Modeling Efficiency Units of Electricity in Production

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Abstract

"Raw energy" in traditional Cobb-Douglas production models is assumed to be homogeneous in both value and productive capacity among producers. In this paper, we describe a new method to model heterogeneous and parsimonious preferences, as well as the constraints of various industries. Simple and versatile, "efficiency units of electricity" is able to significantly model cross-industry variation in energy productivity using principles of statistical physics to mitigate the introduction of several parameters. Our findings demonstrate that the introduction of efficiency units of electricity in production improves the statistical efficiency of estimators for labour and capital. We recommend that supplementary literature should explore the economic significance of the Boltzmann weighted parameter (φ) using alternative proxies and datasets for efficient labour using industry level considerations.

Keywords: Efficiency Units of Electricity, Cobb-Douglas Production Model, Heterogeneous Preferences, Total Factor Productivity, Cross-Industry Variation, Boltzmann distribution

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1. Introduction

The production function is a key economic idea that expresses the relationship between physical inputs and the output produced. Convention dictates that the factors of production feature labour (L) and capital (K), exclusively. However, after extensive research and interest in the field of Econophysics, the goal of this paper is to explore how efficiency units of electricity can account for differences in the use of raw energy in production among industries. The economic question that we are exploring examines how the heterogeneous preferences and constraints, faced by various industries for raw 'energy', can be modeled in production functions.

Our interest in the role of energy stems from the integration of key principles in both economics and physics; whereby the behaviour of matter and properties of energy in physics can describe economic preferences and constraints. Specifically, the idea of energy conservation can mirror the behaviour of industries in cost-minimization problems associated with production. In addition, the variances in productivity among industries will be represented by industry-specific labour force controls that mirror the characteristics of particles in thermodynamics. Howlett, Netherton & Ramesh suggest that the fundamental differences in industry production can be useful to policy makers to incorporate the role of energy when regulating the production of Canadian industries (Howlett, Netherton, and Ramesh 1999, 5-15).

By identifying efficiency units of energy as a significant factor of production, we will estimate a production function that incorporates a productivity parameter using the Boltzmann distribution. To do so, we will draw ideas from our literature survey, as well as the consultation of Professor Saunders and Rui Castro. We will then define our production model and variables using data drawn exclusively from CANSIM. In the last section we intend to outline other considerations we could have made to this model and how we would intend to proceed with the objectives of the paper.

2. Background and Literature Review

2.1 Motivation and Economic Origins

Traditional economic models of production emphasize raw factor data and flows, such as labour, capital, land and technology, in order to describe changes in the output produced by firms and industries. Specifically, in the Cobb-Douglas model, fixed proportions of labour (L) and capital (K) explain how much output (Y) is produced (Williamson 2012). Any unobserved variation is aggregated in Total Factor Productivity (Z). This model is presented below:

$$Y = Z\{f(K, L)\} = Z\{K^{\alpha}L^{\beta}\}$$

such that α = proportion/income share of capital, β = proportion/income share of labour.

The motivation of this paper is to examine the TFP using methods aimed to quantify unobserved factors that generate large variances in output at the industry level. In the traditional Cobb-Douglas model, the TFP aggregates a variety of influences on the growth of output including technology, political, cultural and unobservable economic factors that may be random or unobservable (Jorgenson and Griliches 1967, 276-279). The potential to identify an omitted variable, hidden within the TFP, may cause the estimates of labour and capital to be under or overstated in the existing model. This paper will explore the impact of introducing an additional factor input to the classic Cobb-Douglas model, with the intention of arguing that there is potential for improving the validity and efficiency of the labour and capital estimators.

The classic Cobb-Douglas model emphasizes the role of income shares of capital and labour as α and β respectively. These shares represent ratios of each fixed level of input for labour and capital needed to produce a unit of output, such that:

$$\text{Total Labour Income Share} = \frac{w\{\sum_{i=1}^{n} l_i\}}{Y} = \beta$$

$$\text{Total Capital Income Share} = \frac{\{\sum_{i=1}^{n} \pi_i\}}{Y} = \alpha$$

The derivations of these factor input shares are provided in Figure 1 in the Appendix (Williamson 2012). At the industry level, these shares present the aggregate proportions of labour and capital inputs of all firms. This paper will focus on the national level to interpret these elasticities as aggregate proportions of capital and labour of all industries in a given year to produce GDP. Historically, the shares of capital and labour at the national level were 0.3 and 0.7 respectively according to the Cobb-Douglas Model.

2.2 Preliminary Tests and Parameter Analysis

Preliminary tests on our sample of Canadian industry data were used to test the assumptions of the Cobb-Douglas Model in a few different approaches.

1) An initial regression was conducted as represented below:

$$\ln Y_{it} = \ln Z + \alpha \ln K_{it} + \beta \ln L_{it} + \varepsilon_{it}$$

such that i = industry, t = time period. As shown in Table 1 of the appendix, the results were statistically significant at the 99% confidence level and provided values of 0.253 and 0.232 for α and β respectively. This result provided us with further justification to analyze the inputs of production.

2) Subsequent tests were done to assess the relationship of Total Factor Productivity in relation to the discrepancies in income shares of labour and capital. The Solow Residual was used to measure the TFP indirectly by examining whether the low elasticity values of labour and capital could be attributed to discrepancies in data or whether elasticity estimates could potentially exhibit considerable bias. The Solow Residual is determined as follows for each given year (Williamson 2012):

$$\text{Solow Residual} = TFP_t = Z_t = \frac{Y_t}{K_t^{\alpha} L_t^{\beta}}$$

Table 2 in the appendix compares and contrasts the value of Solow residuals from our approximated values of α and β and that of traditional assumptions, more accurately 0.3 for α and 0.7 for β. The indirect calculations show that the TFP values calculated using the elasticities generated from the first regression of 0.253 and 0.232 for α and β respectively were significantly larger than both the TFP values calculated by CANSIM and the TFP values using the traditional Solow assumptions of 0.7 and 0.3 for α and β respectively. Specifically, the TFP values using our regression-specific elasticities were 4x greater than the TFP values of the sample, while the traditional Solow elasticities were 79x smaller.

This test shows that the *true* elasticities of α and β respectively in our sample are significantly overestimated by the classic assumptions and slightly underestimated by our calculated Solow residuals. The important implication of this preliminary test is to show that the proportions of the TFP from 2002 to 2014 stay fairly consistent, implying that the proportion of output unexplained by labour and capital remains consistent through time.

