Online Appendix: "Early and Late Human Capital Investments, Borrowing Constraints, and the Family"

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O.1 Multiple Investment Inputs Each Period

In this Appendix, we show how our model can be generalized to include multiple investments each period, where i_j would then reflect total investment expenditures in period j given optimal choices about different inputs each period.

Suppose human capital production depends on two inputs each period: purchased goods g_j and parental time τ_j (scaled by effective parental human capital $h_j^p \equiv \Gamma_{j+2}h^p$). Define the child's human capital production function as:

$$h = \theta f(x_1, x_2) \tag{1}$$

where

$$x_j = \chi_j(g_j, \tau_j h_j^p), \quad j = 1, 2.$$

Notice that this technology assumes parental human capital increases the productivity of parental time inputs in the same way it increases productivity in the labor market. This is analogous to the neutrality assumption of Ben-Porath (1967) only with respect to investments in child human capital rather than own human capital.

Next, consider maximizing per period human capital inputs x_j subject to total expenditure i_j that period. We assume input prices (p for the price of goods and w for the price of human capital) are stable across periods and that individuals are at an interior point in their time budget (i.e. $\tau \in (\tau_{min}, \tau_{max})$). Define the following maximized period j input:

$$x_j^*(i_j; p, w) = \max_{g_j, \tau_j} \chi_j(g_j, \tau_j h_j^p) \qquad \text{subject to} \qquad pg_j + w\tau_j h_j^p = i_j \tag{2}$$

for total investment expenditures i_j in each period.

If we assume $\chi_j(\cdot,\cdot)$ are homogeneous of degree 1, then we can write

$$x_j^*(i_j;p,w)=\tilde{x}_j(p,w)i_j,$$

where $\tilde{x}(p,w)$ is the maximized output for a total expenditure of 1. Substituting this into equation (1) yields

$$h=\theta f(\tilde{x}_1(p,w)i_1,\tilde{x}_2(p,w)i_2).$$

Clearly, one can just re-write the production function in terms of total investment expenditures i_1 and i_2 as $\tilde{f}(i_1,i_2)=f(\tilde{x}_1(p,w)i_1,\tilde{x}_2(p,w)i_2)$ where the $\tilde{x}_j(p,w)$ are like technology parameters that depend on prices (p,w).

For the CES production function $f(x_1,x_2)=[ax_1^b+(1-a)x_2^b]^{d/b}$, we have the following:

$$\begin{aligned}
h &= \theta\left[a(\tilde{x}_1(p,w)i_1)^b+(1-a)(\tilde{x}_2(p,w)i_2)^b\right]^{d/b}\\
&= \theta\left\{\left[\left(\frac{a\tilde{x}_1^b}{a\tilde{x}_1^b+(1-a)\tilde{x}_2^b}\right)i_1^b+\left(\frac{(1-a)\tilde{x}_2^b}{a\tilde{x}_1^b+(1-a)\tilde{x}_2^b}\right)i_2^b\right]\left(a\tilde{x}_1^b+(1-a)\tilde{x}_2^b\right)\right\}^{d/b}\\
&= \theta\left(a\tilde{x}_1^b+(1-a)\tilde{x}_2^b\right)^{d/b}\left[\left(\frac{a\tilde{x}_1^b}{a\tilde{x}_1^b+(1-a)\tilde{x}_2^b}\right)i_1^b+\left(1-\frac{a\tilde{x}_1^b}{a\tilde{x}_1^b+(1-a)\tilde{x}_2^b}\right)i_2^b\right]^{d/b}\\
&= \tilde{\theta}\left[\tilde{a}i_1^b+(1-\tilde{a})i_2^b\right]^{d/b}
\end{aligned}$$

where

$$\begin{aligned}
\tilde{\theta} &= \theta[a\tilde{x}_1^b(p,w)+(1-a)\tilde{x}_2^b(p,w)]^{d/b}\\
\tilde{a} &= \frac{a\tilde{x}_1^b(p,w)}{a\tilde{x}_1^b(p,w)+(1-a)\tilde{x}_2^b(p,w)}.
\end{aligned}$$

Thus, if (i) parental time investment is unconstrained (i.e. at an interior point), (ii) parental human capital is equally productive in child development and the labor market, and (iii) within period investment functions $\chi_j(\cdot,\cdot)$ are homogeneous of degree 1, then our CES human capital production function still represents the production process with i_j reflecting total investment expenditures in period j. The 'technology' parameters $\tilde{\theta}$ and $\tilde{a}$ now depend on input prices p and w in addition to true underlying technology parameters.

