. |
|---|---|---|---|
| $n_c^*$ | Fraction of college educated (%) | 35.7 | 38.2 |
| $1 - a_c^*$ | Enrollment rate—rich (%) | 80.0 | 68.9 |
| $1 - a_s^*$ | Enrollment rate—poor (%) | 50.2 | 68.9 |
| $1 - S_c$ | Dropout rate—rich (%) | 32.5 | 27.5 |
| $1 - S_s$ | Dropout rate—poor (%) | 49.2 | 55.0 |
| $\frac{w_c^*}{w_s^*}$ | Skill premium | 1.54 | 1.45 |
| $Y^*$ | Output | 0.509 | 0.515 |
| $\tau$ | Education tax rate—all (%) | 1.9 | 3.6 |
| $s/e$ | Education subsidy rate—poor (%) | 50 | 71.7 |
| $eff$ | Education efficiency measure | 9.64 | 9.25 |

While it is true that only the more able of the rich children now go to college, a larger fraction of lower ability poor children go to college. The increase in efficiency coming from the increased graduation rate of the rich children is more than offset by the decrease in efficiency caused by the decreasing graduation rate of the poor. In terms of (14), the effect of the drop in $S_s$ dominates the increase in $S_c$. In fact, the efficiency goes down throughout the transition to an equal opportunity steady state, since $a_c^*$ and $a_s^*$ monotonically adjust from the old steady state to the new one.

Given Assumption 3, which in effect causes production to increase with the measure of college educated workers, a higher level of output $Y$ is also obtained under this policy, by about 1.2%. Nearly 72% of the college expenses need to be subsidized for the poor to get equality of opportunity. An income tax rate of about 3.6% needs to be levied on all workers in the long run to finance these subsidies, compared to the 1.9% rate that needs to be levied to meet the subsidy expenses in the benchmark. Given (13), this is also the total amount of subsidies given as a fraction of GDP.

We computed the transition paths for all the variables assuming the benchmark as the starting point and an unexpected announcement of the equal opportunity policy. We omit the graphs for sake of brevity. The variables, $n_c$, $Y$, $a_c^*$, and $w_s$ increase monotonically toward their steady-state values, while $a_s^*$ and $w_c$ decrease monotonically. Convergence is rapid, occurring within two generations. With the computed transitions in hand we can compare welfare across the two systems. We summarize this comparison in Table 3.

The first row in Table 3 presents the equivalent increase in consumption each agent would have to be given in the benchmark steady state, in order to make an equally weighted aggregate welfare measure the same as that in an equal-opportunity steady state. Each household needs to be given 1.12% more consumption every period in the current system. In order to address distributional issues, the second and third rows provide the equivalent measures for the rich and the poor; the comparison is across $EV_c$ and $EV_s$ (that is, before the child's ability is revealed). It is clear that it is better to be rich under the current policy, but better to be poor under the new policy. However, the utility gain for the poor outweighs the utility loss for the rich, since

Table 3
Welfare gain in the move to equality of opportunity

| Variable | Equiv. cons. increase (%) |
|---|---|
| *SS* (aggregate) | 1.12 |
| *SS* utility of $c$ | −1.98 |
| *SS* utility of $s$ | 1.97 |
| Including transition (aggregate) | 0.12 |
| Utility of $c$ including transition | −0.77 |
| Utility of $s$ including transition | 0.51 |

there are more of the former. Therefore, the aggregate welfare under the new system is greater.

Skilled wages are higher during the transition than they are at the final steady state. Therefore, in spite of the slightly higher taxes and higher enrollment that prevail during the transition, the utility loss for those who are rich at the time of the policy announcement is less severe when transition is factored in. Lower than steady-state wages and higher taxes have a negative effect on the initially poor; so when transition is accounted for, the poor gain less than they could if the economy jumped to the new steady state right away. In aggregate, the gain in welfare once transition is included is much lower, amounting to an equivalent increase of 0.12% of benchmark consumption.

### 6.2. *Maximization of the skilled labor force*

As mentioned in the introduction, politicians often call for policies that would enable as many students as possible to be college educated. An increase in college attainment in the population is also mentioned as one route to creating a more cohesive society with reduced crime, increased involvement in the political process, etc. In our model, one way to approach the issue of increasing college attainment is to search for a policy that maximizes this attainment in steady state. Given the assumptions made about the production function, steady-state output also increases with the steady-state fraction of college educated labor. Aggregate consumption would also increase.[29] However, as discussed earlier, aggregate steady-state welfare could either increase or decrease with $n_c^*$. We study the problem:

$$\max_{\tau,s} n_c^*,$$

[29] Aggregate consumption is given by $Y - [n_c^*(1 - F(a_c^*)) + (1 - n_c^*)(1 - F(a_s^*))]e$. For the assertion to be true, $(w_c - w_s) > e$ is needed in the steady state (the differences in marginal products is greater than the consumption cost of education). Given that $e$ is a very small fraction of lifetime wages, this is a fairly weak condition.

Table 4
Maximize attainment

| Variable | Interpretation | Benchmark | Max $n_c^*$ |
|---|---|---|---|
| $n_c^*$ | Fraction of college educated (%) | 35.7 | 38.9 |
| $1-a_c^*$ | Enrollment rate—rich (%) | 80.0 | 60.5 |
| $1-a_s^*$ | Enrollment rate—poor (%) | 50.2 | 89.8 |
| $1-S_c$ | Dropout rate—rich (%) | 32.5 | 23.9 |
| $1-S_s$ | Dropout rate—poor (%) | 49.2 | 61.8 |
| $\frac{w_c^*}{w_s^*}$ | Skill premium | 1.54 | 1.43 |
| $Y^*$ | Output | 0.509 | 0.519 |
| $\tau$ | Education tax rate—all (%) | 1.9 | 5.8 |
| $s/e$ | Education subsidy rate—poor (%) | 50 | 91.7 |
| *eff* | Education efficiency measure | 9.64 | 8.26 |

subject to the constraints:

$$n_c^* \int_{a_c^*(n_c^*,\tau,s)}^{1} \pi_c(a)\,\mathrm{d}F(a) + (1-n_c^*) \int_{a_s^*(n_c^*,\tau,s)}^{1} \pi_s(a)\,\mathrm{d}F(a) = n_c^*,$$

$$\tau(n_c^* w_c(n_c^*) + (1-n_c^*) w_s(n_c^*)) = (1-n_c^*)(1-F(a_s^*(n_c^*,\tau,s)))\,s,$$

where, the planner takes as given, the behavior of parents as characterized by the functions $a_c^*(n_c^*,\tau,s)$ and $a_s^*(n_c^*,\tau,s)$. Table 4 summarizes the outcome if a policy of maximizing college attainment were to be pursued.

