NBER WORKING PAPER SERIES

HUMAN CAPITAL FORMATION WITH ENDOGENOUS CREDIT CONSTRAINTS

Lance Lochner
Alexander Monge-Naranjo

Working Paper 8815
http://www.nber.org/papers/w8815

NATIONAL BUREAU OF ECONOMIC RESEARCH
1050 Massachusetts Avenue
Cambridge, MA 02138
March 2002

We are thankful for the comments of seminar participants at the Federal Reserve Bank of Minneapolis, Universitat Autonoma de Barcelona, Northwestern University, the SIEPR conference on Credit Markets at Stanford, the Society of Economic Dynamics meetings in San José, Costa Rica, the Computing Economics and Finance Conference in Universitat Pompeu Fabra, Barcelona, the Information and Aggregate Fluctuations conference in Universita di Roma, La Sapienza, the Wharton School and the University of Iowa. This paper also benefited from discussions with Gadi Barlevy, Marco Bassetto, Cristina de Nardi, Andrés Erosa, Jeremy Greenwood, Rick Hanushek, Jim Heckman, Tim Kehoe, Narayana Kocherlakota, Chris Phelan, Ed Prescott, Hugo Rodríguez and Chris Taber. Monge gratefully acknowledges the financial support of Northwestern University and the National Science Foundation, grant 0112943. Send comments to a-monge1@northwestern.edu and lochner@hoover.stanford.edu. The views expressed herein are those of the authors and not necessarily those of the National Bureau of Economic Research.

Human Capital Formation with Endogenous Credit Constraints
Lance Lochner and Alexander Monge-Naranjo
NBER Working Paper No. 8815
March 2002
JEL No. I2, E0, E2

ABSTRACT

We study the accumulation of human capital and the behavior of consumption and earnings in a life cycle equilibrium model with endogenous borrowing constraints. Constraints arise endogenously from the inalienability of human capital and the limited punishments that creditors are able to impose on those who default. The endogeneity of borrowing constraints produces a number of interesting relationships. First, efficient borrowing limits are functions of individual observable characteristics and choices, especially ability and human capital investments. The connection between human capital investments and borrowing limits creates additional incentives to invest beyond those present in models with exogenous constraints. Second, government policies affect the incentives to default and, hence, the limits on private borrowing. As opposed to exogenous constraint models, additional subsidies for investment in human capital should be accompanied by increases in credit, since borrowers are more able to re-pay higher debts. Finally, general equilibrium considerations have an additional role, since borrowing limits depend on the returns to physical and human capital.

We calibrate the model to U.S. data and are able to replicate key features of the economy regarding human capital investment, earnings, and consumption. The calibrated model is then used to study the steady state impacts of changes in government policies. We find that changes in bankruptcy laws can have sizeable effects on the accumulation of both human and physical capital. At the aggregate level, general equilibrium forces are important and can reverse the results predicted in partial equilibrium. Government subsidies to education (financed with a proportional tax on earnings) cause lenders to increase credit limits and substantially increase aggregate human and physical capital. Most importantly, we show that the implications of our model are very different from those of standard exogenous constraint models. For example, the effects of increases in initial wealth and government subsidies on investment are substantially greater in our model than in a similar model with exogenous constraints.

Lance Lochner
Hoover Institution
Stanford University
Stanford, CA 94305-6010,
University of Rochester,
and NBER
lochner@hoover.stanford.edu

Alexander Monge-Naranjo
Department of Economics
Northwestern University
2003 N. Sheridan Road
Evanston, IL 60208

1 Introduction

This paper develops a framework for analyzing the life cycle behavior of human and physical capital accumulation in general equilibrium environments where credit constraints arise endogenously from individual incentive problems. There are three primary objectives. The first is to understand how credit constraints arise endogenously in an efficient contracting environment with life cycle decisionmaking. Because creditors are limited in the punishments they can impose on borrowers that default, lenders will restrict the amount of credit they extend.[1] In our framework, these limits depend on the future earnings capacity of borrowers. In contrast to models that impose exogenous borrowing constraints, individuals of heterogeneous abilities or those making different schooling choices will face different borrowing limits. Constraints will also vary over the life cycle for similar reasons. In short, borrowing limits are functions of ability, age, and human capital investment choices in an efficient lending environment.[2]

The second objective of the paper is to examine the implications of government policies when borrowing constraints arise endogenously, incorporating the interaction between policies and credit constraints. Policies like public schooling, education subsidies, and income taxation will alter constraint levels in an efficient contracting environment. As a result, previous analyses that exogenously fix constraints are misleading. For example, college subsidies are often discussed as a substitute for student loans. However, we find that with efficient credit markets and endogenously determined borrowing limits, lending should increase in response to an increase in subsidies to schooling. This is because the subsidies encourage additional investments in human capital, which reduces incentives to default. Thus, subsidy and loan policies can be better seen as complements rather than substitutes.

Finally, insights from endogenous constraint models can be used for evaluating and better designing actual student loan programs. For example, in the United States, it is important to understand how the Guaranteed Student Loan (GSL) program affects incentives to invest in human capital, interacts with efficient credit markets, and generates incentives to default. More efficient loan programs may be possible and can only be found with a better understanding of the incentives in the program and in private credit markets.

The framework developed in this paper recognizes that credit laws typically create stronger incentives for individuals with higher earnings potential to repay their debt, because penalties for default disproportionately disrupt their life cycle consumption patterns. (For example, the inability

[1] Fay, Hurst, and White [20] empirically show that households are more likely to declare bankruptcy when the financial benefits of doing so are greater. Gropp, Scholz, and White [22] estimate that households living in states with larger bankruptcy asset exemptions are more likely to have their loan applications rejected and to be discouraged from borrowing. Monge, et al. [40] and the other studies in Pagano [42] examine how incentives to default translate into borrowing constraints in different countries.

[2] See Jappelli [28] and Fay, Hurst, and White [20] for empirical evidence on these relationships.

to borrow additional funds for years after default or bankruptcy can be extremely costly for someone with sharply increasing earnings profiles who would like to smooth consumption over time.) As a result, they can obtain relatively high amounts of credit. Those with low earnings potential have little incentive to repay loans and, consequently, face more stringent constraints. Human capital theory indicates that the more able an individual and the more he invests in his skills, the more earnings potential he will possess. In an efficient credit market, he should, therefore, be allowed to borrow more. That is, efficient credit markets establish a connection between credit limits and the observable characteristics and human capital investments of individuals, an implication consistent with recent empirical work by Keane and Wolpin. [31] The link between human capital investments and the amount of available credit generates additional incentives to invest in human capital–incentives that do not exist in models of exogenously determined borrowing constraints.

Constraints will also depend on the age of borrowers. At early ages, exclusion from lending markets may be costly, while it is not at older ages. In contrast, earnings and savings levels are relatively high near retirement, so the impact of punishments that allow creditors to seize the earnings or assets of defaulters will be particularly acute at those ages. Because the effects of these punishments on borrowers vary over the life cycle, credit limits will also tend to vary with age. Our results suggest that constraints tend to be more stringent for younger individuals.

The endogeneity of credit constraints also captures the response of private markets to public policy through changes in incentive-compatible allocations. Efficient borrowing limits will change in response to policies that alter individual incentives to repay loans by changing their wealth levels or returns to human capital investment. Moreover, as in Kiyotaki and Moore [33], credit constraints depend on asset prices. To the extent that government policies are effective in changing investment decisions, they are likely to have non-trivial general equilibrium effects. Our model incorporates these effects, allowing borrowing constraints to adjust with changes in interest rates and wage rates caused by policy intervention.

It has long been argued that the presence of borrowing constraints induces individuals with low wealth to under-invest in human capital when human capital cannot serve as collateral for financial liabilities (see Becker [6]). In the standard economic framework, credit limits are fixed and independent of the observable characteristics and decisions of individuals. Alternatively, credit 'constraints' are sometimes represented by interest rates that increase with the amount borrowed or that exogenously vary in the population. [6, 10, 11] Based on these ideas, an empirical literature has developed that focuses on two tests of credit constraints in the market for human capital. One branch of the literature tests whether individuals from different family income levels have different college enrollment rates conditional on ability and other variables that may influence tastes for schooling or the ability to attend. [8, 9, 19, 30, 31] The second branch compares the returns to schooling for individuals who are expected to face different interest rates or constraints on their borrowing. [10, 11, 37]

Disagreement about the importance of credit constraints in determining schooling levels in the U.S. abounds. Kane [30] argues that differences in family income are responsible for a sizeable difference in college enrollment rates. However, Cameron and Heckman [8, 9] find that after controlling for cognitive ability and dynamic unobserved self-selection, family income has little effect on enrollment rates. This leads them to conclude that short-term borrowing constraints at the college age are not responsible for much of the difference in college enrollment rates by family income. Estimating a dynamic model of educational choice, Keane and Wolpin [31] find evidence of stringent credit constraints during college years, but they estimate that relaxing those constraints would have little effect on enrollment decisions. While Card [11] argues that individuals most likely to face constraints receive higher returns to schooling (suggesting constraints do exist and prevent constrained youth from pursuing highly productive opportunities), Cameron and Taber [10] find little evidence of differential returns consistent with borrowing constraints.

In the only study that attempts to empirically estimate actual borrowing limits (albeit, in an exogenous constraint framework), Keane and Wolpin [31] find that individuals with more human capital can borrow more than those with fewer skills. While the empirical literature has paid little attention to the forces underlying constraints on borrowing, our calibrated model generates outcomes consistent with this finding. The model embodies the idea that, on average, individuals possess few physical assets at early ages when it is most efficient to invest in human capital. They must, therefore, borrow against future earnings if they want to invest in their skills. Even after schooling, in the early stages of labor market participation, they may want to borrow additional funds to smooth consumption in anticipation of higher future earnings. The degree to which incentives to re-pay early loans line up with the needs of would-be students determines the extent of borrowing constraints at young ages. Because more able and skilled individuals can credibly commit to re-pay higher amounts of debt, our model produces the type of heterogeneity in constraints estimated by Keane and Wolpin [31].

While most human capital studies with borrowing constraints assume that those constraints are exogenously determined (e.g. Aiyagari, Greenwood, and Seshadri [1], Caucutt and Kumar, [13] Hanushek, Yilmaz, and Leung, [23] and Loury [39]), theoretical studies of endogenous credit constraints have ignored issues related to human capital accumulation. Early studies were primarily limited to implications for risk sharing and asset prices, taking household earnings to be an exogenous, and often stationary, process (e.g. Alvarez and Jermann [2], Kehoe and Levine [32], and Kocherlakota, [34]). However, a number of more recent papers have used endogenous constraint models to study the importance of durable goods (Krueger and Fernandez [21]) and pensions (Andolfatto and Gervais [3] and Lambertini [36]) in determining life cycle consumption decisions. Also within endogenous constraint frameworks, Krueger and Perri [35] study the effect of progressive taxation on insurance markets, and Attanassio and Ríos-Rull [5] explore the impact of outside transfers on insurance for villages. None of these studies analyze the role of endogenous credit constraints in determining human capital investment decisions–the primary focus of this paper.

The rest of this paper proceeds as follows. The next section describes our model of endogenous credit constraints, as well as models with perfect credit markets and exogenous borrowing constraints. Section 3 uses data from the U.S. to parameterize the model, and Section 4 analytically and numerically examines the effects of government policies on the repayment incentives of individuals and the resulting implications for credit constraints. We contrast the implications of our model with exogenous constraint models, examining the effects of alternative taxation schemes, public schooling, and education subsidies. We discuss the existing U.S. Guaranteed Student Loan (GSL) program and the possibility of default in Section 5. Section 6 concludes with a summary and discussion of avenues for future research.

2 The Environment

Consider a standard, infinite horizon, discrete time economy, populated by four-period-lived overlapping generations of agents. Each generation contains a continuum (with a mass of unity) of individuals. In each period $t = 0, 1, 2, \ldots$, the economy is populated by young agents, two tiers of mature agents, and old agents. In the second period of adulthood, all individuals have a single child at which time they give them bequests, b.[3] Given this timing, bequest decisions are made when credit constraints are irrelevant; though, parents may still be tempted to default on previous loans at this time. Within each cohort there is two-dimensional intra-generational heterogeneity. Each young generation is composed of agents with different endowments of physical assets, a, (given to them by their parents as bequests) and different endowments of learning abilities, e, which are inherited from their parents following a Markov process described in detail below. The support of these characteristics is $A \times E$. The probability measure λ describes the endogenous distribution of agents in this support.

Preferences at birth are given by

$$U_0 = u(c_0) + \beta u(c_1) + \beta^2 \left[u(c_2) + \rho v(b)\right] + \beta^3 u(c_3),$$

where $\beta \in (0, 1)$ is the discount factor and c_j represents consumption at age j.[4] The function $v(\cdot)$

[3] More generally, the children may be born at any earlier period, but the bulk of expenses paid for by parents are assumed to be paid in the second period of adulthood. Without changing the results, one could instead assume that individuals transfer bequests in the final period of life.

[4] We omit calendar time indices, since we focus on time invariant (stationary) equilibria.

