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and the Dynamics of the Earnings
Distribution**

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Research Report # 2009-1

February 2009



***Department of Economics
Research Report Series***

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Social Security, Differential Fertility, and the Dynamics of the Earnings Distribution

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December 16, 2008

Abstract

Economists and demographers have long argued that fertility differs by income (differential fertility), and that social security creates incentives for people to rear fewer children. Does the effect of social security on fertility differ by income? How does social security change the cross-sectional relationship between fertility and income? Does social security further affect the dynamics of the earnings distribution by changing differential fertility? We answer these questions in a three-period OLG model with heterogeneous agents and endogenous fertility. We argue that given its redistributive property, social security affects people's fertility behavior differentially by income. In the model, earning ability is transmitted from parents to children. Hence, social security can have a significant impact on the dynamics of the earnings distribution through its effects on differential fertility. The mechanism used in the model to generate differential fertility is novel. We follow the line of the "old-age security" hypothesis and assume that children are an investment good in parents' old-age consumption. Thus, the optimal fertility choice depends on how much transfer is expected from children in relation to the cost of rearing these children to adult life. Since the intergenerational earnings process is mean-reverting, poor (rich) parents tend to have more (fewer) children because they have lower (higher) child-rearing cost and expect their children will have higher (lower) earnings than themselves and give back relatively more (less) in transfers. Social security reduces fertility by substituting children out of parents' old-age portfolio. It reduces fertility of the poor proportionally more than it reduces fertility of the rich because social security payments are a larger portion of old-age savings for poor people. These results are consistent with features of the U.S. fertility data. We calibrate the model to the U.S. data and find that social security can explain 32% of the decline in poor-rich fertility differential between the cohort of women born during 1891-1895 and the cohort of women born during 1946-1950.

Keywords: Social Security, Differential Fertility, Earnings Distribution, Growth.

JEL Classifications: E600, H310, J130, O150

*I am deeply indebted to Betsy Caucutt and Karen Kopecky for their continuing guidance and encouragement. I would also like to thank Jim Davies, Paul Klein, Igor Livshits, Jim MacGee, Alice Schoonbroodt, Nathan Sussman, Michele Tertilt, John Whalley, and participants at the Money/Macro Workshop at the University of Western Ontario, the 2008 Canadian Economic Association Annual Meeting, and the 2008 North American Summer Meeting of the Econometric Society for their helpful comments. Address: University of Western Ontario, Department of Economics, Social Science Centre, London, Ontario N6A 5C2, Canada. E-mail: kzhao5@uwo.ca.

1 Introduction

Fertility differs by income (differential fertility). For example, Jones and Tertilt (2007) find a negative cross-sectional relationship between income and fertility within all cohorts of women since 1826 in the United States (see figure 1). This negative relationship between income and fertility has also been found in most other countries.¹ Economists and demographers have also argued that government provided social security creates incentives for people to rear fewer children.² In this paper, we ask whether the effect of social security on fertility varies across the earnings distribution, and how social security changes differential fertility. We further ask how social security affects the dynamics of the earnings distribution through its effects on differential fertility.

To answer these questions, we develop a three-period OLG model with heterogeneous agents and endogenous fertility. In the model, we follow the line of the “old-age security” hypothesis (see Boldrin and Jones (2002) for details) and assume that children are investment goods from the viewpoint of parents. Thus the optimal fertility choice depends on how much transfer is expected from children in relation to the cost of rearing these children to adult life. The cost of rearing children is only the parent’s time. We further assume that earning ability is transmitted from parents to children, and it is mean-reverting over generations. Therefore, in this model, poor (rich) parents tend to have more (fewer) children since they have lower (higher) child-rearing cost and expect that their children will have higher (lower) earnings than themselves and give back relatively more (less) in transfers.

In this setup, social security payments crowd children out of parents’ old-age portfolios. Since government-provided social security is usually very progressive, its payments are a larger portion of old-age savings for poor people.³ Therefore, social security tends to reduce the fertility of the poor proportionally more than it reduces the fertility of the rich. Since earning ability is intergenerationally correlated, a smaller fertility differential between the poor and the rich can lead to a new earnings distribution with a smaller portion of poor people and a higher average earnings level.

Crucial assumptions in the model are that earning ability is intergenerationally correlated and mean-reverting. These assumptions are supported by overwhelming empirical evidence. Gary Becker (1988) has argued that “In every country with data that I have seen... earnings

¹See E. Jones (1982).

²For example, Boldrin, De Nardi, and Jones (2005), Ehrlich and Kim (2007), etc.

³OECD (2007) summarizes the progressivity of pension systems in the OECD countries. It finds that New Zealand and Ireland have a pure flat-rate pension system, while Finland, Italy and Netherlands have a highly earnings-related pension system. The US social security system is somewhere in between these two groups of countries.

strongly regress to the mean between fathers and sons”. Solon (2002) summarizes empirical studies for a number of developed countries, which all find supportive evidence for the argument that earnings are intergenerationally correlated and mean-reverting.

Another important assumption in our model is that the parent’s motivation for child-rearing is old-age security. This is in contrast to the Barro-Becker model, in which parental altruism is the motivation of child-rearing (Barro and Becker, 1989, and Becker and Barro, 1988). This “old-age security” hypothesis was first proposed by Caldwell (1978) in the demography literature. Boldrin and Jones (2002) formalize Caldwell’s idea in a dynamic model of fertility. Our model builds on the Boldrin-Jones model. Sizable empirical evidence has been found supporting the “old-age security” hypothesis.⁴ Boldrin and Jones (2002) provide an excellent review of these empirical studies.