3) Final initial tests were conducted to test whether labour and income shares are also constant through different time periods. Growth accounting was used to examine the growth in output (Y_t) specific to the data sample. The full derivations of the growth factors of L_t, K_t, TFP_t and Y_t are presented in Figure 2 of the appendix (Williamson 2012). Table 3, exhibits the results from conducting regressions of the Cobb-Douglas production function over the short time periods. These results suggest that the income shares of labour and capital are not constant through the time periods, while the shares could not also account fully for the growth rate of GDP per capital from 2002 to 2011. The findings from the regressions in Table 3 of the appendix suggest that the estimates of labour and capital are not consistent due to the large fluctuations in the standard errors. Large variations in standard errors may be caused by endogeneity of the model, which will be tested in subsequent sections.

To better account for discrepancies between the theoretical Cobb-Douglas Model and the empirical data drawn from Statistics Canada, we are proposing a modification to the Cobb-Douglas model, such that a new factor of input is introduced to the model. This factor of input, breaks down the TFP into a quantifiable omitted variable and a random component, with the intention of reducing the bias of existing estimators.

Subsequent sections will describe and analyze the significance of proposing a new input factor that accounts for the role of raw energy in production. What sets our intentions apart from other literature or models that incorporate raw energy data - as energy measured by oil demand or electricity usage among industries - is that we are looking to model energy in accordance with the perceived differences in ability of various industries to use energy as an input. These differences refer to infrastructural and operational differences, as well as parsimonious preferences in energy use.

2.3 The Role of Energy and Econophysics

In economics, the raw energy is traditionally assumed to be homogeneous, whereby the marginal benefit from each additional unit of raw energy is constant (Kümmel, Ayres, and Lindenberger 2010, 147-52). The productivity of each quantity of electricity, for example, is thus considered to be the same. A kilowatt or terajoule of electricity in the agricultural industry has the same productivity capacity as a kilowatt or terajoule of electricity in the manufacturing sector.

However, while raw electricity can be considered homogeneous, the assumption that Kümmel, Ayres, and Lindenberger make that the productive capacity of each unit of raw electricity is also homogeneous is very weak (Kümmel, Ayres, and Lindenberger 2010, 145). In each industry, a certain amount of electricity is needed to keep buildings and equipment running in order to generate heat and light as conditions for labour, etc. These conditions vary according to the specific industry, and thus should not be considered to have the same productive capacity when generating output. Rather, the productivity capacity or "efficiency units" of electricity are heterogeneous.

In order to model the argued heterogeneous behaviour of efficiency units of electricity, this paper will draw on principles from Econophysics. Specifically, thermodynamics can represent economic models of various industry preferences and constraints to derive implicitly the objective functions (Landau 1958, 12). These objective functions consider units of electricity with different efficiency values at the margin. Specifically, industry preferences can be modeled by the non-uniform behaviour of energetic particles in natural equilibrium, while minimized-costing constraint is reflected by the conservation of energy principle.

The primary source for the methodology used in this paper draws from the work of Park, Kim, and Isard (2012). In their paper, the allocation of emission permits is modelled in various countries based on a function of national pollution preferences over time. Rather than allowing for free-trade or the use of social planner, the proportion of permits allocated to each country was argued to be most efficient when distributed according to the Boltzmann distribution. This thermodynamic principle considers the historical emission levels of each country in previous periods against their relative sizes (Park, Kim, and Isard 2012, 4885-890).

The efficiency of a dynamic distribution is upheld by Barbanel and Brams in a purely conceptual cake-cutting problem. In order to allocate the optimal amount of cake to each family member, such that the distribution is Pareto-optimal, envy-free and equitable, the consumption preferences of each family member must be weighted against the caloric intake that is suggested for the relative weight of each individual. That is to say that heavier individuals will require more cake than a thinner individual in order to satisfy each daily caloric requirement. The challenge that Barbanel and Brams found is that as the number of players in the cake-cutting problem increases, the fair distribution of this "cake" becomes far more complicated (Barbanel and Brams 2004, 251-3). Figure 3 below is a pictogram which represents a breakdown of the variables and concepts from statistical physics and how Park, Kim, and Isard used those features as a proxy for their economic model (Park, Kim, and Isard 2012, 4889).

In our paper, the use of the Boltzmann distribution – from physical sciences – will be extended to describe how efficiency units of electricity are assumed to be heterogeneous, modeled on page 26 of Section 3.2, in Figure 6.

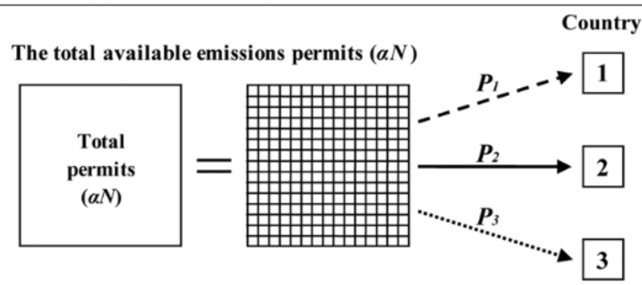
2.4 Incorporation of a Boltzmann Weight

Landau and Lifshitz defined the Boltzmann probability, mentioned in the Emissions Trading paper, as the distribution of energy levels among all particles in a physical system. The distribution is a function of the available energy, relative preferences of energy and the number of particles in the system (Landau and Lifshitz

1958, 11-14). The common model for the Boltzmann distribution is exhibited below in Figure 4.

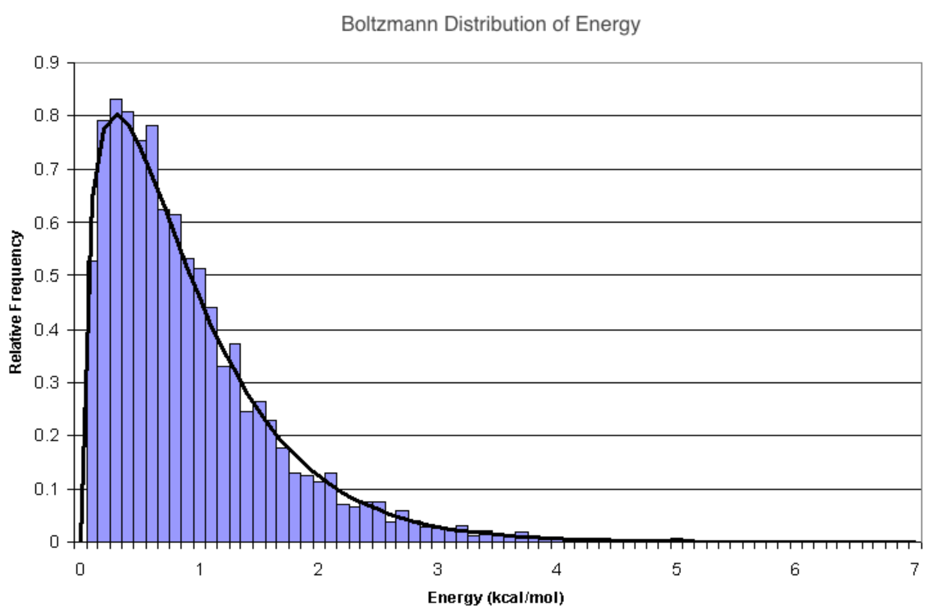
Figure 3: Breakdown of The Park, Kim, and Isard Economic Model

The Boltzmann distribution for permit allocation.

