Growth and Welfare Analysis of Tax Progressivity in a Heterogeneous-Agent Model*

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Abstract

In this paper, we use a general equilibrium model of endogenous growth in which there is heterogeneity in skill, income, and tax rates to evaluate the effect of progressivity of taxes on growth and welfare. In this framework, changes in the progressivity of tax rates can have positive growth effects even in situations where changes in flat rate taxes have no effect. Experiments on a calibrated model indicate that the quantitative effects of moving to a flat rate system are economically significant. The assumption made about the "engine" of growth – an external effect arising from production activities of skilled workers or intentional employment of skilled workers for research and other productivity enhancing activities – has an important effect on the impact of a change in progressivity. Welfare is unambiguously higher in a flat rate system when comparisons are made across balanced growth equilibria; however, when the costs of transition to the higher growth equilibrium are taken into account, only the currently skilled slightly prefer the flat rate system.

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1 Introduction

A justification often given by politicians and policy makers in the U.S. for lowering tax rates is that it would boost economic growth. The U.S. tax system is anything but flat. The offered tax schedule, which governs economic incentives, has always been progressive. For instance, the *Statistical Abstracts of the United States* report that the lowest and highest marginal tax rates for married couples with two dependents when tax brackets are expressed in 1980 dollars, were 0% and 43% in 1960, -10% and 50% in 1975, and 12.5% and 42% in 1984. Pechman (1985) studies the actual incidence of taxes, and concludes that once transfers are taken into account the net tax burden as a percent of adjusted family income varies from -65.5% for the first population decile to 26% for the tenth decile.

Given the prevalence of progressive taxes, considering tax reform in a heterogeneous agent context with progressive taxes seems empirically relevant. Yet the evaluations to date have been of changes in flat rate taxes using a representative agent framework. One of the early formal attempts at studying the growth effects of taxes is Lucas (1990). He uses a representative agent endogenous growth model in which human capital is the engine of growth. He concludes that tax changes do alter long-run growth rates, but that the effect is "quantitatively trivial." This happens because "changes in labor taxation affect equally both the cost and the benefit side of the marginal condition governing the learning decision." In an effort to generalize Lucas' study and to scrutinize the burgeoning tax and growth literature through the lens of a common framework, Stokey and Rebelo (1995) use a general model of endogenous growth to identify model features and parameters that affect the growth rate in a quantitatively significant way when tax rates are changed. They isolate these parameters (factor shares, depreciation rates, elasticity of intertemporal substitution, and elasticity of labor supply), but ultimately conclude that for empirically relevant values of these parameters, Lucas' conclusion of little or no tax effect on the U.S. growth rate is robust.[1]

In this paper, we extend the analysis of Lucas in a relatively unexplored direction and study the growth and welfare effects of changes in the progressivity of taxes in a simple heterogenous-agent, incomplete markets economy.[2] With progressive taxes in a heterogenous agent economy, the result

[1] The studies they consider in detail are Lucas (1990), King and Rebelo (1990), Kim (1992), and Jones, Manuelli, and Rossi (1993). Mendoza, Milesi-Ferretti, and Asea (1997) use cross-country panel regressions and numerical simulations to come to similar conclusions on negligible growth effects of taxes.

[2] Cassou and Lansing (2000) extend Lucas' analysis to incorporate a tax rule that approximates the U.S. tax code by specifying the tax rate as an increasing function of a representative agent's income.

We take the stance that to understand progressivity, with its intrinsic connotation of heterogeneity, one need's to model inequality explicitly. If close to perfect insurance were available – if not within the economy as a whole, at least within a family whose members might be situated in different tax brackets – a representative agent model would be sufficient; however, available evidence does not seem to support the perfect insurance hypothesis. A heterogeneous-agent model also allows the natural modeling of liquidity constraints, which are considered to be widely prevalent in the cited above might not hold; if an agent is taxed at a lower rate while the agent is unskilled and at a higher rate when the agent becomes skilled, the return to becoming skilled, skill accumulation, and growth will be negatively affected. As Heckman and Klenow (1997) note: "For some individuals, the gain in earnings resulting from human capital investment causes them to move up tax brackets. In this case, the returns from investment are taxed at a higher rate, but the cost is expensed at a lower rate. This discourages human capital accumulation."

On the other hand, if the human capital accumulation is done in the presence of liquidity constraints, as several economists believe it is, more progressive taxes will decrease taxes for the poor at the accumulation stage and increase their investment; the effect will be opposite for the rich. The overall effect of progressivity on skill accumulation and growth is therefore an open theoretical and quantitative issue. The main objective of this paper is to quantitatively evaluate the welfare and growth effects of changes in progressivity in the tax schedule.[3]

A natural starting point to study the effect of tax progressivity in a heterogenous agent economy would be a Bewley-type incomplete markets model such as the ones in İmrohoroğlu (1989) and Aiyagari (1994). This would allow for a quantitative examination of the role of market incompleteness in determining growth effects, much like the extension of Lucas's welfare costs of business cycles result to incomplete markets models by İmrohoroğlu (1989). However, to the best of our knowledge, the extension of such an incomplete markets model to endogenize growth has not been developed yet. Therefore, we proceed with a more tractable incomplete markets model which we view as a first step toward a richer heterogenous agent model.[4]

The economic setup is a two-period overlapping generations model with two types of adult workers – skilled and unskilled – who spend resources in educating their children. The probability of the child becoming skilled in the subsequent period depends positively on these expenses. The measure of skilled workers in the economy and the value to being skilled evolve endogenously based on these human capital accumulation decisions. We consider two possibilities for the production sector. In the first, growth arises as a purely external effect on account of production activities of skilled workers. In the second, a portion of the skilled workforce is used to work in research and other productivity

process of human capital accumulation. Hayashi, Altonji, and Kotlikoff (1996), for instance, use PSID data to reject intrafamily (and interfamily) full risk sharing. An earlier micro level study is Zeldes (1989) who uses the Panel Study of Income Dynamics data and studies the behavior of consumption in the presence of liquidity constraints. His findings indicate that these constraints do exist and greatly influence consumption behavior. See Hayashi (1987) for a survey of the empirical tests on liquidity constraints.

[3] In a companion paper (Caucutt, İmrohoroğlu and Kumar (2002)), we construct and analyze the general equilibrium model of endogenous growth with heterogeneity in income and in the tax rates that we use here.

[4] Ventura (1999) and İmrohoroğlu (1998) examine various tax reforms in heterogenous agent, incomplete markets models but they abstract from endogenous growth.

enhancing activities and is compensated for it.[5] There is an infinitely-lived entrepreneur who is the only agent with access to the physical capital markets.

One of the main implications of the model is that a change in the flat rate tax has no effect on growth, while a decrease in the progressivity of taxes has a positive effect. The framework is therefore ideal to isolate the growth effect of a change in progressivity; the *entire* effect can be attributed to the change in tax structure. The model is calibrated to the U.S. economy by considering college-educated workers as skilled. Our main finding is that the quantitative effects of eliminating progressivity are economically significant – in some experiments as high as 0.52 percentage points. Inequality decreases as the reform causes greater mobility for the poor in the long run. The welfare of both agent types is unambiguously higher in a balanced growth path with flat rate taxes. However, when the transition to the higher growth balanced growth path is taken into account only the currently rich slightly prefer a flat rate tax. We also find that the assumption made about the engine of growth makes a quantitative difference on the impact of a change in progressivity. When growth is driven by externality, the effect on growth is stronger than when it is driven by intentional technology adoption.

The rest of the paper is structured in the following way. In Section 2 we provide preliminary calculations for the growth effects of progressivity using the main marginal condition from Lucas (1990) – a standard complete markets model. We argue the effects are significant enough to warrant a more thorough investigation in a fully specified heterogeneous agent model where markets are incomplete. In Section 3 we describe the economic environment and summarize the analytical results from Caucutt, İmrohoroğlu and Kumar (2002). Section 4 is devoted to issues on calibrating the model to the U.S. economy. Section 5 considers various experiments in tax reform and Section 6 presents sensitivity analysis. Section 7 concludes.

2 Preliminary Calculations with Complete Markets

We start by examining the following marginal condition from the Lucas (1990) paper (equation (2.10)), which governs time spent by agents in skill accumulation:

$$w(t)\,h(t) = G'[v(t)] \int_t^{\infty} \exp\left\{-\int_t^{s} (r(\varsigma) - \lambda)\; d\varsigma\right\} u(s)\,w(s)\,h(s)\; ds. \tag{1}$$

Here, w is the rental rate of human capital, h the stock of human capital, G is a human capital production function that governs the evolution of human capital according to $\dot{h}(t) = h(t)\,G[v(t)]$, $v(t)$ is the time spent in accumulating human capital, $u(t)$ is the time spent working, r is the interest rate, and λ is the effective depreciation rate that includes population growth. The left hand side

[5] By considering the main sources of growth that have been extensively discussed in the new growth literature, we are able to conduct a sensitivity analysis of the growth effects to the assumption made about its source.

is the marginal cost of allocating an extra unit of time to human capital accumulation – the wage rate – and the right hand side is the marginal benefit – the marginal product weighted present value of future wages earned on account of this accumulation. If τ is the uniform labor income tax rate, it affects the cost and benefit by the same factor and cancels out of both sides. If human capital accumulation is modeled more realistically, as the process of acquiring skill when one is in a lower tax bracket and then moving to a higher bracket as happens in a progressive tax system, a tax change will not be neutral with respect to growth. By increasing the wedge between present and future tax rates, progressivity will reduce the return to human capital accumulation and decrease growth (i.e. decrease v and hence the growth rate $G[v]$).

We manipulate the above equation in Lucas (1990) in the following way. On a balanced growth path (BGP), h grows at a constant rate g, and w, r, u, and v are all constant. Note that $g = G[v]$. We use Lucas' functional form $G[v] = Dv^{\gamma}$. We also abstract from leisure, a conservative assumption in estimating growth effect of taxes, and assume $u + v = 1$. Suppose, at the time of accumulation, a lower tax rate of τ_s applies and future income is taxed at the higher rate of τ_c. The above condition can be shown to reduce on the BGP to:

$$(1-\tau_s) = G'(v)\ u \left[\frac{1}{(r-\lambda)-g}\right](1-\tau_c).$$

Using the usual Euler condition that characterizes growth, $g = \frac{r-(\rho+\lambda)}{\sigma}$, where ρ is the rate of time preference and σ is the coefficient of relative risk aversion, and using the functional form for G, this condition can be further reduced to the equation:

$$\{\Theta(\sigma-1)+\gamma\}\, Dv^{\gamma} + \Theta\rho = \frac{\gamma D}{v^{1-\gamma}},$$

where $\Theta \equiv \frac{(1-\tau_s)}{(1-\tau_c)}$ is a measure of progressivity of taxes.[6] Progressivity increases when τ_s decreases or τ_c increases. It can be shown that when the progressivity Θ increases, v and hence the growth rate decreases.[7] *It is important to note that this growth effect is purely due to the progressivity of taxes;* when $\Theta = 1$, there is no effect of tax changes on growth, as in Lucas (1990). Our heterogeneous-agent model has this convenient property, which allows all calculated growth effects to be directly attributable to a change in progressivity.