In general, variation in prices (w,p) can affect both total factor productivity $\tilde{\theta}$ and the relative productivity of early vs. late investments, $\tilde{a}$. Two interesting special cases yield variation in $\tilde{\theta}$ alone.

First, variation in price levels (but not relative prices) will only affect $\tilde{\theta}$. For example, consider two sets of prices (p, w) and (p', w') where $\frac{w'}{w} = \frac{p'}{p} = \delta$. In this case, it is easy to see that $\tilde{x}'_j = \tilde{x}_j/\delta$, so $\tilde{\theta}' = \tilde{\theta}\delta^{-d}$ and $\tilde{a}' = \tilde{a}$.

Second, if both within-period production functions are identical, so $\chi_j(\cdot,\cdot) = \chi(\cdot,\cdot)$ and $\tilde{x}_j(p, w) = \tilde{x}(p, w)$ are independent of period j, then differences in input prices (p, w) will generally lead to differences in $\tilde{\theta} = \tilde{x}^d\theta$ but not $\tilde{a}$, which equals a regardless of (w, p).

Special Case: CES $\chi_j(\cdot,\cdot)$

Suppose $\chi_j(g, \tau h^p) = [\psi_{jg} g^\phi + \psi_{j\tau}(\tau h^p)^\phi]^{1/\phi}$. In this case, it is straightforward to show that

$$\tilde{x}_j(p, w) = \frac{\left(\psi_{jg}\left[\left(\frac{\psi_{jg}}{\psi_{j\tau}}\right)\left(\frac{w}{p}\right)\right]^{\frac{\phi}{1-\phi}} + \psi_{j\tau}\right)^{1/\phi}}{p\left(\left[\left(\frac{\psi_{jg}}{\psi_{j\tau}}\right)\left(\frac{w}{p}\right)\right]^{\frac{1}{1-\phi}} + \frac{w}{p}\right)}.$$

O.2 Properties of the Value Function $V_3(\cdot,\cdot)$

In order to apply the proofs contained in CLP to our dynastic structure, we need to demonstrate that the properties of the lifecycle continuation utility are maintained with the dynastic value function. In particular, that $V_3(a_3, h)$ is strictly increasing and strictly concave in both assets and human capital.

It is straightforward to apply the results in Stokey, Lucas, and Prescott (1989) (SLP) to show that the dynastic value function is unique, strictly increasing and strictly concave. We can rewrite the dynastic problem to be consistent with SLP:

$$V(a_3, h) = \max_{c_3, c_4, c_6, a_4, a_5, i'_1, a'_3, h'} \hat{U}(a_3, h, c_3, c_4, c_6, a_4, a_5, i'_1, a'_3, h') + \rho\beta^2 V(a'_3, h') \tag{3}$$

subject to:

$$Ra_3 + W_3(h) + y_3 - a_4 - i'_1 - c_3 - c'_1 > 0,$$

$$Ra_4 + W_4(h) + y_4 + W_2 - a_5 - a'_3 - i'_2(i'_1, h') - c_4 - c'_2 > 0,$$

$$Ra_5 + W_5(h) - R^{-1}c_6 > 0,$$

$$a_4 \geq -L_3,$$

$$a_5 \geq -L_4,$$

$$a_3' \geq -L_2,$$

$$h' = \theta' f(i_1', i_2'),$$

where

$$\hat{U}(a_3, h, c_3, c_4, c_6, a_4, a_5, i_1', a_3', h') = u(c_3) + \beta u(c_4) + \beta^2 u(Ra_5 + W_5(h) - R^{-1}c_6) + \beta^3 u(c_6) +$$

$$\rho[u(Ra_3 + W_3(h) + y_3 - a_4 - i_1' - c_3) + \beta u(Ra_4 + W_4(h) + y_4 + W_2 - a_5 - a_3' - i_2'(i_1', h') - c_4)].$$

Note that $i_2(i_1', h')$ is the i_2' that satisfies, $h' = \theta' f(i_1', i_2')$.

The following assumptions (see SLP, chapter 4) hold for this problem:

A4.3 The state space (a_3, h) is a convex subset of R^2 and the constraint set is non-empty, compact-valued and continuous.

A4.4 The function $\hat{U}$ is bounded and continuous and $\rho\beta^2 < 1$. Because $\hat{U}$ is derived from u it is bounded and continuous. The latter condition holds when $\beta < 1$ and $\rho < 1$.

A4.5 The function $\hat{U}$ is strictly increasing in a_3 and h. It is clear that $\hat{U}$ is strictly increasing in a_3 and h because $u'(\cdot) > 0$, and arguments of u are increasing in a_3 and h.

A4.6 The constraint set is monotone: As either state variable a_3 or h increases, the set of possible choice variables contains the original set.