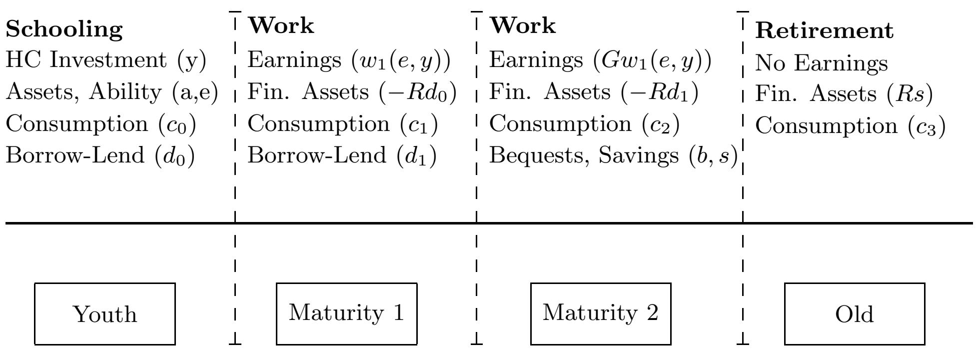
reflects the utility of transferring resources to one's own child. For analytical tractability, we shall assume that $v(\cdot) = u(\cdot)$. The degree of "altruism" will be determined by the parameter $\rho \geq 0$. From the point of view of a young worker, a mature worker, and a retiree, preferences are given by:

$$\begin{aligned} U_1 &= u(c_1) + \beta \left[u(c_2) + \rho v(b) \right] + \beta^2 u(c_3), \\ U_2 &= u(c_2) + \rho v(b) + \beta^2 u(c_3), \\ U_3 &= u(c_3). \end{aligned}$$

Below, we assume the period utility function is given by $u(c) = \frac{c^{1-\sigma}}{1-\sigma}$, where $\sigma > 0$. The typical life cycle of all agents is described in Diagram 1.

Individuals in this economy must optimally convert physical capital into human capital, use their human capital to produce output and earn an income, transfer resources across periods to smooth consumption, and provide bequests to their offspring. In the first period, agents use their initial endowment of assets, a, to finance consumption, c, and investment in human capital, y. Human capital investment is measured in units of the only consumption/capital good and captures not only time in school but also its quality and intensity. They may also borrow or lend. Positive d_0 indicates the net debt of young agents. During the two ages of maturity, individuals work to finance current and future consumption as well as bequests to their children given in the second period of adulthood. Their wages are proportional to their human capital, h, acquired from previous investment and work experience. Let d_1 denote the debt ($d_1 > 0$) or savings ($d_1 < 0$) of individuals in the first age of maturity. Finally, let s denote the savings (always positive in equilibrium) of individuals in the second age of maturity. When old, individuals no longer work, consuming their savings.

Diagram 1: Stages in the Life-Cycle of Agents



We assume that ability and investments are complementary in the production of human capital. Specifically, human capital in the first age of maturity of an agent with ability e who invested y

units during his youth is given by

$$h_1 = ey^{\alpha},$$

where $\alpha \in (0,1)$ is the investment-elasticity of the human capital production function.

Individuals accumulate work experience that augments their human capital, so that human capital at the second age of maturity is given by

$$h_2 = Gh_1 = Gey^{\alpha}, \quad G > 1.$$

Earnings during the first and second maturity ages are given by

$$w_1(e,y) = wh_1 = wey^{\alpha} \quad \text{and} \quad w_2(e,y) = wh_2 = wGey^{\alpha},$$

respectively, where w denotes the equilibrium wage per unit of human capital.

When making decisions, individuals have perfect knowledge of their ability, e, how their earnings depend on e and y, the equilibrium price for labor, w, and the gross interest rate, R. We will assume below that creditors also have this information.

Perfect Capital Markets

In an economy with perfect credit markets, individuals can borrow and lend freely. Implicitly, they can fully commit to repay all debts. Regardless of initial endowments and preferences, investments in human capital maximize the present value (discounted by R) of net earnings. Equating marginal cost and the marginal discounted benefit, the optimal amount of investment is:

$$y^*(e) = (\alpha we)^{\frac{1}{1-\alpha}} \left[\frac{R}{1+G/R} \right]^{\frac{1}{\alpha-1}}.$$

Consumption at each age and bequests satisfy the Euler conditions:

$$\begin{aligned} u'(c_j) &= \beta R u'(c_{j+1}), \\ u'(c_2) &= \rho v'(b), \end{aligned}$$

and the intertemporal budget constraint:

$$c_0 + c_1/R + (c_2 + b)/R^2 + c_3/R^3 = a - y^*(e) + w_1(e, y^*(e))[1 + G/R]/R.$$

Life-cycle consumption profiles are flat or strictly monotone, given by $c_{j+1} = (\beta R)^{1/\sigma} c_j$. This environment can be used as a point of reference for our main model.

Exogenous Credit Constraints

A common assumption in the literature is that agents can borrow only up to a level $\bar{d}_j \geq 0$, which may depend on age j (though it rarely does in previous studies). This borrowing constraint, an exogenous feature of the environment, is normally assumed to be the same across all individuals at all ages.

Let V_0, V_1, V_2 and V_3 denote, respectively, the value functions of agents at the young, mature 1, mature 2, and old ages. The "state" for young agents (i.e. the argument of V_0) is the pair (a, e). Given the formulation of the problem, the arguments for V_1 and V_2 are respectively (w_1, d_0) and (w_2, d_1). Finally, old agents consume all available resources, Rs, and hence $V_3(Rs) = u(Rs)$. More explicitly, these functions satisfy:

$$V_0(a,e) = \max_{\{y, d_0 \colon d_0 \leq \bar{d}_0\}} \left\{ u(a + d_0 - y) + \beta V_1(w_1(e,y), d_0) \right\},$$

$$V_1(w_1, d_0) = \max_{\{d_1 \colon d_1 \leq \bar{d}_1\}} \left\{ u(w_1 + d_1 - Rd_0) + \beta V_2(Gw_1, d_1) \right\},$$

$$V_2(w_2, d_1) = \max_{\{s\}} \left\{ u(w_2 - Rd_1 - b - s) + \rho v(b) + \beta u(Rs) \right\}.$$

If initial wealth is low enough, an individual will under-invest in human capital due to the exogenous borrowing constraint. Among those who are constrained, investments will be strictly increasing in initial assets. Notice, however, that investments will not generally increase one-for-one with increases in the amount of borrowing allowed, since individuals will also want to increase their current consumption. This model is consistent with inverted-U shaped life cycle consumption profiles if $\beta R < 1$.

Endogenous Credit Constraints

The primary focus of this paper is to study an environment in which credit constraints arise endogenously from incentive problems. In this environment, constraints on borrowing will differ across individuals and over the life cycle.

The main insight of the endogenous constraint model is that institutional frameworks protect, albeit imperfectly, the rights of creditors. Institutions often allow creditors to discipline defaulting borrowers by impeding future borrowing, destroying their credit rating, garnishing a fraction of their earnings, and seizing part of their owned assets (currently and in a pre-specified future). We attempt to capture these forces in an admittedly stylized way.

For tractability, assume that defaulting agents are excluded from further borrowing and any savings earn a lower rate of return $R_d = \phi R$, $\phi \in (0, 1)$. The latter punishment captures a number of effects. For example, if creditors can seize all savings deposited in formal markets, individuals will be forced to save in informal sectors or use a 'backyard' technology offering a lower rate of return. Alternatively, creditors may be able to seize a fraction of all physical assets saved by defaulting agents as reflected in Chapter 7 bankruptcy filings. We also assume that individuals must forfeit a fraction $\gamma \in (0, 1)$ of their earnings if they choose to default. Wage earnings can be garnished up to 10% (15% for federal employees) for those who default on federal student loans. Tax refunds can also be seized.[5] More generally, individuals with positive income but zero assets will typically be required to re-pay some of their loan, where that re-payment is likely to depend on their level of income. And, because defaulting borrowers receive a bad credit rating, they face difficulties in borrowing to purchase a home and are typically forced to rent instead. Given the sizeable tax breaks provided for home mortgages, this implies that lucrative tax breaks must be foregone by those who default. Poor credit ratings may also make renting more costly for those who default as property owners may be reluctant to rent to them. To the extent that these costs are positively related to earnings, they are reflected in γ. Overall, these punishment parameters (ϕ, γ) should be viewed as approximations to a more complex system that relates punishments to the income and savings of individuals choosing to default. We assume that all of these punishments apply for only one period.

Clearly, the relevance of these three different punishments varies across the life cycle of agents. Being shut out of the lending market may be extremely costly for young workers who wish to borrow against higher future earnings. Yet, it has little impact on those about to retire. On the contrary, older workers earn higher incomes and hold more assets. For them, the ability of creditors to seize their assets or income if they default can be quite costly. These costs are likely to be smaller for the young worker just out of college.

Creditors foresee the repayment incentives of agents. Therefore, they will only lend up to the maximum amount a borrower will willingly repay. Credit limits should incorporate all the (observable) information of borrowers, which (in this framework) includes their wealth, age, ability, and any human capital investments. As a result, agents with different characteristics will face different credit limits. Furthermore, individuals can affect the amount of credit they receive by choosing to invest more or less in their human capital. Borrowing limits should, therefore, be viewed as functions of age, ability, and investments rather than fixed constants. Because of this, any change in the environment that alters the costs or returns of physical and human capital investment will induce a change in the credit limits faced by everyone.

[5]Student loans receive special treatment in bankruptcy proceedings. In general, they cannot be discharged under Chapter 7 bankruptcy and they must be paid in full under Chapter 13 bankruptcy re-payment plans except under extreme circumstances. Simple failure to make student loan payments, or default, can result in wage garnishments, seizure of tax refunds, and a poor credit rating.

Given preferences, demographics, technologies and the institutional environment (γ, ϕ), it is possible that unrestricted consumption and investment decisions are not incentive compatible for some agents. Credit constraints bind when the unrestricted plan entails levels of debt that are so high that the agent is better off defaulting and enduring the associated punishment. Rational creditors with full information will restrict the amount of credit so that the agent never chooses to default. As a result, the expected discounted utility of repaying one's debt will always be equal to or better than the one attainable by defaulting. With perfect information and without uncertainty, there will be no default in equilibrium.

Letting V_1^d and V_2^d be the value functions obtainable by defaulting in the two maturity periods, and V_0, V_1, and V_2 be the values attainable in equilibrium, the agent's problem is given by

$$V_0(a,e) = \max_{\{y,d_0:\, V_1(w_1(e,y),d_0)\geq V_1^d(w_1(e,y))\}} \left\{ u(a+d_0-y) + \beta V_1(w_1(e,y),d_0) \right\},$$

$$V_1(w_1,d_0) = \max_{\{d_1:\, V_2(w_2(e,y),d_1)\geq V_2^d(w_2(e,y))\}} \left\{ u(w_1+d_1-Rd_0) + \beta V_2(Gw_1,d_1) \right\}, \qquad (1)$$

$$V_2(w_2,d_1) = \max_{\{s,b\}} \left\{ u(w_2-b-Rd_1-s) + \rho v(b) + \beta u(Rs) \right\},$$

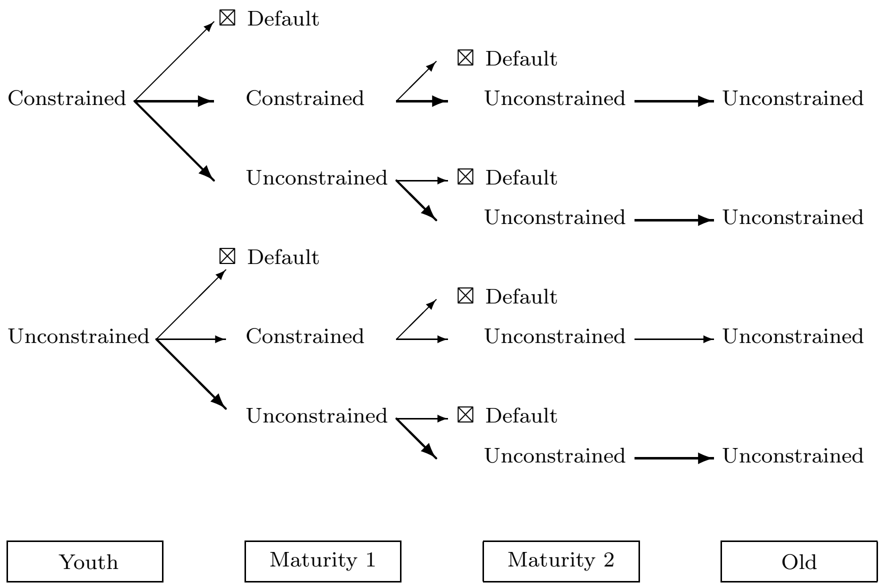
where

$$V_1^d(w_1) = \max_{\{d_1:\, d_1\leq 0\}} \left\{ u(w_1(1-\gamma)+d_1) + \beta V_2(Gw_1,R_d d_1) \right\}, \qquad \text{and} \qquad (2)$$

$$V_2^d(w_2) = \max_{\{s,b\}} \left\{ u(w_2(1-\gamma)-b-s) + \rho v(b) + \beta u(R_d s) \right\}.$$

Diagram 2 displays the possible life cycle investment/consumption plans. The ⊠ s indicate that the market will block any investment plan that triggers default.

