Does the Progressivity of Taxes Matter for Economic Growth?*

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Abstract

In this paper, we develop a general equilibrium model of endogenous growth with heterogeneity in income and tax rates in order to study the effect of progressivity on economic growth. We limit heterogeneity to two types – skilled and unskilled – and posit that the probability of staying or becoming skilled in the subsequent period depends positively on expenses on teacher time. In the production sector, we consider two sources of growth. In the first, growth arises as a purely external effect on account of production activities of skilled workers. In the second, a portion of the skilled workforce is used to work in research and other productivity enhancing activities and is compensated for it. Our analysis shows that changes in the progressivity of tax rates can have positive growth effects even in situations where changes in flat rate taxes have no effect. The assumption made about the engine of growth is important in assessing the effect of changes in progressivity; the response is stronger when growth is driven by externality.

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1 Introduction

The growth effects of tax reform have typically been studied in representative agent models by considering changes in flat rate taxes. The initial influence for this literature is the study by Lucas (1990), who uses an endogenous growth model in which human capital is the engine of growth. He provides a theoretical basis for the small growth effects he finds in his quantitative exercise: " ... changes in labor taxation affect equally both the cost and the benefit side of the marginal condition governing the learning decision."[1]

The tax system in the US and several other countries is progressive.[2] And, as Heckman and Klenow (1997) note, the progressivity of taxes causes individuals to move up tax brackets when they accumulate human capital: "In this case, the returns from investment are taxed at a higher rate, but the cost is expensed at a lower rate." They then verbally argue that this wedge in the return "discourages human capital accumulation" – a conclusion different from the one reached by Lucas for flat rate taxes.[3] In this paper, we develop a general equilibrium model of endogenous growth with heterogeneity in income and tax rates, in order to study the effect of a change in the *progressivity* of

[1] Stokey and Rebelo (1995) use a more general model to assess the studies of Lucas (1990) and others in the literature to conclude that the small tax effect on the U.S. growth rate is robust; see their paper for a comprehensive list of references.

[2] According to the *Statistical Abstracts of the United States* the lowest and highest marginal tax rates for married couples with two dependents when tax brackets are expressed in 1980 dollars, were 0% and 43% in 1960, -10% and 50% in 1975, and 12.5% and 42% in 1984. Pechman (1985) concludes that the *net* of transfers tax burden as a percent of adjusted family income varies from -65.5% for the first population decile to 26% for the tenth decile.

[3] The marginal condition in Lucas (1990) which governs time spent by agents in skill accumulation is:

$$w(t)\,h(t) = G'[v(t)] \int_t^{\infty} \exp\left\{-\int_t^{s} (r(\varsigma)-\lambda)\, d\varsigma\right\} u(s)\, w(s)\, h(s)\, ds,$$

where, w is the rental rate of human capital, h the stock of human capital, G is a human capital production function that governs the evolution of human capital according to $\dot{h}(t) = h(t)\, G[v(t)]$, $v(t)$ is the time spent in accumulating human capital, $u(t)$ is the time spent working, r is the interest rate, and λ is the effective depreciation rate that includes population growth. The left hand side is the marginal cost of allocating an extra unit of time to human capital accumulation – the wage rate – and the right hand side is the marginal benefit – the marginal product weighted present value of future wages earned on account of this accumulation. If τ is the uniform labor income tax rate, it affects the cost and benefit by the same factor and cancels out of both sides. Heckman and Klenow's observation can be accommodated in this condition by using a lower tax rate on the left hand side (while acquiring skills) and a higher one on the right hand side (while earning in a higher tax bracket). By creating a wedge between present and future tax rates, progressivity will reduce the return to human capital accumulation and decrease growth (i.e. decrease v and hence the growth rate $G[v]$).

Progressive taxes can alternately be viewed as a government instituted scheme that provides partial insurance against the risky process of skill accumulation, with accompanying negative effects on *ex ante* incentives. We, however, focus on the first interpretation – that of progressivity driving a wedge in the return to skill accumulation – since this seems more direct.

taxes – rather than a change in the rate of flat taxes – on economic growth.

If the human capital accumulation is done in the presence of liquidity constraints, there is potentially a counter effect to the return effect mentioned above. More progressive taxes will increase income for the poor at the accumulation stage and increase their investment, with the effect being the opposite for the rich. The overall effect of progressivity on skill accumulation and growth therefore needs to be studied in a fully specified model. We construct such a model in this paper and analyze its theoretical implications.[4]

Our economic setup consists of overlapping generations of a large number of two-period lived individuals.[5] In their first period, individuals are children and do not make any decisions. In their second period, they become adults, have their own children, and choose how much to consume and invest in their children's education. The investment in their children's education takes the form of payments for "teacher" time. The educational investment of parents and luck combine to make an adult worker either skilled or unskilled. The fraction of skilled workers in the economy is a key endogenous state variable and the value to being skilled is determined endogenously based on these human capital accumulation decisions. For production, we consider two possibilities for growth in order to broadly capture the flavor of growth engines typically discussed in the new growth literature; this allows us to study the sensitivity of the growth response to the assumption made about its source. In the first case, growth arises purely as an external effect of production activities of skilled workers. In the second, a portion of the skilled workforce is intentionally employed in research and other productivity enhancing activities.

When human capital investment is tax exempt, we find that the long run growth rate is independent of the tax rate in a flat tax system, in line with Lucas' observation above. However, changes in progressivity are not growth neutral; the higher the progressivity, the lower the long-run growth.

We also find that the engine of growth matters crucially in the assessment of a change in progressivity, since the incentive to accumulate human capital is affected in different ways. The external growth case exhibits stronger growth effects than the one in which there is intentional technology adoption. In both cases, an increase in progressivity tends to decrease the human capital investment of both types of agents by creating a wedge in the return to such investment. This in turn increases the equilibrium skill premium which *increases* the incentive to invest. When growth is driven by externality, under mild assumptions on the production elasticities, it is possible to show that the net effect of increased progressivity is decreased human capital accumulation and thus equilibrium growth. When growth is driven by intentional adoption, investment by unskilled agents decreases

[4] In a companion paper (Caucutt, İmrohoroğlu, and Kumar (2002)), we use the model to quantitatively assess the growth and welfare effects of changes in tax progressivity.

[5] As we will discuss later, an infinitely-lived dynastic interpretation can also be given to this model.

unambiguously as in the externality case. If the resulting growth rate is low enough, investment by skilled agents also decreases. However, in the high growth region, the equilibrium effect of increased premium is strong, causing the skilled agents to invest *more* when progressivity increases. The net effect on human capital accumulation and growth is ambiguous.

Overlapping generations models used to study taxes and growth have built-in heterogeneity in age. For example, Uhlig and Yanagawa (1996), consider a two-period overlapping generations model to show that an increase in capital tax shifts income toward the young who have greater propensity to save and thus increases the growth rate. Unlike their model, the heterogeneity in income we are capturing is among agents of the *same* generation. A similar effect is present in Blackenau and Ingram (1999), where higher progressivity increases the saving of the less taxed young who are mostly unskilled. The increase in the capital stock and the return to complementary skilled labor increases the supply of skilled labor. In our model, where capital accumulation plays a secondary role, the effect of progressivity is felt directly on the return to human capital, and has a negative effect on supply of skill. The model in Blackenau and Ingram does not feature sustained endogenous growth and therefore cannot be used to address the key question of our paper.

Yamarik (2001) uses a tax schedule that is a function of income in a representative agent model and finds that increased progressivity reduces transitional growth. He is interested only in the distortionary – not the redistributive effects of taxation on growth – while we are interested in both. In order to capture the differential effects of progressivity and the resulting inequality, we analyze a fully specified heterogeneous agent model in which inequality arises endogenously.[6]

In complementary work, Bovenberg and van Ewijk (1997) develop a heterogeneous-agent model with intergenerational learning spillovers in which progressive taxes hurt long-run growth. Even though there is heterogeneity within and across generations, the growth effect is driven by *inter*generational redistribution – transfers from older to younger generations reduce the growth of after-tax wages and affect the incentive to acquire human capital. Indeed, the imperfectness of the spillover across overlapping generations is necessary for progressivity to affect growth. All households, rich or poor, invest the same amount in learning, which simplifies aggregation. Our model, on the other hand, features *intra*generational heterogeneity in both income and human capital investment, and deals with the concomitant issue of aggregation.[7]

Li and Sarte (2001) also develop a model of progressive taxes and growth, but their principal aim is to argue that the distortionary effects of a higher marginal tax cannot be captured by the share

[6] The mapping of a representative agent framework to a world with progressive taxes would involve the empirically untenable assumption of perfect insurance among agents who are in different tax brackets.

[7] See Caucutt and Kumar (forthcoming) who formally argue that when households are liquidity constrained, educational investments differ by income; they cite empirical empirical evidence consistent with this outcome.

of government expenditure in GDP; in fact economic growth need not fall, and could even increase with this share. Heterogeneity in their model is in the discount factor of households, while in ours it is along the dimensions of skill and income.

Benabou (2002) constructs an analytically tractable dynamic heterogeneous-agent model in order to study the equity-efficiency tradeoff of fiscal and educational redistributions through progressive taxation. He finds that either type of redistribution is effective at substituting for missing insurance and credit markets; however, progressive education finance is more efficient. His main focus is on long-run levels; he then mentions a strategy for accommodating growth in a "heterogeneity-neutral" way using spillovers in the accumulation of human capital. Since our main focus is on the interaction among progressivity, heterogeneity, and long run growth, growth is inherently non-neutral with respect to heterogeneity in our model; we achieve tractability by limiting the number of agent types.