The planner would really like to set policies such that $a_c^* = a_s^* = 0$ since this maximizes $n_c^*$. This is clearly infeasible when the only instrument available is an education tax and subsidy—the wages for both types would have to be simultaneously high to get both $a^*$s to be zero. So, as can be seen in the above table, the planner heavily subsidizes the poor parents, who have a high response to subsidies; their enrollment rate climbs close to 90%. Since $n_c^*$ is the highest it can be under the constraints mentioned (close to 39%), the college premium is the lowest, at 1.43. About 92% of the education expenses of the poor is subsidized, and a tax rate of 5.8% is required to pay for it; alternately, this is the fraction of output given out as subsidies in the steady state. Output is about 2% higher than in the benchmark. The drop in education efficiency is high, corresponding to the steep drop in $a_s^*$. The dropout rate for the poor children goes from 49% in the benchmark to nearly 62%, and the effect of a decrease in the dropout rate of rich children is completely swamped out. Table 5 provides welfare figures.

While steady-state welfare increases relative to the benchmark case, the increase is less than the one with a subsidy that guarantees equality of opportunity. The high tax rate and the decline in skilled wages are particularly severe on the rich; they would be willing to pay up to 3.92% of their consumption to avoid jumping to the steady state where attainment is maximized. Therefore, in spite of the preference of the poor for the new steady state, aggregate increase in steady-state welfare is much lower than the figure given in Table 3. As mentioned in the previous subsection, accounting

Table 5
Welfare gain in the move to maximize attainment

| Variable | Equiv. cons. increase (%) |
|---|---|
| *SS* (aggregate) | 0.51 |
| *SS* utility of $c$ | −3.92 |
| *SS* utility of $s$ | 1.90 |
| Including transition (aggregate) | −0.69 |
| Utility of $c$ including transition | −2.17 |
| Utility of $s$ including transition | 0.35 |

for the transition makes the utility loss for the rich and the gain for the poor less pronounced; however, the loss is still too high for the rich and the aggregate welfare is actually lower than the benchmark value. Each agents would pay close to 0.7% of the steady-state benchmark consumption to avoid the new policy.

This experiment highlights the difficulty with proposals of the sort quoted in the footnote in the introduction. Increased subsidies do help more "enter the doors of college", but by decreasing the marginal ability of entrants cause a strong decline in the efficiency of the education sector. [30] The high tax rates necessary to support the subsidies also decreases aggregate welfare. These losses are only likely to be higher if ability is modeled as an input in production, and labor supply decisions are made endogenous.

### 6.3. Optimal subsidy

In going from the benchmark to equality of opportunity and to maximizing college attainment, there was a steady increase in subsidy, and the effect on (transition-accounted) aggregate welfare was first positive and then negative. This naturally raises the question, "Is there an optimal subsidy level?" Fig. 3 plots the steady-state welfare as a function of the subsidy level, as an aggregate as well as for each type of agent. We can see that the equal opportunity subsidy level is very close to the steady-state aggregate welfare maximizing level; however, recall from Table 3 that transitional gains are significantly lower than the steady-state gain. The subsidy that maximizes attainment is clearly on the decreasing portion of this curve. The positive educational efficiency measure always decreases with the subsidy level. [31] Since inequality declines with the level of subsidy, there is always a tradeoff between equity and educational efficiency. However, when the aggregate steady-state welfare measure is considered, such a

[30] A similar point is made in Cameron and Heckman (1998).

[31] For sake of completeness, we present the results when subsidies are eliminated completely. Attainment drops by 10 percentage points relative to the benchmark in the long run, and the "enrollment gap" is 70%. The premium is higher, at 2.02 and the education efficiency is higher. Factoring transition, agents in the benchmark would be willing to pay upto 2.6% of their steady-state benchmark consumption to avoid elimination of subsidies. As one would expect, the currently poor are willing to pay more, 3.4% of their consumption, as opposed to the rich who would pay only 0.7%.

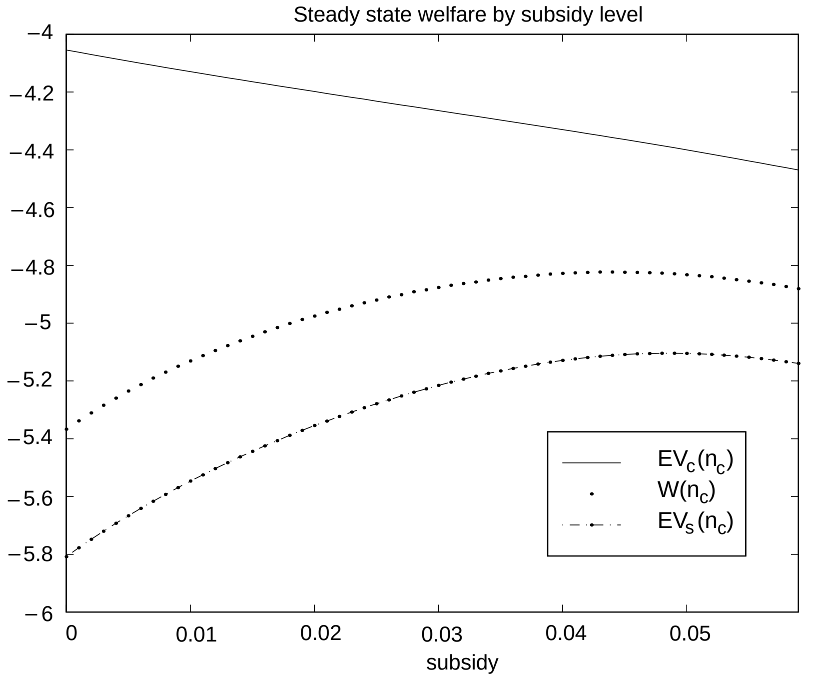


Fig. 3.

tradeoff exists only at high levels of subsidy; in the increasing portion of the welfare curve, equity and efficiency move in the same direction.

From this figure, one can also see that the shape of the aggregate welfare curve is driven by the welfare of the poor agents. The steady-state welfare of rich agents, $EV_c$, declines uniformly with subsidies given to the poor, both by the direct effect of taxes on their income as well as by the indirect effect of lower skill premium. As seen in Tables 3 and 5, they would be willing to pay to avoid the subsidy schemes. The poor on the other hand are helped by the increased enrollment that subsidies allow, till increasing taxes and decreasing value to being educated take a toll on their welfare.

As an aside, the poor form the majority in each of the above experiments. Since they benefit from subsidies and would thus vote for them, the above schemes are politically relevant. [32]

[32] We have abstracted from political economy considerations in our model. However, it is interesting to compare the implication that the majority of poor would prefer the subsidization scheme to the one in Fernandez and Rogerson (1995), where the rich can vote for a tax rate that effectively excludes the poor from education. We have focused only on a need-based subsidy, which seems to be the norm in US higher education. Such a scheme wins the favor of the poor for obvious reasons, while they focus only on a policy that subsidizes anyone who attends college, rich or poor.

### 6.4. Would merit-based subsidies help?

Thus far we have assumed that information on ability $a$ is unobserved by the government, and therefore all poor students get the same amount of subsidy. As mentioned above, by reducing the effective cost of college attendance for all poor students including the less able ones, such a scheme reduces the efficiency of the education sector. One interpretation of the private information assumption is that college aid is need based, rather than merit based. In the US, need-based aid constitutes an overwhelming fraction of total aid. [33] It is useful to ask if some of the deficiencies of the pure need-based system can be avoided if the government is able to observe the ability of poor children and base the subsidy on it.