Diagram 2: Transitions in the Life-Cycle of the Agents



Depending on initial wealth and ability, credit limits may bind during youth (the first period). In subsequent periods, the decisions made in previous periods will determine whether constraints bind. Agents in the first period of maturity with positive debt would typically want to roll-over some of their debt to smooth consumption across their adult life. However, if the debt becomes too high, the amount needed to borrow for the unrestricted consumption plan could trigger default in the second period of maturity. Finally, the presence of retirement (old age) causes agents to save (rather than borrow) during their second period of maturity. Credit constraints are not relevant at that age, so default incentives in old age can be ignored. In summary, loans made during youth and the first age of maturity need to be compatible with individual incentives to default during the first and second periods of maturity. Constraint levels may change between youth and maturity due to changes in incentives to default.

The thick lines in Diagram 2 indicate the only three possible paths in equilibrium. Individuals that are unconstrained during their youth will remain unconstrained throughout the rest of their lives.[6] Intuitively, if a young individual knew he would be constrained the next period, he would borrow and consume less while young. He would continue to reduce his current consumption until either the future constraint was no longer binding or until his current borrowing became

[6]To see why, assume that to the contrary, an agent is unconstrained when young but that he is constrained later in life. Because he is not constrained during his youth, his investment y must equal the unconstrained optimum. Thus, the binding constraints only distort consumption. In the second period of maturity, the agent cannot be credit constrained as he wants to save. Therefore, the only possibility is that he is constrained in the first period of maturity, i.e. $u'(c_1) > \beta R u'(c_2)$, since he is unconstrained during his youth, i.e. $u'(c_0) = \beta R u'(c_1)$. Combining these equations produces $u'(c_0) > (\beta R)^2 u'(c_2)$, which is a contradiction implying that the agent was constrained during the first two periods of life.

constrained. On the other hand, individuals that are constrained in their youth may or may not remain constrained in the first period of maturity.

For any given pair (w, R) fixed, the model can be solved almost entirely in closed form. Regions of (w, R) in which individuals default will never be reached in equilibrium. Still, to solve the model and obtain an expression for the credit constraints, it is necessary to obtain the value functions and the value of default for all nodes of the tree in Diagram 2.

Constraints will be non-binding in the second period of maturity as individuals save for retirement.[7] The value function for an agent in the second period of maturity (third period of life) is

$$V_2(w_2, d_1) = \begin{cases} \Theta_\rho(R_d)\frac{[(1-\gamma)w_2]^{1-\sigma}}{1-\sigma} & \text{if } d_1 > \mu w_2 \text{ (the agent defaults)} \\ \Theta_\rho(R)\frac{[w_2 - Rd_1]^{1-\sigma}}{1-\sigma} & \text{if } d_1 \leq \mu w_2 \text{ (the agent re-pays),} \end{cases}$$

where $\Theta_\rho(R)$ is a strictly positive, strictly increasing function of R:

$$\Theta_\rho(R) = \begin{cases} (\rho + \Theta_0(R))\left[\frac{(\Theta_0(R)/\rho)^{-1/\sigma}}{1+(\Theta_0(R)/\rho)^{-1/\sigma}}\right]^{1-\sigma} & \text{if } \rho > 0 \\ \Theta_0(R) & \text{if } \rho = 0 \end{cases}$$

and the function $\Theta_0(R)$, which would apply in the absence of altruism ($\rho = 0$), is given by

$$\Theta_0(R) \equiv \left(\frac{(\beta R)^{-1/\sigma}R}{1+(\beta R)^{-1/\sigma}R}\right)^{1-\sigma} + \beta\left(\frac{R}{1+(\beta R)^{-1/\sigma}R}\right)^{1-\sigma}.$$

This function is obtained from calculating the optimal retirement savings and bequest decisions of individuals in the second period of maturity. If the agent carried over a large debt relative to his wage earnings, it could be optimal to default on it. In such a case, there are two costs of not repaying the debt. He loses a fraction of his earnings and earns a lower rate of return on his savings. At this age, the restriction on future borrowing is irrelevant. The function μ that defines the default region is given by

$$\mu(R) \equiv \frac{1}{R}\left(1 - (1-\gamma)\left[\frac{\Theta_\rho(R_d)}{\Theta_\rho(R)}\right]^{\frac{1}{1-\sigma}}\right).$$

The level of debt that triggers default is a constant fraction of current earning levels. This fraction is given by $\mu = \mu(R)$, which is a strictly decreasing function of the gross interest rate R. This fraction is also increasing in the punishments creditors can impose on those who default (i.e. μ is increasing in γ and decreasing in ϕ). Because creditors foresee optimal default decisions, the function μ also defines the credit constraints, i.e. the limits of the net debt that agents in the first period of maturity can carry over for the second period of maturity. With perfect information,

[7]Here, we ignore the case of a very generous pay-as-you-go pension system, in which poor agents could wish to borrow even in the second period of maturity.

lenders will never allow the debt of young workers to exceed μw_2 in this environment. While μ only depends on interest rates, punishment parameters γ and ϕ, and preferences, borrowing limits depend on investments and ability through future earnings, w_2. To the extent that earnings are increasing in y and e, so will the amount of borrowing that is allowed.

We can also obtain the amount of bequest transferred by a parent. If $W_2(e, a)$ denotes the net wealth of a mature worker who started life with (e, a), then his child will start life with an initial value of wealth given by

$$b(e, a) = W_2(e, a) \left[\frac{(\Theta_0(R)/\rho)^{-1/\sigma}}{1 + (\Theta_0(R)/\rho)^{-1/\sigma}} \right].$$

With these results, the value function for the first period of maturity can be determined analytically. Assume that the ratio $G/(1-\gamma)$ is high enough with respect to $\beta\phi R$ so that defaulting agents in the first period of maturity will not want to save. Then, agents that default in the first period of maturity will simply consume their current earnings, net of the seized portion.[8]

Characterizing the value function and the optimal decisions in this period is more complex. In addition to determining whether or not an individual defaults on his current debts, it is also necessary to determine whether he is able to accumulate further debts as freely as optimization requires. Borrowing may be restricted from future default incentives, as indicated by μ.

Depending on the "state" (w_1, d_0) at the first age of maturity, the individual will either default on earlier loans (in which case he cannot borrow again), face a constraint on additional borrowing, or make unconstrained choices. The levels of debt that define each of these regions can be expressed as a fraction of current earnings, so

$$V_1(w_1, d_0) = \begin{cases} \delta(R) \frac{[w_1(1+G/R) - Rd_0]^{1-\sigma}}{1-\sigma} & \text{if } d_0 \leq \kappa_0 w_1 \text{ (the agent is unconstrained)} \\ \frac{[w_1(1+\mu G) - Rd_0]^{1-\sigma}}{1-\sigma} + \beta\Theta_\rho(R) \frac{[G(1-\mu R) w_1]^{1-\sigma}}{1-\sigma} & \text{if } \kappa_0 w_1 < d_0 \leq \kappa_1 w_1 \text{ (the agent is constrained)} \\ \frac{w_1^{1-\sigma}}{1-\sigma} [(1-\gamma)^{1-\sigma} + \beta\Theta_\rho(R) G^{1-\sigma}] & \text{if } d_0 > \kappa_1 w_1 \text{ (the agent defaults)} \end{cases}$$

[8]Considering the possibility that defaulting agents want to save does not add much complexity but also does not change the main characteristics and behavior of the credit constraints.

where[9]

$$\delta(R) = \frac{1+\beta(\beta R)^{\frac{1-\sigma}{\sigma}}(1+\rho^{\frac{2\sigma-1}{\sigma}})\beta^2(\beta R)^{\frac{2(1-\sigma)}{\sigma}}}{\left[1+(\beta R)^{(1/\sigma)}(1+\rho^{-1/\sigma})/R+(\beta R)^{(2/\sigma)}/R^2\right]^{1-\sigma}}$$

$$\kappa_0(\mu,R) = \frac{\mu G(1+(\beta\Theta_\rho(R)R)^{-1/\sigma}R)+1-(\beta\Theta_\rho(R)R)^{-1/\sigma}G}{R}$$

$$\kappa_1(\mu,R) = \frac{1+\mu G-\left[(1-\gamma)^{1-\sigma}+\beta\Theta_\rho(R)G^{1-\sigma}[1-(1-\mu)^{1-\sigma}]\right]^{\frac{1}{1-\sigma}}}{R}.$$

In the first region for d_0, debt carried over from youth is relatively low ($d_0 \leq \kappa_0 w_1$) such that re-payment is preferred to default. Furthermore, the agent is not constrained from borrowing additional funds during his first period of work. From that date onwards, he is able to finance the unrestricted consumption profile. At the other extreme, if debt left over from youth is high enough ($d_0 > \kappa_1 w_1$), the individual is better off defaulting and enduring punishment from lenders. In equilibrium, lenders will recognize this and restrict debts to be no greater than $\kappa_1 w_1$ during youth. For mid-level initial debts ($d_0 \in (\kappa_0 w_1, \kappa_1 w_1]$), young workers do not owe enough to make default worthwhile, however, they are constrained from carrying a debt of more than μw_2 into the second period of maturity because of future incentives to default. In this case, the end of period net debt for the worker will be the maximum allowed ($d_1 = \mu w_2$). In equilibrium, debts during youth are limited to $d_0 \leq \kappa_1 w_1$, while debts during the first period of work are constrained to $d_1 \leq \mu w_2$. The factor κ_0 simply characterizes the level of first period debt above which individuals want to borrow more in the second period than will be allowed by lenders.

The rate of growth in earnings, G, plays an important role in determining whether individuals will be constrained from borrowing during their first age of maturity. Given (w_1, d_0), individuals that experience substantial wage growth will want to borrow a lot during their youth and the first age of maturity to smooth consumption. On the other hand, those with high wage growth will face a greater penalty from default – the inability to borrow again is costly since they can no longer smooth consumption and wage garnishments will be greater. These individuals are, therefore, allowed to borrow more than those with less wage growth. Whether or not they are more likely to face binding constraints depends on the balance of these two forces – their greater demand for credit and their greater incentives to re-pay their loans.

Credit constraints will only bind in the first two periods of life, since agents want to *save* and not borrow in the period prior to retirement. Thus, there are only three possibilities for the

[9] The function $\delta(R)$ derives directly from unrestricted optimization. Then, the unrestricted borrowing function, $d_1(d_0, w_1)$, can be calculated. κ_0 is obtained by solving the equality $\mu G w_1 = d_1(d_0, w_1)$, i.e. the maximum level of debt d_0 that is consistent with the credit constraint not binding in that period. Finally, to obtain the value of κ_1, equate the value of default with the value of being credit constrained: $\frac{[w_1(1+\mu G)-Rd_0]^{1-\sigma}}{1-\sigma} + \beta\Theta_\rho(R)\frac{[G(1-\mu)w_1]^{1-\sigma}}{1-\sigma} = \frac{w_1^{1-\sigma}}{1-\sigma}[(1-\gamma)^{1-\sigma}+\beta\Theta_\rho(R)G^{1-\sigma}]$. This is the relevant equality, because $\delta(R)\frac{[w_1(1+G/R)-Rd_0]^{1-\sigma}}{1-\sigma} \geq \frac{[w_1(1+\mu G)-Rd_0]^{1-\sigma}}{1-\sigma} + \beta\Theta_\rho(R)\frac{[G(1-\mu)w_1]^{1-\sigma}}{1-\sigma}$ for all (w_1, d_0).

constraints: (i) the credit constraints bind for the first two periods, (ii) they bind for only the first period, or (iii) they never bind. However, due to the uniformity of preferences, only one of the first two possibilities can arise in any given equilibrium – it is not possible that, in the same stationary equilibrium, some agents are constrained for only one period, while others are constrained for two.

There are two types of equilibria in which at least some individuals are constrained. A Type I equilibrium consists of a group of agents for whom the credit constraints bind in the first two periods and another group (which may not exist) that is always unconstrained. In this equilibrium, $\kappa_0 < \kappa_1$ and individuals who are constrained during their youth will continue to be constrained during their first period of maturity. The borrowing constraints are defined by $d_0 \leq \kappa_1 w_1$ during youth and by $d_1 \leq \mu w_2$ during the first period of work. A Type II equilibrium consists of one group that is only constrained during the first period and another group (which may not exist) that is always unconstrained. In this equilibrium, some youth are constrained to borrow no more than $\kappa_1 w_1$, while all workers are able to borrow freely. Individuals that were constrained during their youth are unconstrained in later periods either because the borrowing constraints they faced initially were very stringent or because constraints during adulthood are quite loose.

The conditions determining which of these two possible types of equilibria arise depends on the interest rate (as we illustrate below). That is, whether or not the inequality $\kappa_0 < \kappa_1$ holds depends on R. Both κ_1 and μ are decreasing with respect to the interest rate. A higher interest rate increases the amount the agent has to repay and, therefore, less debt is needed to trigger default. The factor κ_0 may be increasing or decreasing in R. It captures two effects. On the one hand, the lower the interest rate, the more interested a young worker will be in borrowing. Thus, the punishment of not being allowed to borrow will be more costly. This increases the maximum debt required for the constraint μ to bind. On the other hand, the lower the interest rate, the lower the effective liability of the agent, which makes it more attractive to re-pay any debts. Numerical simulations can produce cases in which either effect dominates. If the punishment of not being able to borrow is so costly that individuals are willing to relinquish resources for the opportunity to increase their consumption smoothing, then κ_0 will be negative and a Type I equilibrium is guaranteed.

General Equilibrium

We now make explicit the set E of different ability types as well as the intergenerational transmission of ability.