As an aside, our modeling of skill acquisition allows for the possibility that the accumulation of all three factors – human capital, physical capital, and technology – is consistent with a balanced growth path. Most existing growth models allow for the endogenous accumulation of only two factors for the technical ease of obtaining a balanced growth path.

The rest of the paper is structured in the following way. In Section 2 we describe the economic environment. The balanced growth path equilibrium is analyzed and characterized in Section 3 and Section 4 concludes.

2 The Model

Households supply labor of differing skills and use the wage for consumption and investment in human capital. They are the only heterogeneous entities.

2.1 Human Capital Accumulation

Following Caucutt and Kumar (forthcoming), we develop a model in which heterogeneity is limited to two types of skill levels.[8] As in Rogerson (1988), we achieve convexification by making the process of skill accumulation probabilistic. At any instant, the economy is populated by two types of adult workers we call "skilled" (subscripted c, for college-educated workers) and "unskilled" (subscripted s, for school-educated workers), with total measure one. There is no population growth. Let n_c denote the fraction of skilled agents in the economy in any given period. Each adult has a child and can hire a skilled teacher for a fraction of the teacher's time, e, to educate her. With this input, the probability that the child of a type-i agent, $i = c, s$, becomes skilled is given by $\pi_i(e_i)$;

[8] The model there is used to study the effect of subsidies in higher education. In addition to adding growth to that model, we abstract from heterogeneity in academic ability and allow varying levels of investment in skill.

with probability $(1 - \pi_i(e_i))$ the child fails and is an unskilled adult in the following period. The probability functions are subscripted because children of skilled agents might have advantages other than just higher education expenditure as they could have better schooling at the earlier levels, better role models, etc. That is, we expect $\pi_c(e) \geqslant \pi_s(e)$, $\forall\ e \in (0,1)$. Additionally, we assume:

$$0 < \pi_i < 1,\ \pi_i' > 0,\ \pi_i'' < 0,\ \pi_i(0) = 0.$$

The increasing nature of π needs little elaboration. We assume concavity in the probability function, in line with the diminishing return to instantaneous investment found in most models of education. When nothing is spent on the child's education, the child remains unskilled with certainty as an adult. We use "skilled" ("unskilled"), "rich" ("poor"), and "college-educated" ("school-educated") interchangeably to refer to the two types of agents.

Note that children make no economic decision and an agent's own lifetime utility depends only on utility from consumption when adult. It is assumed that adults have altruistic preferences. Their lifetime utility consists of the sum of their own lifetime utility and the discounted lifetime utility of their child. This two-period overlapping generations structure is equivalent to an alternative specification under which there are infinitely lived dynasties that switch between skilled and unskilled states based on their investments in a stochastic skill accumulation technology. A skilled agent has to keep updating her skills (accumulate human capital) in order to stay skilled. An agent who is originally unskilled, has to accumulate human capital to *become* skilled. Under this interpretation, when an unskilled agent spends nothing on skill accumulation she will remain unskilled, $\pi_s(0) = 0$. However, when a skilled agent spends nothing she might have a positive probability of staying skilled, $\pi_c(0) > 0$. [9]

We can characterize the choice problems of skilled and unskilled agents recursively. The Bellman equation for a skilled agent, who takes wages as given, is

$$V_c(n_c) = \max_{e_c} \left\{ u\left((1-\tau_c)(1-e_c)\, w_c(n_c)\right) + \beta \pi_c(e_c)\, V_c\left(n_c'\right) + \beta\left(1 - \pi_c(e_c)\right) V_s\left(n_c'\right) \right\}, \quad (1)$$

where n_c is the aggregate (endogenous) state variable, and τ_c and τ_s denote the tax rates on labor incomes of the skilled and unskilled agents, respectively.

The Bellman equation for an unskilled agent is given by

$$V_s(n_c) = \max_{e_s} \left\{ u\left((1-\tau_s)(1-e_s p(n_c))\, w_s(n_c)\right) + \beta \pi_s(e_s)\, V_c\left(n_c'\right) + \beta\left(1 - \pi_s(e_s)\right) V_s\left(n_c'\right) \right\}, \quad (2)$$

[9] While Caucutt and Kumar (forthcoming) connect failure in skill accumulation with college dropout rates, in the present context failure can additionally stand in for idiosyncratic labor productivity shocks, personal and small business bankruptcies, and shocks to the industry for which a particular skill is most suitable.

where $p(n_c) \equiv \frac{w_c(n_c)}{w_s(n_c)}$ is the skill premium. Here, we assume that the unskilled agent also needs to hire a skilled person as a teacher, so the cost is $e_s w_c(n_c) = e_s p(n_c) w_s(n_c)$.[10]

Let $\Theta \equiv \frac{(1-\tau_s)}{(1-\tau_c)}$ denote our measure of progressivity of taxes.[11] Also let $n_c' = \Phi(n_c)$ denote the agents' perceived law of motion for the endogenous state variable.

The first order conditions for skill accumulation for the two types of agent can now be written as

$$\beta \pi_c'(e_c)\Lambda(n_c') = (1-\tau_c) w_c(n_c) u'((1-\tau_c)(1-e_c) w_c(n_c)), \tag{3}$$

$$\beta \pi_s'(e_s)\Lambda(n_c') = p(n_c)(1-\tau_s) w_s(n_c) u'((1-\tau_s)(1-e_s p(n_c)) w_s(n_c)). \tag{4}$$

where $\Lambda(n_c) \equiv V_c(n_c) - V_s(n_c)$ can be viewed as the value to being skilled. Inada conditions on the utility and probability functions ensure $0 < e_i < 1$. The left hand side is the marginal benefit of investing in human capital for either type – the value to being skilled weighted by the discount factor and the marginal productivity of the investment. The right hand side is the cost of accumulating human capital, weighted by the agent's marginal utility. Since the cost of education is the wage of a skilled agent, an increase in the skill premium increases the marginal cost through a direct multiplier in addition to the increase in the marginal utility of the liquidity-constrained agent.

Evaluating the Bellman equations for the two types of agents at the optimal policies $e_c(n_c)$ and $e_s(n_c)$ and subtracting one from the other, we get an expression of how the value to being skilled evolves,

$$\Lambda(n_c) = u(c_c(n_c)) - u(c_s(n_c)) + \beta(\pi_c(e_c(n_c)) - \pi_s(e_s(n_c)))\Lambda(n_c'), \tag{5}$$

where

$$c_c(n_c) \equiv (1-\tau_c)(1-e_c(n_c)) w_c(n_c),$$

$$c_s(n_c) \equiv (1-\tau_s)(1-e_s(n_c) p(n_c)) w_s(n_c).$$

The value to being skilled has two parts – a current (potential) increase in utility from being skilled and a greater chance of realizing the future value of being skilled. The quantity $\beta(\pi_c(e_c) - \pi_s(e_s))$ can be interpreted as an endogenous discount factor which increases with the difference in investment between the two types.

The law of motion for the fraction of skilled workers can be written as

$$\Phi(n_c) \equiv n_c' = n_c \pi_c(e_c(n_c)) + (1-n_c)\pi_s(e_s(n_c)). \tag{6}$$

Equations (3) to (6) characterize the dynamics of the household sector through the four functions $e_c(n_c)$, $e_s(n_c)$, $\Lambda(n_c)$, and $\Phi(n_c)$ for any given wage functions $w_c(n_c)$ and $w_s(n_c)$. The matrix that gives the transition probabilities between the skilled and unskilled states is

[10] Eicher (1996) also assumes that the cost of accumulating human capital is the value of skilled time.

[11] This appears to be a natural way to capture the degree of statutory progressivity when heterogeneity is limited to two types. See Suits (1977) and Kakwani (1977) for indices of progressivity based on the Lorenz curve.

$$\begin{pmatrix} & \textbf{skilled} & \textbf{unskilled} \\ \textbf{skilled} & \pi_c(e_c) & 1-\pi_c(e_c) \\ \textbf{unskilled} & \pi_s(e_s) & 1-\pi_s(e_s) \end{pmatrix}.$$

The modeling decisions made regarding human capital accumulation have important implications for tax policy. For this reason, we discuss these assumptions below.

1. Agents investing in human capital are liquidity constrained. If Lucas' condition were extended to accommodate progressivity, as discussed in footnote 3, the only effect of progressivity is a return effect. If progressivity falls, the opportunity cost of accumulating human capital decreases for poor agents, while the return to being skilled increases, causing their investment in human capital to increase. As we will see, this effect is preserved in our model. We have an additional income effect arising from the liquidity constraint; lower progressivity is likely to decrease investment of the poor relative to that of the rich. The economy-wide investment then depends on the relative strengths of investments made by the two types. The liquidity constraint can be used to motivate governmental interest in differential taxation. However, as is clear from the discussion of the above condition it is not *required* for progressivity to affect growth; the assumption is driven by empirical plausibility, and is indeed a conservative assumption for illustrating the growth effect of progressivity.

2. Human capital investment is subject to diminishing returns, in line with assumptions in most of the literature. The absence of concavity would lead to the empirically unsavory implication that skilled agents find investment so attractive that they are willing to put up with lower current utility than the unskilled agents. Diminishing returns, in the presence of liquidity constraints, also affects the investment by the two types to different degrees when the tax policy changes.