Boltzmann distribution	Description
In physical sciences	$P_i \propto e^{-\beta E_i}$ Where, $P_i =$ probability that a particle stays in substate i $e =$ constant of the exponential function $\approx$2.71828 $\beta = 1/kT$ ($k =$ Boltzmann constant, $T =$ absolute temperature) $E_i =$ energy of substate i
Potential application in permit allocation	$P_i \propto C_i e^{-\beta E_i}$ Where, $P_i =$ probability that emissions permits are allocated to a country i $e =$ constant of the exponential function $\approx$2.71828 $\beta =$ constant ($\geq$0) $E_i =$ allocation potential energy per capita of a country i $C_i =$ total population of a country i
Permit allocation using the Boltzmann distribution*	

* The total available emissions permits (αN) are split into N pieces of unit carbon credit α, and then the emissions permits are allocated to country i (i = 1, 2, and 3) based on the probability distribution (P_i) from the Boltzmann distribution. Note that the number (N) of unit emissions permits can always be made large enough for the Boltzmann statistics by making the unit emissions permit (α) smaller.

Figure 4: Boltzmann Distribution of Energy



More generally, this distribution can describe any set of entities have varying preferences and constraints for energy as mentioned by Banerjee and Yakovenko. Preferences are also constrained to the temperature, or environment conditions, of the system, where the more "energetic" an entity is, the more energy input it requires. The distribution will naturally follow the non-uniform distribution exhibited in Figure 4 (Banerjee and Yakovenko 2010, 755-64). The distribution implies that density of observations is higher at low energy levels, meaning that in any population, the frequency of high-efficiency entities will significantly outweigh observations of "energetic" or low-efficiency entities.

The application of this Boltzmann distribution can show that entities tend to exist in low energy states, since this distribution is more sustainable in the natural equilibrium. When modeling efficiency units of electricity, the Boltzmann weight is the most effective method of modeling heterogeneity in the energy input efficiency among industries for the following reasons:

1) The Boltzmann weight is simple and versatile. This single variable is used to describe industry energy input preferences by weighting relative characteristics and constraints according to their effect on efficiency levels of using electricity to generate output. Thus, the weight is easily calculated based on the unique characteristics of the industry that can be observed, without requiring a specific weight or parameter for each observation by province, industry and year. The weight significantly reduces the amount of terms regressed on output, by requiring no additional parameters on efficiency units of electricity.

2) Energy is inherently a non-linear dynamic flow, according to the assumptions upheld by thermodynamics and argued by other literature analyzed in Section 2.2. Electricity, as a form of energy, varies in volume according to province, industry and year, due to localized and industrial factors associated with energy needs and efficiencies production. The heterogeneous assumption of behaviour in the Boltzmann distribution upholds the argument that efficiency units of electricity follow a similar heterogeneous assumption. It is thus a way to model energy in a parsimonious way, such that there is a relative scarcity of high-energy and low efficient industries compared to high-efficient industries.

3) Lastly, the distribution allows us to interpret the role of energy in production in a meaningful way. Efficiency units of electricity, quantified by electricity input in terajoules required per worker, describe the unique interaction of electricity preferences of an industry with the relative size of efficient workers in the industry.

The specifications for the model for the "Boltzmann weight" will be outlined in Section 3. It is important to note, however, that the first and second reasons listed above outline the importance of including observations at the provincial level, in addition to year and industry, i.e. the subscript of pit. The use of provincial level data can net out the fixed effects of energy regulation at the provincial and federal levels, in order to prevent covariance between the residual of the TFP and the efficiency units of electricity. At the federal level, the National Energy Board oversees the inter-provincial as well as import and export transfer of electricity through power lines, specifically in the energy section (Natural Resources Canada 2016). At the provincial level, utility boards regulate the electricity competition. The provincial government in Alberta has fully privatized the retail competition of electricity, while Ontario is in the process of doing so. All other provinces adhere to public generation and distribution of electricity. The discrepancies in energy regulation among provinces facilitate the need to run fixed-effect using a provincial dummy, or as we did, provincial level data by industry.

Table 4 in the appendix compares the results from conducting a fixed effects model with a random effects models for specific years for each province among specific industries. The results are statistically significant for both models and the Hausman test statistic value of 14.94 in Figure 5 suggests that since unobserved regulation and political decisions at the provincial level are not significant to the findings. However, we will include the subscript of *pit* in our regressions in order to mitigate any fixed effects associated with provincial-level data.

2.5 Hypothesis and Assumptions

The goal of the paper is to estimate a production function that incorporates an efficiency units of electricity parameter. The use of raw energy in production analysis of prior literature does not account for cross-industry variation in productivity.

We are thus using the Boltzmann distribution specifically, because it is a way to model this variation using observables, while reducing the amount of parameters that would necessarily constrain each industry in each province and each year. We are able to reduce the number of parameters to zero parameters, by modeling φ by means of the Boltzmann weight, based on a constant parameter lambda and observables, such that:

$$\ln(\text{Efficiency Units Of Electricity})_{pit} = \ln\{\varphi_{pit}(\text{Raw Energy in TeraJoules})_{pit}\}$$

Very simply stated, our hypothesis tests whether the coefficient for the efficiency units of electricity can significantly add to the share of output not explained in the initial regressions in Section 2.1. The hypothesis aims to account for the discrepancies between the theoretical assumptions with the Cobb-Douglas Model and the empirical model.

$$H_0 : \beta_{\ln(\text{Efficiency Units of Electricity})} = 0$$
$$\text{against: } H_1 : \beta_{\ln(\text{Efficiency Units of Electricity})} \neq 0$$

Our hypothesis is contingent on the following assumptions:

1) We believe energy is used with varying degrees of efficiency across industries.

2) Efficiency of energy use is based on our selected characteristics and proxies.

3) The preferences and constraints of the representative firms within each industry are homogenous, while cross-industry variations in preferences and constraints are heterogeneous.