When the aid is need based, the decision of whether or not to send the child to college depends on the child's ability only through the probability of completion. When the aid is merit based, ability affects this decision by also altering the amount of subsidy the child receives. The problem of a college educated parent is not directly affected; however, $a_s^*$ for the school educated parent is given implicitly now by

$$\beta\pi_s(a_s^*(n_c))\Lambda(\Phi(n_c)) \leqslant u(w_s(n_c)) - u(w_s(n_c) - e + s(a_s^*(n_c))),$$

where $s(\cdot)$ is an increasing function, and the expression holds with equality when $a_s^*$ is interior. We know that a solution exists because the left-hand side of the equation is increasing in $a$, and the right-hand side is decreasing in $a$. The Bellman equation changes in a straightforward way. The merit-based subsidy scheme does not affect $\Phi$, except through it's effect on $a_s^*$. For simplicity, we confine ourselves to a linear subsidy function $s(a) = sa$. The government budget constraint (13) changes—the amount the government spends depends on the ability of the child.

$$(1 - n_c)\int_{a_s^*}^{1} sa\,\mathrm{d}F(a) = \tau(n_c w_c(n_c) + (1 - n_c)w_s(n_c)).$$

The expression for aggregate welfare would change appropriately. Table 6 illustrates the effect of ability-based subsidies.

The third column of Table 6 repeats the benchmark outcomes for convenience. The next column considers the case where the maximum subsidy given is the same as the flat subsidy given, 0.03. The last column considers a "revenue neutral" experiment; the steady-state amount of subsidies is same as that in the benchmark. This involves setting $s = 0.042a$. The role of ability-based subsidies in raising the quality of poor college entrants is clear. Education efficiency improves relative to the flat subsidy scheme under both schemes. Measures of inequality, such as the enrollment gap and the skill premium increase. The move to a merit-based system from the current system entails a welfare loss (including transition) of 1% in the $s = 0.03a$ case, and 0.7% in the revenue neutral case.

[33] Federal aid is currently almost entirely need based. Even if we assume that all state and institutional aid is merit based, the ratio of need based aid to total aid was about 75% in 1988–89. (See Tables 2–4 in McPherson and Schapiro, 1991.)

Table 6
Flat versus ability-based subsidy

| Variable | Interpretation | $s = 0.03$ (flat) | $s = 0.03$a | $s = 0.042$a |
|---|---|---|---|---|
| $n_c^*$ | Fraction of college educated (%) | 35.7 | 31.2 | 32.4 |
| $1 - a_c^*$ | Enrollment rate—rich (%) | 80.0 | 77.0 | 72.8 |
| $1 - a_s^*$ | Enrollment rate—poor (%) | 50.2 | 39.2 | 44.0 |
| $1 - S_c$ | Dropout rate—rich (%) | 32.5 | 31.2 | 29.2 |
| $1 - S_s$ | Dropout rate—poor (%) | 49.2 | 45.8 | 47.3 |
| $\frac{w_c^*}{w_s^*}$ | Skill premium | 1.54 | 1.72 | 1.67 |
| $Y^*$ | Output | 0.509 | 0.497 | 0.501 |
| $\tau$ | Education tax rate—all (%) | 1.9 | 1.3 | 1.9 |
| *eff* | Education efficiency measure | 9.64 | 10.18 | 10.11 |

The results of this experiment indicate that if the government has the political will to use any available signal on ability and target the more able poor students, it can increase educational efficiency with only a slight decrease in welfare. Measured inequality would however increase.

### 6.5. Ability in production

The production function (3) does not take into account ability in production. It seems unrealistic that a person who has a higher probability of completing college ex ante, does not also have a higher ex post probability of completing a task at work successfully. In order to study the robustness of the above results, as well as to gain independent insight, we consider the case with productive efficiency. We make the "brain" term depend on $\bar{a}$, the average ability of college educated workers. We therefore consider the production function:

$$Y = A[\theta g(\bar{a})(N_c + \gamma N_s)^v + (1-\theta)(N_s + \varepsilon N_c)^v]^{1/v},$$

where $g$ is an increasing function. In particular, we use $g(\bar{a}) = \kappa\bar{a}$, where $\kappa$ is a positive constant. For simplicity, the same $a$ matters not only for college completion but also for work ability. [34] There is also some theoretical precedence to this approach. Lucas (1988) uses a specification where the average level of human capital positively affects production. The production structure in Kremer (1993), where an assortative matching of workers with the same human capital occurs, is similar in flavor to the Lucas (1988) specification.

By making the effect purely external, we do not add an extra state variable in the *individual's* problem. However, there will be an extra *aggregate* state variable, $\bar{a}$. The functions $w_c, a_c$, etc. are now dependent on this second aggregate state variable, though

[34] The externality assumption is purely for convenience and not to rationalize subsidization. As will be evident from the results, subsidies in this case have the potential to reduce not only education efficiency, but also output and welfare.

Table 7
Ability in the production function ($s = 0.03$ is benchmark)

| Variable | Interpretation | $s = 0$ | $s = 0.02$ | $s = 0.03$ | $s = 0.04$ | $s = 0.05$ |
|---|---|---|---|---|---|---|
| $n_c^*$ | Fraction of college educated (%) | 23.7 | 30.9 | 33.7 | 35.7 | 36.4 |
| $\bar{a}$ | Average ability of skilled workers | 0.748 | 0.734 | 0.730 | 0.725 | 0.716 |
| $\frac{w_c^*}{w_s^*}$ | Skill premium | 2.05 | 1.66 | 1.54 | 1.46 | 1.42 |
| $Y^*$ | Output | 0.456 | 0.469 | 0.472 | 0.473 | 0.466 |
| $\tau$ | Education tax rate—all (%) | 0 | 1.1 | 1.9 | 3.2 | 5.3 |
| $\omega$ | Welfare (equiv. cons.) (%) | −2.4 | −0.3 | 0 | −0.4 | −2.1 |
| *eff* | Education efficiency measure | 9.97 | 10.02 | 9.97 | 9.64 | 8.77 |

the analysis of the individual's problem is mostly unchanged. The law of motion for $\bar{a}$, is given by

$$\bar{a}'(n_c) = \frac{n_c \int_{a_c^*(n_c,\bar{a})}^{1} a\pi_c(a)\,\mathrm{d}F(a) + (1-n_c)\int_{a_s^*(n_c,\bar{a})}^{1} a\pi_s(a)\,\mathrm{d}F(a)}{n_c \int_{a_c^*(n_c,\bar{a})}^{1} \pi_c(a)\,\mathrm{d}F(a) + (1-n_c)\int_{a_s^*(n_c,\bar{a})}^{1} \pi_s(a)\,\mathrm{d}F(a)}.$$

This formula shows the possibility that any policy that decreases $a_s^*$ more than it increases $a_c^*$, can decrease the average ability and thus output.[35]

When $\kappa$ is 1.3, and all other parameters are at their benchmark levels, we get outcomes that are not too different from the earlier benchmark outcomes. We therefore use that value of $\kappa$, and $s = 0.03$ as our benchmark for this specification. We can see in Table 7 that the average ability of college educated people in the workforce decreases with the level of subsidy. The enrollment rates decrease for the rich and increase for the poor and the dropout rates move in opposite directions (omitted for brevity). The educational efficiency initially increases with the subsidy level due to the decrease in dropout rates for the rich, but the increase in dropout for the poor eventually decreases education efficiency. There is a *decrease* in per capita output when subsidies are increased beyond the 80% level ($s = 0.04$). An increase in college educated people is not necessarily a process that increases per capita output if the new graduates have lower ability than before. At these high levels of subsidies, *both* wages fall. The skill premium continues to decrease with subsidies. Moving from the benchmark to any other subsidy level entails a loss of aggregate (transition-accounted) welfare.