[6]If one were to capture progressivity by specifying the tax rate as a function of the ratio of individual wages to average economywide wages, that is as $\tau \equiv \tau\left(\frac{wh}{WH}\right)$, and derive the analogue of (1) formally, one would obtain $\Theta = \frac{1-\tau\left(\frac{wh}{WH}\right)|_{initial}}{1-\tau\left(\frac{wh}{WH}\right)|_{final}-\left(\frac{wh}{WH}\right)|_{final}\tau'\left(\frac{wh}{WH}\right)|_{final}}$, where "initial" and "final" refer to an agent's situation before and after the learning decision. The Θ specified in the text is therefore a conservative approximation.

[7]For a given Θ, the left hand side is a strictly increasing function of v and the right hand side is a strictly decreasing function of v going from ∞ to zero as v goes from zero to one. A unique v and thus a unique growth rate exist. When the progressivity Θ increases, the left side shifts upward and becomes steeper, both of which serve to decrease v.

We adopt a benchmark of $\sigma = 2$, $\gamma = 0.8$, $\rho = 0.015$, and $D = 0.0595$ (the first three are consistent with the values in Lucas (1990), while D is chosen to pin down an annual per capita growth rate of about 1.8% when the progressivity parameter is 1.5). We consider $\Theta = 1$, 1.5, and 2.[8] We also vary, one at a time from the benchmark, the parameters σ, γ, and ρ. The growth rates for the various parameter combinations are given in Table 1.

Table 1

Tax progressivity and growth rates – Complete markets

Θ	**Benchmark**	**Change σ to 1.5**	**Change γ to 0.65**	**Change ρ to 0.01**
1.0	2.46%	3.14%	2.69%	2.67%
1.5	**1.81%**	2.40%	2.10%	2.05%
2.0	1.38%	1.87%	1.72%	1.63%

There are significant growth effects of changes in progressivity – a movement from a progressivity of 1.5 to flat rate taxes increases the annual growth rate by 0.65 percentage points; a movement from a progressivity of 2.0 to flat rate taxes raises growth by 1.08 percentage points. These are large changes as far as growth rates go, especially when one realizes this is over and above any gains that could be realized by a decrease in capital income taxes or by considering an elastic labor supply.[9]

These calculations show that reducing the progressivity of taxes can have large growth effects even in a representative-agent, complete-markets context. However, it seems more desirable to pose the same question in an economy with heterogenous agents who respond to the tax reform by adjusting on an extensive margin by changing their types. Furthermore, additional quantitative implications can be obtained by examining the impact on inequality and welfare of each type of agent. The current tax and growth literature rarely provides estimates of welfare gains, especially when transitions are taken into account. As we will see, welfare calculations for different agent types bring to light several

[8] A progressivity factor of 2 is not unreasonable in light of the tax rates for 1975 given earlier. More dramatic evidence of progressivity can be found for earlier decades from the *Statistical Abstract of the United States* – the marginal tax rate for single people was 22% at the lowest bracket and 78% at the highest bracket during 1954-63.

[9] Cassou and Lansing's (2000) progressivity specification is essentially $\tau \equiv \overline{\tau}\left(\frac{wh}{WH}\right)^n$. One can then derive $\Theta = \frac{1-\overline{\tau}}{1-(n+1)\overline{\tau}}$. They set $\overline{\tau} = 0.2528$ and $n = 0.2144$, which implies a Θ close to 1.08. When this value of Θ is used with the same benchmark values as in the main text the growth effect of moving to flat taxes is 0.12%. When we use their value of $\sigma = 1$, $\frac{1}{1+\rho} = 0.979$, and recalibrate D to get a growth rate of 1.8% when $\Theta = 1.08$, moving to flat taxes increases growth by 0.13%. In either case, the growth effect we get from this simple analysis concurs well with the results they find in their paper with a more detailed calibration.

However, their calibration of a smooth tax function in a representative agent framework misses the larger, discrete jumps agents are likely to take upon completing education, as well as the *measure* of agents who are likely to take such jumps; i.e. adjustment along the 'extensive margin' of skill categories.

issues that balanced growth rate comparisons alone do not. In the next section we outline the model we use.

3 The Model

If one wants to take the analysis of tax progressivity and growth beyond the realm of representative agent models, the ideal strategy to follow would be to adapt a Bewley-type model, currently the workhorse of the literature on heterogeneous-agent models and inequality, to endogenous growth. In such a framework, agents would be buffeted by uninsured idiosyncratic shocks, would accumulate physical and human capital, and an aggregate balanced growth path would result when individual accumulation decisions are integrated. Unfortunately, this is quite a complicated framework to develop, and to the best of our knowledge it has not yet been done. However, it is still useful to take a step in that direction and explore the effects of progressivity in a tractable growth model with limited heterogeneity and accumulation possibilities. This is the route we have taken. While our study is only a preliminary step, it sheds useful quantitative insights into the intuition discussed in the introduction, that tax progressivity could have a negative effect on human capital accumulation and growth. We first present our model and dedicate a subsequent subsection to summarize its features and argue that it is a relevant one for assessing the effect of progressivity on growth.

The setup is taken from Caucutt, İmrohoroğlu and Kumar (2002) who extend Caucutt and Kumar (forthcoming) to develop a model of growth where the heterogeneity is limited to two types of skill levels. Households supply labor of differing skills and use the wage for consumption and investment in human capital. This is the only component of the economy that features heterogeneity. Producers invest in physical capital and in productivity improvements.

3.1 Households

The economy is populated by two types of adult agents – "skilled" (subscripted c, as in the calibration these agents are identified with college-educated workers) and "unskilled" (subscripted s, for school-educated workers) – with total measure one. We use "skilled" ("unskilled"), "rich" ("poor"), and "college-educated" ("school-educated") interchangeably to refer to the two types of agents. There is no population growth. Let n_c denote the fraction of skilled agents in the economy – this is the only aggregate state variable. Each adult has a child and can hire a skilled teacher for a fraction of the teacher's time, e, to educate the child. With this input, the probability that the child of a type-i agent, $i = c, s$, becomes skilled is given by $\pi_i(e_i)$; with probability $(1 - \pi_i(e_i))$ the child fails and is an unskilled adult in the following period. The π_i's are strictly increasing, lie in the unit interval, and are concave with zero input resulting in zero probability of success. Children of skilled agents might have inherent advantages through better schooling at the earlier levels, better role models, etc. Therefore we specify $\pi_c(e) \geqslant \pi_s(e)$, $\forall\ e \ \epsilon\ (0,1)$.

The Bellman equation for a skilled agent, who takes wages as given, is:

$$V_c(n_c) = \max_{e_c} \left\{ u\left((1-\tau_c)(1-e_c) w_c(n_c)\right) + \beta \pi_c(e_c) V_c\left(n_c^{'}\right) + \beta\left(1-\pi_c(e_c)\right) V_s\left(n_c^{'}\right) \right\}. \quad (2)$$

The tax rate on labor income of the skilled agents is τ_c, and that on the unskilled agent is τ_s. All agents posit a law of motion for the state variable, $n_c^{'} = \Phi(n_c)$. The Bellman equation for the unskilled agent is similar, except the current return term is given by $u\left((1-\tau_s)(1-e_s p(n_c)) w_s(n_c)\right)$. The unskilled agent also needs to hire a skilled person as a teacher, so the cost is $e_s w_c = e_s p w_s$, where $p \equiv \frac{w_c}{w_s}$ is the skill premium.

The first order conditions for skill accumulation for the two types of agent are:

$$\begin{aligned} \beta \pi_c^{'}(e_c) \Lambda\left(n_c^{'}\right) &= (1-\tau_c) w_c(n_c) u'\left((1-\tau_c)(1-e_c) w_c(n_c)\right), & (3) \\ \beta \pi_s^{'}(e_s) \Lambda\left(n_c^{'}\right) &= p(n_c)(1-\tau_s) w_s(n_c) u'\left((1-\tau_s)(1-e_s p(n_c)) w_s(n_c)\right). & (4) \end{aligned}$$

where $\Lambda(n_c) \equiv V_c(n_c) - V_s(n_c)$ can be viewed as the value to being skilled. Inada conditions on the utility and probability functions ensure $0 < e_i < 1$. The left hand side is the marginal benefit of investing in human capital – the value to being skilled weighted by the discount factor and the marginal productivity of the investment. The right hand side is the cost of accumulating human capital, weighted by the agent's marginal utility.

Evaluating the Bellman equations for the two types of agents at the optimal policies $e_c(n_c)$ and $e_s(n_c)$ and subtracting one from the other, we get an expression of how the value to being skilled evolves:

$$\Lambda(n_c) = u(c_c(n_c)) - u(c_s(n_c)) + \beta\left(\pi_c(e_c(n_c)) - \pi_s(e_s(n_c))\right) \Lambda\left(n_c^{'}\right), \quad (5)$$

where: $c_c(n_c) \equiv (1-\tau_c)(1-e_c(n_c)) w_c(n_c)$, and $c_s(n_c) \equiv (1-\tau_s)(1-e_s(n_c) p(n_c)) w_s(n_c)$.The value to being skilled has two parts – a current (potential) increase in utility from being skilled and a greater chance of realizing the future value of being skilled.

The law of motion for the fraction of skilled workers is:

$$\Phi(n_c) \equiv n_c^{'} = n_c \pi_c(e_c(n_c)) + (1-n_c) \pi_s(e_s(n_c)). \quad (6)$$

Equations (3) to (6) characterize the dynamics of the household sector through the four functions $e_c(n_c)$, $e_s(n_c)$, $\Lambda(n_c)$, and $\Phi(n_c)$ for any given wage functions $w_c(n_c)$ and $w_s(n_c)$. The matrix that gives the transition probabilities between the skilled and unskilled states is:

$$\begin{pmatrix} & \textbf{skilled} & \textbf{unskilled} \\ \textbf{skilled} & \pi_c(e_c) & 1-\pi_c(e_c) \\ \textbf{unskilled} & \pi_s(e_s) & 1-\pi_s(e_s) \end{pmatrix}.$$

We calibrate our model to an empirical counterpart of this matrix.

As formulated above, agents investing in human capital are liquidity constrained. In addition to the opportunity cost effect of a progressivity change – lower progressivity decreases the opportunity cost of poor agents and increases their investment in human capital – we also have an income effect – lower progressivity is likely to decrease investment of the poor relative to that of the rich. The effect of progressivity on economy-wide investment then depends on the relative strengths of these effects on both types of agents.[10]

We have assumed human capital investment to be tax exempt. With tax-exempt investment, changes in flat-rate taxes are growth neutral; hence, as discussed earlier, we are able to isolate the effect of tax progressivity.[11] We also consider the formulation where the tax is levied on the entire wage, thereby checking the robustness of our results.

3.2 Production

In addition to the two types of household agents, there is a third type of agent, an infinitely-lived entrepreneur, who carries out production and has preferences identical to the two types of workers. The entrepreneur produces according to the production function:

$$Y = A^{1-\alpha} K^{\alpha} \left[\theta N_c^{\nu} + (1-\theta) N_s^{\nu}\right]^{\frac{1-\alpha}{\nu}}, \tag{7}$$

where N_c is the measure of skilled labor hired and N_s the measure of unskilled labor, and $0 < \nu < 1$. Here K is the physical capital used in production, which we assume is accumulated only by the producer. With this formulation, we limit heterogeneity to skill accumulation and keep physical capital accumulation tractable.