A. 1. *The support of abilities is given by a finite set $E \subset \mathbb{R}+$.*

Let $\mathcal{E}$ be the power set of E. The transmission of abilities from parents to children is given by the transition function $P : E \times \mathcal{E} \to [0, 1]$.

A stationary economy is one in which a time-invariant probability space describes the population at each point in time. Given the structure of the model, the economy is stationary if and only if there is a time invariant distribution for (a, e) among youth. Since bequests are non-negative, the set of possible initial asset levels is contained in $A \subset \mathbb{R}_+$. Let $\mathcal{B}_+$ denote the Borel sets $\mathbb{R}_+$ and $\mathcal{A}$ denote the Borel sets on A. Finally, let $\mathcal{F}$ be the product space $\mathcal{E} \times \mathcal{A}$. For a measure λ defined over the space $(E \times A, \mathcal{F})$ to be invariant, it is required that, given the bequest function $b(e, a)$, λ "reproduces" itself over time. That is, for any set $B \in \mathcal{E} \times \mathcal{B}_+$,

$$\lambda(A) = \int\limits_{E \times A} \chi_{\{(e', b(e,a)) \in B\}}(e, a) P(e', e) \lambda(de \times da). \tag{Agg1}$$

where χ is the indicator function.

Given λ, aggregate human capital H is given by

$$H = (1 + G) \int\limits_{E \times A} e y(a, e)^\alpha \lambda(da \times de). \tag{Agg2}$$

The net savings of all agents determines the aggregate physical capital stock, K:

$$K = \int\limits_{E \times A} [s(a, e) - d_0(a, e) - d_1(a, e)] \lambda(da \times de). \tag{Agg3}$$

Aggregate output, Q, is produced with physical and human capital according to the production function $Q = F(K, H)$, where $F(K, H)$ is increasing and concave in both K and H.

Regardless of the contracting environment, aggregate human and physical capital determine the market-clearing wage and interest rate. Market clearing prices must be

$$w = \frac{\partial F(K, H)}{\partial H}, \qquad R = 1 - \delta + \frac{\partial F(K, H)}{\partial K}. \tag{MC}$$

where $\delta \in (0, 1)$ is the depreciation rate of physical capital.

All the above leads to the following standard definition:

Definition 1. Stationary Equilibrium *Given the demographics $\{E \times A, \mathcal{F}, P\}$, preferences $\{u(\cdot), v(\cdot), \beta, \rho\}$, technologies $\{ey^\alpha, F(\cdot, \cdot), \delta\}$, and the credit market contracting environment (γ, ϕ), a stationary equilibrium is a price pair $\{w, R\}$, an invariant distribution λ, aggregate capital stocks $\{H, K\}$ and individual allocations $\{y(a, e), d_0(a, e), d_1(a, e), s(a, e)\}$ such that*

1. *Given $\{w, R\}$, the allocation $\{y(a, e), d_0(a, e), d_1(a, e), b(a, e), s(a, e)\}$ solves the individual problem for all $(a, e) \in A \times E$;*
2. *The aggregate stocks are consistent with individual decisions (equations Agg1, Agg2 and Agg3); and,*
3. *Prices clear the markets (equations MC).*

Because of their importance in the actual U.S. economy, we include government policies like public schooling and education subsidies in our calibration of the model. Their inclusion affects the decisions and constraints of all agents. It also requires the inclusion of a government budget constraint in the definition of a stationary equilibrium. These changes are straightforward and discussed further in the next section of the paper.

3 Parameterizing the Model

To quantitatively assess the effect of government policies we need to specify empirically grounded parameter values for preferences, intergenerational transmission of ability, technologies, and credit institutions. We parameterize the model using data from the U.S. economy. In particular, we use data on schooling, ability, and wages in the National Longitudinal Survey of Youth (NLSY) and schooling costs from the Digest of Education Statistics [41]. We also generate life cycle consumption patterns that are consistent with the patterns estimated in the literature (e.g. Carroll and Summers [12] and Attanasio, et al. [4]).

In our set up, a period is interpreted as approximately 15 years. Using a yearly discount factor of 1/1.06, the discount factor in the model is $\beta = 1.06^{-15} = 0.4173$. A CES production function is assumed where

$$F(K,H) = A\left[\xi K^{-r} + (1-\xi)H^{-r}\right]^{-1/r}.$$

Our base case assumes complementarity between physical and human capital with $r = 0.2$.[10] An annual depreciation rate of 8% is assumed for physical capital, so $\delta = 1 - 0.92^{15}$. These parameters are reported in Table 1. All other parameters of the model are determined using the data and procedures described in the following subsections.

Table 1: Assumed Parameter Values in the Base Model

Parameter	Base Case Value
β	1.06^{-15}
δ	$1-0.92^{15}$
r	0.2

3.1 Determining Parameters for Wage Growth and Human Capital Production

We empirically estimate the parameters α, G, and the distribution of e using wage and earnings data for men from the National Longitudinal Survey of Youth (NLSY) and average direct schooling expenditures per pupil from the Digest of Education Statistics. [41] The NLSY is particularly useful for our purposes, since it contains often used measures of cognitive ability in the Armed Forces Qualifying Test (AFQT), and it follows the same individuals over the early part (first 17 years) of their careers. To aid in estimation of heterogeneous learning abilities, we categorize individuals in the NLSY into five ability groups (corresponding to each of the five quintiles in the national distribution) according to their scores on the AFQT.

[10] We have also explored a version with $r = -0.2$ in which human and physical capital are substitutes in production. Tables analogous to those presented below show very little substantive difference.

A two-step procedure is used to estimate the parameters of interest. First, total costs for each year of schooling are computed from direct expenditures and foregone earnings. Then, a log wage regression is used to estimate human capital parameters of the model.

In calculating direct expenditures, we use average expenditures per pupil from 1980-89 for primary and secondary schooling, and for universities and colleges, since those years correspond to the years most men in our NLSY sample made their marginal schooling decisions.[11] See Appendix Table A-1 for the present value of direct expenditures for each year of schooling.[12] In 1999 dollars, direct costs for 12 years of schooling amount to almost \$60,000, and expenditures for 16 years of schooling cost about \$100,000. While a substantial fraction of these direct costs are subsidized by the government, they still reflect direct inputs. We include them here because the objective at this moment is to estimate the human capital production function.

Working men in the NLSY are used to compute foregone earnings. We use a standard log earnings regression that controls for experience (age - education - 6), experience-squared, and indicators for each AFQT quintile to compute average annual earnings for men of all schooling and ability combinations. Foregone earnings are set to zero for those with 9 years of schooling under the assumption that individuals generally cannot work before age 14 in most states. For individuals with more than 9 years of schooling, foregone earnings are computed as the earnings for someone with 9 years of schooling (appropriately adjusted each year for experience that would have been acquired) had they chosen to work rather than attend school each year thereafter. Unlike direct expenditures, foregone earnings are specific to the ability group. Not surprisingly, more able individuals forego more earnings for each year of school they attend. The sum of foregone earnings and direct expenditures for each year of schooling are given by AFQT quintile in Table A-1. For someone in the highest ability quintile, total expenditures on schooling for a college graduate are approximately \$136,000.

In step two, we use wage data in the NLSY to estimate α, G, and ability levels e for each of the five types, recognizing that our model produces the following wage relationships:

$$\begin{aligned} log(w_1) &= log(w) + log(e) + \alpha log(y) \\ log(w_2) &= log(w) + log(e) + \alpha log(y) + log(G). \end{aligned}$$

This simple specification motivates the following log wage regression:

$$log(wage) = a + \alpha log(y) + \sum_{j=2}^{5} \eta_j AFQT_j + \gamma_1 X + \gamma_2 X^2 + \varepsilon \qquad (3)$$

[11]Direct expenditures for individuals attending school for twelve or fewer years are simply the present value of annual primary/secondary expenditures for the appropriate number of school years. For those attending 13 or more years of school, direct expenditures include 12 years of primary/secondary expenditures plus college tuition expenditures.

[12]Consistent with our equilibrium calibration, a 4% interest rate was assumed in calculating the present value of all costs. Costs are discounted back to the time of school entry.

where y is taken from total expenditures on schooling for the appropriate schooling-ability classification in Table A-1, $AFQT_j$ represents an indicator for AFQT quintile j, and X represents work experience (defined as age - education - 6). This wage regression maintains the assumption that ability, the logarithm of education expenditures, and experience (incorporated in G in our simplified model) should enter a log wage equation linearly.[13]

The point estimate for α is 0.6146. It directly corresponds to the parameter of interest − the elasticity of human capital with respect to investments − and is used in our simulations below. Since it is impossible to distinguish between $log(w)$ and $log(e_1)$, one must normalize either the price of skills or the lowest ability level. Normalizing $e_1 = 1$, estimates for higher ability learning levels are given by $e_j = exp(\eta_j)$.

Finally, the returns to experience given by γ_1 and γ_2 can be used to determine G. Because log earnings are quadratic in experience, any estimate of G will depend on how we define the two work periods of the model. Since we are interested in studying borrowing and savings behavior for individuals in school and recently out of school, we compute G using the ratio of wages for someone with 15 years of experience relative to zero years of experience yielding a value of 2.1275 for G. These parameter estimates are summarized in Table 2.

Table 2: Estimated Human Capital Parameter Values

Parameter	Estimate
α	0.6146
G	2.1275
e_1	1.0000
e_2	1.2184
e_3	1.3054
e_4	1.3159
e_5	1.3916

Estimates for e_j provide a measure of the quantitative effect of ability on earnings. Someone from the highest ability group will earn 39% more than someone from the lowest ability group conditional on the same human capital investment.

3.2 Intergenerational Transmission of Ability

The estimates of e_j indicate the earnings capacity for agents in different quintiles of cognitive ability. By construction, each of these ability types has a 20% mass, which we take to be the stationary distribution for ability. The intergenerational transmission of ability is assumed to be a five-point

[13]See Heckman, Lochner, and Todd [27] for an empirical discussion of these assumptions over the second half of the 20th Century.

Markov chain with transmission probabilities defined by the matrix

$$P = \begin{pmatrix} 1-(5/2)\pi & \pi & \pi/2 & \pi/2 & \pi/2 \\ \pi & 1-3\pi & \pi & \pi/2 & \pi/2 \\ \pi/2 & \pi & 1-3\pi & \pi & \pi/2 \\ \pi/2 & \pi/2 & \pi & 1-3\pi & \pi \\ \pi/2 & \pi/2 & \pi/2 & \pi & 1-(5/2)\pi \end{pmatrix}.$$

We choose $\pi = 0.1763$ to generate an intergenerational correlation of ability equal to 0.5. See Daniels, et al., [14] for a recent discussion of these estimates. The results we focus on are quite robust to changes in the specification for P or the values for π that we use.

3.3 Calibrating Altruism, Credit Market Punishments, and Aggregate Technology Parameters

We choose the parameters ρ, γ, ϕ, A, and ξ, such that the stationary equilibrium in the economy mimics main features of the U.S. economy. Features we replicate include an equilibrium real interest rate of $R = 1.04$[15], average wage earnings of young workers, $E(w_1)$, average expenditures on education, $E(y)$, and average consumption growth over the life cycle ($E(c_1)/E(c_0)$, $E(c_2)/E(c_1)$, and $E(c_3)/E(c_2)$). Mean wages are calculated using the log wage regression estimates from the NLSY data described above to remove any influences of differential cohort size. We compute a mean wage income for the first 15 years of work to be \$247,145. Using the empirical distribution for schooling by ability and the expenditures reported in Table A-1, we compute the mean investment in the NLSY to be approximately \$92,673. In our calibration, we divide these amounts by 1,000 to normalize units in thousands of dollars. We replicate average consumption growth rates of $E(c_1)/E(c_0) = 2.5$, $E(c_2)/E(c_1) = 1.15$, and $E(c_3)/E(c_2) = 0.8$, which roughly correspond to the patterns reported in Carroll and Summers [12] and Attanasio, et al. [4].[14]

Given the considerable public involvement in the U.S. educational system, we introduce a subsidy schedule for human capital investment in our calibration. Investments are assumed to be subsidized fully through $y_0 = 50$ (roughly grade 10), after which they are subsidized at the rate $\nu = 0.5$ (reflecting the fact that foregone earnings are a sizeable fraction of schooling costs once individuals are old enough to work but that the government also significantly subsidizes the direct costs of high school and college). Flat wage taxes are levied to balance the federal budget (i.e. to pay for the subsidies to investment). See the following section for a more complete discussion of how this policy environment affects the constraint functions.

The calibrated parameters are shown in Table 3 along with the data target values.[15] The intertemporal elasticity of substitution, σ, can be directly determined from β, the desired interest

[14]These (and other) studies report estimated life cycle profiles for consumption that differ considerably. The consumption growth factors we use lie within the range of estimates produced by these studies.

[15]We verify that the steady state is unique within a region around these parameter values.

rate, and the ratio of expected consumption $E(c_3)/E(c_2)$. There is no simple mapping between any of the other parameters and the data targets within each column, as the parameter values needed can only be obtained by solving for the stationary equilibrium repeatedly until all five of those target values are matched.