In summary, the earlier conclusions that any increase in subsidies relative to the benchmark will decrease education efficiency with no appreciable increase in welfare is robust to the production specification with ability. In fact, as conjectured earlier, incorporating the quality of college workers in the production function decreases the welfare maximizing subsidy level; leaving the benchmark is always welfare decreasing.

[35] In a footnote, Autor et al. (1998) note: "The large increases in educational attainment of the U.S. workforce since 1940 may overstate increases in the relative supply of "more-skilled" workers to the extent that the "unobserved" quality of more-educated workers declines with the "relabeling" of "lower productivity" workers into higher education categories". The decrease in average ability in our model is closely related to the point they make.

## 7. Conclusions

In this paper, we have developed a dynamic framework to analyze the effects of higher education subsidies when there is heterogeneity in parental income and student ability. The model is calibrated to the US economy providing us with a sensible benchmark for comparing outcomes from different policies. We focus on three policies that are often proposed by politicians and policy makers. The government can design a tax and subsidy scheme that guarantees equality of opportunity, but only at the expense of a decrease in the efficiency of utilization of education resources. The welfare gain that comes from increasing the fraction of people with higher marginal productivity is minimal. A policy that aims to maximize the fraction of college educated labor by sending as many children as possible to college results in a big drop in the above-mentioned efficiency with little or no welfare gain. If the government has the political will to use any available signal on ability and provide merit-based aid, it can increase this efficiency with little decrease in welfare. These conclusions appear robust to the addition of ability in the production function.

This analysis indicates that any further increases in higher education subsidies to bridge the "enrollment gap" in the US may not be warranted. We do not explicitly model the moral hazard involved in the process of human capital accumulation that causes the incompleteness of markets in the first place. However, for the purpose of arguing that the case for higher education subsidies is overstated, this omission is not serious; the process of completion of the market by the government is only likely to increase the moral hazard and decrease welfare. An omission that might work in the opposite direction is endogenous fertility; poor families tend to have more children, which exacerbates credit constraints they might face.

Our results are broadly consistent with those of the Keane and Wolpin (2001) study mentioned in the introduction. They conclude that educated parents do make larger transfers to their children and this can account for some of the intergenerational correlation of school attainment that is seen in the data; this is also true in our setup. Though they find equalization of parental transfers lead to an equalization of the education distribution as we do, the effect is modest, since rich parents who make most of their transfers contingent on college attendance are the ones most (negatively) affected in their experiment. In contrast, parental transfers are only contingent in our framework and poor parents respond strongly to tax and subsidy schemes. They estimate borrowing constraints to be severe, but easing them leads to little change in enrollment as students respond along other margins such as increased consumption and decreased working time. These results can in fact be used to justify our approach of retaining binding borrowing constraints in the policy experiments we consider.

Since poor parents are situated in the more curved part of the utility function and are more responsive to flat subsidies, as well as for simplicity, we focused on a simple policy where everyone is taxed at the same rate and only poor children get subsidies. It would be useful to see how robust our conclusions are to alternate policy specifications. The case for subsidies is clearly overstated if true college completion ability is *not* i.i.d. This could be implemented by making the distribution function $F$ a Markov distribution. The assumption we have made for our numerical experiments, that children of college educated parents systematically outperform children of school educated parents ($\pi_c > \pi_s$), is only one step in this direction. A high ability person who fails to graduate due to bad luck will not be able to transmit any advantage in our setup, but will be able to in one where there is true persistence. We do not have physical capital in this framework. It will also be useful to study the robustness of our results when an alternate form of bequest is available to parents. Low ability children, especially the rich ones, can be left with capital instead of being sent to college. It will also be interesting to model state run universities, where subsidization is indirectly available to *all* students who are admitted instead of just the poor.


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## Appendix A

### A.1. Proof of Lemma 1

$$\mathrm{E}V_c(n_c) > \mathrm{E}V_s(n_c).$$

Suppose not; i.e. $\mathrm{E}V_c(n_c') \leqslant \mathrm{E}V_s(n_c')$, for any given $n_c'$ that parents take as given. Given $0 \leqslant \pi(a) \leqslant 1$, we have

$$\beta\pi(a)\mathrm{E}V_c(n_c') + \beta(1-\pi(a))\mathrm{E}V_s(n_c') \leqslant \beta\mathrm{E}V_s(n_c')$$

$\forall a$. Given $e > 0$, it follows that for any given state $n_c$:

$$u(w_s(n_c) - e) + \beta\pi(a)\mathrm{E}V_c(n_c') + \beta(1-\pi(a))\mathrm{E}V_s(n_c') < u(w_s(n_c)) + \beta\mathrm{E}V_s(n_c').$$

So, no school educated parent sends her child to college. Likewise one can show that no college educated parent sends her child to college. Therefore, $C, S = \phi$. Since a college educated parent does not send her child to college,

$$V_c(a; n_c) = u(w_c(n_c)) + \beta\mathrm{E}V_s(n_c') \quad \forall a.$$

Since ability $a$ does not appear on the right side of this expression, it follows that $\mathrm{E}V_c(n_c) = u(w_c(n_c)) + \beta\mathrm{E}V_s(n_c')$. Likewise, one can show that $\mathrm{E}V_s(n_c) = u(w_s(n_c)) + \beta\mathrm{E}V_s(n_c')$. Since Assumption 3 implies $w_c(n_c) > w_s(n_c)$, it follows that $\mathrm{E}V_c(n_c) > \mathrm{E}V_s(n_c)$. This is true for any $n_c \in [0,1]$. In particular, it is true for $n_c'$, which contradicts the hypothesis that $\mathrm{E}V_c(n_c') \leqslant \mathrm{E}V_s(n_c')$.

### A.2. Proof of Lemma 2

There exist unique $a_c^*(n_c), a_s^*(n_c) \in (0,1]$.