This entrepreneur's consumption is: $c_e = (1-\tau_e)(Y - w_c N_c - w_s N_s) - I$. Here, τ_e is the tax rate on the entrepreneur's profits, and I is the investment in physical capital, which evolves according to: $K^{'} = I + (1-\delta) K$. Unlike human capital investments, physical capital investment is not tax exempt.[12] The Bellman equation for the third type of agent is:

$$W(K, A) = \max_{N_c, N_s, I} \left\{ u(c_e) + \beta W\left(K^{'}, A^{'}\right) \right\}, \tag{8}$$

subject to (7), the budget constraint and the law of motion for capital listed above, and the law of motion for A to be described below.

[10] In condition (1), only the opportunity cost effect is present. The liquidity constraint is not *required* for progressivity to affect growth; in fact, it is a conservative assumption for quantifying growth effects.

[11] Stokey and Rebelo (1995) note that small effects of taxes on growth follow from the empirically justifiable assumption of high factor shares for human capital in production and relatively light taxation of the human capital producing sector. One interpretation of Lucas (1990) is that the human capital producing sector is completely untaxed.

[12] The implications of this assumption are discussed in Section 3.4.

We consider two sources of growth that are typically considered in the literature – productivity improvement arising as an externality and arising due to intentional use of human capital by the firm.[13] By broadly considering the main sources of growth that have been extensively discussed in the new growth literature, we are able to study the sensitivity of growth effects to the assumption made about the source of growth.

3.2.1 External Growth

In the first specification, growth results due to an external effect which depends on the fraction of skilled workers alone. This is a human capital version of the externality posited by Romer (1986). It is also influenced by the assumption in Lucas (1988) where there is production externality in the average human capital of the *entire* economy, though his model can generate growth even without the externality. We assume that total factor productivity evolves according to:

$$A_{t+1} = (1 + \xi (N_c)) A_t, \tag{9}$$

where ξ is the externality function. That is, the mere hiring of skilled employees in the production process is enough to generate productivity improvements; they will not be compensated for it. Assuming competitive labor markets, optimization by the entrepreneur implies that the skill premium is given by:

$$p = \frac{\theta}{1-\theta} \left(\frac{N_s}{N_c} \right)^{1-\nu}. \tag{10}$$

It is easy to see that before-tax profits are given by αY, from which the entrepreneur consumes, invests, and pays taxes. The first order and envelope conditions for the dynamic program (8) are:

$$\begin{aligned} [I] &: \beta W_1 \left(K', A' \right) = u' (c_e) && (11) \\ [ENV_k] &: W_1 (K, A) = \alpha (1 - \tau_e) \frac{Y}{K} u' (c_e) + \beta (1 - \delta) W_1 \left(K', A' \right). \end{aligned}$$

3.2.2 Intentional Technology Adoption

A second specification for growth features intentional employment and compensation of skilled workers who generate productivity improvements. Each period, the entrepreneur hires a measure N_c of skilled workers, out of which a measure N_{cA} is employed for new technology adoption and productivity improvements. The production function is therefore of the form:

$$Y = A^{1-\alpha} K^{\alpha} \left[\theta (N_c - N_{cA})^{\nu} + (1 - \theta) N_s^{\nu} \right]^{\frac{1-\alpha}{\nu}}. \tag{12}$$

[13] A highly abbreviated list of examples of externality driven models is: Romer (1986), Jones and Manuelli (1992), and Stokey (1991). A few examples of intentional adoption or invention models can be found in Romer (1990), Aghion and Howitt (1998), and Grossman and Helpman (1991).

The productivity parameter is then assumed to evolve according to:

$$A_{t+1} = (1 + \xi(N_{cA}))\, A_t. \tag{13}$$

We use the same ξ for both specifications purely for notational simplicity; they can be different functions.[14] The key difference is that the measure N_{cA} of workers are hired by the firm and are compensated for it. By assuming an infinitely-lived entrepreneur, we are sidestepping industrial organization issues that form a central part of most R&D based models of growth. These are quite important, but do not seem to be of first order importance for the question at hand.[15] The skill premium implied by competitive labor markets and optimizing behavior by the entrepreneur is given in this case by:

$$p = \frac{\theta}{1-\theta}\left(\frac{N_s}{N_c - N_{cA}}\right)^{1-\nu}. \tag{14}$$

Here $w_c(N_c - N_{cA}) + w_s N_s = (1-\alpha)Y$, and out of the remaining αY, the entrepreneur invests in technology improvements by paying $w_c N_{cA}$, invests in physical capital, consumes, and pays taxes. That is, the entrepreneur makes two types of investment – investment in physical capital, and investment in technology improvements. For simplicity, we assume that adoption involves only skilled labor and no physical capital; the implicit assumption is that "R&D costs", $w_c N_{cA}$, are exempt from the tax on profits.[16]

The first order and envelope conditions for the entrepreneur in this case include (11) and the following additional conditions:

$$\begin{aligned}
[N_{cA}] &: \quad \beta A \xi'(N_{cA})\, W_2\left(K', A'\right) = (1-\tau_e)\, w_c u'(c_e) \qquad (15)\\
[ENV_A] &: \quad W_2(K, A) = (1-\tau_e)(1-\alpha)\frac{Y}{A}u'(c_e) + \beta(1+\xi(N_{cA}))\, W_2\left(K', A'\right),
\end{aligned}$$

where $w_c = \frac{\partial Y}{\partial N_c} = \theta(1-\alpha)A^{1-\alpha}K^{\alpha}\left[\theta(N_c - N_{cA})^{\nu} + (1-\theta)N_s^{\nu}\right]^{\frac{1-\alpha}{\nu}-1}(N_c - N_{cA})^{\nu-1}$. The first condition effectively equates the marginal contribution of skilled agents in its two uses, technology adoption and production. The second condition states that the benefit of an extra unit of the

[14] Indeed, the discipline of calibration dictates a different specification.

[15] Besides generating monopoly profits for technology improvements, our simple specification has other features found in technological change models. Our specification for the production of new technology, (13), which uses currently available technology and skilled labor as inputs is a close parallel to the specification used by Romer (1990): $\dot{A} = \delta H_A A$, where H_A denotes human capital used in the technology sector. Note in particular, that it is a constant *level* of human capital that gives rise to growth in his specification, as it does in ours. Romer notes that in his production specification a doubling of all inputs, including endogenous technology, would more than double output. Likewise, in our production specification a doubling of A and K, and a simultaneous improvement in the *composition* of labor has the potential to more than double output. As we note in Section 3.3, technology, capital, and skill are all endogenous in our framework. The level of technology and labor devoted to R&D also enter the specification for productivity improvements in Jones (1995), albeit with intensities different from those in Romer (1990).

[16] The implications of this assumption is discussed in Section 3.4.

technology stock is its contribution to current marginal utility via production and its use in future technology improvements.

3.3 A Discussion of Model Features

As mentioned earlier, we restrict heterogeneity to two agent types principally to make this endogenous growth model more tractable. Given that one of the biggest transitions in observed skill levels and tax brackets in the US occurs when a high school educated worker becomes a college educated worker, the particular type of discreteness we have modeled seems highly relevant to, and even necessary for, the question we are studying. Our proxy for human capital also has the advantage of direct measurability unlike the unobserved stock that grows without bounds in most other growth models; indeed observable college attainment is one of our calibration targets. Accommodating human capital in the way we have done in the production function (12) also allows for the modeling of technology improvements, and physical as well as human capital accumulation in the same framework. Most growth models allow for only *one* of the two usual engines of economic growth – human capital or technology – for the technical ease of obtaining balanced growth.

There is evidence that human capital is associated with growth, especially for developed countries. Benhabib and Spiegel (1994) find that human capital matters for growth for the richest third of their sample of countries.[17] Direct evidence on the positive connection between higher education and growth can also be found in Baumol, Blackman, and Wolff (1989) and McMahon (1993). The *European Competitiveness Report 2001* reports that the correlation of both production growth and labor productivity growth with the percentage of the population that has attained at least tertiary education is high.[18] Our assumption also finds strong support in the argument made by Acemoglu (1998), that an increase in the proportion of college graduates in the US labor force in the 1970s encouraged the adoption of skill-complementary technologies and caused a subsequent increase in technical change. Therefore, our association of skill with college education appears relevant to studying growth in a technologically and economically advanced country such as the U.S.

Any model which endogenously gives rise to a non-trivial distribution of income requires some markets to be incomplete. The Bewley-type model (without growth) typically features uninsured individual income risks. The uninsured risk we specify in the accumulation of human capital, and

[17] Kumar (2002) examines the direction of causality between education and growth, and using a broad cross-section of countries finds evidence that education causes growth.

[18] The correlation coefficients in EU countries for 1998 were 0.4316 and 0.4094 respectively. The corresponding coefficients for correlation with the *working* population with tertiary education are 0.4681 and 0.367. Note that these as well as the earlier-cited correlations are between the *level* of human capital and economic growth, which make them directly relevant to our model.

thus in earnings, serves this purpose. Additionally, several models, especially those that feature human capital accumulation, also impose liquidity constraints. We have taken this assumption one level further by not allowing households to accumulate physical capital. The improvement in tractability that arises from such a move is obvious. Moreover, capital income taxes affect only the richer individuals in the US, and for addressing a discrete and significant jump in tax bracket on completing college, modeling capital income appears to be a secondary issue.[19] For these reasons, and due to the assumption implicit in several growth models that human capital is the real engine of growth and physical capital merely keeps pace, we relegate physical capital accumulation to the entrepreneur who runs the firm. As Stokey (1988) notes in a different context: "The absence of physical capital may at first seem startling. However, ... models built around the accumulation of physical capital alone do *not* give rise to sustained growth. The models that do are those built around the ... accumulation of knowledge ... or around technological change." Making this assumption allows us to calibrate the model to aggregate measures such as the capital-output ratio, while keeping the heterogeneity and the state space as simple as possible for the issue at hand.

3.4 Summary of Theoretical Results

The main analytical findings of Caucutt, İmrohoroğlu and Kumar (2002) of a parametric change in tax progressivity on the balanced growth of the economy are summarized in this section. The primary result is that there is *no* change in the growth rate if taxes are flat and there is a change in the rate of flat taxes; however, if taxes are progressive, a change in the degree of progressivity will affect growth in ways discussed below.[20]

In the above paper, the authors formally define a Balanced Growth Path (BGP) in which stocks, K, A, output, Y, and wages, w_c, and w_s, all grow at a constant rate g, the skill "return" function Λ, and entrepreneur value function W grow at the gross rate $(1+g)^{1-\sigma}$, and human capital investments, e_c^*, e_s^*, the skill premium p^*, and skill attainment n_c^* are all time invariant. The CRRA utility function,

[19] During 1980, capital gains as a percentage of family income was greater than 1% only for the highest income quintile families. Rents, interest, and dividend income though higher than capital gains was 13.9% of family income for the highest quintile and only 6.4% for the next quintile (and even lower for lower quintiles). Even within the highest quintile, it is the top 1% that had a disproportionately high percentage of income from these sources; see Table 6 in Kasten, Sammartino, and Toder (1994). Haliassos and Lyon (1994) likewise find that the highest 1% in the income distribution accounted for 58.12% of capital gains income (but only 12.61% of total income). At the aggregate level, for the fiscal year 1992, capital gains taxes accounted for less than 3% of total revenues. See Figure 2 in Moore and Silvia (1995). Therefore, the omission of capital accumulation by individuals might not be a serious drawback, and might actually understate the degree of progressivity that actually exists.