Lastly, we apply our model to the U.S. data. We find that in our model, social security can explain 32% of the decline in differential fertility between the cohort of women born during 1891-1895 and that born during 1946-1950 in the U.S.⁵

Our paper is motivated by the recent literature that studies the interactions between differential fertility and income and wealth inequality. The main message from this literature is that what really matters is not only the aggregate fertility measures, but also the distribution of fertility across income. For instance, de la Croix and Doepke (2003) argue that differential fertility puts more weight on the poor in the next generation, and thus brings down the weighted average income level of the population. Knowles (1999) argues that accounting for differential fertility is important for understanding the U.S. wealth distribution, since parents treat children’s human capital as part of their wealth. Kremer and Chen (2002) argue that differential fertility increases the proportion of unskilled workers, and reduces their wages. Our study is built on this literature and it focuses on how government-provided public pension affects differential fertility and its interaction with the dynamics of the earnings distribution.

This paper is closely related to papers by Boldrin and Jones (2002) and Boldrin, De Nardi and Jones (2005). In both papers, children are an investment in parents’ old-age consumption and the old-age transfer from children to parents is endogenous. Boldrin and Jones (2002) find that their model can explain the historical correlation between the mortality rate and fertility rate. However, they abstract from distributional issues by only studying a representative-agent model. Boldrin, De Nardi and Jones (2005) study the negative impacts of social security on fertility in the Boldrin-Jones model. They find that social security reduces the period Total

⁴For example, see Willis(1982), Lillard and Willis (1997), Nugent (1985), and Jensen (1990).

⁵In this paper, differential fertility means the negative relationship between fertility and income. The quantitative measurement used for differential fertility is the fertility gap between the women in the top half of income distribution and those in the bottom half of income distribution.

Fertility Rate (TFR) and this effect is quantitatively important, but they do not explore the possibility that social security may affect people's fertility behavior differentially. In fact, the literature on social security and fertility can be dated back to the 1960s. Most studies have found that social security has a negative impact on the fertility rate.⁶ However, no one has studied the differential effects that social security has on fertility.

This paper is also related to the literature that studies the fundamental determinants of the great divergence over the last two hundred and fifty years (see Galor (2005)). Our theory suggests that social security may have contributed to the latter half of the great divergence. A well established fact in this literature is that the previously positive relationship between fertility and income was reversed after the demographic transition of the mid 19th century. Note that after the demographic transition, the negative fertility-income relationship not only held across countries, but also held domestically within each country (see Jones and Tertilt (2007)). As de la Croix and Doepke (2003) argue, the domestic negative fertility-income relationship puts more weight on poor people in future generations, and therefore lowers the average income level in the long run.⁷ However, in the data we do not observe this growth-retarding effect in the Western countries after the demographic transition; instead, these countries quickly completed the transition to the stage of modern economic growth and started an economic take-off. Our theory suggests that this may be due to the social security programs introduced in these Western countries between the late 19th century and the early 20th century. Social security further lowered the total fertility rate in these countries in addition to changes from the demographic transition (Boldrin, De Nardi and Jones (2005)). Also, it lessened the domestic negative relationship between fertility and income, thus offset the growth-retarding effect proposed by de la Croix and Doepke. Meanwhile, in the rest of the world, most countries did not have a social security program until much later. Thus, in these countries, this growth-retarding effect arrived after the demographic transition, which slowed their growth and enlarged their gap with the West.

This paper also fits into the literature that studies the cross-sectional properties of various family decisions and their interactions with inequality, using a heterogeneous-agent model.⁸ Among this literature, there are several papers especially relevant to this paper. Caucutt, Guner and Knowles (2002) argue that both fertility and the timing of fertility differ across income distribution. They develop an equilibrium search model with marriage and fertility and study the interactions between wage inequality and differential marriage and fertility decisions of young

⁶Friedlander and Silber (1967), Hohm (1975), Swilder (1983), Nishimura and Zhang (1992), Cigno and Rosati (1996), Rosati (1996), Zhang (2001), Zhang, Zhang and Lee (2001), etc.

⁷In another paper, they study the welfare results of different education policies when differential fertility is taken into account (de la Croix and Doepke (2004)).

⁸For example, Bar and Leukhina (2007), Caucutt, Guner and Knowles (2002), Greenwood, Guner and Knowles (2003), Schoonbroodt (2003), etc.

women. Another important family decision which may have a significant impact on fertility is female labor participation. Schoonbroodt (2003) studies the cross-sectional relationship between female labor participation and income in the US. She finds that the increase in the female labor participation rate has been independent of income throughout the twentieth century in the U.S.

This paper is distinguished from the existing literature in the following ways. First, we are the first to explore the possibility that social security may affect people's fertility behavior differentially, to quantify the differential effects of social security on fertility, and to derive implications for the dynamics of the earnings distribution. Second, we propose a novel mechanism generating the negative cross-sectional relationship between fertility and income, in which the mean-reversion in earnings over generations plays a key role. We show that this mechanism is robust both theoretically and quantitatively.

The rest of the paper is organized as follows. We describe the model in the second section and present some theoretical analysis on the cross-sectional relationship between income and fertility in the third section. We illustrate the effect of social security on differential fertility and its implication for the dynamics of the earnings distribution in the fourth section. We apply the model to the US data in the fifth section and conclude in the sixth section.

2 The Model

2.1 The Economic Environment

Consider an economy inhabited by overlapping generations of agents who live for three periods: childhood, middle age, and old age. Agents are endowed with one unit of time only in their middle age. They can use it either to work or to rear children. Agents receive a productivity shock ϵ at the beginning of the middle-age period and then jointly make savings, fertility, consumption, and old-age transfers decisions to maximize their lifetime utility. In old age, agents only consume what they have, which include the savings from their middle age and the old-age transfers from their middle-age children. In childhood, agents don't make any economic decision. Let us think about the problem facing a middle-age agent with ϵ_t^i , born in period $t - 1$. Here $i = 1, \dots, n_{t-1}$ and n_{t-1} is the fertility choice of this agent's parent. This agent has the following expected value of his lifetime utility,

$$u(c_t^m) + \beta E[u(c_{t+1}^o) | \epsilon_t^i] + \gamma u\left(\sum_{j \neq i, j=1}^{n_{t-1}} d_t^j + T_t(\epsilon_{t-1}) + d_t^i\right) \quad (1)$$

with

$$u(c) = \log c, \quad (2)$$

where c^m is middle-age consumption and c^o is old-age consumption. Note that d is transfer to the old-age parent, and $d \geq 0$. The social security payment to the old-age parent is denoted by $T_t(\epsilon_{t-1})$, which is a function of the parent's productivity shock, ϵ_{t-1} .⁹ Let j be the index for the middle-age children of the agent's parent, thus d_t^j represents the old-age transfer from the j th child. Here we assume the agent i takes d_t^j , $j \neq i$ and $j = 1, \dots, n_{t-1}$, as given when he makes his own transfer decision.¹⁰