3. Human capital investment is tax exempt. With tax-exempt investment, changes in flat-rate taxes are growth neutral. Hence, this case allows us to *isolate* the effect of the progressivity of taxes.[12] With tax-exemption, the effect of tax changes on marginal costs (which we term the "liquidity effect") is more muted. For instance, an increase in the tax rate on the rich decreases income, but provides an incentive to get a tax exemption by spending more on human capital. We briefly discuss the case where the tax is levied on the entire wage.

4. Both types of agents need skilled teachers' time to accumulate human capital. Having each type of agent use only her own time to become skilled is not only unappealing *a priori*, but also

[12] Stokey and Rebelo (1995) point out that small effects of taxes on growth follow from the empirically justifiable assumption of high factor shares for human capital in production and relatively light taxation of the human capital producing sector. One interpretation of Lucas (1990) is that the human capital producing sector is completely untaxed.

gives the empirically implausible result of poor agents investing more in human capital than the rich, since the poor have a lower opportunity cost. This is a modeling issue unique to a heterogeneous agent formulation of skill acquisition.[13]

2.2 Production and Growth

A third type of agent – an infinitely-lived entrepreneur – carries out production and has preferences identical to the two types of workers, producing the single consumption good according to the production function

$$Y = A^{1-\alpha} K^{\alpha} \left[\theta N_c^{\nu} + (1-\theta) N_s^{\nu}\right]^{\frac{1-\alpha}{\nu}}, \tag{7}$$

where N_c is the measure of skilled labor hired, N_s the measure of unskilled labor, and $0 < \nu < 1$. Here K is the physical capital used in production, which we assume is accumulated only by the producer.[14]

This entrepreneur's consumption is

$$c_e = (1-\tau_e)(Y - w_c N_c - w_s N_s) - I. \tag{8}$$

Here, τ_e is the tax rate on the entrepreneur's profits, and I is the investment in physical capital, which evolves according to

$$K' = I + (1-\delta) K. \tag{9}$$

Unlike human capital investment, physical capital investment is not tax exempt; we elaborate on the effect of this assumption later. The Bellman equation for the third type of agent is

$$W(K, A) = \max_{N_c, N_s, I} \left\{ u(c_e) + \beta W\left(K', A'\right) \right\}, \tag{10}$$

subject to (7), (8), (9), and the law of motion for A to be described below.

[13] Lucas (1988) assumes that human capital accumulation evolves according to $\dot{h} = Buh$, where u is *own* time spent in skill acquisition. This is the lead followed by several tax-and-growth studies.

In the infinite horizon view of our model, while an opportunity cost interpretation can be given for those currently skilled, for the currently unskilled the cost is more than just foregone wages since they hire skilled teachers. The preceding discussion has been couched entirely in terms of teacher time, ignoring time spent by the workers themselves in getting educated. In reality, each type of agent will face a cost that depends both on the value of the agent's own time and the teacher's time. Indeed, the cost of the unskilled agent can be viewed as a composite of opportunity and tuition costs.

[14] This limits heterogeneity to skill accumulation and keeps physical capital accumulation simple. Capital income taxes affect only the richest individuals in the US, and the omission of capital accumulation by individuals might, if anything, understates the degree of progressivity that actually exists.

Also, as is shown during the course of the BGP analysis, skill acquisition is the "engine" of growth and physical capital merely keeps pace with this growth. Capturing the effect of progressivity on skill acquisition therefore seems more important.

We consider two growth specifications that are commonly used in the literature – productivity improvement arising as an externality and arising due to intentional use of human capital by the firm.[15] Broadly considering the sources of growth that have been extensively discussed in the new growth literature, allows us to study the sensitivity of growth response to the assumption made about its source.

2.2.1 Growth Driven by Human Capital Externality

The first specification assumes that growth is a result of externalities from productive activities of skilled workers. That is, the mere hiring of skilled employees in the production process is enough to generate productivity improvements; they will not be compensated for it. We assume that the productivity parameter in the production function evolves according to

$$A_{t+1} = \left(1 + \xi\left(N_c\right)\right) A_t, \tag{11}$$

where ξ is the externality function.[16] Optimization by the entrepreneur implies that the skill premium is given by

$$p = \frac{\theta}{1-\theta}\left(\frac{N_s}{N_c}\right)^{1-\nu}. \tag{12}$$

The entrepreneur's before-tax profits are given by αY, from which consumption, investment, and tax payments have to be met. The first order and envelope conditions for the dynamic program (10) are

$$\begin{aligned} [I] \quad &: \quad \beta W_1\left(K', A'\right) = u'\left(c_e\right) \\ [ENV_k] \quad &: \quad W_1\left(K, A\right) = \alpha\left(1-\tau_e\right)\frac{Y}{K}u'\left(c_e\right) + \beta\left(1-\delta\right)W_1\left(K', A'\right). \end{aligned} \tag{13}$$

2.2.2 Growth Driven by Intentional Technology Adoption

In the second specification, each period the entrepreneur hires a measure N_c of skilled workers, out of which a measure N_{cA} is employed for productivity improvements and new technology adoption. The production function that allows for this possibility is a slight variant of (7)

$$Y = A^{1-\alpha}K^{\alpha}\left[\theta\left(N_c - N_{cA}\right)^{\nu} + \left(1-\theta\right)N_s^{\nu}\right]^{\frac{1-\alpha}{\nu}}. \tag{14}$$

[15] A highly abbreviated list of examples of externality driven models is Romer (1986), Jones and Manuelli (1992), and Stokey (1992), and a few examples of intentional adoption models can be found in Romer (1990), Aghion and Howitt (1998), and Grossman and Helpman (1991). The model in Lucas (1988) features production externality in human capital, though the model can generate growth even without this.

[16] The human capital externality in Lucas (1988) is on the *level* of the output and flows from the activities of the *entire* labor force. In our specification the externality affects productivity growth directly and flows from the activities of only the educated workers.

Current technology and skilled labor are inputs into the production of new technology, as in Romer (1990) and Jones (1995). Productivity evolves according to

$$A_{t+1} = (1 + \xi (N_{cA})) A_t, \tag{15}$$

where ξ is now interpreted as the technology production function instead of an externality function.[17]

Skilled workers are specifically hired by the firm for effecting productivity improvement and are compensated for it, which affects the incentive of agents to accumulate human capital. The skill premium in this case can be shown to be

$$p = \frac{\theta}{1-\theta} \left(\frac{N_s}{N_c - N_{cA}} \right)^{1-\nu}. \tag{16}$$

The elasticity of the premium is increasing in the fraction of the labor devoted to R&D, $\frac{N_{cA}}{N_c}$, an observation that is useful for the balanced growth path (BGP) analysis to follow.

With adoption, since $w_c (N_c - N_{cA}) + w_s N_s = (1-\alpha) Y$, out of the remaining profit, αY, the entrepreneur invests in technology improvements by paying $w_c N_{cA}$, invests in physical capital, consumes, and pays taxes. The entrepreneur's budget constraint (8) implicitly assumes that the R&D cost, $w_c N_{cA}$, is exempt from the tax on profits; we later elaborate on the effect of this assumption.

The first order and envelope conditions for the entrepreneur in this case include (13) and the following additional conditions

$$\begin{aligned} [N_{cA}] &: \beta A \xi' (N_{cA}) W_2 (K', A') = (1-\tau_e) w_c u' (c_e) \qquad (17) \\ [ENV_A] &: W_2 (K, A) = (1-\tau_e)(1-\alpha) \frac{Y}{A} u' (c_e) + \beta (1 + \xi (N_{cA})) W_2 K', A'), \end{aligned}$$

where the skilled wage, $w_c = \frac{\partial Y}{\partial N_c} = \theta (1-\alpha) A^{1-\alpha} K^{\alpha} [\theta (N_c - N_{cA})^{\nu} + (1-\theta) N_s^{\nu}]^{\frac{1-\alpha}{\nu} - 1} (N_c - N_{cA})^{\nu - 1}$. The first order condition equates the marginal contribution of skilled agents in its two uses – technology adoption and production. The envelope condition states that the benefit of an extra unit of the technology stock is its contribution to current marginal utility through production and its use in future technology improvements according to (15).

3 Balanced Growth Analysis

As will be shown below, tax rates affect growth only through the ratio, Θ, of the two retention rates. Therefore, in this section we analyze the effects of a *parametric* change in tax progressivity

[17] Modeling the productive unit as an infinitely-lived entrepreneur allows us to sidestep issues in industrial organization that are typically found in R&D-based models of growth. These issues are important, but for studying the incentives of tax policy on human capital accumulation they do not seem to be of first order importance. Of less significance is the assumption that adoption involves only skilled labor and no physical capital.

on the balanced growth of the economy.[18] The role of the government is limited to collecting taxes; all collected taxes are spent by the government and do not result in any utility or productivity improvements.

3.1 Definition of Balanced Growth Equilibrium

We use the CRRA utility, $u(c) = \frac{c^{1-\sigma}}{1-\sigma}$, which is the only separable preference specification consistent with balanced growth. We envision all three types of agents situated on a balanced growth equilibrium, making decisions optimally.