[20] The role of the government is limited to collecting taxes; all collected taxes are spent by the government and do not result in any utility or productivity improvements.

$u(c) = \frac{c^{1-\sigma}}{1-\sigma}$, is used.

When growth is driven by a human capital externality, equilibrium growth is given by, $g = \xi(n_c^*)$. The skill premium on the BGP, using (10) is:

$$p^* = \frac{\theta}{1-\theta}\left(\frac{1-n_c^*}{n_c^*}\right)^{1-\nu}. \tag{16}$$

The tax rate on the entrepreneur's profits, τ_e, does not affect the long run growth rate. The "engine" of growth is skill acquisition and a tax policy that does not affect that process will have no effect on long run growth. The tax rate on profits will affect the capital-output ratio, and the levels of profits and wages. In particular, the capital-output ratio is given by:

$$\frac{K}{Y} = \frac{\alpha(1-\tau_e)}{(1+\rho)(1+g)^{\sigma} - (1-\delta)}. \tag{17}$$

Higher taxes on profits lower this ratio. If investment in physical capital were tax exempt, even this effect of the profit tax disappears, as higher taxes create an incentive to invest and get a "write-off".

When growth is driven by intentional technology adoption, for any given n_c^*, the entrepreneur in this case has a decision to make about the fraction of the skilled labor force to devote to technology adoption, n_{cA}^*, which affects the growth rate through the equation, $g = \xi(n_{cA}^*)$. The equation determining n_{cA}^*, and hence g, is:

$$(1+\rho)(1+\xi(n_{cA}^*))^{\sigma} = (1-\alpha)\frac{Y}{w_c}\xi'(n_{cA}^*) + (1+\xi(n_{cA}^*)) \tag{18}$$

where $\beta \equiv \frac{1}{1+\rho}$, and $\frac{Y}{w_c}$ can be backed out from $[n_c]$ as:

$$\frac{Y}{w_c} = \frac{(n_c^* - n_{cA}^*)^{1-\nu}\left[\theta(n_c^* - n_{cA}^*)^{\nu} + (1-\theta)(1-n_c^*)^{\nu}\right]}{\theta(1-\alpha)}.$$

The skill premium on the BGP from (10) is:

$$p^* = \frac{\theta}{1-\theta}\left(\frac{1-n_c^*}{n_c^* - n_{cA}^*}\right)^{1-\nu}. \tag{19}$$

Even when growth is driven by technology adoption, the tax rate on profits, τ_e, does not affect the rate of this growth. In a world in which R&D expenses are tax exempt, an increase in this tax rate decreases the marginal cost of hiring a skilled agent to adopt technology and decreases the marginal benefit arising from the improved technology by the same factor. The capital-output ratio continues to be given by (17) and is adversely affected by a tax on profits. When the adoption function is parametrized as $\xi(n_{cA}) = Cn_{cA}^{\varepsilon}$, $0 < \varepsilon < 1$, one can show that when the availability of skilled labor, n_c^*, increases, the portion of that labor devoted to technology adoption, n_{cA}^*, increases in a BGP equilibrium.

Turning next to household behavior, using (3) and (4) we can get the *intra*temporal condition governing investment of the skilled agents relative to those of the unskilled as:

$$\frac{\pi_c'(e_c^*)}{\pi_s'(e_s^*)} = \frac{\Theta^\sigma}{\Theta}\left(\frac{\frac{1}{p^*} - e_s^*}{1 - e_c^*}\right)^\sigma, \tag{20}$$

where $\Theta \equiv \frac{(1-\tau_s)}{(1-\tau_c)}$ is a measure of the progressivity of the tax system. This expression equates the ratio of marginal investment benefit of each type to the ratio of their marginal costs.

Using (5) and (4) we can get the following *inter*temporal condition:

$$\left[\frac{1}{\frac{1}{\beta(1+g)^{1-\sigma}} - (\pi_c(e_c^*) - \pi_s(e_s^*))}\right] \cdot \left[\frac{\left(\frac{1-e_c^*}{\Theta}\right)^{1-\sigma} - \left(\frac{1}{p^*} - e_s^*\right)^{1-\sigma}}{1-\sigma}\right] = \frac{1}{\pi_s'(e_s^*)\left(\frac{1}{p^*} - e_s^*\right)^\sigma}. \tag{21}$$

Evaluating (6) at the BGP equilibrium, we get:

$$n_c^* = \frac{\pi_s(e_s^*)}{1 - (\pi_c(e_c^*) - \pi_s(e_s^*))}. \tag{22}$$

As one would intuitively expect, the higher the investment in skill by any particular type of agent on the BGP, the higher is the level of skill attainment.

Equations (20), (21), and (22) capture the behavior of the household sector on the BGP equilibrium. That is, given the p^* and g arising from production decisions, these three equations determine the investments e_c^* and e_s^*, and thus the skill attainment n_c^*.

The main analytical findings are:

- With flat rate taxes, $\tau_c = \tau_s = \tau$, $\Theta = 1$, and the actual tax rate does not figure in the equations that determine the growth rate. Any effect of tax on growth is because of differences in its *structure*, rather than on its level.

- Using the parametrization $\pi_c = \pi_s = Be^\gamma$, $0 < \gamma < 1$, we show that for a given rate of anticipated growth, no matter its source, when general equilibrium effects of changes in the skill premium are ignored, the BGP investments e_c^* and e_s^*, both decrease with the degree of tax progressivity; the level of skill attainment, n_c^*, thus decreases. Even though an increase in the progressivity could shift the investment in favor of the unskilled through the liquidity effect, it is the intertemporal effect that ultimately dominates and decreases the investments of both types of agents.

- For externality driven growth, an increase in Θ decreases both e_c^* and e_s^*. This tends to increase the premium and thus the value to being skilled, which in turn tends to *increase* the investment by both types now. Under conditions that make this general equilibrium effect mild, we can show that when growth is caused by externalities resulting from activities of skilled labor, an increase in the progressivity of taxes decreases the human capital investment levels of both types of agent on the BGP. The stationary level of skill attainment is lower and thus the growth rate is lower.

- The equilibrium effects are more complicated with intentional technology adoption. The anticipated growth rate directly enters the expression for expected premium (19), since $n_{cA}^{*} = \xi^{-1}(g)$. The higher the fraction of labor in technology adoption, the higher is the premium since skilled workers are fully compensated for their role in generating growth. General equilibrium effects of changes in the skill premium imply that the BGP investment of the unskilled, e_s^*, always decreases with the degree of tax progressivity. If the growth rate is low enough, the investment of the skilled, e_c^*, also decreases, but if the growth rate is high enough, e_c^* can *increase* with progressivity. The general equilibrium effect on the premium at high growth rates makes it attractive for the skilled to invest more; moreover, the premium becomes more sensitive to progressivity at these higher growth rates.[21] Therefore, low equilibrium growth rates are compatible with decreases in human capital investment by both types of agents when progressivity increases and thus in the growth rate itself; for higher growth rates, the effect of increased progressivity is analytically ambiguous.[22]

4 Calibration

We now calibrate the above model to U.S. data assuming that a model period is thirty years. The two types of labor can be readily interpreted as school-educated and college-educated labor and the premium as a college premium.

Our calibration strategy is guided by a desire to make the model economy consistent with the US economy not only along standard dimensions such as the capital-output ratio, but also along those particularly relevant to our study – the fraction of the labor force that is college educated, the annual per capita growth rate, the share of GDP devoted to education, and the two independent conditional transition probabilities in the long-term earnings mobility matrix. We set the capital's share of income α to 0.36. We set ν to 0.35 which implies an elasticity of substitution between skilled and unskilled labor of 1.54. Autor, Katz, and Krueger (1998) report that most estimates for this elasticity fall between 1.4 and 1.5. We use this value of ν, *anticipate* a college attainment of 35% and a college

[21] The increase in p increases "tuition" cost of the unskilled, thus unambiguosly decreasing their investment when progressivity increases.

[22] However, as we will see in the calibrated simulations, growth always decreases with progressivity even in the adoption case; the equilibrium growth rates that obtain are low enough to keep the economy away from the region where the investments by the rich and the poor could move in opposite directions.

premium of 1.75 (both of which are consistent with values reported in the labor literature), and use (16) to get θ as 0.527. We set σ to 2.0, a standard value for the utility curvature parameter.[23] The tax on profits, τ_e, is set to 20%, and the annual physical capital depreciation rate is set to 4.4%.[24] Given these values for σ and τ_e, and an anticipated annual growth rate of 1.8%, the intergenerational discount parameter is set to 0.74 (0.99 in annual terms), so that the capital-to-output ratio given by (17) is close to 3 for annual GDP flows.

Since it is the *ex ante* tax schedule that governs incentives, we prefer to assume values for Θ directly as suggested by this schedule rather than infer them indirectly from equilibrium tax payments. As our benchmark progressivity, we use $\Theta = 1.5$; for example, model tax rates of $\tau_s = 10\%$ and $\tau_c = 40\%$ will yield this level of progressivity. We will see later that this parametrization of progressivity is conservative, since the ratio of tax payments made by the rich to those made by the poor in equilibrium is lower than it is in the data; so we present results for $\Theta = 2.0$ (for example, model tax rates of $\tau_s = 10\%$ and $\tau_c = 50\%$), which appears closer to the U.S. reality.

We parametrize $\pi_c(e) = B_c e^{\gamma_c}$ and $\pi_s(e) = B_s e^{\gamma_s}$. Clearly, B_c, B_s, γ_c, and γ_s are the parameters that are most specific to our model. Of these four parameters, we normalize B_c to 1.

We then calibrate the remaining parameters, those in the human capital production functions and the ξ specification, so that the model outcomes of the key quantities mentioned above match observed values. For the case of external growth, we use the specification $\xi(n_c) = C n_c^{\varepsilon}$. For the benchmark progressivity, our calibration yields: $\gamma_c = 0.1$, $B_s = 0.5$, $\gamma_s = 0.1$; $C = 1.82$, $\varepsilon = 0.9$. For the intentional adoption case, given that the fraction of skilled labor force involved in "R&D" is likely to be small, we use the specification $\xi(n_{cA}) = C + \varepsilon n_{cA}$, so that the constant term can pick up growth arising from other causes. While the education parameters are the same as those given above, the technology adoption parameters are: $C = 0.495$, and $\varepsilon = 3.36$.[25] The model outcomes are compared with their empirical counterparts in Table 2.[26]

[23] Lucas (1990), for instance, uses this value.

[24] See İmrohoroğlu, İmrohoroğlu, and Joines (1999) for the latter.

[25] The annual growth rate, g, can be calculated from ξ, using the relation $(1+g)^{30} = 1+\xi$.

[26] The estimates for the US were obtained from the following sources: fraction of college educated is the full-time equivalent figure for 1990 from Autor, Katz, and Krueger (1998), as is the range for the skill premium. The share devoted to education is for the early 90s from the Digest of Education Statistics (1997); the share of adult population with Master's, Professional, and Doctorate degrees, which we use to compare with the model outcome for n_{cA} is from the same source for the year 1996. The transition probabilities are from Gottschalk and Moffitt (1994). The share of tax payments by the rich is from the tax tables of the Statistical Abstract of the United States (1995); we calculate the percentage of taxes paid by roughly the top 38% to match up with the n_c value.