The first term and the second term in equation (1) are respectively the utility function for the agent's middle age and old age. The expectation is over the uncertainty a middle-age agent has about his children's productivity at the moment he makes his fertility choice. The third term in equation (1) is the altruism function, which says that a middle-age child cares about his old-age parent. It is a function of the agent's own transfer to his parent d_t^i , his siblings' total transfers to his parent $\sum_{j \neq i, j=1}^{n_{t-1}} d_t^j$, and the social security payment to his parent $T_t(\epsilon_{t-1})$. We exclude the parent's own old-age savings from the altruism function in order to prevent the parent from playing strategically with his children.¹¹ This specification of the altruism function has several preferable features. First, the agent's incentive for giving decreases as he has more siblings. Second, the agent's incentive for giving decreases when his parent receives social security transfers. For simplicity the productivity shock is assumed to be the same across all children, which means the productivity shock only refers to the family shock in this paper. Let β be the discount factor and γ be the relative weight on the altruism function. Agents don't derive utility in childhood; they are taken care of by their parents in this period. The cost of child-rearing is parental time, b . Thus the budget constraints for the middle age and the old age are respectively:

$$d_t^i + s_t + c_t^m = W_t \epsilon_t^i (1 - \tau)(1 - bn_t), \quad (3)$$

⁹This implies that social security benefits depend on the agent's own earning ability, such as the one in the U.S.. Strictly speaking, in the U.S., social security benefits depend on actual lifetime earnings, which is a bit different from earning ability in the model. However, we abstract from this difference since labor supply only differ slightly across agents in the model.

¹⁰In other words, we assume that middle-age children play a noncooperative game when they make old-age transfer decisions (see Boldrin and Jones 2002). Note that fertility is a continuous variable in the benchmark model, and a continuous number of children playing a noncooperative game could be a bit problematic. This is a feature of the Boldrin-Jones type model of fertility. Following Boldrin and Jones (2002), we use symmetry for old-age transfers of children when we actually compute the model. In the appendix, we will show that this problem does not significantly affect our results by studying a version of the model with discrete fertility and heterogeneous preference.

¹¹A justification for this assumption is that a parent's saving is private information that is not available to his children. If the parent plays strategically with his children, corner solutions may yield higher utility. The parent may choose either zero savings or zero fertility, which is in stark contrast to the reality. We hypothesize that this extreme result arises because we do not include parental altruism in the model. Lagerlof (1997) analyzes this type of problem in a very similar framework.

$$c_{t+1}^o = R_{t+1}s_t + T_{t+1}(\epsilon_t^i) + n_t D_{t+1}(\epsilon_t^i, n_t, \epsilon_{t+1}), \quad (4)$$

where $D_{t+1}(\cdot, \cdot, \cdot)$ is the children's policy function for old-age transfers in period $t + 1$, n is the fertility choice, and s is saving. They satisfy the conditions: $s \geq 0$, and $n \in [0, \hat{n}]$, where \hat{n} is the upper limit for fertility choice.¹² The agent's problem (P1) can be formulated as a Bellman's equation,

$$V(\epsilon_{t-1}, n_{t-1}, \epsilon_t^i) = \max_{d, n, s} u(c_t^m) + \beta E[u(c_{t+1}^o) | \epsilon_t^i] + \gamma u\left(\sum_{j \neq i, j=1}^{n_{t-1}} d_t^j + T_t(\epsilon_{t-1}) + d_t^i\right)$$

subject to (3) and (4).

The productivity shock $\epsilon_t \in \{\epsilon_1, \epsilon_2, \dots, \epsilon_m\}$, is governed by a Markov chain with transition matrix $\pi(i, j) = \text{Prob}(\epsilon_{t+1} = \epsilon_j | \epsilon_t = \epsilon_i)$. The Markov chain is approximated from the log-normal AR(1) process

$$\ln \epsilon_{t+1} = \rho \ln \epsilon_t + u_{t+1}, u_{t+1} \sim N(0, \sigma_u^2), \forall t \quad (5)$$

where ρ is the intergenerational persistence of productivity, and $\rho \in (0, 1)$.

Production is undertaken in a firm in accordance with

$$Y_t = K_t^\alpha (AL_t)^{1-\alpha}, \quad (6)$$

where K_t is aggregate capital in period t , and L_t is aggregate labor in period t . Let A denote the labor-augmented productivity factor, which is assumed to be constant. In other words, there is no technological progress in the benchmark model.¹³ Let $\alpha \in (0, 1)$, and let capital depreciate at a rate of δ . The firm chooses inputs by maximizing profits, $Y_t - W_t L_t - (R_t - 1 + \delta)K_t$.

Denote the distribution of the middle-age generation by a density function $\phi_t(\epsilon_{t-1}, n_{t-1}, \epsilon_t)$. Then the aggregate population of the middle-aged generation, N_t , evolves over time according to:

$$N_{t+1} = N_t \sum_{j=1}^m \sum_{i=1}^m \int_0^{\hat{n}} \phi_t(\epsilon_j, n, \epsilon_i) G_t(\epsilon_j, n, \epsilon_i) dn, \quad (7)$$

where $G(\cdot, \cdot, \cdot)$ is the middle-age agent's policy function for fertility in period t . Here population growth can be easily derived from equation (7): $\tilde{n}_t = \sum_{j=1}^m \sum_{i=1}^m \int_0^{\hat{n}} \phi_t(\epsilon_j, n, \epsilon_i) G_t(\epsilon_j, n, \epsilon_i) dn$.