A4.7 The function $\hat{U}$ is concave. Because u and f are concave, $\hat{U}$ is concave.

A4.8 The constraint set is convex. Convexity of the constraint set follows because f is concave.

A4.9 The function $\hat{U}$ is continuously differentiable with respect to a_3 and h.

Given these assumptions, we have (see SLP, chapter 4):

Theorem 4.6 If A4.3 and A4.4 hold there exists a unique V.

Theorem 4.7 If A4.3-A4.6 hold, V is strictly increasing.

Theorem 4.8 If A4.3-A4.4 and A4.7-A4.8 hold, V is strictly concave.

Theorem 4.11 If A4.3-A4.4 and A4.7-A4.9 hold, V is continuously differentiable.

Therefore, there exists a unique V, that is strictly increasing, strictly concave, and continuously differentiable. We do not know if V is twice, continuously differentiable. What we need is that V is twice differentiable at an optimum (at least one-sided). If this is the case, then $V_{22} < 0$, due to the concavity of V.

O.3 Calibration Sensitivity Analysis

We conduct a comprehensive sensitivity analysis for our calibration, counterfactual, and policy simulations. In particular, we re-calibrate our model imposing different assumptions about (i) the extent of dynamic complementarity (i.e. different values for b), (ii) greater borrowing opportunities (i.e. $\gamma = 0.5$), (iii) no effect of parental human capital on the child's ability (i.e. $\pi_2 = 0$), and (iv) no unmeasured costs of high school (i.e. $\zeta_1 = 0$). We also re-calibrate our model using a 'full' family income measure that adjusts for the possibility that mothers may work part-time in order to spend time investing in their children. In all cases, we repeat our main counterfactual and policy simulations with the restricted/new parameter sets obtained through the same simulated method of moments procedure.

Table O-1 reports the calibrated parameter values for all cases, while Table O-2 reports the mean weighted squared error (MWSE) for the different subsets of moments. Tables O-3 to O-6 report measures of investment and the fraction of families borrowing up to their limits in the re-calibrated economies. Tables O-7 to O-12 reproduce the main results from our counterfactual and policy simulations for each calibration.

Section 5 of the paper discusses the analysis and findings for our results imposing different values for b (-0.5, 0, 0.5) and using the 'full income' measure in creating our sets of moments (iii)-(v) for investment and period 3 wage outcomes conditional on family income and maternal education. Here, we provide a brief discussion of results for the other three cases.

When re-calibrating the model fixing any single parameter (i.e. $\gamma = 0.5$ or $\pi_2 = 0$

or $\zeta_1 = 0$), most other parameter estimates are quite similar to those of our baseline calibration (Table O-1). One exception is the smaller value for γ when imposing $\zeta_1 = 0$, implying fewer borrowing opportunities than in the baseline case. Given the importance of dynamic complementarity for many of our results, it is also worth noting the somewhat higher values (compared to the baseline calibration) for b when we impose $\pi_2 = 0$ or $\zeta_1 = 0$ and lower value when we set $\gamma = 0.5$. In terms of fit (Table O-2), imposing $\gamma = 0.5$ produces a poor fit for late investments and wage distributions, while imposing $\pi_2 = 0$ leads to a poor fit for early and late investments conditional on parental education and family income. Imposing $\zeta_1 = 0$ fits slightly worse than the baseline for all moments, but is not particularly bad for any subset. In all cases, the investment ratios for children of college graduates vs. high school dropouts are comparable to the baseline calibration (Table O-5). The proportions of families up against their borrowing or transfer constraints (Table O-6) are also quite similar to those reported for the baseline calibration with one exception: far fewer old parents are borrowing constrained when imposing $\gamma = 0.5$. The fraction of young parents up against their borrowing limit is quite similar to the baseline case even with the much higher γ. Other parameters adjust to fit the data in a way that still yields a non-trivial fraction of borrowing constrained young parents.

Table O-7 reports the anticipated and unanticipated short-run effects of a \$10,000/year income transfer to old parents. In all cases, the effects of an anticipated transfer are much greater than an unanticipated transfer; however, the the differences are more modest when $\pi_2 = 0$ or $\zeta_2 = 0$ are imposed. These more muted differences are consistent with the greater substitutability implied by the higher estimated values for b in these cases, much as we see for the case imposing $b = 0.5$.