The UNESCO Statistical Yearbook 1999 reports an R&D share of GNP of 2.52% for the year 1994. The officially reported "R&D" figure is likely to miss the spirit of the widespread productivity enhancing activities captured by the model and thus underestimate resources devoted to such activities. For this reason, and to be consistent with the n_{cA}

Table 2

Comparison of Model Outcomes ($\Theta = 1.5$) with U.S. Data

Model Quantity	Interpretation	U.S. Data	Externality	Adoption
n_c	fraction of college educated	38.6%	35%	37.7%
g	per cap. annual growth rate	1.8%	1.8%	1.81%
$(n_c e_c + (1-n_c) e_s)\frac{w_c}{Y}$	share devoted to education	college: 2.9%; K-12: 7.3%	3.7%	6.90%
$b_s(e_s)^{\gamma_s}$	prob(rich|poor)	0.23 (17 year mobility)	0.24	0.25
$b_c(e_c)^{\gamma_c}$	prob(rich|rich)	0.65 (17 year mobility)	0.56	0.59
n_{cA}	fraction in "R&D"	6.8% PhD+Masters	N/A	6.4%
$n_{cA}\frac{w_c}{Y}$	share devoted to "R&D"	2.52% - 8.87%	N/A	6.16%
$p = \frac{w_c}{\Theta w_s}$	post tax skill premium	1.66-1.73	1.66	1.75
K/Y	capital-output ratio	2.5-3	2.88	2.88
$\tau_c n_c w_c / (\tau_c n_c w_c + \tau_s n_s w_s)$	% tax payments by rich	90.7%	70.4%	74.3%

The model does well in matching our targets, which are listed in the first five rows. It also produces related outcomes that are broadly consistent with the data; these are listed below the targeted outcomes.

5 Policy Experiments

In this section, we consider a change in progressivity when investments in human capital are tax exempt. Recall that this is the case where the actual level of taxes do not matter for long-run growth; only the ratio of retention rates matters. For this reason we do not worry about revenue neutrality while discussing the growth effects; taxing the rich and the poor at different rates in order to meet a given level of government expenditure would still affect growth only through the effect the change has on the retention ratio. However, revenue neutrality will matter for welfare comparisons. We quantify the growth effects of moving from a progressivity of $\Theta = 1.5$, and from $\Theta = 2$, to a flat rate system.

5.1 Externality-driven Growth

Table 3 summarizes the effects of moving to a flat rate system when growth is driven by an externality. A move from a progressivity level of 1.5 to a flat rate system results in an increase of 0.26 percentage points in the long-run growth rate. This is caused by an increase in the level of skill attainment, from

measure reported above, we use data from the U.S. Census Bureau and the BEA to calculate the earnings of those with "Advanced Degrees" as a fraction of GNP for 1994. This is 8.87%. By including graduates with diverse specializations such a measure could overestimate resources devoted to productivity enhancement. Our model outcome of 6.16% lies between these two estimates.

about 35% of the labor force to nearly 43%, which brings down the skill premium. In other words, a move to a less progressive system both increases growth and decreases inequality in the long run. The share of GDP devoted to human capital accumulation increases sharply. The increased level of investment translates to increased mobility for the poor (0.24 to 0.27) and greater persistence for the rich (0.56 to 0.63). The capital-output ratio drops with an increased growth rate by raising the interest rate; here it goes from 2.88 in the benchmark to 2.37 under a flat rate system.

Table 3

Change in Progressivity – Externality

Variable	**Interpretation**	$\Theta = 1.0$	$\Theta = 1.5$	$\Theta = 2.0$
n_c	fraction of college educated	42.6%	35.2%	28.1%
g	growth rate (annualized)	2.06%	1.80%	1.54%
$(n_c e_c + (1 - n_c) e_s)\frac{w_c}{Y}$	share devoted to education	12.8%	3.7%	0.8%
$b_s(e_s)^{\gamma_s}$	prob(rich|poor)	0.27	0.24	0.20
$b_c(e_c)^{\gamma_c}$	prob(rich|rich)	0.63	0.56	0.48
$p = \frac{w_c}{\Theta w_s}$	post tax skill premium	1.35	1.66	2.05
K/Y	capital-output ratio	2.37	2.88	3.56
$\tau_c n_c w_c / (\tau_c n_c w_c + \tau_s n_s w_s)$	% tax payments by rich	50.1%	70.4%	81.6%

The increase in the growth rate and the drop in the premium is more dramatic when we move from the high level of progressivity of 2 to a flat rate system. Growth increases by 0.52 percentage points and the premium drops from 2.05 to 1.35. Education investment by the poor increases by a higher factor than education investment by the rich.

In assessing all these growth rate changes it is important to realize that they are brought about by the effect of progressivity on human capital investments alone. Potential channels for decreased progressivity to increase the labor supply and physical capital investment have been shut down. As a result, the above-mentioned increases seem quite significant, amounting to a 14.4% increase from the current growth rate when the transition is from $\Theta = 1.5$, and a 33.8% increase when the transition is from $\Theta = 2.0$.

In Table 4, we present results on welfare gains from eliminating progressivity. The derivation of the dynamic equations used to compute transition, the nature of the transition paths, and details on welfare computation are presented in the appendix. In order to conduct revenue neutral experiments, we set the tax rate in the flat rate regime to $\tau_c = \tau_s = 30\%$, which yields labor tax revenues that are 19.2% of GDP. The tax rates in the progressive regimes that yield the same tax revenues as a share of GDP are, $\tau_c = 44.6\%$, $\tau_s = 16.9\%$, when $\Theta = 1.5$, and $\tau_c = 55\%$, $\tau_s = 10\%$, when $\Theta = 2.0$.

Table 4

Welfare Gain from Eliminating Progressivity – Externality

Variable	$\Theta = 1.5$ **to** $\Theta = 1.0$	$\Theta = 2.0$ **to** $\Theta = 1.0$
BGP (Aggregate)	0.55%	1.31%
BGP utility of c	0.82%	1.68%
BGP utility of s	0.35%	1.04%
Including Transition (Aggregate)	−0.047%	−0.057%
Utility of c including transition	0.002%	0.061%
Utility of s including transition	−0.088%	−0.10%

The first row in Table 4 presents the compensating variation in annual consumption each agent would have to be given in a BGP indexed by $\Theta > 1$, in order to make an equally weighted aggregate welfare measure the same as that in a flat rate BGP. If the move to flat rate taxes is from a progressive system with parameter Θ of 1.5, each consumer has to be given more than 0.5% of current consumption annually to make aggregate welfare the same across regimes. If the move is from a Θ of 2.0, each consumer has to be given more than 1.3%. As seen in Table 3, the growth rate increases when progressivity decreases, which increases aggregate welfare in the BGP of the flat rate regime and households in the progressive regimes have to be compensated to equate welfare.

The second and third rows attempt to address the following question. "Is it better to be rich (poor) in the BGP of the progressive regime or the flat rate regime?" From the compensating variations given above, it is clear that the welfare gain for the rich is higher than that of the poor in going to a flat rate system. In addition to the increased growth rate in a flat rate system, the rich are being taxed less. The growth gains of the poor are partly, but not completely, negated by their increased taxes. The aggregate numbers lie in between the individual numbers. Thus, looking across balanced growth paths alone, it is clearly beneficial to everyone in the model economy to move to a flat rate system.

The last three rows, which provide the equivalent variation taking the transition into account, tell a different story. From the fourth row, we can see each consumer will be willing to pay to stay in the progressive regime in order to equate aggregate welfare across regimes. To understand this result, we look at the welfare of the rich and the poor at the time the regime change occurs given in the last two rows. The college-educated rich slightly prefer the move to the flat rate system even when the transition is accounted for, but their gain is much smaller than the one indicated by the BGP comparisons. While they are helped by the decreased tax rate, there are two other forces that work to decrease their consumption and temper the gain – first is their increased investment in education and a drop in the equilibrium skill premium; second is the discounting of the gain from increased long run growth. For the school-educated poor, the decrease in consumption from the increased tax rate and increased investment early on seem to outweigh the later gains from a decreased skill premium, and they prefer the progressive system. Their willingness to pay to stay in the progressive system, and the fact that there are nearly twice as many of them, account for the (slight) overall negative effect on aggregate welfare.

Two points are worth noting in this regard. The liquidity constraint on households clearly contributes to the initial consumption loss they endure when investment increases. If in the real economy there are households that are not constrained, the above welfare measures will underestimate the gains in going to the flat rate regime. Second, it might be possible to design a scheme in which the government issues debt to finance the transition costs of households, especially the poor, and starts repaying it when the growth gains are realized, thereby making the transition more palatable. Or one could phase out the progressive system gradually to soften the direct effect of increased taxes for the poor. The exploration of such schemes is a topic for future research.

5.2 Adoption-driven Growth

Table 5 summarizes the effects of changes in progressivity in the adoption case.

Table 5

Change in Progressivity – Adoption

Variable	**Interpretation**	$\Theta = 1.0$	$\Theta = 1.5$	$\Theta = 2.0$
n_c	fraction of college educated	44.6%	37.7%	30.6%
g	growth rate (annualized)	1.93%	1.81%	1.66%
$(n_c e_c + (1 - n_c) e_s) \frac{w_c}{Y}$	share devoted to education	20.8%	6.9%	1.7%
$b_s (e_s)^{\gamma_s}$	prob(rich|poor)	0.28	0.25	0.22
$b_c (e_c)^{\gamma_c}$	prob(rich|rich)	0.65	0.59	0.51
n_{cA}	fraction in "R&D"	8.4%	6.4%	4.3%
$n_{cA} \frac{w_c}{Y}$	share devoted to "R&D"	7.2%	6.2%	4.7%
$p = \frac{w_c}{\Theta w_s}$	post tax skill premium	1.47	1.75	2.09
K/Y	capital-output ratio	2.61	2.88	3.22
$\tau_c n_c w_c / (\tau_c n_c w_c + \tau_s n_s w_s)$	% tax payments by rich	54.1%	74.3%	84.4%

The increase in the growth rate is more modest when we move from the benchmark progressivity to a flat rate system, amounting to 0.12 percentage points. Here growth does not increase automatically as more people become skilled; the adopting firm has to absorb a higher fraction of the labor force into R&D, and given the cost of doing this it treads lightly. And as discussed in Section 3.4, the increase in the growth rate exerts an upward pressure on the premium and negatively affects the investment of the poor; this numbs the positive effect of an increase in the return to being skilled. Indeed, unlike the externality case, here we find that the investment of the rich increases by more than the increase in the investment of the poor.[27] The share devoted to education is driven up nevertheless. The level of skill attainment increases from 37.7% to 44.6% which brings the premium down from 1.75 to 1.47. The percentage of labor involved in adoption goes up from 6.4% to 8.4%. And, as in the externality case, the mobility of the poor increases.

The increase in the growth rate is higher when one goes to a flat rate system from a higher progressivity, amounting to 0.27 percentage points. Changes along other dimensions are also higher.