Definition: A Balanced Growth Path (BGP) equilibrium is a collection of the quantities $\{K,\ A,\ Y,\ c_c,\ c_s,\ c_e,\ e_c^*,\ e_s^*,\ n_c^*,\ N_c^*,\ N_s^*,\ N_{cA}^*,\ \Lambda,\ W\}$, prices $\{w_c,\ w_s,\ p^*\}$, and a tax policy $\{\tau_c,\ \tau_s,\ \tau_e\}$ such that:

- K, A, Y, c_c, c_s, c_e, w_c, and w_s all grow at a constant rate g. The skill "return" function Λ, and entrepreneur value function W grow at the gross rate $(1+g)^{1-\sigma}$.[19]
- Human capital investments, e_c^*, e_s^*, the skill premium p^*, and skill attainment n_c^* are all time invariant.
- Given the constant growth paths of w_c, w_s, Λ, the time-invariant p^*, and taxes τ_c, τ_s, the human capital investments e_c^*, and e_s^* solve the household problems given in (1) and (2).
- When growth is driven by externality, given τ_e, w_c, and w_s, the quantities N_c^*, N_s^*, c_e, and I, solve (10) subject to (7), (8), (9), and (11). When growth is driven by intentional technology adoption, given τ_e, w_c, and w_s, the quantities N_c^*, N_s^*, N_{cA}^*, c_e, and I, solve (10) subject to (14), (8), (9), and (15).
- The skill return function, Λ, that households posit is consistent with the human capital investment decisions; that is, (5) is satisfied at the BGP quantities.
- The law of motion for the skilled fraction of labor force, Φ, that households posit is consistent with household decisions; that is, (6) is satisfied at the BGP quantities.
- The labor market clears; i.e. $N_c^* = n_c^*$, and $N_s^* = (1-n_c^*)$.[20]

[18] In Caucutt, İmrohoroğlu, and Kumar (2002) where revenue-neutral welfare comparisons are made, the actual tax rates, in addition to the ratio Θ matter.

[19] The utility function is the same for all agents and is homogeneous of degree $(1-\sigma)$. It follows that $\Lambda\left(n_c'\right) = (1+g)^{1-\sigma}\Lambda(n_c)$, and $W(K', A') = (1+g)^{1-\sigma} W(K, A)$ on the BGP.

[20] Note that we have not accounted for the skilled labor used in "teaching". This is done for simplicity; we expect the labor involved in teaching, $e_s(1-n_c) + e_c n_c$, to be a small fraction of the labor force. In data, according to *Education*

- In the externality case, $g = \xi(N_c^*)$ and in the technology adoption case, it is given by, $g = \xi(N_{cA}^*)$.

Even though both types of wages grow at the gross rate $(1+g)$, for individuals transiting between the two states, the gross growth rates will be given by

$$\begin{pmatrix} & \textbf{skilled} & \textbf{unskilled} \\ \textbf{skilled} & (1+g) & (1+g)/p \\ \textbf{unskilled} & (1+g)\,p & (1+g) \end{pmatrix}.$$

3.2 Production on the Balanced Growth Path

Since the production entity features no heterogeneity, it is easiest to start our BGP analysis there. It is convenient to think of the production sector (the entrepreneur) as taking the supply of skill as given, and making production decisions that result in a particular growth rate. The entire n_c^* versus g schedule derived in this fashion can be thought of as the human capital "demand" curve.

3.2.1 External Growth

Given the n_c^* that results from the skill acquisition of households, the balanced growth is given in this case by (15) as

$$g = \xi(n_c^*). \tag{18}$$

The n_c^* vs g "demand curve" for the external growth case is trivial. It just follows from (18); it does not hinge on any entrepreneurial decision. The higher the availability of skilled labor, the higher is the spillover and therefore the growth rate. The "demand" curve slopes upward in g. It does not directly depend on the progressivity parameter, Θ.

On the BGP, given the homogeneity of W (of degree $1-\sigma$), we can write $W_1(K', A') = (1+g)^{-\sigma} W_1(K, A)$. Use this and the conditions in (13) to get

$$(1+\rho)(1+g)^{\sigma} = \alpha(1-\tau_e)\frac{Y}{K} + (1-\delta),$$

where the discount rate $\rho \equiv (1-\beta)/\beta$. This is the analogue of the continuous-time growth condition $\rho + \sigma g = r$, where the interest rate, gross of depreciation, is given as $\alpha(1-\tau_e)\frac{Y}{K}$. But given the growth rate determined by skill acquisition, the interest rate and capital-output ratio merely adjust according to the above condition; they do not determine growth. It follows that the tax rate on the

at a Glance: OECD Indicators 1997, US teaching staff involved in the all levels of education was only 3.2% of the total employed population in 1995, out of which 0.7% was involved in tertiary education. What appears to be of first order importance is to have the cost of teacher's time enter the marginal conditions for human capital accumulation.

entrepreneur's profits, τ_e, does not affect the long run growth rate. The "engine" of growth is skill acquisition and a tax policy that does not affect that process will have no effect on long run growth.

The capital-output ratio, which will be affected by τ_e, is given by

$$\frac{K}{Y} = \frac{\alpha(1-\tau_e)}{(1+\rho)(1+g)^{\sigma} - (1-\delta)}. \tag{19}$$

The higher the tax on profits is, the lower this ratio. If investment in physical capital were tax exempt, even this effect of the profit tax disappears, as higher taxes create an incentive to invest and get a "write-off". This tax rate will also affect the the levels of profits and wages.

Finally, the skill premium on the BGP (12), which is used together with the household conditions to derive the "supply" schedule, is given by

$$p^* = \frac{\theta}{1-\theta}\left(\frac{1-n_c^*}{n_c^*}\right)^{1-\nu}. \tag{20}$$

3.2.2 Technology Adoption

In this case, for any given n_c^*, the entrepreneur has to make a decision about the fraction of the skilled labor force to devote to technology adoption, n_{cA}^*. This decision affects the growth rate through the equation, $g = \xi(n_{cA}^*)$. Again, using $W_1(K', A') = (1+g)^{-\sigma} W_1(K, A)$ and using (17), we can write the equation determining n_{cA}^*, and hence g, as

$$(1+\rho)(1+\xi(n_{cA}^*))^{\sigma} = (1-\alpha)\frac{Y}{w_c}\xi'(n_{cA}^*) + (1+\xi(n_{cA}^*)) \tag{21}$$

where $\frac{Y}{w_c}$ is derived from $[n_c]$ as

$$\frac{Y}{w_c} = \frac{(n_c^* - n_{cA}^*)^{1-\nu}\left[\theta(n_c^* - n_{cA}^*)^{\nu} + (1-\theta)(1-n_c^*)^{\nu}\right]}{\theta(1-\alpha)}.$$

The skill premium on the BGP, as derived from (16), is

$$p^* = \frac{\theta}{1-\theta}\left(\frac{1-n_c^*}{n_c^* - n_{cA}^*}\right)^{1-\nu}. \tag{22}$$

What happens when the available pool of skilled labor, n_c^*, increases? Using (15), (21), and (22), and parametrizing $\xi(n_{cA}) = C n_{cA}^{\varepsilon}$, $0 < \varepsilon < 1$, we prove the following lemma in the appendix.

Lemma 1 *The adoption firm's policy function $n_{cA}^*(n_c^*)$ is strictly increasing. That is, when the availability of skilled labor increases, the portion of skilled labor devoted to technology adoption increases in a BGP equilibrium.*

It follows that the "demand" function $g(n_c^*)$ is a strictly increasing function as it is in the externality case, and does not directly depend on Θ. As in the externality case, and as in any growth model, the growth rate decreases with σ or ρ for any given n_c^*.

Note that even when growth is driven by intentional technology adoption, the tax rate on profits, τ_e, does not affect the rate of growth. In a world in which R&D expenses are tax exempt, an increase in this tax rate decreases the marginal cost of hiring a skilled agent to adopt technology and decreases the marginal benefit arising from the improved technology by the same factor, as is evident from (17). This is similar to the tax neutrality seen in the Lucas (1990) condition discussed in footnote 3.[21]

The capital-output ratio continues to be given by (19) and is adversely affected by a tax on profits if physical capital investment is not tax exempt.

3.3 Households on the Balanced Growth Path

Analogous to the preceding analysis of production decisions, it is convenient to think of the household sector as anticipating a particular growth rate g, (as well as the resulting skill premium p^*), and making human capital investment decisions that imply a supply of skill n_c^*. This n_c^* versus g schedule can then be thought of as the human capital "supply" curve. The intersection of this curve with the "demand" curve derived earlier gives the balanced growth rate and the stationary skill level on the balanced growth path. Unlike the "demand" curve, the "supply" curve will depend on Θ; indeed, the main purpose of the paper is to extend the analysis of Lucas (1990) and Heckman and Klenow (1997), to study the effect of tax reform (a change in Θ) on the supply of human capital and thus growth. Therefore, we will examine the supply of skill when Θ changes for a given anticipated growth rate g, and when g changes for a given Θ. This method of characterizing the BGP will allow us to understand the impact of tax reform on each type of agent and ultimately on the equilibrium growth rate.

We begin by listing the various forces that govern skill acquisition. An increase in progressivity decreases investment by the rich *relative* to that of the poor when $\sigma > 1$ and increases it if $\sigma < 1$; in the former case the poor benefit more by the easing of their liquidity constraint, while in the latter the rich benefit more by getting a tax write-off on human capital investments. An increase in the equilibrium premium decreases investment by the poor relative to the rich as education becomes costlier for them. On the intertemporal front, an increase in the equilibrium premium or a decrease in the progressivity increases the return to skill and tends to increase investment by *both* types of agents. An increase in anticipated growth increases the effective discount factor when $\sigma < 1$ (a substitution effect of increased growth), increasing investment of both types, and works in the opposite direction

[21] If R&D expenses are not deductible, the entrepreneur's consumption becomes

$$c_e = (1 - \tau_e)(Y - w_c N_c - w_s N_s) - I - \tau_e w_c n_{cA}^*.$$

Now the first term of the right side of (21) will have a factor $(1 - \tau_e)$, and the tax rate on profit will have a negative effect on the growth rate.

when $\sigma > 1$ (an income effect of increased growth).