Tables O-8 and O-9 reproduce the counterfactual analyses aimed at understanding the importance of ability transmission and market frictions for intergenerational mobility. In all cases, child ability accounts for a comparable share of the investment gaps by parental income, while eliminating lifecycle borrowing constraints would have similar or stronger effects (compared to the baseline calibration). There is a greater discrepancy between calibration cases in the implied role of ability vs. market frictions when we simulate the economy with zero intergenerational ability correlation (Table O-9). Assuming greater opportunities for borrowing than estimated by our baseline calibration (imposing $\gamma = 0.5$)

produces a much greater role for ability transmission relative to market frictions.

As shown in Tables O-10 and O-11, we obtain very similar effects of relaxing borrowing constraints (one-by-one or completely eliminating all constraints) for all of our restricted calibration sets, even when $\gamma = 0.5$ is assumed. In all cases, completely eliminating all life-cycle borrowing constraints has substantial effects on investments and post-school earnings – much greater than the effects of relaxing any single borrowing limit by itself.[1]

Finally, Table O-12 reports the short-run effects of fiscally equivalent early and late investment subsidies. In all calibration cases, we consider the impacts of increasing s_1 to 0.1, as well as increasing s_2 by an amount that produces the same total expenditure on all investment subsidies. Our main conclusions hold for all parameterizations: (i) early investment subsidies have greater effects than late subsidies, and (ii) the effects of late subsidies are much greater when the subsidies are announced early so early investment can respond. Perhaps surprisingly, the effects of subsidies are greatest when $\gamma = 0.5$. They are smallest when $\pi_2 = 0$.

References

Ben-Porath, Y. (1967). The production of human capital and the life cycle of earnings. *Journal of Political Economy 75*(4), 352–365.

Stokey, N. L., R. E. Lucas Jr., with E. C. Prescott (1989). *Recursive Methods in Economic Dynamics*. Cambridge, MA: Columbia University Press.

[1] Note that Table O-10 studies the short-run effects of increasing borrowing limits by \$1,500 rather than \$2,500 as in the paper, because increasing borrowing limits (at one stage) by the latter amount (in two calibration cases) would extend them beyond the natural borrowing limits for some families due to subsequent constraints.

Table O-1: Calibrated Parameter Values under Different Restrictions and Data Assumptions

Parameter	Baseline	b = 0	b = 0.5	b = -0.5	γ = 0.5	π_2 = 0	ζ_1 = 0	full income
a	0.58	0.53	0.62	0.50	0.52	0.55	0.60	0.59
b	0.26	**0**	**0.5**	**-0.5**	0.15	0.37	0.33	0.32
d	0.82	0.82	0.82	0.82	0.80	0.80	0.81	0.81
θ_1	4.85	4.60	5.10	4.77	5.35	5.00	5.20	5.46
θ_2	12.03	12.16	12.44	12.06	13.79	13.16	14.52	14.18
π_0	-0.88	-0.57	-0.90	-0.76	-0.89	-1.07	-0.66	-0.69
π_1	0.15	0.10	0.13	0.14	0.12	0.15	0.12	0.12
π_2	0.000019	0.000014	0.000019	0.000018	0.000016	**0**	0.000017	0.000010
ζ_1	47.49	61.72	30.86	88.19	57.81	29.97	**0**	39.85
ζ_2	760.73	726.46	719.28	571.99	857.38	808.31	809.12	888.53
m	9.90	9.96	9.90	9.92	9.93	9.85	9.81	9.90
s	0.71	0.70	0.71	0.75	0.74	0.71	0.77	0.71
ρ	0.86	0.85	0.84	0.87	0.84	0.86	0.83	0.85
γ	0.22	0.11	0.07	0.15	**0.5**	0.17	0.05	0.09

Table O-2: Weighted Average Mean Squared Error for Calibration Sensitivity Analysis

	Baseline	b = 0	b = 0.5	b = -0.5	$\gamma = 0.5$	$\pi_2 = 0$	$\zeta_1 = 0$	full income
Moment subset:								
$Pr(i_2)$	0.0001	0.0002	0.0001	0.0026	0.0008	0.0000	0.0001	0.0002
$E(W_3 \| i_2)$, $Var(W_3 \| i_2)$, $Cov(W_3,W_4)$	0.6490	0.8221	0.6410	1.0850	0.8426	0.6545	0.6924	0.6485
$E(\Phi \| i_2, W_3, W_4)$	0.0478	0.0597	0.0630	0.0442	0.0453	0.0882	0.0544	0.0650
$Pr(i_2' \| i_2, W_4)$	0.0046	0.0049	0.0058	0.0044	0.0047	0.0066	0.0055	0.0053
$E(W_3' \| i_2, W_3, W_4)$	0.0282	0.0302	0.0233	0.0391	0.0237	0.0248	0.0321	0.0278
$Pr(a_4<0)$	0.0336	0.0335	0.0339	0.0325	0.0260	0.0326	0.0347	0.0339
All moments	0.0089	0.0099	0.0093	0.0114	0.0106	0.0107	0.0097	0.0089

Notes: Values for subsets of moments reflect the weighted average MSE over that subset of moments only.