The intention of the hypothesis test is to compare the standard errors of the original Cobb-Douglas parameters against the new parameters set by our model. The comparison of standard errors allowed us to analyze whether our model could more efficiently explain the growth in industry output through time and account for the discrepancies in the initial tests conducted in Section 2.1.

3. Model

Our model incorporates the role of efficiency units of electricity (E) as an additional factor of production. We will firstly account for raw energy (Raw E), in addition to labour (L) and capital (K) as the factors of production. The model draws on empirical data in Section 4, using Canadian industry data over multiple time periods and cross-sectional variables.

3.1 Model of Production

In contrast to the conventional Cobb-Douglas production function, we have proposed an alternative model that incorporates energy as a factor of production in aggregate industry output:

$$\ln Y_{pit} = \ln Z + \alpha \ln K_{pit} + \beta \ln L_{pit} + \gamma \ln RawE_{pit} + \varepsilon_{it}$$

such that p = province, i = industry, t = time period

In this model: $\ln Y_{pit}$ represents the aggregate output in industry and year; $\ln Z$ represents the total factor productivity, $\ln K_{pit}$ and $\ln L_{pit}$ are the natural logarithms of labour and capital inputs respectively broken down by province, industry and year, while α and β are their income shares. Ln $RawE_{pit}$ represents the natural logarithm of raw electricity input, while γ measures its respective income share or elasticity.

3.2 Boltzmann Considerations in the Model of Production

Ln E_{pit} represents the natural log of efficiency units of electricity, is also broken down by industry, province and year. This transformation is outlined as follows:

$$lnE_{pit} \;=\; ln\{\varphi_{pit}(RawE_{pit})\}$$

where

$$\varphi_{pit} \;=\; Boltweight_{pit} \;=\; \frac{1}{Employ_{pit}}\{PT_{pit}(e^{-\lambda\{AgeCohort_{pit}\}}\}$$

$$RawE_{pit} \;=\; Raw \;\; Electricity \;\; In \;\; Terajoules$$

$$such \;\; that \;\; p \;=province, \;\; i \;=industry, \;\; t \;=time \;\; period$$

The Boltzmann weight is a relative weight of electricity units required per worker. It is directly proportional to the fraction of part time employed and inversely proportional to the exponential function of the number of workers within the age range of 25-54 in the economy. In this subsection we would like to explain the rationale behind the suitability of using these economic variables as counterparts for the statistical physics variables.

We modeled the heterogeneous nature of efficiency units of electricity by applying the aforementioned Boltzmann weight. These efficiency units are proportional to the fraction of part-time employed as, in a given province, industry and year, part-time workers, which will be used as a proxy for unskilled workers, would have a higher marginal propensity to consume a unit of electricity input due to their burdens of more energy-intensive tasks. While unskilled labour may be measured with other proxies such as education attainment and work experience, in the context of energy, we are incorporating part-time employment to account for the fraction of employed that requires more energy-intensive tasks with greater allocation of energy to certain tasks in order to meet deadlines and output quotas (Hirsch 2005, 547-51). An industry with abundant part-time workers can thus be considered to be an energy-intensive industry, since output requires more productivity per labour hour to obtain hourly wages, compared to the productivity levels per labour hour of salaried employees.

Another important parameter of the Boltzmann weight is the age cohort which is similarly broken down by province, industry and year. The age cohort proxies the “velocity” of a particle – which is a characteristic of its energy – in a physical system to our economic system through the concept of efficiency. Based on economic research and intuition, age has an effect on productivity levels, as in the proportion of workers within 25-54 years have the most mobility between roles and positions within the industry allowing them to be more efficient in converting factors of inputs to outputs (Skirbekk 2003, 2004-6). This allows us to convert efficiency units of electricity from an exogenous to an endogenous variable that is a function of the non-homogenous nature of the efficiencies of various industries (particularly due to the age cohort). This framework forms a key role in understanding and modeling the heterogeneous nature of the efficiency units of electricity.

An additional feature of this Boltzmann weight is the Greek constant lambda 'λ', whose variation has a consequence on the actual value of the weight and thus the efficiency units of electricity. In a thermodynamic system, λ is a constant that is inversely related to temperature of the system, a higher temperature (lower λ) relates to a higher internal energy for a particle and therefore the overall system. There isn't a well-defined or developed economics equivalent concept for the idea of a temperature or λ but references have been made in the work of Landau and Lifshitz where they describes that at temperatures close to zero in a negative temperature state, the economy "corresponds to an allocation of all workers to a state of the highest productivity" (Landau and Lifshitz 1958, 45-6).

We would like to apply a similar ideology, which assumes that a lower temperature system (thus a higher 'λ') signals an economy that requires more efficiency units of electricity input thus a relative scarcity of highly-energy intensive or low-efficient industries. Varying the value of λ between 0 and 1 affords us the ability to examine the effects of these efficiency units of electricity on the elasticity of labour and capital. A 'λ' value of close to 0 suggests that there is a larger fraction of highly efficient industries, whereas a value of 1 suggests otherwise. In line with the suggestions of Park, Kim, and Isard, the calculation of the optimum value of λ is not the focus of the paper and recommends that the value at which the least square has a minimum can be used as a reference point (Park, Kim, and Isard 2012, 4890).

Figure 6: Boltzmann Weight for Efficiency Units of Electricity

The Boltzmann Weight for Efficiency Units of Electricity

Boltzmann Weight	Description	
Potential application for heterogeneous production efficiencies among industries	$\mathbf{Boltweight_{pit} \propto (1/employ_{pit})(PT_{pit})^{*}e^{-\lambda(AgeCohort_{pit})}}$	
	Where,	
	$Boltweight_{pit}$	= relative weight of electricity required per worker; terajoules of electricity per worker per province, industry and year
	$Employ_{pit}$	= Number of workers employed in the industry per province, industry and year
	e	= constant of exponential function ~ 2.71828
	Λ	= constant ≥ 0
	$AgeCohort_{pit}$	= number of workers within 25-54 years per province, industry and year; proxy for max velocity/ mobility to engage in new tasks
	PT_{pit}	= total amount of part-time employees in a given province, industry and year; proxy for less skilled or 'energy-intensive' labour;

The economic significance of using the Boltzmann-weight, as opposed to any other weight or the lack thereof, is the idea that the labour force follows similar discrepancies in behavior as energy, which is inherently non-uniform. Since both the characteristics concerning full-time and age statistics are measured in the number of workers, we are able to compare the energy input with capital and labour inputs by analyzing electricity input as the amount of additional energy contributed by industry-specific labour force characteristics.