Consider any $n_c, n_c' \in [0,1]$. The school portion of the Bellman equation, $u(w_i(n_c)) + \beta \mathrm{E}V_s(n_c')$ is independent of $a$. Therefore, when plotted against $a$ it is a horizontal line. The college portion of the Bellman equation (1) is $u(w_i(n_c) - e) + \beta \mathrm{E}V_s(n_c') + \beta\pi(a)(\mathrm{E}V_c(n_c') - \mathrm{E}V_s(n_c'))$. When evaluated at 0, it is $u(w_i(n_c) - e) + \beta \mathrm{E}V_s(n_c')$ (assuming $\pi(0)=0$), which is less than the school portion. The college portion is strictly increasing in $a$, assuming $\pi'(a) > 0$ and given the above result that $\mathrm{E}V_c(n_c') > \mathrm{E}V_s(n_c')$. These two curves either intersect at a unique $a_i^* > 0$, or do not intersect at all, in which case $a_i^* = 1$.

These imply that there exist threshold ("reservation") abilities $a_i^* \in (0,1]$, $i = \mathrm{s,c}$, such that a parent of type $i$ finds it profitable to send her child to college only if the child's ability is $\geqslant a_i^*$.

### A.3. Proof of Lemma 3

$$a_c^*(n_c) < a_s^*(n_c).$$

Write the Euler-like equations (4) for the marginal children of both parent types, for a given combination of aggregate state $n_c$ and anticipated next-period state $n_c'$:

$$\beta\pi(a_s^*(n_c))\Lambda(n_c') \leqslant g_s(n_c),$$

$$\beta\pi(a_c^*(n_c))\Lambda(n_c') \leqslant g_c(n_c),$$

with equality if the relevant threshold ability is $< 1$. We have used the notation introduced in the main text, where the right-hand side is the utility cost of college education, and the left-hand side is the discounted expected benefit of sending the child to college, weighted by the probability of college completion.

The key to proving this lemma is the fact that $g_c(n_c) < g_s(n_c)$, which follows from the strict concavity of the utility function and $w_c(n_c) > w_s(n_c)$. That is college educated parents always have a lower utility cost of sending their children to college. We first rule out the possibility that $a_c^* = 1$, and $a_s^* < 1$. Suppose this were true, we have $\beta\pi(1)\Lambda(n_c') \leqslant g_c(n_c)$ and $\beta\pi(a_s^*(n_c))\Lambda(n_c') = g_s(n_c)$. These imply the following inequality:

$$\frac{\pi(1)}{\pi(a_s^*(n_c))} < \frac{g_c(n_c)}{g_s(n_c)}.$$

Given that $\pi(\cdot)$ is strictly increasing and the above fact on relative utility costs, this cannot be true—the left-hand side is $> 1$ while the right-hand side is $< 1$. Since we are considering only equilibria in which some college education takes place, we do not analyze the case $a_c^*(n_c) = a_s^*(n_c) = 1$. (At this stage we are analyzing the parents' problem and we have to explicitly rule out an equilibrium of this type. Once we consider the whole equilibrium, it will become obvious that such a situation cannot arise. Only $n_c' = 0$ is consistent with this situation, and the anticipated gain to college education will be the highest in this case, which will induce some college educated parents to send their children to college.)

Now consider the case where $a_c^*, a_s^*$ are interior. From the Euler-like equations, we get:

$$\frac{\pi(a_c^*(n_c))}{\pi(a_s^*(n_c))} = \frac{g_c(n_c)}{g_s(n_c)} < 1.$$

Given that $\pi(\cdot)$ is strictly increasing, it follows that $a_c^* < a_s^*$.

Since the maximum $a_s^*$ can be 1, it is immediately apparent that $a_c^* < 1$.

### A.4. Derivation of Eq. (9)

Simplify the equation system (7) by first integrating out terms that do not depend on $a$ to get:

$$\begin{aligned} \mathrm{E}V_c(n_c) = {} & F(a_c^*(n_c))[u(w_c(n_c)) + \beta \mathrm{E}V_s(n_c')] \\ & + (1 - F(a_c^*(n_c)))[u(w_c(n_c) - e) + \beta \mathrm{E}V_s(n_c')] \\ & + \beta[\mathrm{E}V_c(n_c') - \mathrm{E}V_s(n_c')] \int_{a_c^*(n_c)}^{1} \pi(a)\,\mathrm{d}F(a), \end{aligned}$$

$$\begin{aligned} \mathrm{E}V_s(n_c) = {} & F(a_s^*(n_c))[u(w_s(n_c)) + \beta \mathrm{E}V_s(n_c')] \\ & + (1 - F(a_s^*(n_c)))[u(w_s(n_c) - e) + \beta \mathrm{E}V_s(n_c')] \\ & + \beta[\mathrm{E}V_c(n_c') - \mathrm{E}V_s(n_c')] \int_{a_s^*(n_c)}^{1} \pi(a)\,\mathrm{d}F(a). \end{aligned}$$

Subtract one from the other to get:

$$\begin{aligned} & (\mathrm{E}V_c - \mathrm{E}V_s)(n_c) \\ & \quad = F(a_c^*(n_c))u(w_c(n_c)) + (1 - F(a_c^*(n_c)))u(w_c(n_c) - e) \\ & \qquad - [F(a_s^*(n_c))u(w_s(n_c)) + (1 - F(a_s^*(n_c)))u(w_s(n_c) - e)] \\ & \qquad + \beta[(\mathrm{E}V_c - \mathrm{E}V_s)(n_c')] \left[ \int_{a_c^*(n_c)}^{1} \pi(a)\,\mathrm{d}F(a) - \int_{a_s^*(n_c)}^{1} \pi(a)\,\mathrm{d}F(a) \right]. \end{aligned}$$

Using the notational definitions for $x$ and $\Lambda$ defined in the main text, and the transition function $\Phi$, we get Eq. (9).

### A.5. Theorem 1 (existence and uniqueness)

We first write Eqs. (8)–(12) for the specialization $\pi(a) = a$, $F(a) = a$. We define the ratio of utility costs, $J(n_c) \equiv g_s(n_c)/g_c(n_c)$. Given the concavity of the utility function and the fact $w_c'(n_c) < 0$, and $w_s'(n_c) > 0$, we get $J(\cdot)$ to be strictly decreasing and bounded below by 1. It also follows that $g_c(n_c) < g_s(n_c)$, $g_c'(n_c) > 0$, and $g_s'(n_c) > 0$. We will use these facts repeatedly in the proof. The relevant equations then are (note that we have dropped the "$*$" from the notation for threshold abilities for the sake of convenience):

$$\beta a_c(n_c)\Lambda(n_c') = g_c(n_c) \tag{15}$$

$$a_s(n_c) = J(n_c)a_c(n_c) \tag{16}$$

$$n_c' = \frac{1}{2} - \frac{1}{2}[n_c a_c(n_c)^2 + (1-n_c)a_s(n_c)^2] \tag{17}$$

$$\Lambda(n_c) = x(n_c) + \frac{\beta}{2}[a_s(n_c)^2 - a_c(n_c)^2]\Lambda(n_c') \tag{18}$$

$$\begin{aligned} x(n_c) \equiv\ & [a_c(n_c)u(w_c(n_c)) + (1-a_c(n_c))u(w_c(n_c)-e)] \\ & - [a_s(n_c)u(w_s(n_c)) + (1-a_s(n_c))u(w_s(n_c)-e)], \end{aligned} \tag{19}$$

where $n_c' \equiv \Phi(n_c)$.[36] We will derive a set of sufficient conditions that will guarantee existence and uniqueness as we proceed through the various steps of the proof, and also provide an example of a set of parameters that satisfy the sufficient conditions. The sufficient conditions will be stringent given our strategy of using the contraction mapping argument on a single function, $\Lambda$, in a system of four functional equations. However, any other strategy, for instance using the contraction argument on a pair of functions, would make the proof much more complicated. The assumptions made are on the utility-wage composite functions $g_s$, $g_c$, $J$, and hence and are likely to be satisfied for a wide variety of individual utility and production functions. These assumptions are, therefore, not too restrictive. The main thrust of these assumptions are to make the strength of the income effect mentioned in the main text, as captured by $J'/J$, strong enough. Equivalently, the assumptions set a bound on the elasticity of $a_c$, which is an indication of the strength of the substitution effect.