Table O-3: Calibrated Education Distribution (Senstivity Analysis)

Education	Baseline	$b = 0$	$b = 0.5$	$b = -0.5$	$\gamma = 0.5$	$\pi_2 = 0$	$\zeta_1 = 0$	full income
HS Graduate or More	0.83	0.81	0.83	0.83	0.82	0.82	0.82	0.81
Some College or More	0.44	0.44	0.42	0.54	0.41	0.41	0.42	0.41
College Graudate	0.21	0.19	0.21	0.22	0.15	0.19	0.21	0.20

Table O-4: Avg. Early Investment Factor Scores and Educational Attainment by Parental Education

	Parental Education	Early investment Score	HS Grad. or More	Some College or More	College Grad.
Baseline	HS Dropout	-0.49	0.64	0.20	0.08
	HS Graduate	-0.40	0.81	0.27	0.08
	Some College	0.11	0.90	0.57	0.15
	College Graduate	0.67	0.94	0.82	0.63
$b = 0$	HS Dropout	-0.49	0.68	0.23	0.08
	HS Graduate	-0.40	0.78	0.30	0.08
	Some College	0.13	0.87	0.58	0.14
	College Graduate	0.67	0.91	0.76	0.58
$b = 0.5$	HS Dropout	-0.48	0.66	0.20	0.08
	HS Graduate	-0.39	0.80	0.27	0.08
	Some College	0.12	0.91	0.54	0.14
	College Graduate	0.67	0.93	0.79	0.62
$b = -0.5$	HS Dropout	-0.51	0.64	0.27	0.09
	HS Graduate	-0.41	0.78	0.35	0.09
	Some College	0.09	0.89	0.64	0.12
	College Graduate	0.67	0.94	0.84	0.65
$\gamma = 0.5$	HS Dropout	-0.47	0.65	0.20	0.07
	HS Graduate	-0.38	0.80	0.27	0.07
	Some College	0.14	0.92	0.56	0.12
	College Graduate	0.67	0.91	0.78	0.54
$\pi_2 = 0$	HS Dropout	-0.48	0.70	0.22	0.09
	HS Graduate	-0.39	0.78	0.29	0.09
	Some College	0.13	0.91	0.53	0.13
	College Graduate	0.67	0.91	0.69	0.56
$\zeta_1 = 0$	HS Dropout	-0.50	0.66	0.19	0.08
	HS Graduate	-0.40	0.79	0.23	0.07
	Some College	0.13	0.89	0.57	0.13
	College Graduate	0.67	0.93	0.85	0.66
full income	HS Dropout	-0.49	0.65	0.19	0.08
	HS Graduate	-0.40	0.79	0.24	0.07
	Some College	0.13	0.88	0.55	0.15
	College Graduate	0.67	0.93	0.79	0.63

Table O-5: Average Investment Amounts by Parental Education (Sensitivity Analysis)

	Parental Education	i_1	i_2	$i_2 + \zeta(i_2) - S_2(i_2)$
	All Levels	1888	8744	5629
Baseline	**HS Dropout**	770	4351	2671
	College Graduate	4600	18687	12304
	All Levels	1921	8369	5374
b = 0	**HS Dropout**	877	4660	2875
	College Graduate	4721	17305	11306
	All Levels	1296	8464	5375
b = 0.5	**HS Dropout**	523	4329	2612
	College Graduate	3463	18325	11946
	All Levels	3389	9641	6078
b = -0.5	**HS Dropout**	1521	4952	3039
	College Graduate	7315	19149	12186
	All Levels	1229	7524	4924
$\gamma = 0.5$	**HS Dropout**	575	4181	2604
	College Graduate	3200	16910	11345
	All Levels	736	8137	5246
$\pi_2 = 0$	**HS Dropout**	363	4842	2984
	College Graduate	2028	16547	10907
	All Levels	2055	8483	5462
$\zeta_1 = 0$	**HS Dropout**	877	4279	2586
	College Graduate	5227	19347	12857
	All Levels	1702	8337	5471
full income	**HS Dropout**	738	4362	2721
	College Graduate	4312	18388	12371

Table O-6: Fraction Borrowing and Transfer Constrained (Sensitivity Analysis)