Formally, we start by using (3) and (4), to get the *intra*temporal condition governing investment of the skilled agents relative to those of the unskilled as

$$\frac{\pi_c'(e_c^*)}{\pi_s'(e_s^*)} = \frac{\Theta^\sigma}{\Theta}\left(\frac{\frac{1}{p^*} - e_s^*}{1 - e_c^*}\right)^\sigma. \tag{23}$$

This expression equates the ratio of marginal investment benefit of each type to the ratio of their marginal costs.

It can be shown that an increase in the anticipated skill premium increases the cost of skill acquisition for the unskilled relative to their income and increases e_c^* relative to e_s^* – a "tuition" effect.

The Θ^σ in the numerator of the right hand side captures the "liquidity effect" mentioned above. An increase in Θ lowers the income of the skilled relative to that of the unskilled, causing investment by skilled people to fall relative to that of the unskilled; i.e. $\frac{e_s^*}{e_c^*}$ tends to increase given the concavity of π.

The Θ in the denominator captures the effect of tax-exemption alluded to earlier. An increase in Θ decreases the marginal cost for the skilled and increases their incentive to invest relative to that of the unskilled in order to get a tax "write-off".

The net effect of Θ in determining the relative investment levels clearly depends on σ. When $\sigma > 1$, the liquidity constraint effect dominates and an increase in progressivity lowers the investment by the skilled relative to that of the unskilled.[22] With log utility, both cancel out.

We next turn to the return effect of progressivity. Using the fact, $\Lambda\left(n_c'\right) = (1+g)^{1-\sigma}\Lambda(n_c)$, on the BGP, we can use (5) and write a normalized version of the return to being skilled – an *inter*temporal condition – as

$$\frac{\Lambda(n_c^*)}{[(1-\tau_s)w_s^*]^{1-\sigma}} = \frac{1}{\left[1 - \beta(1+g)^{1-\sigma}(\pi_c(e_c^*) - \pi_s(e_s^*))\right]} \cdot \frac{\left[(1-e_c^*)\frac{p^*}{\Theta}\right]^{1-\sigma} - [1 - e_s^* p^*]^{1-\sigma}}{1-\sigma}.$$

The first part of the right side is an effective discount factor and the second is an excess utility term. The discount factor increases with β as well as with increased investment by the skilled. The

[22] If each agent could use her own time for skill accumulation (i.e. hire an agent of her own type as the teacher), the relative investment condition for $\pi(e) = Be^\gamma$ becomes:

$$\left(\frac{e_s^*}{e_c^*}\right)^{1-\gamma} = \left(\frac{p^*}{\Theta}\right)^{1-\sigma}\left(\frac{1-e_s^*}{1-e_c^*}\right)^\sigma.$$

If the net of tax skill premium $\frac{p^*}{\Theta} > 1$, as we would expect in a BGP equilibrium, we will have $e_c^* < e_s^*$. This outcome is highly counterfactual. Making investment by skilled more productive or assuming that the cost of education relative to income is higher for the unskilled, as we have done, is sufficient to avoid this outcome.

discount factor increases with anticipated growth, g, if $\sigma < 1$ and decreases if $\sigma > 1$. In the former case, the substitution effect of an increase in anticipated growth dominates, increasing the incentive to invest, while in the latter case, the income effect of increased growth dominates, decreasing the incentive to invest.

The excess utility term, and thus the normalized return, increases with p^* and decreases with the progressivity parameter Θ, no matter what σ is. One can think of $\frac{p^*}{\Theta}$ as the effective premium that determines the returns to human capital. An increase in the return Λ will tend to increase both e_s^* and e_c^*, as can be seen from (3) and (4).

With flat rate taxes, $\tau_c = \tau_s = \tau$, $\Theta = 1$, and the actual tax rate does not figure in the above equations. The production conditions are independent of these tax rates. Therefore, any effect of tax on growth can only be due to differences in its *structure*, rather than on its level. We therefore have the following proposition.

Proposition 1 *With flat rate taxes and tax-exempt human capital investment, a change in the rate of tax has no effect on the long run growth rate of the economy.* •

Taking into account the evolution of Λ, we can rewrite (4) as

$$\frac{\Lambda\left(n_c^*\right)}{\left[\left(1-\tau_s\right) w_s^*\right]^{1-\sigma}} = \frac{1}{\beta\left(1+g\right)^{1-\sigma}} \frac{p^*}{\pi_s'\left(e_s^*\right)\left(1-e_s^* p^*\right)^{1-\sigma}}.$$

Equating the right hand sides of the above two expressions yields the following important equation

$$\left[\frac{1}{\frac{1}{\beta(1+g)^{1-\sigma}} - \left(\pi_c\left(e_c^*\right) - \pi_s\left(e_s^*\right)\right)}\right] \cdot \left[\frac{\left(\frac{1-e_c^*}{\Theta}\right)^{1-\sigma} - \left(\frac{1}{p^*} - e_s^*\right)^{1-\sigma}}{1-\sigma}\right] = \frac{1}{\pi_s'\left(e_s^*\right)\left(\frac{1}{p^*} - e_s^*\right)^{\sigma}}. \tag{24}$$

Evaluating (6) at the BGP equilibrium, we get

$$n_c^* = \frac{\pi_s\left(e_s^*\right)}{1 - \left(\pi_c\left(e_c^*\right) - \pi_s\left(e_s^*\right)\right)}. \tag{25}$$

As one would intuitively expect, the higher the investment in skill by any particular type of agent on the BGP, the higher is the level of skill attainment; that is, n_c^* is increasing in e_s^*, e_c^*.

Equations (23), (24), and (25) capture the behavior of the household sector on the BGP equilibrium. That is, given the p^* and g arising from production decisions, these three equations determine the investments e_c^* and e_s^*, and thus the skill attainment n_c^*. To analyze the human capital "supply" curve it is convenient to consider the expression for the premium, (20) or (22), also in this system.

We are now in a position to assess how the above-mentioned forces affect human capital investment response and thus skill attainment to a change in tax policy. First consider the response of investments to a change in progressivity for a given anticipated growth rate (fix g, vary Θ). An increase in progressivity decreases the value to being skilled, Λ, and tends to decrease investment by both types. But can an increase in investment by the unskilled through the liquidity effect offset this decrease?

To investigate this, suppose for a moment that agents do not have foresight about changing wages; that is, ignore the general equilibrium effects of investment decisions of agents. (Alternatively one could think of ν being 1; the two types of labor are perfectly substitutable and the premium stays constant for a given growth rate.) The analysis that ignores general equilibrium is common to both types of growth. Later, we will incorporate the general equilibrium effect, as well as consider what happens when progressivity is fixed but anticipated growth changes, for each type of growth. In the appendix, using the parametrization $\pi_c = \pi_s = Be^{\gamma}$, $0 < \gamma < 1$, we prove

Lemma 2 *For a given rate of anticipated growth, no matter its source, when general equilibrium effects of changes in the skill premium are ignored, the BGP investments e_c^*, and e_s^*, both decrease with the degree of tax progressivity; the level of skill attainment, n_c^*, thus decreases.* •

This result holds for any $\sigma > 0$. Even though an increase in the progressivity could cause a relative shift of investment in favor of the unskilled through the liquidity effect, the decrease in the return to human capital decreases the investments of *both* types of agents and thus the supply of skill. Next we consider what happens when we take into account the general equilibrium effects of changes in the skill premium. Since the premium depends on the type of growth, we will analyze the effects separately.

3.3.1 General Equilibrium Effects: External Growth

Use (20) and (25) to get the premium for the chosen parametrization as

$$p^* = \frac{\theta}{1-\theta}\left(\frac{1 - B\left(e_c^*\right)^{\gamma}}{B\left(e_s^*\right)^{\gamma}}\right)^{1-\nu}. \tag{26}$$

As shown in Lemma 2, an increase in Θ decreases both e_c^* and e_s^*. But from the above equation we can see this tends to increase the premium and thus the value to being skilled. This in turn tends to *increase* the investment by both types now. However, the effect of increased tuition reinforces the original effect for the unskilled agent, and decreases e_s^*. The general equilibrium effect is mildest when ν is high enough (high enough substitution between the two types of labor) and when γ is low enough (there is enough diminishing returns in the human capital accumulation process to keep elasticities of response low). In the appendix, we give sufficient conditions on σ, γ, and ν to prove

Lemma 3 *For a given rate of anticipated external growth, even when general equilibrium effects of changes in the skill premium are accounted for, the BGP investments e_c^*, and e_s^*, both decrease with the degree of tax progressivity, and therefore, so does the level of skill attainment n_c^*.* •

Next consider the response of human capital investments to a change in the anticipated growth rate g, for a given level of progressivity, Θ (fix Θ; vary g). The only expression g enters is the intertemporal condition (24). As mentioned above, the effect of anticipated growth depends on the value of σ. When $\sigma < 1$ the substitution effect of an increase in anticipated growth dominates, increasing the incentive to invest (e_c^* and e_s^* increase), and when $\sigma > 1$, the income effect of increased growth dominates, decreasing the incentive to invest (e_c^* and e_s^* decrease).[23]

In summary, we conclude that the n_c^* versus g schedule shifts down with increased progressivity and is upward or downward sloping, depending on $\sigma < 1$ or $\sigma > 1$.