Lastly, before testing of efficiency units of electricity, it is important to understand the conceptual implications of its elasticity γ. According to the derivations in Figure 7 in the appendix, γ represents the proportion of efficiency units of time to industry output, such that:

$$\boldsymbol{\gamma} = \frac{\Phi(\mathbf{RawE}_{pit})}{\mathbf{Y}_{pit}} = \frac{\mathbf{E}_{pit}}{\mathbf{Y}_{pit}}$$

This elasticity represents the industry level output response to a change in efficiency units of electricity. It is interesting to note that the derivative for β does not change and still remains a proportion of labour wages in output. α however, has a noticeable decrease in its elasticity with the introduction of γ. These conceptual changes will be addressed when analyzing the data and results in Section 4.

4. Data, Results and Analysis

As mentioned earlier in the paper, a majority of our data was collected from statistics provided on CANSIM. We proceeded to collect data on provincial GDP, industry GDP, labour force characteristics, productivity, and energy measured in Tera Joules. We used these measures to create variables to fit our model, such as the Boltzmann weights and the factor intensity of industries. Further details of these variables are provided in Figure 8 and Table 5 of the Appendix.

We began by conducting a regression of the natural log of industry GDP on the natural log of labour and capital. The results are represented in Figure 1 of the Appendix and suggest that the total contribution of these factors to output does not equal one, suggesting there might be some other variables which could significantly contribute to remaining share of output. Based on our results, a one percent increase in the flow of capital and one percent increase in the flow of labour leads to 23.2% and 25.3% increases in industry GDP. These coefficients are also statistically significant at a 99.9% confidence level with low standard errors.

Encouraged by these results, we compared results from conducting regressions of the Cobb-Douglas production function, incorporating raw electricity input as energy flow (measured in terajoules), to understand whether energy could be the factor which could help explain the remaining share of output.

The results shown in Table 6 below express that raw energy input (in the form of electricity flow in Terajoules) – as well as labour input and capital input - is economically and statistically significant at 99.9%, with a one percent increase in the flow of energy leading to an approximately 3.45% increase in industry GDP, without any significant economic changes to the other factors of input.

Table 6: Comparison of Regression Results from Cobb-Douglas & Cobb-Douglas (With Raw Electricity Input)

ln IndustryGDP		
ln Labour	0.253***	0.251***
	(3.67)	(3.64)
ln Capital	0.232***	0.239***
	(5.80)	(6.03)
ln Raw E		0.0345***
		(3.47)
_cons	6.270***	5.803***
	(7.75)	(6.65)
N	1210	1210

t statistics in parentheses
* p<0.05, ** p<0.01, *** p<0.001

Although, the overall contribution of factors of inputs still does not entirely represent the growth in GDP we were successful in rejecting the null hypothesis (based on our sample) that energy might be insignificant. The results from our data confirm, to some extent, the findings of Kümmel, Ayres, and Lindenberger (2010) where they suggested that neo classical economic models regard the returns from energy flow as an input to be insignificant.

We then proceeded to test the hypothesis mentioned in section 2.4, with the intention of examining the influence of introducing a new parameter on the standard errors of the model. Using the Boltzmann weights – which were functions of certain labour force characteristics – we suggested a combination of raw electricity flow and labour flow (as inputs) could help generate statistically and economically efficient estimates of labour and capital.

The results of our regressions using the efficiency units of electricity inputs represented in Table 7 below show that most of our variables are statistically significant at the 99.9% level and all are definitely significant at the 95% level, thus allowing us to reject the null hypothesis of statistical insignificance of the parameters in our modified version of the Cobb-Douglas production function.

Table. 7: Comparison of Regression Results from the Boltzmann Weighted Energy Input (0.1<λ<1.0)

ln Industry GDP	(λ = 0.10)	(λ = 0.25)	(λ = 0.50)	(λ = 0.75)	(λ = 1.00)
ln Labour	0.277**	0.265**	0.258**	0.251**	0.246*
	(2.90)	(2.87)	(2.82)	(2.62)	(2.54)
ln Capital	0.222***	0.211***	0.203***	0.210***	0.209***
	(4.37)	(4.19)	(3.98)	(3.79)	(3.59)
ln E	-0.0279***				
	(-5.35)				
ln E1		-0.0197***			
		(-6.95)			
ln E2			-0.0124***		
			(-8.06)		
ln E3				-0.00914***	
				(-3.89)	
ln E4					-0.00862**
					(-2.85)
_cons	6.302***	6.357***	6.445***	6.406***	6.458***
	(9.90)	(10.33)	(10.66)	(10.20)	(10.24)
N	992	960	890	801	751

t statistics in parentheses
* p<0.05, ** p<0.01, *** p<0.001

As the lambda value increases from 0.1 to 1 the economic significance of the returns from efficiency units of electricity input decreases as exhibited by a decrease in industry GDP by 2.4% and 0.83% when lambda was 0.1 and 1 respectively. These efficiency units are a function of the labour force characteristics and indicate that a more efficient industry would require lower levels of energy inputs to produce a change in the overall output. This is evident in the fact that as the lambda value increases from 0.1 to 1 – acting as a proxy for the overall efficiency level of industries from higher to lower levels – the returns from labour and capital input due to these efficiency units of electricity increases when compared to the Cobb-Douglas production function. There is a maximum return from labour input of approximately 27% and return from capital input of 22% at lower levels of lambda.

It can be observed from the derivations of elasticities in Figure 6 in the Appendix, with the introduction of efficiency units of electricity we expected the returns from labour to stay constant and the returns from capital to decrease. While the returns from capital certainly have decreased in every variation of the efficiency units we have applied, there is also an increase of the returns from labour. This result although contrary to our expected derivations – possibly due to discrepancies in the data we have collected – might have grounds in an economic intuition. When observed independently, the premium of the efficiency units of electricity does not signify much economically or statistically (as they are negative). As the functional form of these units relies on the characteristics of the labour force, they would optimize, the share of labour and capital in national accounts according to the proportion of efficient firms within the given industry, in the province, in that period of time.

However, returning to our principal reason for the use of heterogeneous efficiency units as outlined in the introduction results exhibited in Tables 6 and 7 above shows that the standard errors of returns from labour and capital inputs from regressions using the efficiency units of electricity as a control variable appear to be more statistically efficient. The standard errors are reduced significantly when compared with the results from the standard Cobb-Douglas regression. These results help us reject the null hypothesis that the returns from the efficiency units of electricity inputs are insignificant and also comment on the variation in the standard errors of returns from labour and capital inputs. Conceptually, these results are similar to the findings of Kümmel, Ayres, and Lindenberger, where they experienced an increase in returns from inputs of labour and capital by incorporating a cost share theorem of energy input (Kümmel, Ayres, and Lindenberger 2010, 178-9).