The density function $\phi_t(\epsilon_{t-1}, n_{t-1}, \epsilon_t)$ evolves according to:

$$\phi_{t+1}(\epsilon_j, n_t, \epsilon_{t+1} = \epsilon_i) = \frac{\sum_{l=1}^m \pi(j, i) \int_0^{\hat{n}} \phi_t(\epsilon_l, n, \epsilon_j) G_t(\epsilon_l, n, \epsilon_j) I(G_t(\epsilon_l, n, \epsilon_j) = n_t) dn}{\sum_{l=1}^m \int_0^{\hat{n}} \phi_t(\epsilon_l, n, \epsilon_j) G_t(\epsilon_l, n, \epsilon_j) dn}. \quad (8)$$

Here, $I(\cdot)$ is the indicator function.

¹²Here \hat{n} can be understood as the physical limit, which satisfies $\hat{n} \leq 1/b$.

¹³We add technological progress in the fifth section where we calibrate the model to the US data.

Denote the policy function for saving by $H(., ., .)$, the market clearing conditions for capital and labor are:

$$K_{t+1} = N_t \sum_{j=1}^m \sum_{i=1}^m \int_0^{\hat{n}} \phi_t(\epsilon_j, n, \epsilon_i) H_t(\epsilon_j, n, \epsilon_i) dn, \quad (9)$$

and:

$$L_{t+1} = N_{t+1} \sum_{j=1}^m \sum_{i=1}^m \int_0^{\hat{n}} \phi_{t+1}(\epsilon_j, n, \epsilon_i) \epsilon_i (1 - bG_{t+1}(\epsilon_j, n, \epsilon_i)) dn. \quad (10)$$

The social security system is characterized by a payroll tax rate τ_t , and a benefit formula $T_t(\epsilon_{t-1})$. Note that $T_t(\epsilon_{t-1})$ is the social security payment to the old-age agents in period t , which is a function of the old-age agent's productivity, and the social security payments are financed via the payroll tax τ on the middle age agents. The government's balanced budget constraint is as follows,

$$N_{t-1} \sum_{i=1}^m \sum_{j=1}^m \int_0^{\hat{n}} \phi_{t-1}(\epsilon_j, n, \epsilon_i) T_t(\epsilon_i) dn = W_t L_t \tau_t. \quad (11)$$

The left-hand side is the total social security payments in period t . The right-hand side is the total social security revenue in period t .

Definition 1 a *competitive equilibrium*: Given an initial distribution of a middle-age generation $\phi_0(\epsilon_{-1}, n_{-1}, \epsilon_0)$, an initial stock of physical capital K_0 , and an initial population of the middle-age generation N_0 , a competitive equilibrium consists of sequences of prices $\{W_t, R_t\}$, government parameters $\{\tau_t, T_t(\cdot)\}$, aggregate quantities $\{L_t, K_{t+1}, N_{t+1}\}$, distributions $\phi_{t+1}(\epsilon_t, n_t, \epsilon_{t+1})$ and policy functions $\{D_t(., ., .), G_t(., ., .), H_t(., ., .)\}$ such that:

1. the policy functions $\{D(., ., .), G(., ., .), H(., ., .)\}$ solve the agent's problem (P1);
2. the firm's choices L_t and K_t maximize profits;
3. the prices W_t and R_t are such that markets clear, i.e. conditions (9) and (10) are satisfied;
4. the distribution evolves according to (8) ; and population, N_t , evolves according to (7); and
5. the government budget constraint (11) is satisfied.

Definition 2 a *stationary equilibrium* is a competitive equilibrium where the density function, $\phi(., ., .)$, prices W , R and W , and social security parameters, τ and T , are all constant over time, and given the same prices and government parameters, agents of different generations share the same policy function for the transfer to old-age parents $D(., ., .)$. Thus population grows at a constant rate.

In the rest of the paper, we focus on stationary equilibria.

3 Key Forces Underlying Differential Fertility

In this section, we analytically show why the poor choose to have more children than the rich in the model. The first order conditions (FOC) for the individual's problem are the following,

$$u'(c_t^m) = \gamma u' \left(\sum_{j \neq i, j=1}^{n_{t-1}} d_t^j + T_t(\epsilon_{t-1}) + d_t^i \right), \quad (12)$$

$$u'(c_t^m) = \beta R_{t+1} E[u(c_{t+1}^o) | \epsilon_t^i], \quad (13)$$

$$u'(c_t^m) b W_t \epsilon_t^i (1 - \tau) = \beta E[u'(c_{t+1}^o) (D_{t+1}(\cdot, \cdot, \cdot) + n_t D'_{t+1}(\cdot, \cdot, \cdot)) | \epsilon_t^i], \quad (14)$$

where $u'()$ represents the first order derivative. Equation (12) is the FOC for the transfer choice, d . The left-hand side is the marginal cost of the transfers, and the right-hand side is the marginal benefit of the transfers. The FOC clearly implies that agents reduce their transfers when their parents receive more transfers from other sources, such as their siblings or the social security program. Agents also reduce their transfers when they have lower earnings. Equation (13) is the FOC for the saving choice, which is a standard Euler equation. Equation (14) is the FOC for the fertility choice. The left-hand side of the equation is the marginal loss of having children. The right-hand side is the marginal benefit of having children. As can be seen, the uncertainty surrounding children's productivity makes it difficult for us to see how people's earnings affect their fertility choices from the FOCs. Therefore, to illustrate the key forces underlying the fertility differential between the poor and the rich in the model, we use a simplified version of the model.