		Fraction of Young Parents Constrained	Fraction of Old Parents Constrained	Fraction of Parents Transfer Constrained
Baseline	All Levels	0.12	0.14	0.00
	HS Dropout	0.13	0.06	0.01
	HS Graduate	0.20	0.17	0.00
	Some College	0.06	0.17	0.00
	College Graduate	0.01	0.14	0.00
$b = 0$	All Levels	0.13	0.18	0.00
	HS Dropout	0.13	0.05	0.00
	HS Graduate	0.21	0.17	0.00
	Some College	0.10	0.22	0.00
	College Graduate	0.03	0.25	0.00
$b = 0.5$	All Levels	0.13	0.18	0.00
	HS Dropout	0.16	0.06	0.00
	HS Graduate	0.23	0.18	0.00
	Some College	0.06	0.25	0.00
	College Graduate	0.00	0.21	0.00
$b = -0.5$	All Levels	0.10	0.16	0.00
	HS Dropout	0.09	0.03	0.00
	HS Graduate	0.18	0.13	0.00
	Some College	0.11	0.26	0.00
	College Graduate	0.01	0.18	0.00
$\gamma = 0.5$	All Levels	0.11	0.04	0.01
	HS Dropout	0.15	0.03	0.02
	HS Graduate	0.17	0.05	0.00
	Some College	0.05	0.05	0.00
	College Graduate	0.00	0.01	0.00
$\pi_2 = 0$	All Levels	0.12	0.13	0.00
	HS Dropout	0.16	0.06	0.01
	HS Graduate	0.19	0.14	0.00
	Some College	0.05	0.17	0.00
	College Graduate	0.00	0.14	0.00
$\zeta_1 = 0$	All Levels	0.15	0.21	0.00
	HS Dropout	0.10	0.03	0.00
	HS Graduate	0.27	0.22	0.00
	Some College	0.08	0.29	0.00
	College Graduate	0.01	0.25	0.00
full income	All Levels	0.13	0.19	0.00
	HS Dropout	0.11	0.04	0.00
	HS Graduate	0.24	0.21	0.00
	Some College	0.06	0.24	0.00
	College Graduate	0.01	0.25	0.00

Table O-7: Short-Run Effects (% Change) of $10,000 One-Time Transfer to Old Parents (Senstivity Analysis)

	Unanticipated or Anticipated	Avg. i_1	Avg. i_2	Some Coll+	Avg. W_3
Baseline	**unanticipated**	0.0	1.4	3.0	0.2
	anticipated	8.0	6.2	7.2	1.3
b = 0	**unanticipated**	0.0	2.8	4.9	0.3
	anticipated	7.5	6.5	6.7	1.2
b = 0.5	**unanticipated**	0.0	8.9	11.4	1.0
	anticipated	9.2	7.7	8.2	1.3
b = -0.5	**unanticipated**	0.0	0.2	0.0	0.0
	anticipated	5.6	4.7	3.6	1.1
γ = 0.5	**unanticipated**	0.0	0.9	0.5	0.1
	anticipated	7.8	6.2	7.3	1.0
π_2 = 0	**unanticipated**	0.0	6.6	11.1	0.8
	anticipated	10.5	6.9	7.5	1.1
ζ_1 = 0	**unanticipated**	0.0	6.0	7.3	0.7
	anticipated	8.1	7.0	6.3	1.4
full income	**unanticipated**	0.0	5.5	6.9	0.6
	anticipated	8.8	7.4	7.6	1.4

Table O-8: Decomposition of Investment Gaps between Parental Income Quartiles 1 and 4 (Sensitivity Analysis)