It is quite evident that modeling a heterogeneous nature of efficiency units of electricity inputs on labour force characteristics would provide us with larger returns from labour input than capital, and our results in Tables 6 and 7 confirm as such.

5. Future Work

This section provides possible comments for future work on the economic significance of the Boltzmann weighted parameter (φ) and the resulting labour decisions. Also mentioned are a few general remarks on possible future research to contribute towards the field of Econophysics.

1) A more comprehensive formulation of the elasticity of the efficiency units of electricity might better represent the empirical results. Modeling the cost of the efficiency units as a function of the wages – due to the nature of inherent labour characteristics – in the maximization problem shown in the appendix might mitigate theoretical and empirical discrepancies.

2) Trying to implement further aspects of Kümmel, Ayres, and Lindenberger (2010) cost share theorem into our model of the Boltzmann weighted energy (Kümmel, Ayres, and Lindenberger 2010, 146). Along with a more nuanced application of Landau's concepts of 'negative temperature' might provide more statistically efficient and economically significant explanations for the efficiency units (Landau and Liftshitz 1958, 4-6).

3) Continue on the path illuminated by Park, Kim, and Isard by applying these weights to the problem of efficient resource allocation (Park, Kim, and Isard 2012, 4889). Without invoking the role of energy into the production function, one could allocate resources based on the Boltzmann weights described in the model. The objective would be to maximize entropy by minimizing the sum of errors squared when conducting tests on the factors of inputs with the Boltzmann weights. The null hypothesis we would then like to reject, suggests that the coefficient of a weighted production function would be equal to the coefficients produced without weights. In addition there would be an overall reduction in the sum of residuals squared. There would be changes to the parameters of the model, with the number of people employed replacing the total amount of full time employees and the aggregate industry proportion of GDP replacing the age cohort variable.

4) Our model can be subject to refinement, both, in terms of the amount of sample data and the actual specification of the weights. It is important to mention that there is no other research that has been conducted in this field that incorporates a three dimensional panel data, especially conducted over 13 years, 10 provinces and 15 major industries. Even Park, Kim, and Isard conducted their research over a conservative data set of eight countries and two time periods (Park, Kim, and Isard 2012, 4885). The other aspect of the refinement would involve a readjusted definition of the Boltzmann weights as there may potentially exist other viable variables which would act as a better proxy to their counterparts in statistical physics as well as the application of better quality human capital variables, which act as efficient proxies for labour flow as input.

6. Conclusion

In conclusion, we attempted to enhance our understanding of the neo-classical Cobb-Douglas production function by augmenting it with efficiency units of electricity as a factor of production. We assumed the efficiency units of electricity input to be heterogeneous in nature due to the heterogeneous nature of efficiency within Canadian industries. Heavily inspired by literature by Kümmel, Ayres, and Lindenberg, as well as Park, Kim, and Isard, we decided to model the nature of these efficiency units using the statistical physics concept of Boltzmann distribution.

We were able to successfully test – based on our data – the initial null hypothesis in demonstrating statistically and economically significant results. The results suggested that raw electricity flow has a role in the Cobb Douglas function as a factor of input. The results weren't highly economically significant and agreed with neo-classical economics that energy – as a factor of production – has a relatively small contribution to overall output (about 5%).

The Boltzmann weights and efficiency units of electricity were a function of the labour force characteristics of Canadian industries, chiefly the proportion of part-time workers, the optimum age cohort and a constant signaling of the overall productivity level of the industry. We then conducted a Cobb-Douglas regression, but applying these weighted efficiency units of the electricity as a factor of input. The results were statistically significant at the 95% confidence level, although we weren't able to conclusively prove an economic significance, namely weighted efficiency units of electricity contribute to more than 5% of total industry GDP.

What we were able to observe from the results was that the contribution of the other factors of inputs increased with the addition of these new variables, more specifically the return from labour input seemed to be higher than previously achieved in a normal Cobb-Douglas regression. This could – to some extent – verify our assumptions about modeling the heterogeneity of efficiency units of electricity through a Boltzmann Distribution.

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Appendix

Figure 1: Derivative of Standard Labour and Capital Income Shares

Within each industry, a representative firm i faces the profit function:

$$\pi\ (k,l) = z(k^{\alpha} l^{\beta}) - wl \qquad \text{given } w$$

Firms maximize profit with respect to labour supplied (l) by each firm

$$\underset{l}{Max}\ \pi \quad \{z(k^{\alpha} l^{\beta})\ - wl\}$$

$$(\beta)k^{\alpha} l^{\beta-1} = w = 0 \quad \Rightarrow \quad l^{*} = \left\{ \frac{(\beta z k^{\alpha})}{w} \right\}^{\frac{1}{\beta-1}} \quad \Rightarrow \text{ optimal level of labour supplied represents the total hours worked}$$

$$w = (\beta)zk^{\alpha} l^{\beta-1} \qquad \Rightarrow \quad wl = (\beta)zk^{\alpha} l^{\beta} = (\beta)Y$$

$$\text{Total Profits } (\Pi) = \sum_{i=1}^{n} \pi_i\ ,$$

$$\text{Total Industry Labour Supplied } (L) = \sum_{i=1}^{n} l_i\ ,$$

$$\text{Total Industry Capital Supplied } (K) = \sum_{i=1}^{n} k_i$$

$$\Rightarrow \Pi = Y - wL = Y - (\beta)Y = \alpha Y$$

$$\text{Total Labour Income Share} = \frac{wL}{Y} = \beta = 1-\alpha$$

$$\text{Total Capital Income Share} = \frac{\Pi}{Y} = \alpha$$

Table. 1: Initial Cobb Douglas Production Function Regression

	ln IndustryGDP
ln Labour	0.253***
	(3.67)
ln Capital	0.232***
	(5.80)
_cons	6.270***
	(7.75)
N	1210

t statistics in parentheses
* p<0.05, ** p<0.01, *** p<0.001

Table. 2: Comparison of TFP Based on Regression and Historical Estimates

Year	2002	2003	2004	2005	2006
Solow Residual	52766.67	53526.46	52925.34	51695.59	52331.91
TFP	14000.00	13980.60	13918.20	14057.30	14038.20
Historical Solow	178.57	177.11	167.18	153.32	151.15

Year	2007	2008	2009	2010
Solow Residual	56641.11	58126.79	56026.34	57621.27
TFP	13994.50	14100.60	13865.60	13995.60
Historical Solow	151.15	170.89	167.04	168.59

Figure 2: Derivation of the Full Growth of GDP

Given:

$$Labour\ Share = \frac{wL}{Y} = \beta\ ,\ Capital\ Share = \frac{\Pi}{Y} = \alpha$$