*Step* 1: Consider the mapping $T$ defined by

$$T\Lambda(n_c) = x(n_c) + \frac{\beta}{2}[a_s(n_c)^2 - a_c(n_c)^2]\Lambda(\Phi(n_c)), \tag{20}$$

in conjunction with (15), (16), (17), and (19). Let $C[0,1]$ be the space of bounded, continuous, non-increasing functions $\Lambda : [0,1] \to R$, with the sup norm. $T$ maps $C[0,1]$ into itself, $T : C[0,1] \to C[0,1]$.

The strategy will be to start with a weakly decreasing $\Lambda$, and find sufficient conditions on the utility and production function composite for each of the three components of the right-hand side of the mapping to be decreasing in $n_c$.

*Sub-step* (i): $a_c(n_c)$ is increasing. That is, if $n_{c1} < n_{c2}$, $a_c(n_{c1}) < a_c(n_{c2})$.

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[36] We have assumed interior $a^*$s in writing these expressions. As seen in the main text, $\pi(0) = 0$ and Lemma 3 guarantee $a_c^*$ is interior. While $a_s^* > 0$, it can be =1. It is assumed to be interior in the proof for simplicity; extending the proof to a corner solution for $a_s^*$ should be straightforward.

We can use (15) and (17) to write:

$$a_c(n_c) = \frac{g_c(n_c)}{\beta \Lambda\{\frac{1}{2} - \frac{1}{2}[n_c + (1-n_c)J(n_c)^2]a_c(n_c)^2\}}.$$

For a given $n_c$, we can plot the two sides of this equation as functions of $a_c$. The left side is the 45° line. The right side is a weakly decreasing function, given the hypothesis of a weakly decreasing $\Lambda$, that starts at a positive number. Therefore a unique $a_c^*(n_c)$ exists. Suppose $n_c$ increases, say from $n_{c1}$ to $n_{c2}$. The left side is unchanged. The quantity $[n_c + (1-n_c)J(n_c)^2]$ is a convex combination of 1 and $J(n_c)^2 > 1$; $J(n_c)^2$ is decreasing in $n_c$; moreover, a greater weight is placed on the smaller quantity, 1, as $n_c$ increases. Therefore, the entire term within the square brackets is decreasing in $n_c$. Using this fact, and the hypothesis on $\Lambda$, we can see that the right side shifts out. Even with a weakly decreasing $\Lambda$, we therefore get $a_c(n_{c1}) < a_c(n_{c2})$.

*Sub-step* (ii): Suppose $[n_c g_c^2(n_c) + (1-n_c)g_s^2(n_c)]$ is strictly decreasing in $n_c$. Then $\Phi(n_c)$ (that is, $n_c'$) is increasing. That is, if $n_{c1} < n_{c2}$, $n_{c1}' < n_{c2}'$.

Suppose not. That is, $n_{c2}' < n_{c1}'$. The hypothesis on $\Lambda$ implies $\Lambda(n_{c2}') \geqslant \Lambda(n_{c1}')$. It also follows from (17) that $[n_{c1} + (1-n_{c1})J_1^2]a_{c1}^2 < [n_{c2} + (1-n_{c2})J_2^2]a_{c2}^2$. We will use subscripts 1 and 2 to denote the values of a function evaluated at $n_{c1}$ and $n_{c2}$. Use (15) to then get:

$$\begin{aligned}[n_{c1} + (1-n_{c1})J_1^2]g_{c1}^2 &\leqslant \frac{\Lambda(n_{c1}')}{\Lambda(n_{c2}')}[n_{c2} + (1-n_{c2})J_2^2]\,g_{c2}^2 \\ &\leqslant [n_{c2} + (1-n_{c2})J_2^2]\,g_{c2}^2,\end{aligned}$$

which implies $[n_{c1}\,g_{c1}^2 + (1-n_{c1})g_{s1}^2] \leqslant [n_{c2}\,g_{c2}^2 + (1-n_{c2})\,g_{s2}^2]$, contradicting the assumption that this quantity is strictly decreasing. This quantity is a sum of the square of the utility costs weighted by the fraction of people that constitute each type—the first term is increasing and the second decreasing. The assumption guarantees that the decrease in the weighted square utility costs for the poor parents dominates the increase for the rich, a condition satisfied for any reasonable specification of the production and utility functions.

*Sub-step* (iii): Suppose $[n_c\,g_c^2(n_c) + (1-n_c)\,g_s^2(n_c)]/(g_s^2(n_c) - g_c^2(n_c))$ is strictly increasing in $n_c$. Then $(a_s^2(n_c) - a_c^2(n_c))$ is strictly decreasing.

Use (16) to write $a_s^2 - a_c^2 = (J^2-1)a_c^2$. Therefore, we need to show for $n_{c1} < n_{c2}$:

$$\frac{a_{c1}^2}{a_{c2}^2} > \frac{J_2^2 - 1}{J_1^2 - 1}. \tag{21}$$

We know from the previous sub-step that

$$\frac{a_{c1}^2}{a_{c2}^2} > \frac{[n_{c2} + (1-n_{c2})J_2^2]}{[n_{c1} + (1-n_{c1})J_1^2]}.$$

A sufficient condition for (21) to be true is

$$\frac{[n_{c2} + (1-n_{c2})J_2^2]}{[n_{c1} + (1-n_{c1})J_1^2]} > \frac{J_2^2 - 1}{J_1^2 - 1},$$

that is, when $[n_c+(1-n_c)J^2]/(J^2-1)$ is strictly increasing, which is what is assumed. The previous sub-step assumed that the numerator is decreasing. Since $g_c$ is increasing, and $g_s$ is decreasing, the above assumption holds when the difference in the square of utility costs is decreasing faster. It is easy to verify that this condition is satisfied when $-J'/J > \frac{1}{2}((J^2-1)/J)^2$, and as mentioned earlier is an assumption on the strength of the income effect. The result that $(a_s^2 - a_c^2)$ is decreasing is weaker than claiming that $a_s$ is decreasing, which we find in all our computations. (We know from sub-step (i) that $a_c$ is increasing). However, it does capture the a flavor of the intuitive result that the difference in educational investment between the rich and the poor decreases with $n_c$.