		Investment Gaps			% Change Relative to Benchmark		
		Avg. i1	Avg. i2	Some Coll+	Avg. i1	Avg. i2	Some Coll+
Baseline	**Benchmark:**						
	Unconditional	3057	7743	0.38			
	Conditional on child ability	2615	5924	0.28	-14.5	-23.5	-26.6
	Relax all borrowing limits:						
	Unconditional	3555	8174	0.33	16.3	5.6	-14.1
	Conditional on child ability	2480	3757	0.12	-18.9	-51.5	-67.6
b = 0	**Benchmark:**						
	Unconditional	3038	7447	0.42			
	Conditional on child ability	2591	5755	0.32	-14.7	-22.7	-23.8
	Relax all borrowing limits:						
	Unconditional	3452	6375	0.21	13.6	-14.4	-49.4
	Conditional on child ability	2254	2014	0.00	-25.8	-73.0	-100.0
b = 0.5	**Benchmark:**						
	Unconditional	2226	10351	0.54			
	Conditional on child ability	1904	8518	0.44	-14.5	-17.7	-18.5
	Relax all borrowing limits:						
	Unconditional	2837	7119	0.26	27.4	-31.2	-52.7
	Conditional on child ability	1818	1722	0.00	-18.3	-83.4	-100.0
b = -0.5	**Benchmark:**						
	Unconditional	4671	7613	0.34			
	Conditional on child ability	3855	5430	0.20	-17.5	-28.7	-39.5
	Relax all borrowing limits:						
	Unconditional	5270	8706	0.32	12.8	14.4	-6.5
	Conditional on child ability	3632	4260	0.10	-22.2	-44.0	-69.1
$\gamma = 0.5$	**Benchmark:**						
	Unconditional	1798	5079	0.26			
	Conditional on child ability	1519	3581	0.17	-15.5	-29.5	-36.3
	Relax all borrowing limits:						
	Unconditional	2285	6736	0.27	27.1	32.6	5.2
	Conditional on child ability	1658	3382	0.11	-7.8	-33.4	-59.6
$\pi_2 = 0$	**Benchmark:**						
	Unconditional	1195	7685	0.44			
	Conditional on child ability	984	5815	0.34	-17.7	-24.3	-23.6
	Relax all borrowing limits:						
	Unconditional	1462	5640	0.22	22.4	-26.6	-49.7
	Conditional on child ability	805	919	0.00	-32.6	-88.0	-100.0
$\zeta_1 = 0$	**Benchmark:**						
	Unconditional	3569	9730	0.47			
	Conditional on child ability	3136	8087	0.39	-12.1	-16.9	-17.7
	Relax all borrowing limits:						
	Unconditional	4595	8943	0.34	28.8	-8.1	-27.5
	Conditional on child ability	3418	4499	0.15	-4.2	-53.8	-68.5
full income	**Benchmark:**						
	Unconditional	2870	9010	0.45			
	Conditional on child ability	2514	7435	0.37	-12.4	-17.5	-18.1
	Relax all borrowing limits:						
	Unconditional	3325	6838	0.23	15.8	-24.1	-49.0
	Conditional on child ability	2245	2262	0.01	-21.8	-74.9	-97.8

Table O-9: Intergenerational Ability and Investment Transmission (Sensitivity Analysis)

		Baseline	No effect of parental h_3 on child θ	No correlation between parent and child θ	Perfect correlation between parent and child θ
	Intergen. corr. in θ	0.31	0.31	0.00	1.00
Baseline	**Intergen. corr. In i_2**	0.52	0.46	0.29	0.85
	Intergen. corr. In lifetime earnings	0.29	0.26	0.19	0.44
	Intergen. corr. in θ	0.24	0.24	0.00	1.00
b = 0	**Intergen. corr. In i_2**	0.45	0.40	0.26	0.84
	Intergen. corr. In lifetime earnings	0.27	0.25	0.17	0.47
	Intergen. corr. in θ	0.29	0.29	0.00	1.00
b = 0.5	**Intergen. corr. In i_2**	0.50	0.44	0.27	0.85
	Intergen. corr. In lifetime earnings	0.29	0.26	0.18	0.44
	Intergen. corr. in θ	0.30	0.30	0.00	1.00
b = -0.5	**Intergen. corr. In i_2**	0.50	0.44	0.25	0.85
	Intergen. corr. In lifetime earnings	0.30	0.27	0.18	0.46
	Intergen. corr. in θ	0.29	0.29	0.00	1.00
γ = 0.5	**Intergen. corr. In i_2**	0.46	0.34	0.13	0.83
	Intergen. corr. In lifetime earnings	0.20	0.10	0.05	0.21
	Intergen. corr. in θ	0.28	0.28	0.00	1.00
$\pi_2 = 0$	**Intergen. corr. In i_2**	0.42	0.42	0.24	0.82
	Intergen. corr. In lifetime earnings	0.23	0.23	0.14	0.45
	Intergen. corr. in θ	0.31	0.31	0.00	1.00
$\zeta_1 = 0$	**Intergen. corr. In i_2**	0.55	0.49	0.33	0.84
	Intergen. corr. In lifetime earnings	0.33	0.30	0.22	0.47
	Intergen. corr. in θ	0.27	0.27	0.00	1.00
full income	**Intergen. corr. In i_2**	0.51	0.47	0.33	0.83
	Intergen. corr. In lifetime earnings	0.30	0.28	0.21	0.48

Table O-10: Short-Run Effects (% Change) of Increasing Borrowing Limits by $1,500 (Sensitivity Analysis)