The simplified version is different in two ways. First, the uncertainty over individual productivity is assumed away. For each family i , the intergenerational productivity process follows the linear equation $\epsilon_{t+1}^i = g^i \epsilon_t^i$, where g^i is the intergenerational persistence of productivity (or earnings), and $g^i > 0$. Second, it is assumed that prices, R and W , are exogenously given and constant over time. In other words, this is a partial equilibrium model. We also abstract from social security in this simplified model. Thus the middle-age agent's problem for family i is the following:

$$\max u(c_t^m) + \beta u(c_{t+1}^o) + \gamma u \left(\sum_{j \neq i, j=1}^{n_{t-1}} d_t^j + d_t^i \right) \quad (15)$$

subject to

$$c_t^m = W \epsilon_t^i - s_t - d_t - \theta_t n_t \quad (16)$$

$$c_{t+1}^o = R s_t + D_{t+1}(n_t) n_t. \quad (17)$$

Here $D_{t+1}(n_t)$ is children's decision rule for old-age transfers, which is a function of n_t . Note that $\theta_t = W \epsilon_t^i b$. It is easy to see that in a steady state it must be that $n_t = n_{t+1}$, $s_{t+1} = g^i s_t$,

and $d_{t+1} = g^i d_t$ for each family i . Proposition 1 highlights the mechanism underlying differential fertility in this model.

Proposition 1: *Suppose that there are two families i and j , with productivity ϵ_t^i and ϵ_t^j respectively. They face intergenerational persistence of productivity, g^i and g^j , so that $\epsilon_{t+1}^i = g^i \epsilon_t^i$ and $\epsilon_{t+1}^j = g^j \epsilon_t^j$, for all t . Then the following two statements are true:*

1. *If $g^i = g^j$, then in the steady state the two families have the same fertility rate, $n^i = n^j$, regardless of their productivity levels.*
2. *If $g^j < g^i < R(1 + \beta)$, then in the steady state family i has a higher fertility rate than family j , $n^i > n^j$, regardless of their productivity levels.*

Proof: in appendix 1.

The key message of this proposition is that the intergenerational persistence of earnings is all that matters for the family's fertility choice. The first statement implies that a poor household has the same fertility rate as a rich household if it has the same intergenerational persistence of productivity as the rich household. The second statement says that a household facing a steeper upward intergenerational productivity process has more children, regardless of the productivity levels. The two statements together tell us that the poor have more children not because they are poor, but because they are expecting an increasing intergenerational earnings process.¹⁴

4 Differential Effects of Social Security

In this section, we first demonstrate that the negative relationship between fertility and earnings remains when we incorporate the general equilibrium effects and uncertainty. We then illustrate the effect of social security on differential fertility and its further implications for the dynamics of the earnings distribution. We rely on numerical methods. Specifically, we answer the following two questions:

1. How does the cross-sectional relationship between fertility and earnings change as the size of social security increases?
2. How does the earnings distribution change as the size of social security increases?

¹⁴Proposition 1 tells us that the mean reversion of earnings over generations can generate a negative cross-sectional relationship between fertility and earnings. This is not only true in the “old-age security” type model of fertility. Zhao (2008) shows that the mean reversion of earnings over generations can also generate a negative relationship between fertility and earnings in the Barro-Becker type model of fertility.

4.1 Parameterization

First of all, we need to parameterize the model. There are in total 8 model parameters (see table 1). We choose their values either based on empirical estimates or directly from previous related literature.

Fixed Parameter	Value	Source
A	1.0	normalization
α	0.33	capital share
Depreciation rate: δ	0.08	BDJ (2005)
Discount factor: β	0.99 (yearly)	BDJ (2005)
Time cost : b	0.03	BDJ (2005)
Intergen. corr.: ρ	0.667	Zimmerman (1992)
σ_μ	0.45	lifetime earnings Gini: 0.31
Altruism weight: γ	1.0	normalization

Note: BDJ (2005) refers to Boldrin, De Nardi and Jones (2005).

As was shown in Proposition 1, an important parameter in this model is the intergenerational persistence of productivity (ability), ρ , which affects the cross-sectional relationship between fertility and earnings. We set it to 0.667 according to Zimmermann (1992). Another important parameter is the time cost of child-rearing, b , which is set to 0.03 based on Boldrin, De Nardi and Jones (2005). Though a value of $b = 0.03$ seems surprisingly low, it is actually appropriate since b refers to the fraction of the total time available to work during the entire working life (see Boldrin, De Nardi and Jones (2005) for detailed explanation). The standard deviation of lifetime productivity σ_μ is set to 0.45 so that the Gini coefficient of lifetime earnings is 0.31 in the model.¹⁵ For simplicity, we set γ to one, and normalize A to one. Since the rest of the parameters α , β , and δ are all fairly standard, we choose their values to be consistent with previous literature (see Boldrin, De Nardi and Jones (2005)). To get the transition matrix of ϵ , we discretize the AR(1) process and convert it to nine-state Markov chains according to Tauchen (1986). A model period is set to 20 years.

In this section, the social security benefit formula, $T(\cdot)$, is assumed to be constant across the distribution of earnings. This means that the old-age agents get lump-sum payments from the social security system. This benefit formula is very close to the pension system in New Zealand and Ireland (see OECD 2007), but a bit different from the U.S. one. The latter is

¹⁵See Bowlus and Robin (2004) for a detailed description of the empirical literature on lifetime earnings inequality.

less progressive, which will be discussed in next section in detail. All the parameter values are summarized in table 1. This set of parameter values implies a yearly interest rate of 6.0%, and a capital-output ratio of 2.36 in the stationary equilibrium (when $\tau = 0$). We also find that the negative cross-sectional relationship between fertility and earnings remains in the complete model (see figure 3).

To understand the effects of social security, we simply compare stationary equilibria with different levels of social security (more specifically, different levels of the social security tax rate τ). Figure 2 plots the relationships between τ and the several aggregate variables: the interest rate, the capital-out ratio, the TFR, and the average earnings. Figure 2(a) and 2(b) show that when τ increases from 0 to 20%, the interest rate (yearly) slightly decreases from 6.0% to 5.0%, and the capital-output ratio increases 0.18 from 2.36 to 2.54. Figure 2(c) and 2(d) show that the TFR drops 0.9 from 2.79 to 1.89 and the average earnings increase by 35% from 0.22 to 0.3. One important thing worth mentioning here is that, in contrast to the previous literature on social security (see Martin Feldstein 1974), the interest rate slightly decreases as the size of the social security program increases. The traditional wisdom says the social security payments crowd out the private life-cycle savings, and increase the interest rate. However, in this model, the social security payments reduce not only the private life-cycle savings, but also the fertility rate which is directly related to the labor supply in the next period. Furthermore, the social security payments also weaken children's incentives to give old-age transfers, which cause the parents to further substitute some children for their own savings in their old-age portfolios. Thus the social security payments may reduce the next period's labor supply (by reducing the fertility rate) proportionately more than the private life-cycle savings, which drives down the interest rate.