Apply Growth Accounting to $t\ \varepsilon\ [2002{,}2011]$:

$$Y_t = Z_t K_t^{\alpha} L_t^{\beta} \Rightarrow \frac{Yt}{Population_t} \Rightarrow y_t = z_t k_t^{\alpha} l_t^{\beta}$$

Decompose growth in y_t *over time into growth rates of* $z_t,\ k_t^{\alpha},\ l_t^{\beta}$

$$\frac{y_{t+1}}{y_t} \Leftrightarrow \left(\frac{z_{t+1}}{z_t}\right)\left(\frac{k_{t+1}}{k_t}\right)^{\alpha}\left(\frac{l_{t+1}}{l_t}\right)^{\beta}$$

$$ln\left(\frac{y_{t+1}}{y_t}\right) = ln\left(\frac{z_{t+1}}{z_t}\right) + \alpha ln\left(\frac{k_{t+1}}{k_t}\right) + \beta ln\left(\frac{l_{t+1}}{l_t}\right)$$

$\rightarrow$ *Let* g_{yt} = *growth rate of* y *from* t *to* $t+1$, $t\ \varepsilon\ [2002{,}2011]$

then $\frac{y_{t+1}}{y_t} = 1+g_{yt}$

$\rightarrow$ *Let* g_{zt} = *growth rate of* z *from* t *to* $t+1$, $t\ \varepsilon\ [2002{,}2011]$

then $\frac{z_{t+1}}{z_t} = 1+g_{zt}$

$\rightarrow$ *Let* g_{kt} = *growth rate of* k *from* t *to* $t+1$, $t\ \varepsilon\ [2002{,}2011]$

then $\frac{k_{t+1}}{k_t} = 1+g_{kt}$

$\rightarrow$ *Let* g_{zt} = *growth rate of* l *from* t *to* $t+1$, $t\ \varepsilon\ [2002{,}2011]$

then $\frac{l_{t+1}}{l_t} = 1+g_{lt}$

$$\therefore\ ln(1+g_{yt}) = ln(1+g_{zt}) + \alpha ln(1+g_{kt}) + \beta ln(1+g_{lt})$$

$\rightarrow$ *For small values of* g $ln(1+g) \approx g$

$$\therefore\ g_{yt} \approx g_{zt} + \alpha g_{kt} + \beta g_{lt}$$

$\Rightarrow$ *The growth rate of GDP per capita* (g_{yt}) *over one time period (ie 1 year), should be appoximately equal to the sum of the three components.*

Table. 3: Results from Cobb-Douglas Production Function Over Short Time Periods

	(2002-2003)	(2003-2004)	(2004-2005)	(2005-2006)
	ln Industry GDP			
ln Labour	0.0803	0.103	0.188***	0.153***
	(0.97)	(1.27)	(4.75)	(3.92)
ln Capital	0.331***	0.121	0.251***	0.195***
	(4.00)	(1.85)	(4.06)	(4.82)
_cons	6.863***	8.676***	6.673***	7.566***
	(9.35)	(10.04)	(9.71)	(16.31)
N	260	260	260	260

t statistics in parentheses
* p<0.05, ** p<0.01, *** p<0.001

	(2006-2007)	(2007-2008)	(2008-2009)	(2009-2010)
	ln Industry GDP			
ln Labour	0.140*	0.0300	0.298	0.263*
	(2.10)	(0.27)	(1.75)	(2.51)
ln Capital	0.220***	0.103	0.458**	0.110
	(3.47)	(0.92)	(2.62)	(1.27)
_cons	10.21***	10.32***	12.66***	7.385***
	(18.89)	(8.64)	(7.33)	(9.16)
N	270	280	280	280

t statistics in parentheses
* p<0.05, ** p<0.01, *** p<0.001

Table. 4: Comparison of Regression from a Fixed Effects and a Random Effects Model

ln IndustryGDP	(Fixed Effects)	(Random Effects)
ln Labour	0.253***	0.251***
	(8.77)	(8.71)
ln Capital	0.236***	0.239***
	(11.16)	(11.26)
ln RawE	0.0178	0.0345**
	(1.35)	(2.74)
_cons	5.998***	5.803***
	(22.51)	(21.81)
N	1210	1210

t statistics in parentheses
* p<0.05, ** p<0.01, *** p<0.001

Figure. 5: Results from Hausman Test

```
b = consistent under Ho and Ha; obtained from xtreg

B = inconsistent under Ha, efficient under Ho; obtained from xtreg


Test:  Ho:  difference in coefficients not systematic


chi2(3) = (b-B)'[(V_b-V_B) ^ (-1)] (b-B)

        =       14.94

Prob>chi2 =      0.0019

(V_b-V_B is not positive definite)
```

Figure 6: Derivations of Elasticities with Efficiency Units of Electricity

Industry Homogeneous Maximization Problem

$$Max_{L,E} \quad \sum_{i=1}^{n} \Pi(L,K,E) = n\{Y - wL - \varphi E\}$$

Derive β using the industry maximization problem for representative firm i, given w, φ

$$\Rightarrow Max_L \{ zK^{\alpha}L^{\beta}(\varphi E)^{\gamma} - wL - \varphi E \}$$

$$\beta z[K^{\alpha}L^{\beta-1}(\varphi E)^{\gamma}] - w = 0$$

$$\beta z[K^{\alpha}L^{\beta-1}(\varphi E)^{\gamma}] = w$$

$$\beta L z[K^{\alpha}L^{\beta-1}(\varphi E)^{\gamma}] = wL \Longrightarrow \beta z[K^{\alpha}L^{\beta}(\varphi E)^{\gamma}] = wL$$

$$\beta Y = wL \Longleftrightarrow \beta = \frac{wL}{Y} \Longleftrightarrow \beta = \beta$$

Derive γ using the industry maximization problem for representative firm i, given w, φ

$$\Rightarrow Max_E \{ zK^{\alpha}L^{\beta}(\varphi E)^{\gamma} - wL - \varphi E \}$$

$$\gamma z[K^{\alpha}L^{\beta}(\varphi E)^{\gamma-1}] - \varphi = 0$$

$$\gamma z[K^{\alpha}L^{\beta}(\varphi E)^{\gamma-1}] = \varphi$$

$$\gamma \varphi E z[K^{\alpha}L^{\beta}(\varphi E)^{\gamma-1}] = \varphi^2 E \Longrightarrow \gamma \varphi z[K^{\alpha}L^{\beta}(\varphi E^{\gamma})] = \varphi^2 E$$

$$\gamma \varphi Y = \varphi^2 E \quad \Longrightarrow \gamma = \frac{\varphi E}{Y}$$