Table 3: Calibrated Parameter Values and Target Data (Base Case)

Parameter	Value	Target Data
σ	1.2804	$\sigma = log(\beta R)/log(E(c_3)/E(c_2))$, $E(c_3)/E(c_2) = 0.8$
ρ	0.2286	$E(y) = 93$
A	13.593	$E(w_1) = 247$
ξ	0.0664	$R = 1.04^{15} = 1.80$
γ	0.1285	$E(c_1)/E(c_0) = 2.5$
ϕ	0.9834	$E(c_2)/E(c_1) = 1.15$

The calibration implies that individuals who default face costs equal to 13% of their earnings and 2% of their savings. These cost factors correspond well to official policies toward individuals who default on federal student loans, which specify that wage earnings can be garnished up to 10% (15% for federal employees). The role of assets in determining punishments for those defaulting on student loans is less clear from a legal standpoint, reflecting the fact that most recent graduates have little or no assets to seize. As we show below, the equilibrium is quite insensitive to changes in ϕ.

3.4 The Base Case Economy

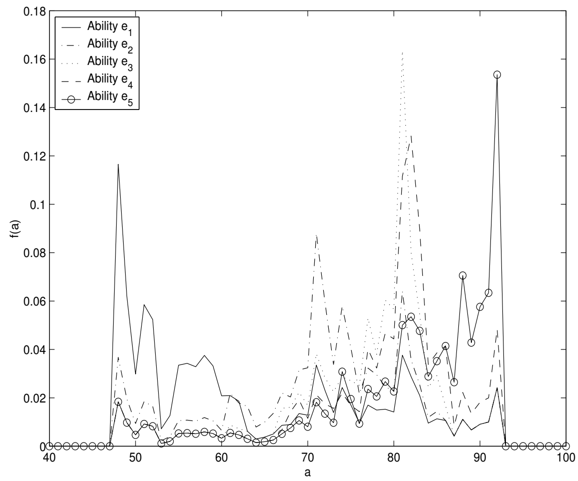
This section describes important characteristics of the calibrated base economy. Equilibrium interest rates and skill prices are presented in Table 4 along with aggregate human and physical capital levels, the wage tax rate, and the average educational subsidy amount. A wage tax rate of 9.23% is needed to pay for education subsidies, which average $71 per person.

Table 4: Base Economy

Variable	Value
w	12.2102
R	1.8009
τ	0.0923
H	63
K	40
$E(subsidy)$	71

Figure 1 displays the stationary distribution of initial assets in equilibrium by ability. Because ability is correlated across generations, more able individuals tend to receive larger bequests from

Figure 1: Distribution of Initial Assets by Ability



their parents, who were also likely to have been more able. While most lower ability individuals begin with assets of less than \$70, most high ability individuals begin with assets above that amount. The distribution of initial assets is summarized in column 1 of Table 5. The difference in average initial assets for the most able and least able is about \$17.

Table 5: Base Case Initial Assets, Investment, Earnings, Debt, and Consumption

Statistic	a	y	w_1	d_0	d_1	c_0	c_1	c_2
$E(\cdot)$	74	93	247	30	35	83	206	235
$E(\cdot\|e_1)$	64	77	176	21	25	71	147	168
$E(\cdot\|e_2)$	71	89	235	28	33	80	196	224
$E(\cdot\|e_3)$	76	96	263	32	37	85	219	251
$E(\cdot\|e_4)$	78	98	269	32	38	87	224	256
$E(\cdot\|e_5)$	81	103	293	35	42	90	244	279
$Var(\cdot)$	162	153	1,796	26	36	114	1,247	1,628
$E(Var(\cdot\|e))$	125	74	198	3	4	73	138	180

Figure 2 shows how human capital investments increase with ability and initial assets. For any given level of initial assets, the least able individuals invest about \$5 less than the second lowest ability group and about \$10 less than the most able. The affects of ability on investment appear to be fairly constant across initial asset levels. For every \$1 increase in initial assets, investment tends to increase by nearly \$.80 for all ability groups.

The overall distribution of investment in the economy depends on the joint distribution of initial assets and ability. The positive correlation between ability and initial assets results in much higher investment among the more able, as seen in Figure 3, which graphs the conditional distribution of investment for each ability type. As Table 5 shows, average investment differences by ability (column 2) are substantial. This is also clearly reflected in Figure 3, which shows that

Figure 2: Human Capital Investment as a Function of Ability and Initial Assets

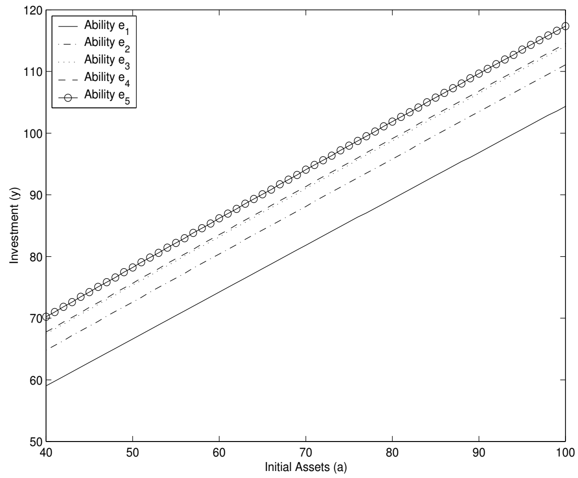
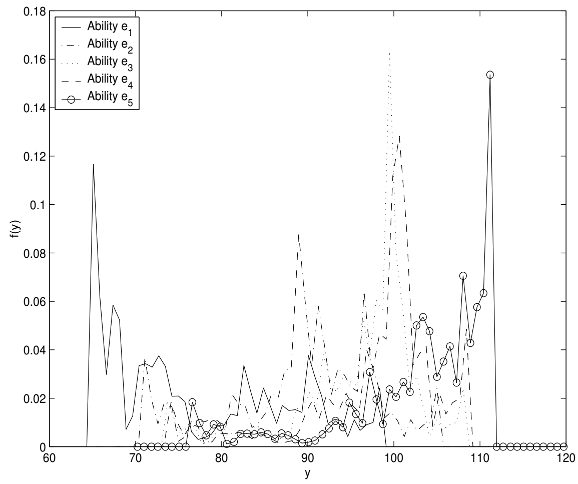


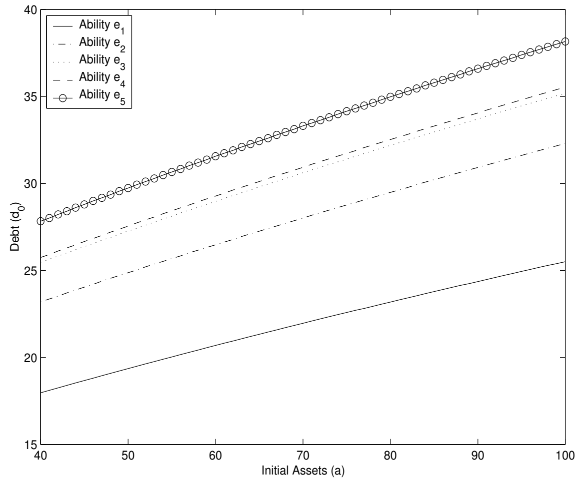
Figure 3: Distribution of Human Capital Investment by Ability



the distributions of investment for the lowest and highest ability groups barely overlap. Much of the investment difference is due to the correlation between ability and initial assets. Recall that Figure 2 implies that, conditional on initial assets, the difference in investment between the least and most able should be about \$10. This reflects the direct effect of ability through it's effect on the marginal returns to investment (and the ability to borrow as we discuss more below). The \$26 average difference in investment between the most and least able individuals reported in Table 5 reflects this direct relationship as well as the fact that more able individuals also tend to have higher initial assets.

Table 5 reports a number of other interesting statistics for the economy. For example, column 3 reports average earnings in the population, average earnings conditional on ability, the variance in earnings in the population, and the average conditional variance (i.e. the average of each variance

Figure 4: Debt for Youth (d_0) as a Function of Ability and Initial Assets



conditional on ability) for individuals during the first period of work. Differences in earnings across ability types reflect both differences in investment and differences in the return to investment. Thus, more able individuals earn substantially more than their less able counterparts.

Columns 4 and 5 of Table 5 report the same statistics on debt for youth and young workers. In equilibrium, all youth and young workers are constrained, so the entries in the table reflect the amount of borrowing allowed. On average, youth are able to borrow up to \$30 and young workers \$35, but there is considerable variation related to initial asset levels and ability as shown in Figures 4 and 5. Allowable debt increases in initial assets (by about \$1 for every \$6 increase in assets), because individuals with higher initial assets will invest more in their human capital. This is also true for ability; however, allowable debt would also increase with ability if investment were held constant since more able individuals will earn higher returns on their investments. Holding initial assets constant, the most able can borrow about \$10-15 more than the least able during their youth and first period of work – these differences are sizeable given the relatively low levels of average debt. Thus, more able individuals not only have an advantage in that they can produce more human capital for any given investment, but they also tend to begin life with higher initial assets and greater access to credit.

The maximum allowable debt depends on future earnings power relative to the factors κ_1 and μ, which are reported in Table 6 along with κ_0. Notice that $\kappa_0 < 0$, which implies that the economy is in a Type I equilibrium – constraints bind during youth and the first period of work. Creditors will allow debt during youth to reach 13.2% of (after-tax) earnings next period, and they will allow young workers to borrow up to 7.3% of their (after-tax) earnings the following period.

As discussed earlier, these constraint factors depend on the equilibrium interest rate. Figure 6 shows this relationship for R corresponding to annual interest rates ranging from 2-6%. Since

Figure 5: Debt for Young Workers (d_1) as a Function of Ability and Initial Assets

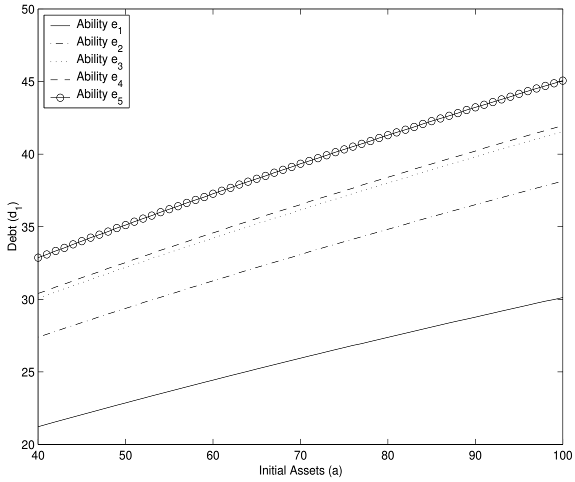


Table 6: Base Economy

Variable	Value
κ_0	-0.0860
κ_1	0.1323
μ	0.0734

$\frac{d_1}{d_0} = \left(\frac{\mu}{\kappa_1}\right) G$ and $\frac{\kappa_1}{\mu} < G$ over the entire range of R, the figure suggests that individuals will generally be allowed to borrow more during the first period of work than during their youth. The fraction of future earnings that youth can borrow tends to be more responsive to changes in interest rates, so we might expect greater general equilibrium effects on borrowing at this time than during the first period of work.

Finally, we can examine consumption in the economy. The final three columns of Table 5 report summary statistics on consumption for the first three periods (note that $c_3 = 0.8c_2$ for everyone).

Figure 6: Dependence of Solvency Constraints on the Interest Rate

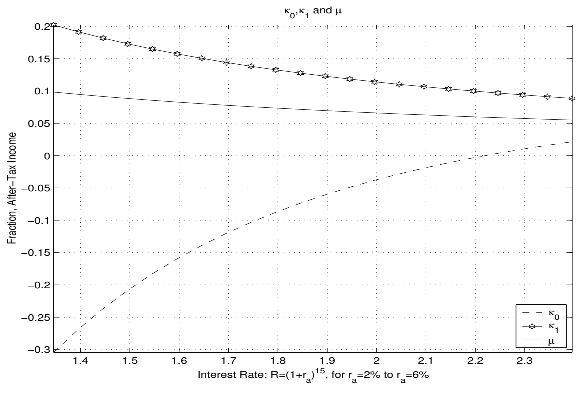
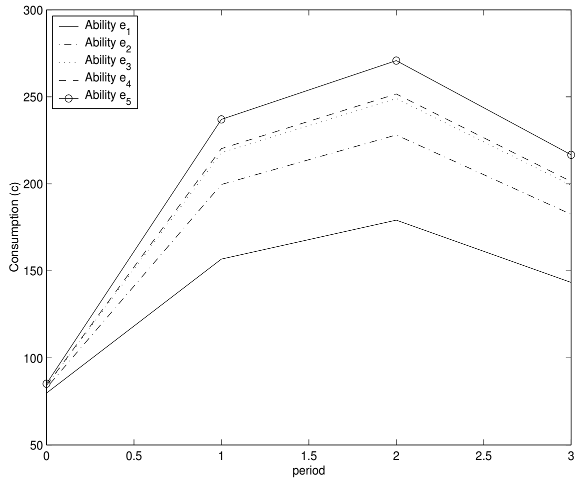


Figure 7: Lifecycle Consumption Profiles for Individuals with Initial Assets of $75



Consumption increases substantially (by a factor of 2.5, on average) from youth to the first period of work. This is by design, since we calibrate the economy to produce this increase. As shown in the table, the increase is substantially greater (in percentage terms as well as levels) for the more able types. This is largely because more able individuals earn a higher return on their investments and are, therefore, willing to give up more consumption in the short run to finance those investments. Figure 7 graphs some representative consumption profiles for individuals with identical initial assets ($a = 75$) but who differ in ability. While consumption varies very little by ability for youth with the same level of initial assets, there are sizeable differences in consumption among workers and retirees.