Table 6

Welfare Gain from Eliminating Progressivity – Adoption

Variable	$\Theta = 1.5$ **to** $\Theta = 1.0$	$\Theta = 2.0$ **to** $\Theta = 1.0$
BGP (Aggregate)	0.38%	0.97%
BGP utility of c	0.67%	1.41%
BGP utility of s	0.15%	0.64%
Including Transition (Aggregate)	−0.052%	−0.053%
Utility of c including transition	0.026%	0.072%
Utility of s including transition	−0.094%	−0.105%

We present the welfare estimates in Table 6. In these revenue neutral experiments, the tax rate in the flat rate regime is set to $\tau_c = \tau_s = 30\%$, which yields labor tax revenues that are 21.4% of GDP. The tax rates in the progressive regimes that yield the same tax revenues as a share of GDP are, $\tau_c = 44.1\%$, $\tau_s = 16.1\%$, when $\Theta = 1.5$ and $\tau_c = 54.7\%$, $\tau_s = 9.3\%$, when $\Theta = 2.0$.

These numbers are very similar to those for the externality case. However, the gains are smaller reflecting the smaller increases in growth rates. In summary, a decrease in progressivity increases growth, mobility for the poor, and decreases inequality as measured by the skill premium. The increase in growth rates can be large, ranging from 0.13 to 0.53 percentage points depending on the experiment considered. The assumption made about the engine of growth matters, both qualitatively and quantitatively. There are significant welfare gains across BGPs, but transitional costs are large resulting in little change in overall welfare.

[27] If we had made the empirically implausible assumption that each type of agent needs only her own time to accumulate human capital, the growth effect would have only been higher. Therefore, our assumption that skilled time is required for human capital accumulation is a conservative one.

6 Sensitivity Analysis

The analysis above was conducted with tax-exempt human capital investment, given the existing precedence for this assumption and the convenience it lends to isolating the effect of progressivity. In this section we verify that deviating from this assumption does not alter our main conclusions about the growth rate and welfare. We also study the effect of changes in model parameters and specifications.

6.1 The Non-Exempt Case

When investments in human capital are not tax exempt, the actual tax levels matter even for studying the long-run growth effects; it is no longer the case that progressivity affects allocations only through the ratio of the retention rates, Θ. This can be seen from the analogues of (20) and (21), the only conditions that change for the non-exempt case:[28]

$$\frac{\left[\left((1-\tau_c)-e_c^*\right)^{1-\sigma}(p^*)^{1-\sigma}-\left((1-\tau_s)-e_s^*p^*\right)^{1-\sigma}\right]}{(1-\sigma)\left[1-\beta\left(1+g\right)^{1-\sigma}\left(\pi_c\left(e_c^*\right)-\pi_s\left(e_s^*\right)\right)\right]} = \frac{p^*\left((1-\tau_s)-e_s^*p^*\right)^{-\sigma}}{\beta\left(1+g\right)^{1-\sigma}\pi_s'\left(e_s^*\right)}, \tag{23}$$

$$\frac{\pi_c'\left(e_c^*\right)}{\pi_s'\left(e_s^*\right)} = \frac{\left((1-\tau_s)-e_s^*p^*\right)^{\sigma}}{\left((1-\tau_c)-e_c^*\right)^{\sigma}(p^*)^{\sigma}}. \tag{24}$$

The non-exempt case also allows us to examine the issue of whether changes in the level of flat rate taxes indeed matter less than changes in the progressivity of taxes, one of the motivations of this paper. We consider a decline in the flat rate tax from 30% to 10%, as well as changes from a progressive system to a flat rate system. The tax rates are set in the progressive system so as to match the pre-specified progressivity parameter Θ, and raise the same tax revenues (as a fraction of GDP) as a 30% flat rate system.

6.1.1 Externality-driven Growth

The results for growth driven by externality are given in Table 7. The tax rate in the flat rate regime is set to 30%, which yields labor tax revenues that are 19.2% of GDP.[29] The tax rates in the progressive regimes that yield the same tax revenues as a share of GDP are, $\tau_c = 44.7\%$, $\tau_s = 17\%$, which corresponds to $\Theta = 1.5$ and $\tau_c = 55\%$, $\tau_s = 10\%$, which corresponds to $\Theta = 2.0$.

[28] The current utility terms in (2) now read $u\left((1-\tau_c)\,w_c\left(n_c\right)-e_c w_c\left(n_c\right)\right)$ and $u\left((1-\tau_s)\,w_s\left(n_c\right)-e_s p\left(n_c\right)w_s\left(n_c\right)\right)$. Following the same steps as in the exempt case, we can derive (23) and (24).

[29] While we go from the 30% flat rate system to the 10% flat rate system, the taxes collected as a fraction of income drops; it is no longer 19.2%. One way to make this a revenue neutral change would be to increase the capital tax rate by the appropriate amount; in this model, such a change would be growth neutral.

Table 7

Change in Progressivity (non-exempt) – Externality

Variable	**Interpretation**	$\Theta = 1.0$	$\Theta = 1.0$	$\Theta = 1.5$	$\Theta = 2.0$
		$\tau = 10\%$	$\tau = 30\%$		
n_c	fraction of college educated	42.1%	40.9%	34.0%	27.6%
g	growth rate (annualized)	2.04%	2.01%	1.76%	1.52%
$(n_c e_c + (1 - n_c) e_s) \frac{w_c}{Y}$	share devoted to education	11.79%	9.99%	2.83%	0.68%
$b_s (e_s)^{\gamma_s}$	prob(rich|poor)	0.27	0.27	0.24	0.21
$b_c (e_c)^{\gamma_c}$	prob(rich|rich)	0.63	0.62	0.53	0.44
$p = \frac{w_c}{\Theta w_s}$	post tax skill premium	1.37	1.41	1.71	2.09
K/Y	capital-output ratio	2.40	2.47	2.98	3.63
$\tau_c n_c w_c / (\tau_c n_c w_c + \tau_s n_s w_s)$	% tax payments by rich	49.8%	49.5%	69.9%	81.3%

The first two columns in Table 7 show the effects of a change in a flat rate system. There is little change in growth or other quantities in going from a 30% to a 10% system in our heterogenous agent setup, in line with the findings of Lucas (1990) and Stokey and Rebelo (1995). There is a small positive effect due to increased income of the liquidity constrained agents but the effect of lower progressivity in eliminating the wedge in human capital return is absent. Hence, comparing flat rate regimes to evaluate growth is misleading even in our heterogeneous-agent setup. As can be seen from the last three columns, which are discussed below, the change in the degree of progressivity has more telling effects.

The findings in Table 7 also serve as a check of the robustness of the results presented in Table 3. When the baseline progressivity is 1.5 going to a flat rate system increases growth by 0.25 percentage points; when the progressivity is 2.0, the increase is 0.49 percentage points. The quantities, n_c, n_{cA}, education and "R&D" share of GDP, mobility of the poor, and persistence of the rich all increase and the premium decreases. These results are similar to those in Table 3.

For the sake of brevity, we do not report the welfare figures for this experiment, but they are very comparable to those in Table 4.

6.1.2 Adoption-driven Growth

For completeness, the results for growth driven by adoption in the non-exempt case are given in Table 8. The tax rate in the flat rate regime is again set to 30%, which yields labor tax revenues that are 21.3% of GDP. The tax rates in the progressive regimes that yield the same tax revenues as a share of GDP are, $\tau_c = 44.2\%$, $\tau_s = 16.3\%$, which correspond to $\Theta = 1.5$ and $\tau_c = 54.8\%$, $\tau_s = 9.7\%$, which correspond to $\Theta = 2.0$. The points discussed for Table 7 are applicable here too – reform in the tax progressivity has more of an effect than a decrease in flat rate taxes, and the results are very similar to the exempt case presented in Table 5.

Table 8

Change in Progressivity (non-exempt) – Adoption

Variable	**Interpretation**	$\Theta = 1.0$	$\Theta = 1.0$	$\Theta = 1.5$	$\Theta = 2.0$
		$\tau = 10\%$	$\tau = 30\%$		
n_c	fraction of college educated	44.0%	42.7%	36.1%	29.6%
g	growth rate (annualized)	1.92%	1.90%	1.78%	1.64%
$(n_c e_c + (1 - n_c) e_s) \frac{w_c}{Y}$	share devoted to education	19.2%	15.9%	5.0%	1.2%
$b_s(e_s)^{\gamma_s}$	prob(rich|poor)	0.28	0.27	0.25	0.22
$b_c(e_c)^{\gamma_c}$	prob(rich|rich)	0.65	0.64	0.56	0.46
n_{cA}	fraction in "R&D"	8.2%	7.8%	6.0%	4.0%
$n_{cA} \frac{w_c}{Y}$	share devoted to "R&D"	7.2%	7.0%	5.9%	4.4%
$p = \frac{w_c}{\Theta w_s}$	post tax skill premium	1.49	1.54	1.82	2.15
K/Y	capital-output ratio	2.63	2.68	2.94	3.28
$\tau_c n_c w_c / (\tau_c n_c w_c + \tau_s n_s w_s)$	% tax payments by rich	53.9%	53.4%	73.6%	83.7%

6.2 Changes in Parameters

We first study the sensitivity of our results to a change in the preference parameter σ from its benchmark value of 2. In particular we consider values of 1.5 and 2.5 (this entails a change in β from 0.74 to 0.56 and 0.97 respectively, in order to get a reasonable capital-output ratio for $\Theta = 1.5$). These results are presented in Table 9.

Table 9

Sensitivity to σ – Externality

	$\sigma = 1.5$			$\sigma = 2.5$		
Variable	$\Theta = 1.0$	$\Theta = 1.5$	$\Theta = 2.0$	$\Theta = 1.0$	$\Theta = 1.5$	$\Theta = 2.0$
fraction of college educated	42.7%	35.2%	28.1%	42.5%	35.2%	28.2%
growth rate (annualized)	2.06%	1.81%	1.54%	2.06%	1.81%	1.54%
share devoted to education	12.7%	3.7%	0.8%	13.0%	3.8%	0.8%
prob(rich|poor)	0.28	0.24	0.20	0.27	0.24	0.20
prob(rich|rich)	0.63	0.55	0.48	0.63	0.56	0.48
post tax skill premium	1.35	1.66	2.05	1.36	1.66	2.05
capital-output ratio	2.45	2.84	3.33	2.27	2.88	3.78

When compared to Table 3, there is practically no change in most of the quantities, especially in the growth rates. The capital-to-output ratio is the most sensitive quantity, and only to a small degree. In order to conserve space, we do not present the table for the adoption case. As with the externality case, there is little change in the results relative to those in Table 5.

We also perturbed the human capital production parameters, γ_c and γ_s, one at a time from their benchmark value of 0.1. The changes in growth rates were virtually identical to those reported in Table 3. For instance, when γ_s was set to 0.08, the growth rate increases from 1.56% to 2.11% when we move from $\Theta = 2.0$ to flat rate taxes; when γ_s was set to 0.12, the corresponding growth rates are 1.51% and 2.01%. The entries in the mobility matrix change in expected directions and only by small magnitudes – lowering γ_s increases the probability of the poor becoming rich relative to that in Table 3. Growth results are similar when γ_c alone is varied, with a lower γ_c decreasing the probability of the poor becoming rich.

6.3 Changes in Specifications

The specifications for ξ in Section 4 are chosen for reasons of parsimony and to ensure a good "fit" of targets; the targets themselves are chosen so that they correspond to central outcomes of the model and on which widely accepted data are available. An alternative strategy involves the use of scant micro evidence to calibrate the function parameters directly, making the goal of matching other empirical outcomes a secondary one. Given the lack of direct connection between available empirics and our model, this process necessarily entails making heroic assumptions to go from regression coefficients to model parameters.