4.2 Differential fertility

Figure 3 plots the fertility-earnings relationships for different levels of social security. It demonstrates that the fertility of those at the low end of the distribution declines much more than other people when the social security tax rate increases. The difference in fertility change makes the fertility-earnings curve flatter when τ increases. This point can be best seen in Table 2, which summarizes the fertility changes of agents with different levels of earning shock as τ increases from 0 to 10%. We can see that fertility declines the most for the agents with the lowest productivity, and the drop in fertility gets smaller as ϵ increases. There is no drop in fertility at all for the agent with the highest level of earnings shock.

Table 2: Fertility changes by earning shock ϵ (τ : 0 \rightarrow 10%)

Earning shock ($\epsilon_1 < \epsilon_2$)	ϵ_1	ϵ_2	ϵ_3	ϵ_4	ϵ_5	ϵ_6	ϵ_7	ϵ_8	ϵ_9
Fertility drop (%)	34.9%	25.6%	21.6%	17.6%	12.5%	10.0%	7.1%	3.8%	0.0%

4.3 Dynamics of the earnings distribution

When taking into account differential fertility, social security affects the earnings distribution through two channels. First, social security changes the composition of the population in the stationary equilibrium by reducing the fertility differential between the rich and the poor. This compositional effect can be best observed by looking at the earnings distributions corresponding to different values of social security tax rate τ (see figure 4). As can be seen, the distribution shifts to the right when the social security tax rate τ increases. The density of poor people decreases as the density of rich people increases. In other words, there are fewer poor people in the population as the size of social security expands. The intuition behind this compositional change is as follows. Since earnings are correlated over generations, the poor tend to have children who are also relatively poor. Therefore, when fertility of the poor drops proportionally more than the rich, the portion of poor people in the whole population goes down.

The second channel is that social security increases people's work time by reducing their time devoted to child-rearing. Due to social security's differential effects on fertility, the work time of poor people increases more than that of the rich, and so do their earnings. Table 3 shows this effect clearly. When τ increases from 0 to 20%, the earnings of the least productive people increase by 3.7%, and the earnings of the most productive people increase 0.1%.

We can see that social security does not have significant impacts on the earnings distribution through the second channel, and its main effect on the earnings distribution is the compositional effect via changing differential fertility.

Table 3: Earnings changes by earning shock ϵ (τ : 0 \rightarrow 20%)

Earning shock ($\epsilon_1 < \epsilon_2$)	ϵ_1	ϵ_2	ϵ_3	ϵ_4	ϵ_5	ϵ_6	ϵ_7	ϵ_8	ϵ_9
Earnings increase (%)	3.7%	2.6%	2.1%	1.4%	1.0%	0.8%	0.5%	0.4%	0.1%

The changes in the earnings distribution lead to corresponding changes in the average earnings of the economy. Since there are fewer low income people as τ rises, the average earnings increase as the social security tax τ increases (see figure 2(d)).¹⁶

¹⁶Note that part of the increase in average earnings is also due to the small increases in wage and labor supply.

Since we abstract from technological progress, there is no economic growth in stationary equilibria in this model. However, an increase in the size of social security can generate sizable economic growth on the transition path between two stationary equilibria. We can see in figure 2(d), when the social security tax rate increases from 0% to 10%, the average earnings increase by 17%. A further increase from 10% to 20 % will lead to a 15% increase in average earnings. Furthermore, since one model period corresponds to one generation, the growth on the transition path may last over decades.

5 Compression of Fertility: an experiment

In this section, we apply our model to the U.S. data. Jones and Tertilt (2007) document a negative cross-sectional relationship between fertility and income for all five-year cohorts of women since 1826 in the U.S. One of their main findings is that there is a dramatic compression in fertility by income since the cohort of women born at the end of the 19th century. In other words, the fertility differential between the rich and the poor experienced a massive decline over this period. They call this fact “compression of fertility”, and it can be clearly seen in figure 5 (taken from Jones and Tertilt (2007)). In figure 5, the bottom half curve represents the average fertility rate of women at the bottom half of the income distribution, and the top half curve represents the average fertility rate of women at the top half of the income distribution.¹⁷ The horizontal axis is the birth year of women. Here each year represents a five-year period (i.e. 1893 represents 1891-1895). We can see the gap between the two curves started to shrink dramatically since the cohort of women born between 1891-1895.

We hypothesize that the “compression of fertility” is partially caused by the implementation of the government-provided social security program in the U.S. since 1935. We can see this clearly in figure 6, in which we use 35 years old as the cohort year and plot the fertility gap between the top half and the bottom half of the earnings distribution for all cohorts of women born after 1826. We can see clearly that the fertility gap starts to drop dramatically in the 1930s, which coincides with the beginning of the U.S. social security program in 1935 (denoted by the red line). Since 1935, the size of the U.S. social security program gradually increased to above 7% of the GDP in 2000 (see Boldrin, De Nardi and Jones (2005)), while the fertility gap dropped from around one child to 0.2 child.¹⁸

We run a computational experiment with our model to see how much social security can

¹⁷Income is measured by the husband’s Occupational Income (OI), which is a variable available in the census data. OI is a good proxy for lifetime earnings compared to other one-year earnings measurements.