4 Government Policies

A wide variety of government policies present in actual economies are likely to affect human capital accumulation and consumption/savings decisions. Some of these policies are institutionalized with the explicit aim to foster human capital. Others are not, though they are likely to affect the life cycle decisions of individuals.[16] Each of these policies have been the subject of extensive research. Our contribution is to investigate them explicitly recognizing the response of private financial markets.

In an efficient credit market, government policies will affect investment through three channels. First, they may directly alter the costs and benefits of investment. Second, they may alter borrowing limits placed on individuals. Finally, general equilibrium changes in the price of human capital and interest rates will affect investment through changes in the costs and benefits of investment as well as the amount of lending creditors will provide. We analyze all three of these channels.

[16]See Heckman, Lochner, Smith, and Taber [25] for a discussion of government policy and human capital formation.

4.1 Bankruptcy Policy

First, we investigate the effects of changing the bankruptcy code and its enforcement. In our model, this can take the form of changes in the parameters (ϕ, γ). Given (w, R), tougher punishments on default increase the feasible credit for each agent. Unconstrained agents will not change their plans, while constrained agents, facing stricter punishments, can commit to increased borrowing and investment allowing greater human capital investment and smoother consumption profiles. Thus, as long as w and R remain unchanged, all individuals will be better off.

Figures 8 and 9 show the equilibrium constraint factors $(\mu, \kappa_0, \kappa_1)$ as functions of ϕ and γ given R. As the amount of earnings and savings that can be seized increase, individuals can borrow a greater fraction of their future earnings. This increase is much greater during the schooling period than the first period of work when γ is increased. As a result, increases in the amount of earnings that can be seized will greatly enhance borrowing opportunities among youth in school. On the other hand, increases in the amount of savings that can be seized (represented by an increase in ϕ) has a greater impact on the borrowing constraints of young workers. Overall, the amount of earnings that can be seized appears to play a much more important role in determining borrowing constraints. Completely eliminating the ability of creditors to seize any saved assets (i.e. setting $\phi = 1$) would not eliminate borrowing, whereas eliminating the ability of creditors to seize earnings would.

Figure 8: Response of Constraints to Changes in ϕ

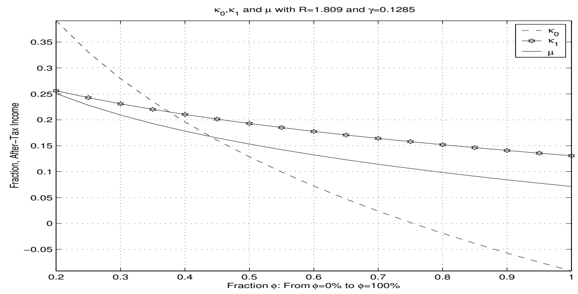
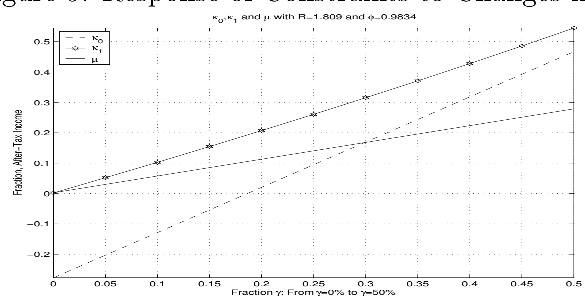


Figure 9: Response of Constraints to Changes in γ



In general equilibrium, this is not necessarily true. Changes in human capital investment decisions will affect interest rates and the equilibrium price of human capital, which also determine borrowing constraints. We numerically analyze the impacts of increasing punishments imposed on those who default in both partial and general equilibrium. Table 7 reports the impacts of increasing the amount of earnings that can be seized by creditors to $\gamma = 0.15$. It also shows the impacts of reducing the rate of return on savings to $\phi = 0.95$. Both changes give creditors more power to punish borrowers, so that constraints should loosen given w and R. This is indeed the case as seen from the partial equilibrium responses in the constraint functions μ and κ_1 and average levels of debt.

We begin with a discussion of changes in γ. Holding interest rates constant, human capital investment increases substantially in response to a rise in γ due to the greater availability of credit. This results in higher wages and aggregate levels of human capital. But, the large increase in human capital and decline in physical capital suggests that the price of human capital is likely to decline while interest rates increase once general equilibrium effects are accounted for. This is confirmed in column 3. The increase in interest rates has only modest effects on μ, consistent with Figure 6. Overall, individuals can borrow more during their first period of work for any given level of future (after-tax) earnings. In contrast, the increase in R has substantial effects on κ_1, causing the new equilibrium value to be lower than in the base case economy. Because κ_1 determines the amount of debt youth can take on, its decline causes investment in human capital to decline, thereby reducing subsequent earnings. The reduction in investment also lowers wages in the second period of adulthood, which causes the actual amount of debt young workers can incur to decline even though κ_1 increases. Overall, an increase in γ from the base case value reduces human capital investment, earnings, and consumption once general equilibrium factors are considered.

The final two columns of Table 7 show the impacts of reducing ϕ to 0.95, in which case individuals who default can only retain 95% of the assets they save. As expected, κ_1 and μ increase when R is held fixed. But, as with the increase in γ, the indirect effects of an increase in R more than offset the direct effect on κ_1 in general equilibrium. After incorporating price changes, the average amount of debt decreases slightly among youth and young workers, as does investment in human capital. Earnings and consumption also decline at all ages.

These policy experiments highlight the importance of considering general equilibrium impacts when analyzing changes in bankruptcy policy. A partial equilibrium analysis suggests that increases in the punishments creditors can impose on individuals who default should increase allowable debt, human capital investment, earnings, and consumption. While this is generally true when punishment levels are very low (e.g. increasing γ from zero or reducing ϕ from one), it may not be true at current punishment levels. Based on our calibrated economy, we find that allowable debt, investment, earnings, and consumption all decline in response to strengthened default punishments

Table 7: Increased Punishments

Variable	Base Case ($\gamma = 0.1285$, $\phi = 0.9834$)	$\gamma = 0.15$ Partial Equilibrium	$\gamma = 0.15$ General Equilibrium	$\phi = 0.95$ Partial Equilibrium	$\phi = 0.95$ General Equilibrium
w	12.2102	12.2102	12.0646	12.2102	12.1690
R	1.8009	1.8009	2.0492	1.8009	1.8678
τ	0.0923	0.0929	0.0932	0.0923	0.0925
κ_0	-0.0860	-0.0541	-0.0025	-0.0747	-0.0571
κ_1	0.1323	0.1546	0.1287	0.1356	0.1287
μ	0.0734	0.0853	0.0749	0.0776	0.0748
H	63	69	61	64	62
K	40	29	33	37	37
$E(subsidy)$	71	78	68	72	70
$E(y)$	93	107	87	94	89
$E(y(e_1))$	77	87	73	78	75
$E(y(e_2))$	89	102	84	91	86
$E(y(e_3))$	96	111	90	98	92
$E(y(e_4))$	98	113	92	100	94
$E(y(e_5))$	103	120	96	105	99
$V(y)$	153	225	128	161	136
$E(V(y\|e))$	74	98	63	77	67
$E(w_1)$	247	270	235	250	240
$E(w_1(e_1))$	176	190	168	178	172
$E(w_1(e_2))$	235	256	223	237	229
$E(w_1(e_3))$	263	288	250	266	257
$E(w_1(e_4))$	269	294	255	271	261
$E(w_1(e_5))$	293	321	278	296	285
$V(w_1)$	1,796	2,274	1,590	1,853	1,673
$E(V(w_1\|e))$	198	236	171	203	182
$E(d_0)$	30	38	27	31	28
$E(d_1)$	35	44	34	37	35
$E(c_0)$	83	89	78	83	80
$E(c_1)$	206	221	191	209	200
$E(c_2)$	235	250	220	236	227

when general equilibrium changes in interest rates and skill prices are taken into account.

4.2 Labor Income Taxation

The effect of labor income taxation on human capital formation is the subject of an extensive literature, which generally assumes that perfect capital markets exist.[17] Taxes affect the net return and net cost of human capital allocation as well as the resources available to agents in each point of their life cycle. In our model, taxes also affect the credit constraints.

It is well known that if all the costs of human capital investment are in the form of foregone earnings, then the partial equilibrium effect of a proportional tax on earnings is neutral. Tax impacts in general equilibrium models with perfect credit markets are also negligible. [15, 17, 26, 44, 45] This is because a proportional tax on earnings reduces both the costs and benefits of human capital at the same rate. More generally, the response of unconstrained agents to a tax on labor income depends on the relative importance of foregone earnings versus other investments costs. In our model, all costs are from foregone consumption goods, so an increase in labor taxes reduces the returns to investment without affecting the costs (ignoring general equilibrium effects through changes in the return on capital and labor); labor income taxes should, therefore, reduce investment and consumption for unconstrained agents. Among constrained individuals, consumption and investment profiles will be affected by the tightening of credit constraints (assuming individuals cannot default on the tax due to the government). The fact that the government captures part of the returns from human capital investment reduces individual incentives to repay financial liabilities.

Holding interest rates constant, wage taxes will not change the constraint factors μ and κ_1. However, allowable debt will decline by the factor $1-\tau$ since future earnings capacity is reduced by that amount. To the extent that investment also declines, future earnings capacity is reduced even further causing additional restrictions on credit. The final general equilibrium effects will depend on the form of the production function and the relative importance of income and substitution effects on savings, which determines the responsiveness of w and R.

The impacts of an increase in tax rates can easily be examined in our calibrated base economy. In this economy, a 1% increase in the wage tax rate produces negligible effects on investment and borrowing limits when w and R are held constant.[18] However, once the price of human capital and interest rates are allowed to equilibrate, the model predicts a \$1 decline in borrowing limits for youth and young workers and a \$3 decline in investment. Thus, general equilibrium effects are important, and once considered, they suggest that an increase in the wage tax rate leads to important declines in borrowing and investment.

[17] See, e.g. [7, 15, 16, 17, 18, 24, 26, 29, 43, 44, 45]

[18] We no longer balance the budget in this experiment, since we want to identify the impacts of a change in tax rates alone.

4.3 Public Provision of Human Capital

We briefly consider two separate forms of government schooling interventions that combine to create a more general schooling policy that roughly corresponds to that of the United States. In the first intervention, government freely provides a lump-sum amount of schooling to everyone. In the second, the government provides a proportional subsidy to the expenses incurred by young agents in human capital production. The current U.S. education system contains features from both of these policies, as reflected in our base economy.

4.3.1 Public Schooling

To consider the case for public schooling, assume that labor income taxes (τ) are allocated to provide all young agents with a minimum level of human capital, y_0. In this case, private investments are additions to y_0. In a stationary equilibrium, a self-financed public investment provision program requires that $y_0 = \tau w H$. The young agent's problem becomes

$$V_0(a, e) = \max_{\{(d_0, y):\, d_0 \leq \kappa_1 (1-\tau) w e (y_0 + y)^\alpha\}} \left\{ u(a + d_0 - y) + \beta V_1((1-\tau) w e (y_0 + y)^\alpha, d_0) \right\}. \tag{4}$$

As previously discussed, proportional taxes reduce the unconstrained optimal investment amounts. As a result, those agents that were not credit constrained and invested more than y_0 will reduce their human capital. The direct effect of an increase in y_0 on other agents, poorer and/or less able, is to increase their human capital. The endogeneity of the credit constraints also has an effect on them: since the government program increases their earnings potential, these agents are more able to obtain credit, which further increases their human capital.

The redistribution across agents can be significant. The inequality in earnings and wealth is reduced. The effect on aggregate capital stocks depends on the distribution of agents on $A \times E$.

4.3.2 Subsidies to Education

Now, assume that labor income taxes are used to finance a proportional subsidy, ν, on the schooling expenses of young agents. Thus, the self-financing constraint requires that $\nu \int_{A \times E} y(a, e) \lambda(da \times de) = \tau w H$. Young agents deciding how much to invest in human capital must solve the problem

$$V_0(a, e) = \max_{\{(d_0, y):\, d_0 \leq \kappa_1 (1-\tau) w e y^\alpha\}} \left\{ u(a + d_0 - y(1-\nu)) + \beta V_1((1-\tau) w e y^\alpha, d_0) \right\}. \tag{5}$$

Abstracting from general equilibrium effects, the regime may increase or reduce the unconstrained investment depending on whether $\nu \lesseqgtr \tau$. Among constrained youth, subsidies encourage investment, much like an increase in assets – they increase the amount of investment that can be supported for a given level of debt. A more unique result of this model concerns the effects of the policy on the constraints themselves. As above, labor taxes serve to reduce the amount of allowable debt, counteracting the increase in available funds from the subsidy. However, by encouraging human capital investments over consumption during youth, subsidies increase future earnings power producing an indirect effect on the amount of borrowing allowed. The final impact of the combined tax and subsidy policy on lending depends entirely upon whether after-tax earnings increase or decrease.

We numerically analyze the impacts of an increase in the rate of subsidy to schooling to show how our model of endogenous constraints responds. The base model is described in the calibration procedure, and roughly corresponds to the current U.S. educational system. Investments are assumed to be subsidized fully through $y_0 = 50$ (i.e. the tenth grade) and at the rate $\nu = 0.5$ above that level. Flat wage taxes are levied to balance the federal budget (i.e. to pay for the subsidies to investment).