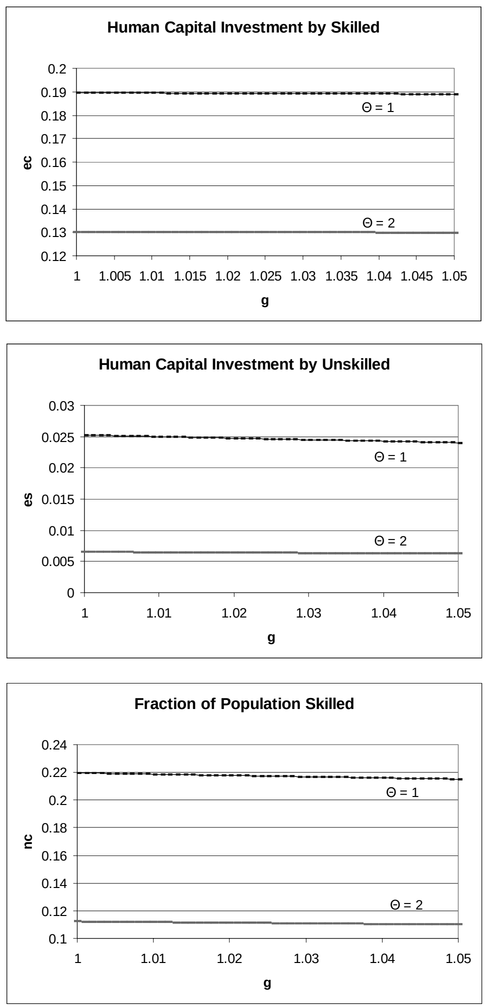

Fig 1: Externality Driven Growth

[23] The fact that this simple intuition carries through when equilibrium effects of a change in premium are taken into account can be shown, analagous to Lemmas 2 and 3. However, we do not present the results here, as this is not our main focus. As we will see the results are insensitive to which effect of an increase in anticipated growth dominates. Kumar (forthcoming) also discusses the two effects of anticipated growth and finds cross-country evidence that the income effect dominates.

To illustrate the effects discussed above with a numerical example, we set annual $\beta = 0.98$, $\pi(e) = e^{0.5}$, $\alpha = 0.5$, $\theta = 0.5$, $\nu = 0.35$, and $\xi(n_c) = 0.08 n_c^{0.95}$. In Figure 1, we plot the investment by the two types of agents and the 'n_c^* versus g' "supply" schedule, when $\sigma = 1.5$. The investment curves, especially the one for the poor, are slightly downward sloping, and therefore so is n_c^* versus g. The graphs for $\sigma = 0.5$ (not shown) are very similar. The only difference is that the investment curves and n_c^* versus g curve are now all upward sloping. However for any given anticipated growth rate, the investment levels and skill attainment decrease with the progressivity.

When the household responses are put together with the upward sloping "demand" curve $n_c^* = \xi^{-1}(g)$, the following proposition emerges.

Proposition 2 *When growth is caused by externalities resulting from activities of skilled labor, an increase in the progressivity of taxes decreases the human capital investment levels of both types of agents on the BGP. The stationary level of skill attainment is lower and thus the growth rate is lower for all $\sigma > 0$; the skill premium is higher.* •

For instance, in the above numerical example, the growth rates are 1.88%, 1%, and 0.65% for a progressivity parameter, Θ, of 1, 2, and 3, respectively.

3.3.2 General Equilibrium Effects: Technology Adoption

As mentioned earlier, Lemma 2 continues to hold when growth arises from adoption. That is, when the equilibrium effect of investment changes on the premium is ignored, an increase in the progressivity unambiguously decreases investment by both types of agents and the level of skill attainment. However, the expression for the skill premium is different from the one under externality, and we will see this alters the incentive to accumulate human capital significantly. The premium is now given by

$$p^* = \frac{\theta}{1-\theta} \left(\frac{1 - B(e_c^*)^\gamma}{B(e_s^*)^\gamma - n_{cA}^*} \right)^{1-\nu}, \tag{27}$$

where $n_{cA}^* = \xi^{-1}(g)$. In addition to affecting the effective discount factor, the anticipated growth rate directly enters the expression for the expected premium; the higher the fraction of labor in technology adoption, the higher the premium. Recall that skilled workers are fully compensated for their role in generating growth.

The direction of change in investment with progressivity, for a given anticipated growth rate, is not as clear-cut as it was with external growth. In the appendix, we show the following.

Proposition 3 *When growth is driven by technology adoption, general equilibrium effects of changes in the skill premium imply that the BGP investment of the unskilled, e_s^*, always decreases with the* *degree of tax progressivity. If the anticipated growth rate is low enough, the investment of the skilled, e_c^*, also decreases, but if the growth rate is high enough, e_c^* can increase with progressivity. Therefore, for low enough growth rates, the level of skill attainment n_c^* decreases with progressivity, but for higher growth rates the effect of progressivity on n_c^* is ambiguous.* •

The equilibrium effect is stronger in the high growth region. Recall that the elasticity of the skill premium is high in this region; the decrease in e_c^*, e_s^*, arising from the forces outlined in Lemma 2 could increase p enough to reverse the overall effect on the investment of the skilled while reinforcing it through the tuition effect on the investment of the unskilled.

We use the same parameters that we used for the external growth case (interpreting $\xi(n_c)$ as $\xi(n_{cA})$) and illustrate the above-mentioned effects with the following graphs. For a given progressivity, the discount factor effect of an anticipated change in growth is swamped out by the return and tuition effects arising from the increased premium that accompanies increased growth. Therefore, σ plays a minimal role and we show the graphs only for $\sigma = 1.5$; those for $\sigma = 0.5$ are very similar.

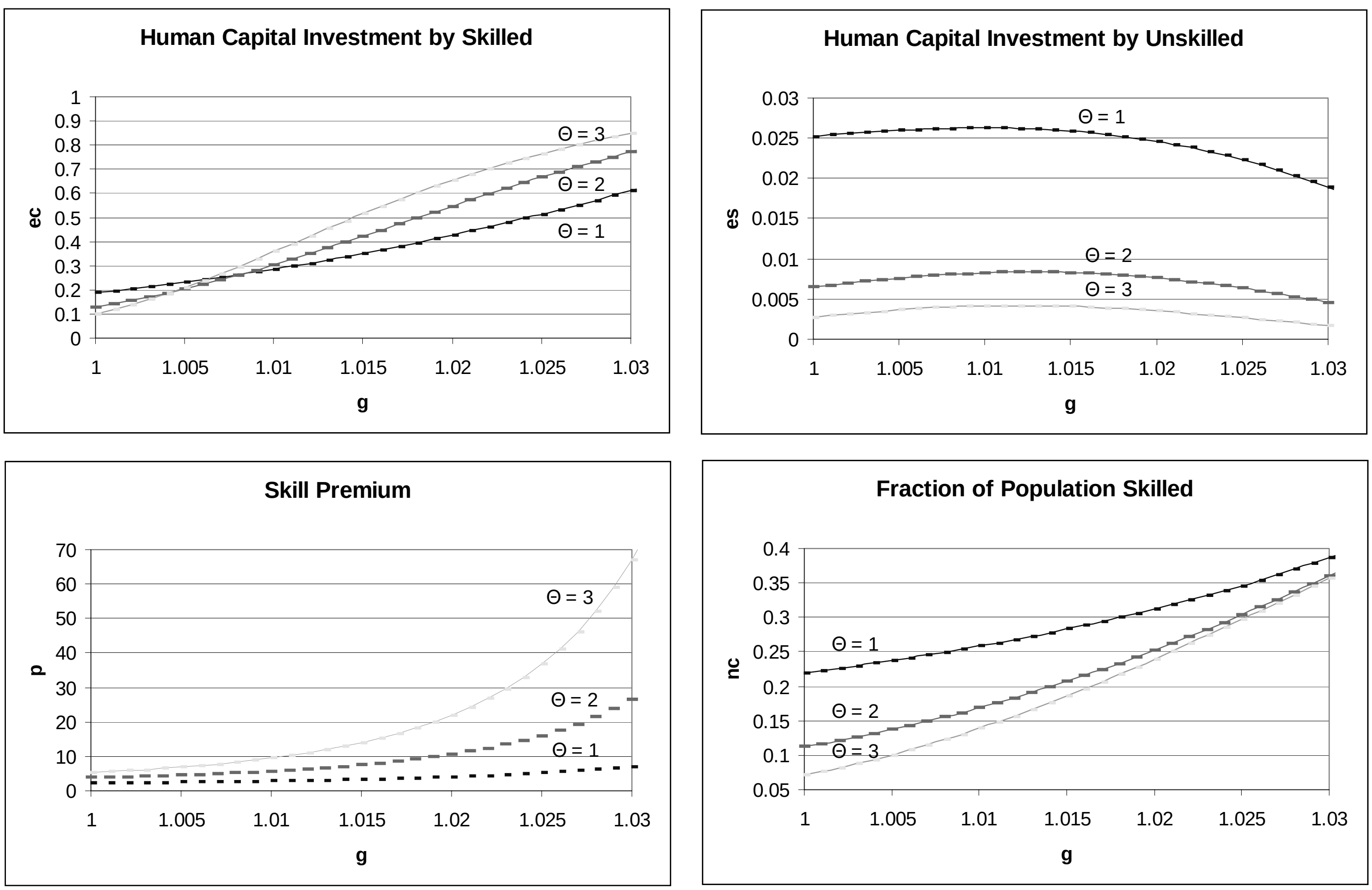

Fig 2: Adoption Driven Growth

Figure 2 shows investments made by the two types of agents, the behavior of the premium, and the human capital "supply" curve. Both investments decrease with the degree of progressivity when the anticipated growth rate is low, similar to the external growth case. But as discussed in Proposition 3, when the anticipated growth rate is high enough, the general equilibrium effect on the premium makes it attractive for the skilled to invest more when progressivity *increases*; the unskilled investment decreases as in the externality case. The premium becomes more sensitive to progressivity at these higher growth rates, which explains the differing effects for high growth rates.