¹⁸As a measurement of differential fertility, the bottom-top fertility gap has one problem: it is sensitive to the TFR of the cohort, which also falls over the last several decades. Therefore, we also look at the income elasticity of fertility in our computational experiment. Income elasticity of fertility is measured by the regression coefficient

Table 4: Fertility changes between two cohorts: 1891-1895 VS 1946-1950

Two cohorts of women	All	Bottom half	Top half	Bottom-top fertility differential	Income elas. of fertility
1891-1895	3.048	3.512	2.584	0.928	-0.44
1946-1950	2.218	2.348	2.088	0.260	-0.20
Decline (#)	0.83	1.16	0.50	0.67	-0.24
Decline(%)	27%	33%	19%	72%	55%

Bottom half: the average fertility rate of women in the bottom half of the income distribution;

Top half: the average fertility rate of women in the top half of the income distribution;

Income elasticity of fertility: the regression coefficient of the log fertility on the log income.

(Data source: Jones and Tertilt (2007))

account for this “compression of fertility”. Specifically, we focus on the fertility changes between two cohorts of women: 1891-1895 and 1946-1950.¹⁹ Table 4 demonstrates the fertility changes between these two cohorts of women. We can see that while the fertility of women in the bottom half of income distribution declines by 33%, the fertility of the top half only declines 19% between these two cohorts of women. The differential changes in fertility by income lead to the result that the top-bottom fertility differential shrinks from 0.928 to 0.260 between these two cohorts, and income elasticity of fertility declines by 0.24. Our experiment aims at understanding quantitatively how much social security can explain these changes between cohorts. We assume that the 1891-1895 cohort of women live in an economy without social security, and the 1946-1950 cohort of women live in an economy with a social security system similar to the current one in the U.S.²⁰ Our strategy in the computational experiment is as follows. First, we recalibrate our model so that the stationary equilibrium captures the key characteristics of the cohort of

from the following regression:

$$\log(F_i) = b_0 + b_1 \log(I_i) + \epsilon_i, \quad (18)$$

where F_i is the fertility rate and I_i is the income. Jones and Tertilt calculated the income elasticities of fertility for all cohorts of women since 1826. These are plotted in figure 7. We can also see a clear drop in income elasticity of fertility since 1935. By definition, income elasticity of fertility is not sensitive to proportional changes in fertility across the earnings distribution. Therefore, the drop in income elasticity of fertility means that the “compression of fertility” is not due to the drop in TFR in the twentieth century.

¹⁹Following the suggestion of Jones and Tertilt (2007), we do not consider the latest two cohorts of women. The reason is that their fertility are computed from the 1990 census, and they may not have completed their fertility yet.

²⁰We argue that what really matters to women’s lifetime fertility decision is the social security system at the end of their fecund period.

Table 5: Model-data comparisons

Cohort year/SS tax rate	1891-1895/0%		1946-1950/10%		The changes b/w two cohorts		
	Data	Model	Data	Model	Data	Model	Model/Data
Bottom half	3.512	3.060	2.348	2.336			
Top half	2.584	2.580	2.088	2.070			
The bottom-top gap	0.928	0.481	0.260	0.266	0.67	0.21	32%
Income elasticity of fertility	-0.44	-0.17	-0.20	-0.11	-0.24	-0.06	25%

Bottom half: the average fertility rate of women in the bottom half of the income distribution;

Top half: the average fertility rate of women in the top half of the income distribution;

The bottom-top gap: the fertility difference between the bottom half and the top half;

Income elasticity of fertility: the regression coefficient of the log fertility on the log income.

(Data source: Jones and Tertilt (2007))

women born during 1946-1950. Then, we take away the social security system and compute the new stationary equilibrium. We compare the changes between two equilibria in the model with the changes between two cohorts of women in the data.

To model the current social security system in the U.S., we follow Hugget and Ventura (1999) and Fuster, Imrohorglu and Imrohorglu (2007). The social security tax rate, τ , is set to 10%. The social security benefit formula is as follows: the marginal replacement rate is 90% for earnings lower than 20% of the economy's average earnings. From 20% to 125% of the economy's average earnings, the marginal replacement rate is 33%, and the marginal replacement rate is 15% from 125% to 246% of the economy's average earnings. Above 246% of the economy's average earnings, the marginal replacement rate is zero.²¹ We also match the top-bottom fertility gap within the 1946-1950 cohort of women: 0.26, by resetting ρ to 0.79. Then we choose the altruism weight, γ , to 0.78 so that the model matches the average fertility rate of the whole cohort: 2.2. Lastly, we assume that the labor-augmented productivity factor, A , grows at the rate of 2% yearly to match the postwar U.S. data.

Table 5 summarizes the results of the experiment. We find that 32% of the decline in differential fertility between the two cohorts can be explained by social security. When looking at the income elasticity of fertility, 25% of its drop can be accounted for by social security.

²¹Note that the demographic structure in our model is different from the data. In the model, agents' retirement period is as long as their working period. This difference makes it impossible to balance the government's budget if we want to match both the social security tax rate and its benefit levels. Therefore, we choose to only match the social security tax rate, and then scale down the social security benefit levels to balance the government's budget. By doing so, we leave the social security system's progressivity constant.

6 Conclusion

In this paper, we study the effects that government-provided social security has on differential fertility (by income) and its implications for the dynamics of the earnings distribution. As shown in previous literature, social security has negative effects on fertility when children are treated as parents' old-age security (see Boldrin, De Nardi and Jones (2005)). We find that given its redistributive property, social security reduces the fertility of the poor proportionally more than that of the rich, and therefore reduces the fertility differential between the poor and the rich. We further show that this reduction in differential fertility generates a new stationary distribution with a smaller proportion of poor people and raises the economy's average earnings. We apply the model to the U.S. data and find that in the model social security can account for 32% of the decline in the top-bottom fertility differential between the cohort of women born in 1891-1895 and the cohort of women born in 1946-1950 in the U.S. It is easy to see that in this framework, rearing children and saving are two alternative ways to secure people's old age. A change in fertility distribution should be strongly correlated to changes in the wealth distribution. Hence, we expect that the impact of social security on differential fertility will also have significant implications for the dynamics of the wealth distribution. We leave this hypothesis for future research.