The base case equilibrium prices (w, R), wage tax rate (τ), aggregate human and physical capital stocks (H, K), credit constraint functions $(\mu, \kappa_0, \kappa_1)$, and average subsidy amount are shown in the first column of Table 8. The table also shows average investment amounts and earnings for individuals of different ability (e_j) levels. Average debt, savings, and consumption are also shown at different ages.

Table 8 reports the impacts of increasing the rate of subsidy, ν, to 0.55 in both partial and general equilibrium. Column 1 reflects the base case equilibrium. Column 2 shows the partial equilibrium changes when the prices of human and physical capital are held constant (the tax rate is allowed to adjust to maintain a balanced budget). These results reflect the direct impacts of an increased subsidy on decisions and credit constraints. The final column of the table reports the new general equilibrium values once w and R are allowed to change, showing the added impact of changing prices on decisions and constraints.

In response to the increased rate of subsidization, human capital investment increases by nearly 20% resulting in a 13% increase in aggregate human capital. Holding w and R (and, therefore, κ_1 and μ) constant, the amount of debt held by youth and young workers increases by about 10%. This is because individuals respond to the subsidy by increasing their investment in human capital, which increases their future earnings power (by about 12%) and relaxes limits placed on their borrowing. The small increase in the tax rate is not enough to offset these forces. The relaxation of borrowing constraints also serves to increase investment in addition to early consumption.

The indirect effect of increased borrowing on investment is difficult to measure, since credit limits themselves are affected by investment. However, we suggest that this effect is not trivial. Consider that a \$1 increase in initial assets causes investment to increase by about \$.80. Simply treating the increase in credit as an increase in initial assets would imply an average increase in investment of about \$2.50, nearly one-third of the observed increase. We further discuss the role

Table 8: Increased Subsidy on Investment

		$\nu = 0.55$	
Variable	Base Case ($\nu = 0.5$)	Partial Equilibrium	General Equilibrium
w	12.2102	12.2102	12.2081
R	1.8009	1.8009	1.8045
τ	0.0923	0.0967	0.0967
κ_0	-0.0860	-0.0860	-0.0850
κ_1	0.1323	0.1323	0.1319
μ	0.0734	0.0734	0.0733
H	63	71	71
K	40	44	44
$E(subsidy)$	71	83	83
$E(y)$	93	111	110
$E(y(e_1))$	77	91	91
$E(y(e_2))$	89	107	106
$E(y(e_3))$	96	115	115
$E(y(e_4))$	98	118	117
$E(y(e_5))$	103	124	123
$V(y)$	153	247	244
$E(V(y\|e))$	74	121	119
$E(w_1)$	247	276	275
$E(w_1(e_1))$	176	195	195
$E(w_1(e_2))$	235	262	261
$E(w_1(e_3))$	263	294	294
$E(w_1(e_4))$	269	301	300
$E(w_1(e_5))$	293	328	327
$V(w_1)$	1,796	2,357	2,341
$E(V(w_1\|e))$	198	282	280
$E(d_0)$	30	33	33
$E(d_1)$	35	39	39
$E(c_0)$	83	88	88
$E(c_1)$	206	229	228
$E(c_2)$	235	262	261

of endogenous debt constraints in the next subsection.

The increase in human capital investment drives down the price of human capital in column 3, though the change is quite small. Interest rates also increase slightly. Because prices change very little in general equilibrium, changes in the constraint factors κ_1 and μ are negligible. As a result, changes in allowable debt are almost entirely due to changes in investment and not changes in prices or the amount of debt allowed for any given amount of future earnings. General equilibrium effects play little role in the measured responses to an increase in the rate of subsidy.

It is significant that these results imply that borrowing limits should be relaxed when education subsidies are increased. This is in direct contrast to the standard discussion of federal student loan and subsidy policy, which generally assumes there is a trade-off between the two.

4.4 Comparing Exogenous and Endogenous Borrowing Constraints

We next analyze the role of the endogenous nature of debt constraints in determining levels of human capital investment. Table 9 compares the role of ability, initial assets, wage taxes, and investment subsidies in our model of endogenous constraints with a model that assumes exogenous borrowing constraints. To focus on the role of the constraints, all parameters, including τ, R, and w, are held fixed at their base case levels. The first column reports optimal investment, levels of debt, dy/da, dy/de, $dy/d\nu$, and $dy/d\tau$ for someone with the median ability ($e = 1.3054$) and average amount of initial assets ($a = 74.30$) in the base case endogenous constraint economy. Column two reports the optimal level of investment along with the same comparative statics in an exogenous constraint model with d_0 and d_1 set to the same levels of debt in the endogenous constraint model. Thus, the only difference between the two columns is the endogenity of the credit constraints.[19] Notice that investment is nearly 25% higher in the endogenous constraint model. Treating debt constraints as exogenous would cause one to overstate the degree of under-investment in human capital caused by credit market imperfections. Differences in investment behavior are entirely due to the additional incentives to invest created by the link between endogenous debt constraints and the amount of investment.

Investment also increases much more with initial assets (34% more) in the endogenous constraint model. Again, this difference reflects the fact that constrained individuals with higher initial assets will, all else equal, invest more in their human capital. This relaxes borrowing constraints in the endogenous constraint framework, which further increases investment.

While investment also increases with ability in the endogenous constraint framework, it is decreasing in ability when the constraints are exogenous. The exogenous constraint framework predicts that more able individuals will invest *less* in their human capital given any level of initial

[19] While not shown, the same patterns emerge for all other combinations of a and e in our model economy.

Table 9: Comparison of Endogenous and Exogenous Constraint Models

	Endogenous Constraint	Exogeous Constraint
a	74.30	74.30
e	1.3054	1.3054
d_0	31.30	31.30
d_1	36.96	36.96
y	94.34	75.73
dy/da	0.78	0.58
dy/de	34.41	-14.63
$dy/d\nu$	171.09	98.98
$dy/d\tau$	-49.48	-113.62

Notes: Levels of debt for the exogenous constraint economy are set to be the the same as those determined in the endogenous credit market economy.

assets. This is because a higher ability improves future earnings and consumption, which makes consumption while young relatively more valuable. When debt constraints are endogenous, this effect is more than offset by the increased capacity to borrow resulting from a greater future earnings capacity.

Finally, investment responses to education subsidies are 73% greater in the endogenous constraint model, while responses to wage taxes are less than half those of the exogenous constraint framework. Thus, taxing income to subsidize investment will have much more positive impacts on human capital investment when borrowing constraints are endogenously determined from default incentives than when they are assumed to be exogenous.

One can also use these comparisons to decompose the effects of policy or endowments on investment in the endogenous constraint model. For example, the endogenous constraint model predicts that a one dollar increase in initial assets raises investment in human capital by about 78 cents. From the exogenous constraint comparative statics, we observe that about 58 cents of that increase would occur if debt levels were held constant. Therefore, about 25% of the total increase in investment can be traced to the endogenous nature of the constraints. The same decomposition is useful when analyzing the response to education subsidies, which suggests that about 42% of the increase in investment comes from the endogeneity of debt constraints. These contrasts are stark and clearly argue for more carefully analyzing the sources of borrowing constraints. Not only do the two models predict very different levels of investment for any given amount of observable debt, but they also predict very different correlations between investment and individual ability and asset endowments. Policy responses to investment subsidies and wage taxes are also substantially different in the two models.

5 The Guaranteed Student Loan (GSL) Program

This type of model can also be used to study the implications of government supported loan programs such as the U.S. GSL program. The GSL program includes some features of the efficient contracts in our model. Indeed, individuals are allowed to borrow more if they attend school for a longer time period. To this extent, student loan initiatives offer greater borrowing limits to those acquiring more human capital. But limits should also depend on other factors that affect future earnings capacity. In practice, the loan amount offered to a student will loosely depend on his future earnings capacity, since government loan amounts are tied to the tuition cost of the university attended, which depends on both the student's ability and the quality of the institution. However, the GSL program does not directly adjust loans to individual ability, which could be assessed (albeit, imperfectly) with previous grades or standardized tests like the SAT or GRE. One could imagine larger available loan amounts for individuals with better high school grades or SAT scores. Other factors that affect future earnings capacity, such as choice of major, gender, race, and family background do not factor into borrowing limits or terms, perhaps reflecting other objectives of policymakers. Overall, there is little reason to expect that borrowing limits adjust optimally in response to differential incentives to default.

To examine the relationship between human capital investment, ability, and default under the GSL program, we provide a brief analysis in a stylized government lending environment when individuals face incentives to default as in the previous sections.[20] In this section, we simply model the GSL program by assuming that young agents can borrow a fraction of their expenditures on human capital investment up to a specified upper limit. Lending is tied to investments which are fully observable, as before.[21]

More specifically, for an upper loan limit of d_{max}, assume that youth who invest y in their human capital can receive a government loan of $d_g = min\{(1-\nu)(y-y_0), d_{max}\}$. Thus, all investments up to $y_d = y_0 + d_{max}/(1-\nu)$ can be fully financed through government subsidies and loans. The government may have special rights to impose different punishments (γ_g, ϕ_g) on borrowers that default and can set the interest rate at its discretion (R_g), funding any losses from the loan program through taxes on earnings. In this respect, the program may simultaneously provide both loans and subsidies to investment, much like the federal GSL program. For simplicity, assume that $(\gamma, \phi) = (0, 1)$ for the private sector, so the government program is the only source of loans for

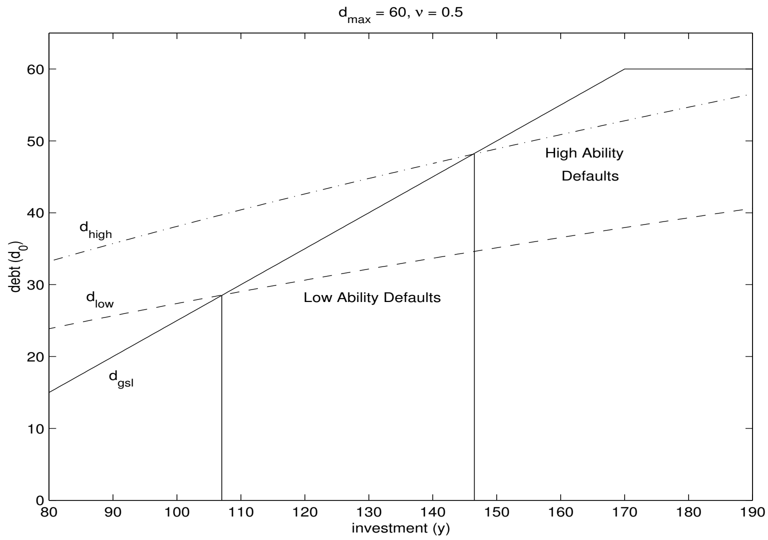
[20]A more detailed analysis of federal loan programs is carried out in Lochner and Monge [38].

[21]If investments are not fully observable, moral hazard problems arise in addition to ex-post default incentives.

youth.

Because this system does not adjust lending for ability or other relevant observable information, it will be subject to default. Figure 10 reveals the default regions for high and low ability borrowers when government punishments and interest rates are set at the base case values and $d_{max} = 60$. The figure displays the maximum level of debt that low and high ability agents (d_{low}, d_{high}) can commit to re-pay for each level of total investment. The d_{gsl} line represents the loan amount individuals can receive from the government system for each level of investment. At lower investments, d_{gsl} is below both d_{high} and d_{low}, implying that individuals of all ability levels can commit to re-pay small loans for which the amount of debt accrued is greater than d_{gsl}. However, low ability agents choosing to invest more than \$108 and high ability agents choosing to invest more than \$147 will take out loans which they do not intend to re-pay. They will default.

Figure 10: Default Under the GSL Program



The government can reduce default rates by lowering the maximum loan amount. By setting $d_{max} \leq 48$, the government could avoid default by more able youth, though less able youth investing more than \$108 will still choose to default. Further lowering d_{max} below \$29 could eliminate default entirely but at the price of severely limiting the effectiveness of the program. More generally, default may occur for all ability groups when maximum loan amounts are high, while for lower limits, more able individuals will never choose to default.

It is important to recognize that default decisions will depend on the level of government subsidies for schooling. Higher subsidies imply that larger amounts of debts will be re-paid for any given level of investment, reducing default rates. For the parameterization in Figure 10, increasing the subsidy rate by 5% would increase the investment amounts above which low and high ability agents default by about \$10 and \$20, respectively.

In this stylized model, as long as $\gamma_g < 1$, anyone who plans on defaulting will invest y_d, taking out the maximum loan amount. This is easily understood by considering someone who decides to default and must determine whether to invest $y < y_d$ or to invest more. He can invest more at no cost to current consumption, since any expenditures are offset by additional lending. However, if $\gamma_g < 1$, future consumption unambiguously increases with investment. The restriction on future borrowing does not depend on the amount he defaults on, and he will have no assets to seize during his first period of work. The only punishment that varies with the default amount is the wage garnishment, which increases with the loan amount and investment since future earnings increase. But, this financial punishment increases less than future earnings in response to any investment, making it optimal to invest and borrow as much as possible up to the limits y_d and d_{max}.