Kahn and Lim (1998) regress industry level TFP growth on skilled labor's share of income, among other variables, which allows them to provide estimates of how much TFP growth would increase when that share increases. We used their regression coefficient of 0.1969.[30] In our model the share corresponds to $\frac{w_c n_c}{Y}$. Under the simplifying, but empirically implausible, assumption that $\frac{w_c}{Y}$ is a constant, independent of n_c, and calculating a value for it from the data in Table 1, we can convert this regression coefficient to a corresponding one on n_c. For the externality specification, we start with $\xi = (B + Cn_c)^{\varepsilon}$, linearize the expression, and connect the coefficient on n_c to the value obtained above, to derive a restriction connecting the three parameters, B, C, and ε. The two free parameters are then calibrated to obtain outcomes as close as possible to the US data.[31] With this specification, we obtain the increase in growth rate to be 0.16 percentage points while we move from a progressivity of 1.5 to flat rate taxes and an increase of 0.33 percentage points from a progressivity of 2.0.

We attempted the calibration using Kahn and Lim (1998) for the adoption case as well. There is an extra step involved since the regression coefficient has to be converted to a corresponding one on n_{cA} instead of on n_c. We write $n_c = fn_{cA}$ and calculate f from data in Table 1; this again involves a useful but heroic assumption that f is a constant. We use the same model specification as in the externality case, but with n_{cA} replacing n_c – that is, $\xi = (B + Cn_{cA})^{\varepsilon}$ – and proceed as before.[32] With this specification, we obtain the increase in growth rate to be 0.07 percentage points while we move from a progressivity of 1.5 and an increase of 0.13 percentage points from a progressivity of 2.0. Relative to the benchmark, we overstate the fraction of labor in R&D, and share of GDP devoted to R&D by about one percentage point and the share devoted to education by 0.5 percentage point. Thus, while the growth effects are lower with this calibration, it must be borne in mind that the calibration involved a tortuous extraction of parameters from available empirical results, that the fit of the targets is worsened, especially in the adoption case, and that the final result is in the same direction as the earlier results.

7 Conclusions

Our analysis of tax progressivity in a heterogeneous-agent, endogenous growth framework is a step toward analyzing the structural linkages between growth, inequality, and welfare. It is interesting that a less progressive tax system, which is rarely perceived as an egalitarian measure, gives rise to increased growth, decreased inequality, and greater mobility for the poor in the long run, especially in light of

[30] This is reported in their Table V for the 1974-91 time period.

[31] The parameter values used are: B = .0485, C = 1.4869, and ε = .6147. When B is set to zero, we retrieve the benchmark specification with one less free parameter.

[32] The parameter values used are: B = .0313, C = 5.1003, and ε = .381. When ε is set to 1, we retrieve the benchmark specification with one less free parameter.

contradicting claims in the literature regarding the connection between growth and inequality.[33]

Experiments on a calibrated model indicate the quantitative effects are economically significant, ranging from 0.13 to 0.53 percentage points. Reform in the *structure* of taxes has more of an effect than reform in the *level* of taxes alone. We also find that the assumption made about the engine of growth matters, both qualitatively and quantitatively. When progressivity decreases, welfare unequivocally increases across balanced growth paths; however, once the transition is taken into account, only the currently rich slightly prefer the flat-rate system. While the long-run welfare gains of moving to a flat rate system are high, so are the costs of transition, resulting in little change in aggregate welfare; the effect, if anything, is slightly negative. The exploration of debt-based schemes and gradual phase out of progressivity to soften the blow for the initially poor and make them "buy into" the flat rate system seem useful avenues for future research.

A Appendix

A.1 Transitions

We seek a system of difference equations in the following variables when growth is driven by externality: n_{ct}, the share of skilled labor in the economy, $\frac{\Lambda_{t+1}}{[(1-\tau_s)w_{st}]^{1-\sigma}}$, a normalized version of the value to being skilled, and $z_t \equiv \frac{K_t}{A_t}$, the capital stock normalized by the productivity level. These normalizations help in analyzing the system in terms of variables that settle down to stationary values on the BGP; on the BGP, Λ grows at the gross rate $(1+g)^{1-\sigma}$, w_s, K, and A grow at the gross rate $(1+g)$, so the last two variables will indeed settle down.

The steady state values of the normalized quantities in terms of the four BGP quantities determined in the main text, p^*, e_s^*, g, and n_c^*, are:

$$\begin{aligned}\left(\frac{\Lambda}{[(1-\tau_s)\,w_s]^{1-\sigma}}\right)^* &= \frac{p^*}{\beta\gamma_s B_s}\frac{(e_s^*)^{1-\gamma_s}}{(1-p^*e_s^*)^{\sigma}},\\ z^* &= [\theta\,(n_c^*)^{\nu}+(1-\theta)\,(1-n_c^*)^{\nu}]^{\frac{1}{\nu}}\left(\frac{K}{Y}\right)_*^{\frac{1}{1-\alpha}},\end{aligned}$$

where the capital-to-output ratio is given on the BGP as before: $\left(\frac{K}{Y}\right)_* = \frac{\alpha(1-\tau_e)}{(1+\rho)(1+g)^{\sigma}-(1-\delta)}$. Note we use the parametrization $\pi_c(e) = B_c e^{\gamma_c}$; $\pi_s(e) = B_s e^{\gamma_s}$

The law of motion for n_c is:

$$n_{ct+1} = n_{ct}B_c e_{ct}^{\gamma_c} + (1-n_{ct})\,B_s e_{st}^{\gamma_s}. \tag{25}$$

The difference equation for the value to being skilled can be derived similar to the equations in the main text

[33] For instance, Forbes (2000) concludes that a reassessment of the linkages between inequality, growth, and their determinants is warranted. Our study points to the progressivity of taxes as one such structural linkage.

as:

$$
\begin{aligned}
\frac{\Lambda_t}{\left[(1-\tau_s) w_{st-1}\right]^{1-\sigma}}\left(\frac{w_{st-1}}{w_{st}}\right)^{1-\sigma} &= \frac{(1-e_{ct})^{1-\sigma}\left(\frac{p_t}{\Theta}\right)^{1-\sigma}-(1-p_t e_{st})^{1-\sigma}}{1-\sigma} \\
&\quad +\beta\left(B_c e_{ct}^{\gamma_c}-B_s e_{st}^{\gamma_s}\right)\frac{\Lambda_{t+1}}{\left[(1-\tau_s) w_{st}\right]^{1-\sigma}}. \qquad (26)
\end{aligned}
$$

To get the wage growth term in this equation in terms of the system variables, one needs to take derivatives of the production function, use the definition of z, and use the law of motion for A to get:

$$
\frac{w_{st-1}}{w_{st}}=\frac{1}{1+\xi(n_{ct-1})}\left(\frac{z_{t-1}}{z_t}\right)^{\alpha}\left[\frac{\theta(n_{ct-1})^{\nu}+(1-\theta)(1-n_{ct-1})^{\nu}}{\theta(n_{ct})^{\nu}+(1-\theta)(1-n_{ct})^{\nu}}\right]^{\frac{1-\alpha}{\nu}-1}\left(\frac{1-n_{ct}}{1-n_{ct-1}}\right)^{1-\nu}.
$$

To get the difference equation for z we start with the entrepreneur's Euler equation:

$$
\left(\frac{c_{et+1}}{c_{et}}\right)^{\sigma}=\beta\left[1+(1-\tau_e)\alpha\frac{Y_{t+1}}{K_{t+1}}-\delta\right],
$$

where we can write $c_{et}=(1-\tau_e)\alpha Y_t-K_{t+1}+(1-\delta)K_t$. Divide throughout by A_t, use the definition of z to get:

$$
\frac{c_{et}}{A_t}=(1-\tau_e)\alpha\frac{Y_t}{A_t}-z_{t+1}(1+\xi(n_{ct}))+(1-\delta)z_t.
$$

One can therefore rewrite the above Euler equation and get the equation for z as:

$$
\frac{(1-\tau_e)\alpha\frac{Y_{t+1}}{K_{t+1}}z_{t+1}-z_{t+2}(1+\xi(n_{ct+1}))+(1-\delta)z_{t+1}}{(1-\tau_e)\alpha\frac{Y_t}{K_t}z_t-z_{t+1}(1+\xi(n_{ct}))+(1-\delta)z_t}(1+\xi(n_{ct}))=\beta^{\frac{1}{\sigma}}\left[1+(1-\tau_e)\alpha\frac{Y_{t+1}}{K_{t+1}}-\delta\right]^{\frac{1}{\sigma}}. \qquad (27)
$$

In terms of the system variables, the Y/K variable used above is given as:

$$
\frac{Y_t}{K_t}=\frac{\left[\theta(n_{ct})^{\nu}+(1-\theta)(1-n_{ct})^{\nu}\right]^{\frac{1-\alpha}{\nu}}}{z_t^{1-\alpha}}.
$$

Equations (25) through (27) completely characterize the dynamics of the system when growth is driven by externality.

The transition path we are interested in will be from the BGP of one tax regime to the BGP of the new tax regime. The starting and ending BGP values can be computed in a straightforward way, given those the above difference equations can be used to compute the transition paths using a standard relaxation method.

When growth is driven by intentional adoption, one has an additional state variable in $n_{cA,t}$. This gives rise to a fourth difference equation; this is the main change when compared to the earlier transitions. The steady state quantities are n_{cA}^*, n_c^*, and:

$$
\begin{aligned}
\left(\frac{\Lambda}{\left[(1-\tau_s) w_s\right]^{1-\sigma}}\right)^* &= \frac{p^*}{\beta\gamma_s B_s}\frac{(e_s^*)^{1-\gamma_s}}{(1-p^* e_s^*)^{\sigma}}, \\
z^* &= \left[\theta(n_c^*-n_{cA}^*)^{\nu}+(1-\theta)(1-n_c^*)^{\nu}\right]^{\frac{1}{\nu}}\left(\frac{K}{Y}\right)_*^{\frac{1}{1-\alpha}},
\end{aligned}
$$

where the capital-to-output ratio is given on the BGP as before: $\left(\frac{K}{Y}\right)_*=\frac{\alpha(1-\tau_e)}{(1+\rho)(1+g)^{\sigma}-(1-\delta)}$.To derive the difference equation for $n_{cA,t}$, use the equations in (15) to get:

$$
\beta\left[(1-\alpha)\frac{Y_t}{w_{ct}}\xi'(n_{cA,t})+(1+\xi(n_{cA,t}))\right]\frac{w_{ct}u'(c_{et})}{A_t\xi'(n_{cA,t})}=\frac{w_{ct-1}u'(c_{et-1})}{A_{t-1}\xi'(n_{cA,t-1})}.
$$

Use the entrepreneur's Euler equation to eliminate consumption and get:

$$\left(\frac{w_{ct}}{w_{ct-1}}\right)\left(\frac{A_{t-1}}{A_t}\right)\frac{\xi'\left(n_{cA,t-1}\right)}{\xi'\left(n_{cA,t}\right)}\left[(1-\alpha)\frac{Y_t}{w_{ct}}\xi'\left(n_{cA,t}\right)+\left(1+\xi\left(n_{cA,t}\right)\right)\right]=1+\left(1-\tau_e\right)\alpha\frac{Y_t}{K_t}-\delta.$$