	Relaxing Constraint on Young Parents					Relaxing Constraint on Old Parents				
	Avg. i_1	Avg. i_2	HS+	Some Coll+	Avg. W_3	Avg. i_1	Avg. i_2	HS+	Some Coll+	Avg. W_3
Baseline	1.7	1.1	-0.2	2.9	0.3	7.2	6.3	1.7	3.8	1.2
b = 0	2.3	1.6	0.5	2.2	0.3	5.7	5.8	3.6	4.5	1.0
b = 0.5	1.0	1.2	0.5	1.8	0.2	7.2	6.8	3.2	3.6	1.1
b = -0.5	1.2	0.9	0.6	1.8	0.2	4.7	4.3	0.0	1.3	1.0
$\gamma = 0.5$	2.2	1.2	-0.8	2.7	0.2	8.4	7.0	0.8	0.5	1.0
$\pi_2 = 0$	0.9	1.1	0.8	2.1	0.2	11.1	6.3	1.8	2.4	1.0
$\zeta_1 = 0$	2.5	1.6	0.6	2.6	0.4	6.6	5.4	0.7	2.9	1.1
full income	2.5	1.6	0.7	2.8	0.3	7.1	6.7	4.3	4.2	1.1

Table O-11: Short-Run Effects (% Change) of Fully Relaxing All Borrowing Limits (Sensitivity Analysis)

	Avg. i_1	Avg. i_2	HS+	Some Coll+	Avg. W_3
Baseline	72.5	63.2	12.5	31.0	11.7
b = 0	80.7	71.5	17.1	44.9	12.8
b = 0.5	96.9	82.4	15.7	54.7	13.7
b = -0.5	48.7	45.8	4.8	12.6	10.1
$\gamma = 0.5$	68.1	59.5	11.9	25.8	8.5
$\pi_2 = 0$	116.6	69.9	21.4	41.5	11.4
$\zeta_1 = 0$	76.9	66.9	4.7	35.8	13.1
full income	88.7	86.0	16.9	68.0	14.9

Table O-12: Short-Run Effects (% Change) of Early and Late Investment Subsidies (Sensitivity Analysis)

	Policy	Avg. i_1	Avg. i_2	HS+	Some Coll+	Coll Grad	Avg. W_3
	Announced early:						
	$s_1 = 0.10$	63.6	22.5	0.8	13.5	43.2	6.5
Baseline	**$s_2 = 0.026$**	13.0	25.9	15.9	17.7	39.3	3.6
	Announced late:						
	$s_2 = 0.026$	0.0	15.4	15.9	15.4	15.1	1.6
	Announced early:						
	$s_1 = 0.10$	48.7	16.8	0.6	7.7	36.2	4.9
b = 0	**$s_2 = 0.028$**	10.5	19.3	17.6	10.4	29.8	2.6
	Announced late:						
	$s_2 = 0.028$	0.0	9.6	17.6	8.3	6.2	0.9
	Announced early:						
	$s_1 = 0.10$	72.3	8.8	0.4	3.3	18.7	4.2
b = 0.5	**$s_2 = 0.015$**	3.8	13.4	12.9	5.7	20.9	1.5
	Announced late:						
	$s_2 = 0.015$	0.0	12.6	12.9	5.6	19.0	1.3
	Announced early:						
	$s_1 = 0.10$	43.1	21.8	-0.4	8.4	48.0	6.9
b = -0.5	**$s_2 = 0.048$**	16.6	22.5	18.7	6.1	42.5	4.1
	Announced late:						
	$s_2 = 0.048$	0.0	3.8	18.6	0.0	0.0	0.3
	Announced early:						
	$s_1 = 0.10$	107.2	44.8	0.5	16.6	112.5	8.7
$\gamma = 0.5$	**$s_2 = 0.024$**	38.9	46.4	16.8	24.4	95.3	6.0
	Announced late:						
	$s_2 = 0.024$	0.0	13.0	16.8	21.4	0.0	1.3
	Announced early:						
	$s_1 = 0.10$	86.6	13.1	-0.2	3.0	30.9	3.9
$\pi_2 = 0$	**$s_2 = 0.012$**	9.4	12.7	14.1	4.1	20.1	1.6
	Announced late:						
	$s_2 = 0.012$	0.0	8.9	14.1	3.9	10.5	0.9
	Announced early:						
	$s_1 = 0.10$	52.9	13.0	0.2	9.4	23.7	5.2
$\zeta_1 = 0$	**$s_2 = 0.022$**	6.5	18.7	17.0	11.8	26.1	2.4
	Announced late:						
	$s_2 = 0.022$	0.0	15.5	17.0	11.0	18.9	1.6
	Announced early:						
	$s_1 = 0.10$	59.1	14.8	0.8	9.8	27.2	5.1
full income	**$s_2 = 0.021$**	8.2	18.6	17.5	10.7	26.4	2.4
	Announced late:						
	$s_2 = 0.021$	0.0	14.5	17.5	9.4	17.5	1.4