In our numerical example, it appears the decline of investment by the unskilled outweighs the increase by the skilled so that the overall supply of skill decreases with progressivity, though by smaller amounts.[24] Taken with the upward sloping "demand" curve resulting from Lemma 1, we get the result that the growth rate decreases with progressivity in this numerical example. An increase in equilibrium skill premium is immediate. The growth rates are 1.42%, 0.54%, and 0.15% for a progressivity parameter, Θ, of 1, 2, and 3, respectively, when $\sigma = 1.5$.

3.4 A Discussion of the Non-Exempt Case

When investments in human capital are not tax exempt, the actual tax levels matter even for studying the long-run growth effects; it is no longer the case that progressivity affects allocations only through the ratio of the retention rates, Θ. The current utility terms in the non-exempt case are $u\left(\left((1-\tau_c)-e_c\right)w_c\left(n_c\right)\right)$ and $u\left(\left((1-\tau_s)-e_s p\left(n_c\right)\right)w_s\left(n_c\right)\right)$. The right hand sides of (3) and (4) now become $w_c\left(n_c\right)u'\left(\left((1-\tau_c)-e_c\right)w_c\left(n_c\right)\right)$ and $p\left(n_c\right)w_s\left(n_c\right)u'\left(\left((1-\tau_s)-e_s p\left(n_c\right)\right)w_s\left(n_c\right)\right)$ respectively. Proceeding as we did in the exempt case, the analogues of (23) and (24) for the non-exempt case – the only conditions that will be different – can be derived as

$$\frac{\left[\left((1-\tau_c)-e_c^*\right)^{1-\sigma}\left(p^*\right)^{1-\sigma}-\left((1-\tau_s)-e_s^*p^*\right)^{1-\sigma}\right]}{(1-\sigma)\left[1-\beta\left(1+g\right)^{1-\sigma}\left(\pi_c\left(e_c^*\right)-\pi_s\left(e_s^*\right)\right)\right]} = \frac{p^*\left((1-\tau_s)-e_s^*p^*\right)^{-\sigma}}{\beta\left(1+g\right)^{1-\sigma}\pi_s'\left(e_s^*\right)}, \tag{28}$$

$$\frac{\pi_c'\left(e_c^*\right)}{\pi_s'\left(e_s^*\right)} = \frac{\left((1-\tau_s)-e_s^*p^*\right)^{\sigma}}{\left((1-\tau_c)-e_c^*\right)^{\sigma}\left(p^*\right)^{\sigma}}. \tag{29}$$

Since Θ does not factor out conveniently as it did earlier, it becomes difficult to characterize this case analytically. However, in Caucutt, İmrohoroğlu, and Kumar (2002), we pursue this case numerically in a calibrated setup and find that the effects of progressivity are similar to those in the exempt case. The strengthening of the liquidity constraint effect, mentioned in Section 2, seems to be countered by a milder effect of a decrease in progressivity on the value to being skilled. Moreover, we find that changes in progressivity have a more telling effect than a change in the rate of flat taxes.

[24] Caucutt, İmrohoroğlu, and Kumar (2002) arrive at the same quantitative conclusion in a calibrated model.

4 Conclusions

The heterogeneous-agent, endogenous growth framework developed in this paper demonstrates that higher progressivity has the potential to decrease investment in human capital and growth in the long run, and increase the skill premium, a measure of inequality in our setup. This paper points to the importance of studying the effect of a reform in the *structure* of taxes on growth rather than a reform in the *level* of taxes alone. It also suggests tax progressivity as a structural factor linking growth and inequality.

The model developed here, with idiosyncratic risk and household heterogeneity limited to two types of agents and only along the dimension of human capital, is highly tractable. It will be of interest to analyze progressivity in a more general model with a continuum of agents, heterogeneous in both their human and physical capital stocks, which could be more robustly matched up with data. The challenge of extending a Bewley-type model to accommodate endogenous growth is a useful one to pursue.

A Appendix

A.1 Lemma 1 ($n_{cA}^*\left(n_c^*\right)$ is strictly increasing for the adoption firm)

The left hand side of (21), which is the analogue of $\rho + \sigma g$ in continuous time growth models, is increasing in n_{cA}^* for any increasing ξ function and any $\sigma > 0$. It is a marginal cost term that captures the utility costs of lost production by diverting labor to technology adoption on the BGP. The right hand side captures the marginal benefit on the BGP of increasing labor in adoption; the first term reflects an increase in current output arising from an increased productivity level and the second term reflects an increase in the level of future productivity improvements given the law of motion (15). The first term decreases with n_{cA}^* for any concave adoption function ξ; an increase in R&D labor is less effective in increasing A on the margin, and is further compounded by a diminishing effect on output due to a reduction in the skilled labor force available for production ($\frac{Y}{w_c}$ is decreasing). The second term is clearly increasing in n_{cA}^*. Therefore, to determine the sign of the right hand side, we parametrize the adoption function to be $\xi\left(n_{cA}\right) = Cn_{cA}^{\varepsilon}$, $0 < \varepsilon < 1$. Some algebra now shows that the right side hand decreases unambiguously (noting $\nu < 1$). In summary, the left hand side when plotted against n_{cA}^* is increasing, starting from $(1+\rho)$. The right hand side is decreasing from ∞. And it is possible to find parameters for which a unique intersection exists.

For any given n_{cA}^*, the only term in (21) that is affected by n_c^* is the $\frac{Y}{w_c}$ term. This term increases in n_c^*, provided the p^* in (22) is greater than one. We expect the post-tax premium on a BGP equilibrium, $\frac{p^*}{\Theta}$, to be greater than one; otherwise there will be no incentive for anyone to accumulate skills. For a given level of adoption activity, higher the skilled labor force, higher the output will be (though increasing at a diminishing rate) and lower will the skilled wages be, both of which increase $\frac{Y}{w_c}$ and the production benefit of increasing adoption. In terms of the graphical analysis outlined in the preceding paragraph, the right side shifts up and the intersection point, n_{cA}^*, increases ■ .

A.2 Lemma 2 (Ignoring equilibrium effects for both types of growth)

Consider the parametrization $\pi_c = \pi_s = Be^{\gamma}$, $0 < \gamma < 1$, to characterize the household sector on the BGP. For this parametrization, (23) becomes

$$\left(\frac{e_s^*}{e_c^*}\right)^{1-\gamma} = \frac{\Theta^{\sigma}}{\Theta}\left(\frac{\frac{1}{p^*} - e_s^*}{1 - e_c^*}\right)^{\sigma}. \tag{30}$$

Eliminate $\Theta^{1-\sigma}$ from (30) in (24), use the parametrization chosen for π, and simplify to get

$$\frac{\left[\frac{1-e_c^*}{(e_c^*)^{1-\gamma}} - \frac{\frac{1}{p^*}-e_s^*}{(e_s^*)^{1-\gamma}}\right]}{1-\sigma} = \frac{1}{\gamma B}\left[\frac{1}{\beta\left(1+g\right)^{1-\sigma}} - B\left((e_c^*)^{\gamma} - (e_s^*)^{\gamma}\right)\right]. \tag{31}$$

Take logs and differentiate (30) with respect to Θ, and simplify to get

$$\left[\frac{1-\gamma}{e_s} + \frac{\sigma}{\frac{1}{p} - e_s}\right]\frac{\partial e_s}{\partial \Theta} - \left[\frac{1-\gamma}{e_c} + \frac{\sigma}{1-e_c}\right]\frac{\partial e_c}{\partial \Theta} = -\frac{(1-\sigma)}{\Theta} - \frac{\frac{\sigma}{p^2}}{\frac{1}{p} - e_s}\frac{\partial p}{\partial \Theta}, \tag{32}$$

where we have dropped asterisks from all variables for simplicity of notation. Differentiate (31) with respect to Θ, and wade through some algebra to get

$$\left[\sigma + (1-\gamma)\frac{\left(\frac{1}{p} - e_s\right)}{e_s}\right]\frac{\partial e_s}{\partial \Theta} - \left(\frac{e_s}{e_c}\right)^{1-\gamma}\left[\sigma + (1-\gamma)\frac{(1-e_c)}{e_c}\right]\frac{\partial e_c}{\partial \Theta} = -\frac{1}{p^2}\frac{\partial p}{\partial \Theta}. \tag{33}$$

When we ignore the effects on the skill premium, $\frac{\partial p}{\partial \Theta} = 0$. In this case, we can see from (33) $\frac{\partial e_c}{\partial \Theta}$ and $\frac{\partial e_s}{\partial \Theta}$ have the same signs. This is important in understanding the final result; the intertemporal effect of an increase in Θ is of the same sign for both agents' investments. And given that we expect $e_c > e_s$, the above equation indicates the effect is stronger for e_c. Substitute for $\frac{\partial e_s}{\partial \Theta}$ from (33) into (32), using (30), and wading through more algebra, one can get

$$\left\{\frac{\left[\frac{1-e_c}{\Theta}\right]^{1-\sigma} - \left(\frac{1}{p} - e_s\right)^{1-\sigma}}{1-\sigma}\right\}\frac{1}{\left(\frac{1}{p} - e_s\right)^{1-\sigma}}\left[\frac{1-\gamma}{e_c} + \frac{\sigma}{1-e_c}\right]\frac{\partial e_c}{\partial \Theta} = -\frac{1}{\Theta}. \tag{34}$$

The term within curly braces is an excess utility term which needs to be positive in any BGP equilibrium with positive investment; otherwise there will be no incentive for agents to become skilled (Λ will become negative). This means $\frac{\partial e_c}{\partial \Theta} < 0$ and as discussed above $\frac{\partial e_s}{\partial \Theta} < 0$. From (25) it follows that n_c decreases with Θ.