Using the law of motion for A and simplifying, we get the difference equation in n_{cA} as:

$$\left(\frac{w_{ct}}{w_{ct-1}}\right)\frac{\xi'\left(n_{cA,t-1}\right)}{1+\xi\left(n_{cA,t-1}\right)}\left[(1-\alpha)\frac{Y_t}{w_{ct}}+\frac{\left(1+\xi\left(n_{cA,t}\right)\right)}{\xi'\left(n_{cA,t}\right)}\right]=1+\left(1-\tau_e\right)\alpha\frac{Y_t}{K_t}-\delta. \tag{28}$$

Note that this will reduce to condition (18) in the paper on the BGP. In the above expression:

$$\begin{aligned}\frac{Y_t}{w_{ct}} &= \frac{\left(n_{ct}-n_{cA,t}\right)^{1-\nu}\left[\theta\left(n_{ct}-n_{cA,t}\right)^{\nu}+(1-\theta)\left(1-n_{ct}\right)^{\nu}\right]}{\theta(1-\alpha)}\\ \frac{w_{ct}}{w_{ct-1}} &= \left(1+\xi\left(n_{cA,t-1}\right)\right)\left(\frac{z_t}{z_{t-1}}\right)^{\alpha}\left[\frac{\theta\left(n_{ct}-n_{cA,t}\right)^{\nu}+(1-\theta)\left(1-n_{ct}\right)^{\nu}}{\theta\left(n_{ct-1}-n_{cA,t-1}\right)^{\nu}+(1-\theta)\left(1-n_{ct-1}\right)^{\nu}}\right]^{\frac{1-\alpha}{\nu}-1}\left(\frac{n_{ct-1}-n_{cA,t-1}}{n_{ct}-n_{cA,t}}\right)^{1-\nu}.\end{aligned}$$

Most of the household equations do not change. The only other change of any significance is now:

$$c_{et}=\left(1-\tau_e\right)\left(\alpha Y_t-n_{cA,t}w_{ct}\right)-K_{t+1}+(1-\delta)K_t.$$

This will be reflected in the difference equation in z which is:

$$\frac{\left(1-\tau_e\right)\frac{Y_{t+1}}{K_{t+1}}z_{t+1}\left(\alpha-n_{cA,t+1}\frac{w_{ct+1}}{Y_{t+1}}\right)-z_{t+2}\left(1+\xi\left(n_{cA,t+1}\right)\right)+(1-\delta)z_{t+1}}{\left(1-\tau_e\right)\frac{Y_t}{K_t}z_t\left(\alpha-n_{cA,t}\frac{w_{ct}}{Y_t}\right)-z_{t+1}\left(1+\xi\left(n_{cA,t}\right)\right)+(1-\delta)z_t}\left(1+\xi\left(n_{cA,t}\right)\right)=\beta^{\frac{1}{\sigma}}\left[1+\left(1-\tau_e\right)\alpha\frac{Y_{t+1}}{K_{t+1}}-\delta\right]^{\frac{1}{\sigma}}. \tag{29}$$

The inverse of $\frac{w_c}{Y}$ was given earlier, and $\frac{Y}{K}$ is given as:

$$\frac{Y_t}{K_t}=\frac{\left[\theta\left(n_{ct}-n_{cA,t}\right)^{\nu}+(1-\theta)\left(1-n_{ct}\right)^{\nu}\right]^{\frac{1-\alpha}{\nu}}}{z_t^{1-\alpha}}.$$

Equations (25), (26), (28), and (29) are the four difference equations that govern the dynamics of the economy when growth is driven by technology adoption.

The equations governing the dynamics when education investments are non-exempt are identical except for (26), which will now be derived from (23) and (24).

We only summarize the qualitative aspects of the transition paths here; graphs of actual paths are available from the authors. In the externality case, the endogenous state variables, n_c and z, do not jump at the time of the unexpected policy change. The share of skilled labor increases monotonically to its final level in response to a decrease in Θ, since the wedge in return to skill created by progressivity now vanishes and more people acquire skills. The increase in skilled labor is preferentially beneficial to technological change; therefore, the ratio of capital to technology decreases steadily to its final level. The (normalized) value to being skilled jumps at the time of the policy change. In fact, it overshoots its final value on account of the initial fixed nature of the stock of skilled labor. It takes time for the households to respond to the increase in value to being skilled by acquiring skills; as n_c increases over time, the value to being skilled drops to its final level. This is reflected in the skill premium, p, which does not jump given that n_c does not jump, but declines over time as the fraction of skilled increases. The investment in education by both types jump at time zero, with a prominent overshooting for the rich. The interest rate increases over time to its final value consistent with a decreasing z.

In the adoption case, the behavior of n_c, z, and the value to being skilled are similar to those in the externality case. The share of labor devoted to "R&D" does not jump significantly at the time of the policy change. The intuition for this can be gleaned by examining (19). If n_c is unchanged initially, a steep increase in n_{cA} will cause the premium to shoot up; the firm will have to pay more for its skilled labor. As households respond to the decreased progressivity and become more educated, n_c increases, which makes an increase in n_{cA} more affordable to the firm. In (19) as n_c increases, n_{cA} can increase without causing the premium to shoot up. Indeed, the time path for n_{cA} is found to be remarkably similar to that for n_c. The fact that increasing A is now costly, and it is optimal for firms to increase it sluggishly means that K will adjust faster than A, causing z to initially increase slightly before it settles down to its final value. Corresponding to this, the interest rate initially drops and increases over time to its final value on the BGP. Given that increased growth exerts an upward pressure on the premium, the premium drops less dramatically than it does in the externality case.

A.2 Welfare Measures

We can compute two types of welfare measures to evaluate progressivity policies – one an equally weighted aggregate measure, which will help us see in an overall sense how a policy fares, and individual welfare measures for those initially skilled or unskilled to study the distributional implications. We compute these measures only for the households and not the entrepreneur – there is no heterogeneity in the latter, making the welfare computation of the entrepreneur a bit uninteresting. In all cases we will assume that the BGP and transition quantities have been obtained.

A.2.1 Aggregate welfare

This measure is fairly straightforward to compute. First denote the individual consumptions as:

$$\begin{aligned} c_{ct} &= (1-\tau_c)(1-e_{ct})\, w_{ct} \\ c_{st} &= (1-\tau_s)\left(\frac{1}{p_t} - e_{st}\right) w_{ct}. \end{aligned}$$

At time t aggregate welfare is $W_t = n_{ct}\frac{c_{ct}^{1-\sigma}}{1-\sigma} + (1-n_{ct})\frac{c_{st}^{1-\sigma}}{1-\sigma}$, which amounts to:

$$W_t = \left\{ n_{ct}\left[(1-\tau_c)(1-e_{ct})\right]^{1-\sigma} + (1-n_{ct})\left[(1-\tau_s)\left(\frac{1}{p_t} - e_{st}\right)\right]^{1-\sigma} \right\} \frac{w_{ct}^{1-\sigma}}{1-\sigma}.$$

The aggregate welfare on a BGP is given by:

$$\begin{aligned} W^{BGP,0} &= \sum_{t=0}^{\infty} \beta^t (1+g)^{(1-\sigma)t} \left\{ n_c^*\left[(1-\tau_c)(1-e_c^*)\right]^{1-\sigma} + (1-n_c^*)\left[(1-\tau_s)\left(\frac{1}{p^*} - e_s^*\right)\right]^{1-\sigma} \right\} \frac{w_{c0}^{1-\sigma}}{1-\sigma} \\ &= \frac{\left\{ n_c^*\left[(1-\tau_c)(1-e_c^*)\right]^{1-\sigma} + (1-n_c^*)\left[(1-\tau_s)\left(\frac{1}{p^*} - e_s^*\right)\right]^{1-\sigma} \right\}}{\left[1-\beta(1+g)^{(1-\sigma)}\right]} \cdot \frac{w_{c0}^{1-\sigma}}{1-\sigma}. \end{aligned} \tag{30}$$

This welfare can be computed only relative to a starting wage; hence the superscript zero in $W^{BGP,0}$ to indicate w_{c0} is being used. If we use $W^{BGP,T}$ it means w_{cT} is being used. To compare two regimes the same starting wage needs to be used; we normalize w_{c0} to one.

Suppose transition from one BGP to another is almost completed at time T. Then the aggregate welfare including transition is given by:

$$W^{new} = \sum_{t=0}^{T-1} \beta^t W_t + \beta^T W^{BGP,T}.$$

This assumes we know $w_{c0} \cdots w_{cT}$, which is where the normalization $w_{c0} = 1$ is helpful. Use that and use expressions for $\frac{w_{ct}}{w_{ct-1}}$ from the earlier section, to get the sequence of wages, including the w_{cT} needed to get $W^{BGP,T}$.

The aggregate welfare on the BGP of the pre-policy change regime, W^{old}, can be computed using (30); all the starred quantities will correspond to the old BGP and $w_{c0} = 1$. The consumption equivalence ω can be obtained from:

$$(1+\omega)^{1-\sigma} W^{old} = W^{new}.$$

A.2.2 Individual welfare

To first get individual welfare on the BGP at an arbitrary time zero, start with the maximized Bellman equations:

$$\begin{aligned} V_c &= u(c_{c0}) + \beta\pi_c(e_c^*)(1+g)^{1-\sigma} V_c + \beta(1-\pi_c(e_c^*))(1+g)^{1-\sigma} V_s \\ V_s &= u(c_{s0}) + \beta\pi_s(e_s^*)(1+g)^{1-\sigma} V_c + \beta(1-\pi_s(e_s^*))(1+g)^{1-\sigma} V_s. \end{aligned}$$

Simplifying, we get the following linear system:

$$\begin{bmatrix} 1-\beta\pi_c(e_c^*)(1+g)^{1-\sigma} & -\beta(1-\pi_c(e_c^*))(1+g)^{1-\sigma} \\ -\beta\pi_s(e_s^*)(1+g)^{1-\sigma} & 1-\beta(1-\pi_s(e_s^*))(1+g)^{1-\sigma} \end{bmatrix} \begin{bmatrix} V_c^{BGP,0} \\ V_s^{BGP,0} \end{bmatrix} =$$

$$\begin{bmatrix} [(1-\tau_c)(1-e_c^*)]^{1-\sigma} \\ \left[(1-\tau_s)\left(\frac{1}{p^*}-e_s^*\right)\right]^{1-\sigma} \end{bmatrix} \frac{w_{c0}^{1-\sigma}}{1-\sigma},$$

which can be solved for the BGP welfare given all the other quantities.

Since the time T, when the economy is almost on the new BGP, is usually small in our case, we can "draw" the event tree for each type of agent starting at time zero, when the policy change occurs. The probabilities of each branch, the discount factor weighted utility at each node, are all known; the terminal nodes at time T will have value $\beta^T V_c^{BGP,T}$ or $\beta^T V_s^{BGP,T}$. One can compute welfare at time zero as a probability weighted sum of node utilities. In other words, we do not have to simulate the economy to compute the individual welfare measures, given the discreteness of types and rapid convergence to steady states.

We can then compute the consumption equivalences as above:

$$
\begin{aligned}
(1+\omega_s)^{1-\sigma} V_s^{BGP,old} &= V_s^{BGP,old} \\
(1+\omega_c)^{1-\sigma} V_c^{BGP,new} &= V_c^{BGP,new}.
\end{aligned}
$$

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