To further study the decision to default, consider agents with low initial wealth levels, a, such that their own assets are used only for consumption and their human capital investments are financed entirely from government subsidies and loans. All individuals who would choose to invest less than y_d fall into this category. These agents will choose an investment and consumption profile that does not involve default on their student loans as long as

$$u(a) + \beta \max_{y:y \leq y_d} V_1((1-\tau)wey^{\alpha}, (1-\nu)(y-y_0)) < u(a) + \beta V_1^d((1-\tau)wey_d{}^{\alpha}), \qquad (6)$$

where $V_1(w_1, d_0)$ and $V_1^d(w_1)$ are described by equations (1) and (2), respectively. Because poorer individuals consume all of their initial wealth and use government subsidies and loans to finance investment, variations in a will have no effect on their future incentives to default. Thus, among the poor, changes in initial assets have no affect on their decision to default (as long as they remain poor). This need not be the case for wealthier individuals who finance some of their investment from their own initial assets. The relationship between ability and default is more complicated and need not be monotonic.[22]

A number of conclusions can be drawn. First, a program such as the GSL is subject to default by not linking borrowing limits to investment returns. Second, it is possible to predict ahead of time who is likely to default. (With no risk and full information, one can perfectly forecast who will default.) Third, with low maximum loan amounts, only the least able are at risk of default. Finally, decisions to default will depend on the rate of government subsidy to investment. Limitations of this analysis include the implications that all defaults are fully predictable and that all individuals who default choose to borrow the maximum. To address these shortcomings and better explain empirical patterns in default, Lochner and Monge [38] introduce uncertainty in the returns to

[22]The condition in (6) divides individuals into ability regions in which it is optimal to default and into regions in which it is not. For most reasonable parameterizations, we find a single cutoff level for ability below which all agents default. However, for more extreme parameterizations, it is possible that the condition defines multiple regions of default, such that default is not monotone in ability. It is also possible to choose parameters so that everyone chooses to default or so that everyone re-pays.

human capital investment in a more extensive study of the GSL program.

6 Conclusions

This paper has developed a framework for studying human capital decisions and the endogenous formation of credit constraints that arise out of individual incentives to default and the capacity of lenders to punish those who do. The amount of debt lenders will provide, therefore, depends on the costs imposed by available punishments. The individual costs of any given punishment strategy are likely to vary across the population and over the life cycle. Those who face greater costs from any type of punishment will be given greater access to credit, since lenders understand that those borrowers are more likely to repay their debts in order to avoid punishment. We demonstrate that if lenders can seize a fraction of defaulting borrowers earnings and/or savings, then the amount of debt lenders will extend to borrowers depends on their future earnings capacity. More able individuals will be extended more credit, because potential punishments are more costly for them. Lenders will also extend greater credit to those who invest more in their human capital for the same reason. Furthermore, credit limits will vary over the life cycle, reflecting changes in the individual cost of default punishments. In our calibrated economy, allowable debt increases with age as both savings and earnings increase.

Not only do borrowing constraints vary in the population, but they are also influenced by government policy and macroeconomic forces. Policies that increase the returns to investment (e.g. education subsidies) should increase the amount of credit lenders extend, since individuals will optimally choose to invest more in their human capital, increasing their future earnings capacity. Thus, more extensive loan programs should go hand-in-hand with increased subsidies for schooling. This sheds new light on the standard discussion that posits a tradeoff between lending and subsidies. Interest rates and the return on human capital are also important determinants of credit limits. Higher interest rates make it more difficult for borrowers to re-pay their loans, so lenders should respond by restricting credit. Higher wage rates have the opposite effect. Thus, it may be important to consider general equilibrium effects of government policy on credit constraints. While we find this channel relatively unimportant in our study of school subsidy policy, it is extremely important for analyzing the impacts of changes in bankruptcy laws.

The insights learned from studying the endogenous formation of borrowing constraints in efficient credit markets can be useful for improving federal student loan programs. They may also be useful for predicting who is likely to default in our current college finance system. While the GSL program has some features of an efficient credit market (e.g. the amount of loans offered is tied to the quantity and, indirectly, quality of education), it does not fully adjust credit limits to reflect differences in future earnings capacity. Whether this results in over- or under-investment in human capital is not immediately obvious. It is possible that limits are too stringent on more able individuals choosing lucrative careers and too lax on less able students choosing careers with little financial return. This is an open empirical question worth studying.

Much can be learned from taking the origin of borrowing constraints more seriously. In comparing our endogenous constraint model with a model assuming exogenous borrowing constraints, we find that for any level of observable debt, human capital investment is substantially greater when constraints are endogenous. The model with exogenous constraints predicts that ability and investment should be negatively correlated, in sharp contrast to empirical findings and the predictions of the endogenous constraint model. Finally, the endogenous constraint model predicts much larger impacts from an education subsidy and smaller impacts from a wage tax on human capital investment. Assuming constraints are exogenous is not an innocuous simplification.

Future research should focus on better understanding the precise punishments creditors have at their disposal. It is also important to consider the role of uncertainty about the future and, perhaps, private information in determining the optimal structure for credit markets. In addition to adding realism, incorporating these factors will be helpful for determining the optimal system of punishment as well as who chooses to default.

References

[1] S. R. Aiyagari, J. Greenwood, and A. Seshadri. Efficient Investment in Children. *Journal of Economic Theory*, forthcoming.

[2] F. Alvarez and U. Jermann. Asset Pricing when Risk Sharing is Limited by Default. Working Paper, University of Chicago and The Wharton School, 1997.

[3] D. Andolfatto and M. Gervais. Social Security and the Payroll Tax with Endogenous Debt Constraints. Working Paper, 1999.

[4] O. Attanasio, J. Banks, C. Meghir, and G. Weber. Humps and Bumps in Lifetime Consumption. NBER Working Paper 5350, 1995.

[5] O. Attanasio and V. Rios-Rull. Consumption Smoothing in Island Economies: Can Public Insurance Reduce Welfare. University of Pennsylvania Working Paper, 2001.

[6] G. Becker. *Human Capital*. Columbia University Press, New York, 1964.

[7] M. Boskin. Notes on the Tax Treatment of Human Capital. In *Conference on Tax Research 1975*. Dept. of the Treasury, Washington DC, 1975.

[8] S. Cameron and J. Heckman. Life Cycle Schooling and Dynamic Selection Bias: Models and Evidence for Five Cohorts of American Males. *Journal of Political Economy*, 106(2):262–333, April 1998.

[9] S. Cameron and J. Heckman. The Dynamics of Educational Attainment for Blacks, Whites and Hispanics. Working Paper, 1999.

[10] S. Cameron and C. Taber. Borrowing Constraints and the Returns to Schooling. Working Paper, 1999.

[11] D. Card. Earnings, Schooling, and Ability Revisited. *Research in Labor Economics*, 14:23–48, 1995.

[12] C. Carroll and L. Summers. Consumption Growth Parallels Income Growth: Some New Evidence. NBER Working Paper 3090, 1989.

[13] E. Caucutt and K. Kumar. Higher Education Subsidies and Heterogeneity. Working Paper, 2000.

[14] M. Daniels, B. Devlin, and K. Roeder. Of Genes and IQ. In B. Devlin et al., editors, *Intelligence, Genes, and Success: Scientists Respond to The Bell Curve*. Springer-Verlag, New York, 1997.

[15] J. Davies and J. Whalley. Taxes and Human Capital Formation: How Important is Human Capital? In B. Bernheim and J. Shoven, editors, *National Saving and Economic Performance*. University of Chicago Press, Chicago, 1991.

[16] E. Driffill and H. Rosen. Taxation and Excess Burden: A Life-Cycle Perspective. 24(3):671–83, Oct. 1983.

[17] W. Dupor, L. Lochner, C. Taber, and M. B. Wittekind. Some Effects of Taxes on Schooling and Training. *American Economic Review*, 86(2):293–7, 1996.

[18] J. Eaton and H. Rosen. Taxation, Human Capital, and Uncertainty. *American Economic Review*, 70(4):705–15, Sept. 1980.

[19] D. Elwood and T. Kane. Who is Getting a College Education? Family Background and the Growing Gaps in Enrollment. In S. Danziger and J. Waldfogel, editors, *Securing the Future: Investing in Children from Birth to College*, chapter 10. Russell Sage Foundation, New York, 2000.

[20] S. Fay, E. Hurst, and M. White. The Household Bankruptcy Decision. Working Paper, 2000.

[21] J. Fernandez-Villaverde and D. Krueger. Consumption and Savings over the Life Cycle: How Important are Consumer Durables? Working Paper, 2001.

[22] R. Gropp, J. K. Scholz, and M. White. Personal Bankruptcy and Credit Supply and Demand. *Quarterly Journal of Economics*, 112(1):217–51, Feb. 1997.

[23] E. Hanushek, K. Yilmaz, and C. Leung. Redistribution Through Education and Other Transfer Mechanisms. Working Paper, 2001.

[24] J. Heckman. A Life-Cycle Model of Earnings, Learning, and Consumption. *Journal of Political Economy*, 84(4):s11–s44, August 1976.

[25] J. Heckman, L. Lochner, J. Smith, and C. Taber. The Effects of Government Policies on Human Capital Investment, Unemployment and Earnings Inequality. In GAAC, editor, *Third Public GAAC Symposium - Labor Markets in Germany and the USA*, pages 305–36. 1998.

[26] J. Heckman, L. Lochner, and C. Taber. Tax Policy and Human Capital Formation. *American Economic Review*, 88(2):293–97, May 1998.

[27] J. Heckman, L. Lochner, and P. Todd. Fifty Years of Mincer Earnings Regressions. Working Paper, 2001.

[28] T. Jappelli. Who is Credit Constrained in the U.S. Economy. *Quarterly Journal of Economics*, 105(1):219–34, Feb. 1990.

[29] L. Jones and R. Manuelli. On the Taxation of Human Capital. Working Paper, 1999.

[30] T. Kane. College Entry by Blacks Since 1970: The Role of College Costs, Family Background, and the Returns to Education. *Journal of Political Economy*, 102(5):878–911, Oct. 1994.

[31] M. Keane and K. Wolpin. The Effect of Parental Transfers and Borrowing Constraints on Educational Attainment. Working Paper, 1999.

[32] T. Kehoe and D. Levine. Debt-Constrained Asset Markets. *Review of Economic Studies*, 60(4):865–88, Oct. 1993.

[33] N. Kiyotaki and J. Moore. Credit Cycles. *Journal of Political Economy*, 105(2):211–48, 1997.

[34] N. Kocherlakota. Implications of Efficient Risk Sharing without Commitment. *Review of Economic Studies*, 63(4):595–609, 1996.

[35] D. Krueger and F. Perri. Risk Sharing: Private Insurance Markets or Redistributive Taxes? Working Paper, 2001.

[36] L. Lambertini. Social Security with Endogenous Debt Limits. Working Paper, 1999.

[37] K. Lang. Ability Bias, Discount Rate Bias, and the Return to Education. Unpublished Manuscript, 1994.

[38] L. Lochner and A. Monge-Naranjo. Human Capital Investment and Default Incentives with Government Student Loan Programs. Working Paper, 2002.

[39] G. Loury. Intergenerational Transfers and the Distribution of Earnings. *Econometrica*, 49(4):843–67, 1981.

[40] A. Monge-Naranjo, J. Cascante, and L. J. Hall. Enforcement, Contract Design and Default: Exploring the Financial Markets of Costa Rica. In M. Pagano, editor, *Defusing Default: Incentives and Institutions*. John Hopkins University Press, New York, 2001.

[41] U.S. Department of Education. *Digest of Education Statistics*. USGPO, Washington, DC, 1999.

[42] M. Pagano. *Defusing Default: Incentives and Institutions.* John Hopkins University Press, New York, 2001.

[43] H. Rosen. Taxation and On-the-Job Training Decisions. *Review of Economics and Statistics*, 64(3):442–49, Aug. 1982.

[44] C. Taber. Tax Reform and Human Capital Accumulation: Evidence from an Empirical General Equilibrium Model of Skill Formation. Working Paper, 2000.

[45] P. Trostel. The Effect of Taxation on Human Capital. *Journal of Political Economy*, 101(2):327–50, 1993.

Table A-1: Total Schooling Costs by Year of School and AFQT Quintile (1999 dollars)

Years of School	Direct Expenditures	Total Expenditures Quintile 1	Quintile 2	Quintile 3	Quintile 4	Quintile 5
8	41,505	41,505	41,505	41,505	41,505	41,505
9	45,836	45,836	45,836	45,836	45,836	45,836
10	50,001	52,444	53,166	53,705	53,708	53,939
11	54,005	59,214	60,754	61,903	61,908	62,401
12	57,856	66,162	68,617	70,450	70,458	71,244
13	68,372	80,110	83,579	86,169	86,181	87,292
14	78,485	93,982	98,563	101,983	101,999	103,465
15	88,208	107,781	113,567	117,886	117,906	119,758
16	97,558	121,094	128,579	133,862	133,887	136,152
17	106,547	134,719	143,575	149,881	149,911	152,614
18	115,191	148,227	158,520	165,900	165,935	169,099
19	123,503	161,587	173,373	181,866	181,906	185,548
20	131,495	174,762	188,080	197,717	197,763	201,894

Notes:

1) Direct expenditures assume average expenditure per pupil in primary and secondary schooling through grade 12. Additional expenditures for higher grades are taken from average expenditures per student in all colleges and universities. [41]

2) Foregone earnings are calculated from a regression of log wage income on AFQT quintile, education indicators, experience and experience-squared. Foregone earnings are based on someone with 9 years of schooling plus the corresponding level of experience. Total costs equal direct expenditures plus foregone earnings. See text for details.

3) Cost measures are discounted at a 4% annual interest rate beginning with school entry.