Even though an increase in the progressivity could shift the investment in favor of the unskilled through the liquidity effect, it is the intertemporal effect that ultimately dominates and decreases the investments of both types of agents. ■

A.3 Lemma 3 (With equilibrium effects for external growth)

For the external growth case, we can take logs and differentiate (26) to get

$$\frac{1}{p}\frac{\partial p}{\partial \Theta} = -\frac{\gamma B\left(1-\nu\right)}{e_c^{1-\gamma}\left(1-Be_c^{\gamma}\right)}\frac{\partial e_c}{\partial \Theta} - \frac{\gamma\left(1-\nu\right)}{e_s}\frac{\partial e_s}{\partial \Theta}.$$

Using this in (33) we get

$$\left\{\sigma + (1-\gamma)\frac{\left(\frac{1}{p}-e_s\right)}{e_s} - \frac{\gamma\left(1-\nu\right)}{pe_s}\right\}\frac{\partial e_s}{\partial \Theta} = \left\{\left(\frac{e_s}{e_c}\right)^{1-\gamma}\left[\sigma + (1-\gamma)\frac{(1-e_c)}{e_c}\right] + \frac{\gamma B\left(1-\nu\right)}{pe_c^{1-\gamma}\left(1-Be_c^{\gamma}\right)}\right\}\frac{\partial e_c}{\partial \Theta}. \tag{35}$$

The result is that $\frac{\partial e_s}{\partial \Theta}$ is made stronger relative to $\frac{\partial e_c}{\partial \Theta}$, than when the effect of tuition increase is ignored as in Lemma 2. Doing the same for (32), using (35) and (30), and after some messy algebra we can get

$$\Bigg[\frac{1}{\left(\frac{1}{p}-e_s\right)^{1-\sigma}}\left[\sigma + (1-\gamma)\frac{(1-e_c)}{e_c}\right]\left\{\frac{\Psi\left[\frac{1-e_c}{\Theta}\right]^{1-\sigma} - \left(\frac{1}{p}-e_s\right)^{1-\sigma}}{1-\sigma}\right\} +$$
$$\left[\frac{\gamma B\left(1-\nu\right)}{\left(\frac{1}{p}-e_s\right)pe_c^{1-\gamma}\left(1-Be_c^{\gamma}\right)}\right]\left\{\frac{\sigma + (1-\gamma)\frac{\left(\frac{1}{p}-e_s\right)}{e_s}}{\sigma + (1-\gamma)\frac{\left(\frac{1}{p}-e_s\right)}{e_s} - \frac{\gamma(1-\nu)}{pe_s}}\right\}\Bigg]\cdot\frac{\partial e_c}{\partial \Theta} = -\frac{1}{\Theta},$$

where,

$$\Psi \equiv \frac{\left[\sigma + (1-\gamma)\frac{\left(\frac{1}{p}-e_s\right)}{e_s} - \frac{\sigma\gamma(1-\nu)}{pe_s}\right]}{\left[\sigma + (1-\gamma)\frac{\left(\frac{1}{p}-e_s\right)}{e_s} - \frac{\gamma(1-\nu)}{pe_s}\right]}.$$

When compared to (34), the factor Ψ within the first set of curly braces and the second term are new. It is straightforward to show that the first term within curly braces is positive no matter what σ is, as long as $\frac{1-e_c}{\Theta} > \frac{1}{p} - e_s$, which as we argued in Lemma 1 needs to be the case on the BGP for Λ to be positive. So, we can unambiguously sign $\frac{\partial e_c}{\partial \Theta}$ provided the numerator and denominator of Ψ are positive. The sufficient conditions that guarantee this are

$$(1-\sigma) < \gamma < \min\left\{\frac{1}{2-\nu}, \frac{1}{1+\sigma\left(1-\nu\right)}\right\}.$$

The second part of the inequality is automatically satisfied if $\nu = 1$. In other words, as alluded to in the main text, we want high enough elasticity of substitution in production, sufficient diminishing returns in human capital accumulation, and high enough σ so that elasticities of investment to premium changes are low and general equilibrium effect does not overturn the result in Lemma 1. These seem relatively mild conditions. For instance, $\gamma = \nu = 1/2$, will work for $\sigma = 1/2$, and $\nu = 1/2$, $\gamma = 1/4$ will work for $\sigma = 2$. The same sufficient conditions guarantee the sign of $\frac{\partial e_s}{\partial \Theta}$ is the same as $\frac{\partial e_c}{\partial \Theta}$. ■

A.4 Proposition 3 (With equilibrium effects for adoption-driven growth)

For the external growth case, we can take logs and differentiate (27) to get

$$\frac{1}{p}\frac{\partial p}{\partial \Theta} = -\frac{\gamma B\left(1-\nu\right)}{e_c^{1-\gamma}\left(1-Be_c^{\gamma}\right)}\frac{\partial e_c}{\partial \Theta} - \frac{\gamma B\left(1-\nu\right)}{e_s^{1-\gamma}\left(Be_s^{\gamma}-n_{cA}\right)}\frac{\partial e_s}{\partial \Theta}.$$

The second term is different from the analogous expression in Lemma 3. One can derive an expression analogous to (35) in the previous lemma

$$\left\{\sigma+(1-\gamma)\frac{\left(\frac{1}{p}-e_s\right)}{e_s}-\frac{\gamma B(1-\nu)}{pe_s^{1-\gamma}\left(Be_s^{\gamma}-n_{cA}\right)}\right\}\frac{\partial e_s}{\partial\Theta}=\left\{\left(\frac{e_s}{e_c}\right)^{1-\gamma}\left[\sigma+(1-\gamma)\frac{(1-e_c)}{e_c}\right]+\frac{\gamma B(1-\nu)}{pe_c^{1-\gamma}\left(1-Be_c^{\gamma}\right)}\right\}\frac{\partial e_c}{\partial\Theta}.$$

Again, only the second term within the curly braces in the left side is different. However the earlier sufficient conditions to guarantee e_s and e_c move in the same direction will not be enough here. In particular, for high enough anticipated growth (high enough n_{cA}), the second term can become negative enough to cause the two types of investment to move in the opposite direction. Proceeding as we did in the previous lemma, we can get

$$\Bigg[\frac{1}{\left(\frac{1}{p}-e_s\right)^{1-\sigma}}\left[\sigma+(1-\gamma)\frac{(1-e_c)}{e_c}\right]\left\{\frac{\widehat{\Psi}\left[\frac{1-e_c}{\Theta}\right]^{1-\sigma}-\left(\frac{1}{p}-e_s\right)^{1-\sigma}}{1-\sigma}\right\}+$$

$$\left[\frac{\gamma B(1-\nu)}{\left(\frac{1}{p}-e_s\right)pe_c^{1-\gamma}\left(1-Be_c^{\gamma}\right)}\right]\left\{\frac{\sigma+(1-\gamma)\frac{\left(\frac{1}{p}-e_s\right)}{e_s}}{\sigma+(1-\gamma)\frac{\left(\frac{1}{p}-e_s\right)}{e_s}-\frac{\gamma B(1-\nu)}{pe_s^{1-\gamma}\left(Be_s^{\gamma}-n_{cA}\right)}}\right\}\Bigg]\cdot\frac{\partial e_c}{\partial\Theta}=-\frac{1}{\Theta},$$

where,

$$\widehat{\Psi}\equiv\frac{\left[\sigma+(1-\gamma)\frac{\left(\frac{1}{p}-e_s\right)}{e_s}\right]-\frac{\sigma\gamma B(1-\nu)}{pe_s^{1-\gamma}\left(Be_s^{\gamma}-n_{cA}\right)}}{\left[\sigma+(1-\gamma)\frac{\left(\frac{1}{p}-e_s\right)}{e_s}\right]-\frac{\gamma B(1-\nu)}{pe_s^{1-\gamma}\left(Be_s^{\gamma}-n_{cA}\right)}}.$$

When $n_{cA}\longrightarrow 0$, these expressions reduce to the ones in the previous lemma and the same results hold. The numerator and denominator of $\widehat{\Psi}$ can be negative for large enough growth, leaving $\widehat{\Psi}$ positive. But the more important point is when the term $\frac{\gamma B(1-\nu)}{pe_s^{1-\gamma}(Be_s^{\gamma}-n_{cA})}$ is large enough to make the denominator positive, it makes $\frac{\partial e_c}{\partial\Theta}>0$. This *same* force makes $\frac{\partial e_s}{\partial\Theta}$ and $\frac{\partial e_c}{\partial\Theta}$ moves in opposite directions and causes $\frac{\partial e_s}{\partial\Theta}<0$. In other words, while e_c can increase with progressivity for large enough growth rates, e_s always decreases with progressivity.

■

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