*Sub-step* (iv): Suppose $u(w_c) - u(w_s) - J(g_s - g_c)$ is strictly decreasing in $n_c$. Then $x(n_c)$ is strictly decreasing.

Use (16) and the definition of $x$ in (10) to see that $x(n_{c2}) < x(n_{c1})$, for $n_{c2} > n_{c1}$ if

$$\begin{aligned}&[u(w_{c1}-e) - u(w_{s1}-e)] - [u(w_{c2}-e) - u(w_{s2}-e)]\\ &\quad > a_{c1}\left(\frac{g_{s1}^2 - g_{c1}^2}{g_{c1}}\right) - a_{c2}\left(\frac{g_{s2}^2 - g_{c2}^2}{g_{c2}}\right).\end{aligned}$$

Since, from sub-step (i), $a_{c2} > a_{c1}$, a sufficient condition can be obtained by seeing that the right-hand side is bounded by

$$a_{c1}\left(\frac{g_{s1}^2 - g_{c1}^2}{g_{c1}} - \frac{g_{s2}^2 - g_{c2}^2}{g_{c2}}\right),$$

which in turn is bounded by

$$\left(\frac{g_{s1}^2 - g_{c1}^2}{g_{c1}} - \frac{g_{s2}^2 - g_{c2}^2}{g_{c2}}\right).$$

Therefore, we need

$$\begin{aligned}[u(w_{c1}-e) - u(w_{s1}-e)] - \left(\frac{g_{s1}^2 - g_{c1}^2}{g_{c1}}\right) > {} & [u(w_{c2}-e) - u(w_{s2}-e)]\\ & - \left(\frac{g_{s2}^2 - g_{c2}^2}{g_{c2}}\right).\end{aligned}$$

A few algebraic steps will show that this is exactly the sufficient condition assumed. The assumption guarantees that the unweighted utility difference between the rich and the poor decrease fast enough with $n_c$, so that there are no perverse effects of a wage decrease for the rich that would cause them to send far fewer children to college and actually get a greater weighted utility $x$. In other words, this is an assumption on the upper bound on the increase in $a_c$.

*Sub-step* (v): If $n_{c1} < n_{c2}$, and $\Lambda$ is weakly decreasing, $T\Lambda(n_{c2}) \leqslant T\Lambda(n_{c1})$. That is, $T$ maps $C$ into itself.

Consider the mapping $T$ defined in (20). Sub-step (ii) provided conditions under which $\Phi(n_c)$ increases. Given the weakly decreasing $\Lambda$, we can see that $\Lambda(\Phi(n_c))$ is weakly decreasing. Sub-step (iii) provided conditions under which $(a_s^2 - a_c^2)$ decreases, and sub-step (iv) provided conditions under which $x(n_c)$ decreases. Therefore every component in $T$ decreases, two of them strictly. Therefore, $T$ maps $C$ into itself. (In fact, $T$ maps $C$ into the space of strictly decreasing continuous functions.)

An example of a set of parameters that satisfies the three sufficient conditions listed above is: $A = 1$, $\theta = 0.525$, $v = 0.5$, $\varepsilon = 0.25$, $\gamma = 0.02$, $e = 0.05$, and $\sigma = 1$ (log utility).

*Step* 2: $T$ satisfies the monotonicity property. That is, if $\Lambda_2(n_c) \geqslant \Lambda_1(n_c)$ $\forall n_c \in [0,1]$, $T\Lambda_2(n_c) \geqslant T\Lambda_1(n_c)$.

Fix $n_c$. Use (15) and (17) to write

$$a_c(n_c) = \frac{g_c(n_c)}{\beta\Lambda\{\frac{1}{2} - \frac{1}{2}[n_c + (1-n_c)J(n_c)^2]a_c(n_c)^2\}}.$$

The left-hand side is the 45° line when plotted against $a_c$ and given the non-increasing property of $\Lambda$, the right-hand side is a decreasing function of $a_c$ that starts at a value $>$ zero. There is a unique intersection that yields $a_c(n_c)$. When the entire function $\Lambda$ increases, the right-hand curve shifts inward, causing $a_c(n_c)$ to decrease. Therefore, $a_{c,2}(n_c) < a_{c,1}(n_c)$, where the additional subscript now refers to the relevant function $\Lambda$ (as opposed to step 1, where the subscripts referred to different $n_c$'s for the same function). Use (15) and (18) to write

$$(T\Lambda_2 - T\Lambda_1)(n_c) = x_2(n_c) - x_1(n_c) + \frac{(J(n_c)^2 - 1)}{2} g_c(n_c)(a_{c,2}(n_c) - a_{c,1}(n_c)).$$

Using the definition of $x$ in (19), $F(a) = a$, and (16) we can write

$$x_2(n_c) - x_1(n_c) = (a_{c,2}(n_c) - a_{c,1}(n_c))(g_c(n_c) - J(n_c)g_s(n_c)).$$

Using this in the above equation, we get

$$(T\Lambda_2 - T\Lambda_1)(n_c) = (a_{c,2}(n_c) - a_{c,1}(n_c))\left(\frac{(J(n_c)^2 + 1)}{2} g_c(n_c) - J(n_c)g_s(n_c)\right).$$

We showed that the first factor on the right side is negative. It is easy to show that the second factor is negative if $g_c(n_c)^2 < g_s(n_c)^2$, which is true for any $n_c$ under the assumptions already made in the main text on the utility and production functions. Therefore, $(T\Lambda_2 - T\Lambda_1)(n_c) > 0$. That is, the mapping $T$ satisfies the monotonicity property.

*Step* 3: $T$ satisfies the discounting property. If we consider $(\Lambda + \delta)(n_c) \equiv \Lambda(n_c) + \delta$ $\forall n_c$, it follows that $T(\Lambda + \delta)(n_c) \leqslant T\Lambda(n_c) + \rho\delta$, for some $0 < \rho < 1$.

Again, fix $n_c$. Let the $\Lambda_2$ considered in step 2 now refer to $\Lambda + \delta$, and let $\Lambda_1$ refer to $\Lambda$. We saw in step 2 that $a_{c,2}(n_c) < a_{c,1}(n_c)$, but in this step we need to set a bound on this change. From this fact and (17), we can deduce that $n'_{c,2} > n'_{c,1}$, which given the space $C[0,1]$ assumed, implies

$$\Lambda(n'_{c,2}) \leqslant \Lambda(n'_{c,1}) \Rightarrow -\frac{1}{\Lambda(n'_{c,2})} \leqslant -\frac{1}{\Lambda(n'_{c,1})}.$$

Use this fact, and (15) for the two functions to get

$$\begin{aligned} a_{c,1}(n_c) - a_{c,2}(n_c) &= \frac{g_c(n_c)}{\beta}\left\{\frac{1}{\Lambda(n'_{c,1})} - \frac{1}{\Lambda(n'_{c,2})+\delta}\right\} \\ &\leqslant \frac{g_c(n_c)}{\beta}\left\{\frac{1}{\Lambda(n'_{c,1})} - \frac{1}{\Lambda(n'_{c,1})+\delta}\right\} \\ &= \frac{g_c(n_c)}{\beta}\frac{\delta}{\Lambda(n'_{c,1})[\Lambda(n'_{c,1})+\delta]} \\ &= \frac{\delta\beta a_{c,1}(n_c)^2}{g_c(n_c)+\delta\beta a_{c,1}(n_c)}. \end{aligned}$$