Derive α given β, γ, w, φ

$$\Pi = Y - wL - \varphi E \Rightarrow \frac{\Pi}{Y} = 1 - \frac{wL}{Y} - \frac{\varphi E}{Y}$$

$$\frac{\Pi}{Y} = 1 - \beta - \gamma \Rightarrow \frac{\Pi}{Y} = \alpha$$

$$\alpha = 1 - \beta - \gamma \Longleftrightarrow 1 = \alpha + \beta + \gamma$$

Figure 7: Variable Summary

Variable	Obs	Mean	Std. Dev.	Min	Max
Year	3250	2008	3.742233	2002	2014
Industry	3250	10.56	8.622603	0	25
Province	3250	5.5	2.872723	1	10
LF	2340	206.0422	515.5626	.2	5722.8
Employ	2340	158.0796	386.9576	.2	4241.4
PT	2340	38.17363	111.7924	0	1254.5
AgeCohort	2337	66.75627	106.1986	.2	916.1
Y_Annual	3250	1584161	234788.4	1189452	1973043
Y	3250	11721.87	34416.01	0	554833.9
L	1260	15316.08	18189.18	5841.837	481357.1
K	1260	16644.94	7498.856	8098.594	136252.7
TFP	1260	99.96079	15.63845	10.2	216.1
lnY	2850	11.26034	.9569581	9.340505	14.12529
lnL	1260	9.563359	.2609403	8.672801	13.08436
lnK	1260	9.668686	.2886822	8.999446	11.82227
lnRawE	1756	2.998837	11.22657	-80.23475	12.05883
Phi	2077	.0315416	.0508655	0	.3479315
Phi1	2077	.0132406	.0333946	0	.2503878
Phi2	2077	.0061116	.021295	0	.2155107
Phi3	2077	.0035844	.0149871	0	.1854918
Phi4	2077	.0023119	.0111183	0	.1596543
lnE	1753	2.935786	10.72305	-71.79883	11.85816
lnE1	1679	-3.449839	17.18033	-78.62382	11.58816
lnE2	1538	-8.782712	21.10222	-76.23698	11.4705
lnE3	1377	-9.661809	20.18285	-78.80045	11.3955
lnE	1753	2.935786	10.72305	-71.79883	11.85816

Table 5: Variable Descriptions

Variable Definition				
Variable	**Type**	**Unit**	**Description**	**CANSIM Table**
Year	**Numeric**	**Year**	**Panel Data for the years [2002-2014]**	**All specified below**
Industry	**String; converted into numeric Industry1**	**N/A; Identifier;**	**20 Industries Identified: Agriculture, forestry, fishing and hunting; Mining, quarrying, and oil and gas extraction; Utilities; Construction; Manufacturing; Wholesale trade; Retail trade; Transportation and warehousing; Information and cultural industries Finance and insurance; Real estate and rental and leasing; Professional, scientific and technical services; Management of companies and enterprises; Administrative and support, waste management and remediation services; Educational services; Health care and social assistance; Arts, entertainment and recreation; Accommodation and food services; Other services (except public administration); Public administration**	**All specified below**
Province	**String; converted into numeric Province1**	**N/A; Identifier (Province1 given values of 1-10)**	**10 Provinces Specified; Territories not included to provide a more balanced panel;**	**All Specified Below**
Employ	**Numeric**	**Persons x 1000**	**Number of Persons Employed; Number of persons who, during the reference week, worked for pay or profit, or performed unpaid family work or had a job but were not at work due to own illness or disability, personal or family responsibilities, labour dispute, vacation, or other reason. Those persons on layoff and persons without work but who had a job to start at a definite date in the future are not considered employed. Estimates in thousands, rounded to the nearest hundred.**	**Table 282-0008**
FT	**Numeric**	**Persons in thousands**	**Number of Persons Employed who work 30 hours or more per week at their main or only job. Estimates in thousands, rounded to the nearest hundred.**	**Table 282-0008**
PT	**Numeric**	**Persons in thousands**	**Number Of Part Time Employed = Number Of Persons Employed - Full Time Workers. Estimates in thousands, rounded to the nearest hundred.**	
AgeCohort	**Numeric**	**Persons x 1000**	**Total persons, between 25-54 years of age in the labour force, broken down by province and industry**	**Table 282-0008**
Y_Annual	**Numeric**	**GDP in Current Dollar**	**Total aggregate GDP annually**	**Table 384-0038**

Y	**Numeric**	**GDP in current Dollars**	**The product of Provincial GDP(in Dollars) * [Industry Share of GDP(Provincial)/100] to calculate the contribution that each industry within each province has to GDP**	**Calculated**
L	**Numeric**	**Hours Worked**	**= (1/Labour Productivity)*GDP in current prices**	**Calculated**
K	**Numeric**	**Dollars**	**= (1/Capital Productivity)*GDP in current prices**	**Calculated**
TFP	**Numeric**	**GDP /(Capital + Labour Inputs)**	**Multifactor productivity, as known as total factor productivity, measures the efficiency with which all inputs are used in production. It is the ratio of real gross domestic product (GDP) to combined labour and capital inputs.**	**Table 383-0026**
RawE	**Numeric**	**Terajoules**	**Measured the physical flow of energy use annually; aggregated by industry, consistent among provinces; consolidated data using a terminated data set and current dataset; all industry classifications are the same between the two sets except for manufacturing, transportation, education and other services.**	**Table 153-0032(terminated); Table 153-0013**
lnY	**Numeric**	**Log Dollars**	**Natural log of the Industry GDP = ln(Industry GDP)**	**Calculated**
lnL	**Numeric**	**Log Hours Worked**	**Natural log of Labour input = ln(L)**	**Calculated**
lnK	**Numeric**	**Log Dollars**	**Natural log of Capital input = ln(K)**	**Calculated**
lnRawE	**Numeric**	**Log TeraJoules**	**Natural log of Raw Electricity Flow as a factor of input = ln(RawE)**	**Calculated**
Phi	**Numeric**	**TeraJoules per worker**	**Efficiency Units Of Electricity = Fraction Of Part Time Workers in total Employed*(2.71828)^(-λ*AgeCohort); where values of the constant λ are tested at [0.1,0.25,0.5,0.75,1.0]. Lambda = 0.1; Lambda1 = 0.25 etc.**	**Calculated**
lnE	**Numeric**	**Log Of Weighted Energy Input**	**Natural Log of the Efficiency units of electricity weighted energy input = phi*EnergyInput; ln(E) is calculated at phi = 0.1, ln(E1) is calculated at phi = 0.25 etc.**	**Calculated**