Using the definition of $J(n_c)$, we can get

$$J(n_c)g_s(n_c) - \frac{(J(n_c)^2+1)}{2}g_c(n_c) = \frac{g_s(n_c)^2 - g_c(n_c)^2}{2g_c(n_c)}.$$

Using the expression for $T\Lambda_2 - T\Lambda_1$ from the previous step, and the above two expressions, we can write

$$T(\Lambda+\delta)(n_c) - T\Lambda(n_c) \leqslant \delta\left(\frac{\beta a_{c,1}(n_c)^2}{g_c(n_c)+\delta\beta a_{c,1}(n_c)}\right)\left(\frac{g_s(n_c)^2 - g_c(n_c)^2}{2g_c(n_c)}\right).$$

We would like the product of the last two terms to be $<1$. A sufficient condition is

$$\left(\frac{\beta.1}{g_c(n_c)+\delta\beta.0}\right)\left(\frac{g_s(n_c)^2 - g_c(n_c)^2}{2g_c(n_c)}\right) < 1,$$

which is satisfied when $J(n_c)^2 < 1+(2/\beta)$, which we assume holds. That is, by making $\beta$, the intergenerational discount factor, small enough, we can ensure that discounting holds.

*Step* 4: $T$ thus satisfies Blackwell's sufficient conditions for a contraction. The contraction mapping theorem (see Stokey et al., 1989) guarantees the existence of a unique function $\Lambda(n_c) \in C[0,1]$ that satisfies the system of functional equations.

*Step* 5: From step 1 it is clear that $T$ actually maps $C[0,1]$ into the space of strictly decreasing functions, $C'[0,1]$. That is $T(C) \subseteq C' \subseteq C$. Using Corollary 1, (p. 52), in Stokey et al. (1989), we get that the fixed point of $T \in C'$. In other words, the equilibrium $\Lambda(n_c)$ is strictly decreasing. From (15) through (18), and the monotonicity of $g_s(n_c)$ and $g_c(n_c)$, we get that the other functions that are part of the equilibrium, $a_c(n_c)$, $a_s(n_c)$, and $\Phi(n_c)$ exist and are uniquely determined once $\Lambda(n_c)$ is uniquely determined.

### A.6. Proposition 1 (linear utility)

The fact that a unique steady state is reached in a single step with linear utility can be proved for any $\pi$ and $F$. With linear utility, the utility cost of college education is the same for both types, $g_s(n_c)=g_c(n_c)=e$, $\forall n_c \in [0,1]$. It can then be seen from Eqs. (11) and (12) that $a_s^*(n_c) = a_c^*(n_c) \equiv a^*(n_c)$. From (10), $x(n_c) = w_c(n_c) - w_s(n_c)$. It follows that $\Lambda'(n_c) < 0$. The law of motion (8) reduces to

$$\Phi(n_c) = \int_{a^*(n_c)}^{1} \pi(a)\,\mathrm{d}F(a).$$

We can rewrite (12) for the linear utility case which can be written as

$$\beta\pi(a^*(n_c))[w_c(\Phi(n_c)) - w_s(\Phi(n_c))] = e.$$

Suppose $a^*$ increases with $n_c$, which implies that $\Phi$ decreases with $n_c$. These imply that the left side is increasing in $n_c$, while the right side is constant. Thus, it is not possible to have $a^*$ increasing in $n_c$. Similarly, if $a^*$ decreases with $n_c$, $\Phi$ increases with $n_c$, and the left side is decreasing in $n_c$, while the right side is constant. Thus, the possibility of $a^*$ decreasing in $n_c$ is also ruled out. The only possibility is that $a^*$ is independent of $n_c$—that is, $a^*(n_c) \equiv a^*$. The unique $a^*$, which is common to both types of parents, is given by

$$\beta\pi(a^*)\left[w_c\left(\int_{a^*}^{1} \pi(a)\,\mathrm{d}F(a)\right) - w_s\left(\int_{a^*}^{1} \pi(a)\,\mathrm{d}F(a)\right)\right] = e,$$

with $n_c^*$ then given by $\int_{a^*}^{1} \pi(a)\,\mathrm{d}F(a)$. Thus, when utility is linear, the economy ends up at this unique steady state in one step.

### A.7. Computation

The computational strategy that we use involves approximating the two functions $\Phi$ and $\Lambda$, defined in Eqs. (8) and (9). The function $\Phi$ depends on $a_i^*(n_c;\Phi)$, which in turn depends on the function $\Lambda$ (refer to Eqs. (11) and (12)). Consequently, we need to first approximate $\Lambda$. We approach this by nesting the estimation of $\Lambda$ within the estimation of $\Phi$. The algorithm that we use is a Chebyshev interpolation algorithm. (See Judd, 1998 for details on constructing the Chebyshev nodes and coefficients.)

1. We start by selecting $N$ Chebyshev interpolation nodes, $n_c^k$, $k = 1,\dots,N$. These nodes will be used in approximating both $\Phi$ and $\Lambda$. We can do this because both are functions of the same variable.
2. Next, we choose an initial guess for $\Phi(n_c^k)$ $\forall k$.

3. We can then compute the Chebyshev coefficients, $bp_i$. We can then construct an approximation of $\Phi$ at any point $n_c$: $\Phi(n_c) = \sum_1^N bp_i T_i(n_c)$.
   (i) This is the inner loop devoted to approximating $\Lambda$. We start here by choosing an initial guess for $\Lambda(n_c^k)\ \forall k$.
   (ii) We construct the Chebyshev coefficients $bl_i$ in the same manner as we constructed the $bp_i$.
   (iii) We then approximate $\Lambda$ at $\Phi(n_c^k)\ \forall k$: $\Lambda(\Phi(n_c^k)) = \sum_1^N bl_i T_i(\Phi(n_c^k))$.
   (iv) Given these approximations we can construct $a_c^{k*}\ \forall k$ and $a_s^{k*}\ \forall k$, following Eqs. (11) and (12). Then we can update $\Lambda(n_c^k)\ \forall k$ following Eq. (9).
   (v) Lastly we recalculate the $bl_i$ and check for convergence. If the $bl_i$ have converged, we continue on to step 4. If the $bl_i$ have not converged, we return to step iii, with the updated $bl_i$.
4. Given the approximation of $\Lambda$, we can update $\Phi$ at all $k$ nodes, following Eq. (8).
5. Lastly, we recalculate the $bp_i$ and check for convergence. If the $bp_i$ have converged, we are finished. If the $bp_i$ have not converged, we return to step $i$, with the updated $\Phi$, and reapproximate $\Lambda$.
6. Newton's algorithm can then be used on $\Phi(n_c^*) = n_c^*$ to compute the steady state.

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