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7 Appendix 1: proof of proposition 1

At the steady state, the FOCs of the middle-age agent's problem are:

$$\frac{1}{\sum_{j \neq i, j=1}^{n_{t-1}} d_t^j + d_t^i} = \frac{1}{c_t^m} \quad (19)$$

$$R\beta \frac{1}{c_{t+1}^o} = \frac{1}{c_t^m} \quad (20)$$

$$\frac{\partial c_{t+1}^o}{\partial n_t} \beta \frac{1}{c_{t+1}^o} = \frac{\theta_t}{c_t^m} \quad (21)$$

Imposing symmetry in siblings' transfer decisions, and rearranging the FOCs above gives:

$$n_{t-1}d_t = c_t^m \quad (22)$$

$$R\beta c_t^m = c_{t+1}^o \quad (23)$$

$$\frac{\partial c_{t+1}^o}{\partial n_t} \beta c_t^m = \theta_t c_{t+1}^o \quad (24)$$

Substituting in the budget constraint (17) and solving equation (22) for d_t gives:

$$d_t = \frac{1}{1 + n_{t-1}} (W\epsilon_t^i - s_t - \theta_t n_t) \quad (25)$$

Substituting this into the old age budget constraint gives:

$$c_t^o = R s_{t-1} + \frac{n_{t-1}}{1 + n_{t-1}} (W\epsilon_t^i - s_t - \theta_t n_t) \quad (26)$$

After some algebra, we obtain the rate of return on children:

$$\frac{\partial c_{t+1}^o}{\partial n_t} = \frac{1}{(1 + n_t)^2} (W\epsilon_{t+1}^i - s_{t+1} - \theta_{t+1} n_{t+1}) \quad (27)$$

Remember that at steady state we have the following conditions: $n_t = n_{t+1}$, $s_{t+1} = g^i s_t$, and $d_{t+1} = g^i d_t$ for each family i . Dropping the time index, substituting in the rate of return on children, and combining equations (23) and (24) gives:

$$\theta^i R = g^i \frac{1}{(1 + n^i)^2} (W\epsilon^i - s^i - \theta^i n^i) \quad (28)$$

Combining equations (22) and (23), and some algebra gives:

$$s^i = \frac{\frac{n^i}{n^i+1} (W\epsilon^i - \theta^i n^i) (\beta R - g^i)}{R + \frac{n^i}{n^i+1} (\beta R - g^i)} \quad (29)$$

By substituting (29) into (28), we cancel out s^i and get the following equation:

$$(R^2 + R(\beta R - g^i))(n^i)^2 + (2R^2 + R^2\beta)n^i + R^2 - \frac{g^i R}{b} = 0 \quad (30)$$

Equation (30) is the equation determining the fertility choice of the family i . We can see equation (30) does not contain ϵ^i . The only family-specific parameter it contains is g^i . Thus it is obvious that, for any two families i and j with $g^i = g^j$, we have $n^i = n^j$, regardless of the levels of ϵ^i and ϵ^j . The first statement is proved.

The second statement says if $R + R\beta > g^i$ is satisfied, n^i increases as g^i increases. To prove it, we first need to derive $\frac{\partial n^i}{\partial g^i}$. We get $\frac{\partial n^i}{\partial g^i}$ by applying implicit function theorem to equation (30).

$$\frac{\partial n^i}{\partial g^i} = -\frac{\frac{\partial F(\cdot)}{\partial g^i}}{\frac{\partial F(\cdot)}{\partial n^i}} = \frac{R(n^i)^2 + \frac{R}{b}}{2(R^2 + R(\beta R - g^i))n^i + 2R^2 + R^2\beta} \quad (31)$$

It can be easily seen that, $\frac{\partial n^i}{\partial g^i} > 0$, if $R^2 + R(\beta R - g^i) > 0$, or $R + R\beta > g^i$. Q.E.D.

8 Appendix 2: discrete fertility choice

As we said in footnote 10, assuming continuous fertility may cause a measurement problem. We take a shortcut by using symmetry for old-age transfers of children when we actually solve the model. In order to show that this problem does not significantly affect our results, we study a version of the model with discrete fertility choice and heterogenous preference. In this version of the model, agents also differ by altruism toward their parents (γ). The middle-age agent's problem can be written as follows:

$$V(\epsilon_{t-1}, n_{t-1}, \epsilon_t^i, \gamma_t^i) = \max_{d,n,s} u(c_t^m) + \beta E[u(c_{t+1}^o) | \epsilon_t^i, \gamma_t^i] + \gamma_t^i u\left(\sum_{j \neq i, j=1}^{n_{t-1}} d_t^j + T_t(\epsilon_{t-1}) + d_t^i\right)$$

subject to

$$d_t^i + s_t + c_t^m = W_t \epsilon_t^i (1 - \tau)(1 - bn_t), \quad (32)$$

$$c_{t+1}^o = R_{t+1} s_t + T_{t+1}(\epsilon_t^i) + n_t D_{t+1}(\epsilon_t^i, n_t, \epsilon_{t+1}, \gamma_{t+1}). \quad (33)$$

Here the altruism weight $\gamma_t \in \{\gamma_1, \gamma_2, \dots, \gamma_m\}$, is governed by a Markov chain with transition matrix $\pi_\gamma(i, j) = \text{Prob}(\gamma_{t+1} = \gamma_j | \gamma_t = \gamma_i)$. The Markov chain is approximated from the AR(1) process

$$\gamma_{t+1} = (1 - \rho_\gamma)\mu + \rho_\gamma \gamma_t + \nu_{t+1}, \nu_{t+1} \sim N(0, \sigma_\nu^2), \forall t, \quad (34)$$

where μ is the unconditional mean of γ , ν is the standard deviation, and ρ is the time persistence coefficient. We set μ to one, ν to 0.2, and ρ_γ to 0.667, and compute this version of the model (the rest of parameter values are the same as those in the benchmark parameterization). Figure 8 and 9 show the analogs of the results presented in figure 3 and 4. We find that this version of the model produces very similar results as the benchmark model does. This means that assuming continuous fertility does not significantly affect our results.

Figure 1: Differential fertility by income in the U.S.: 1826-1960. (from Jones and Tertilt (2007))

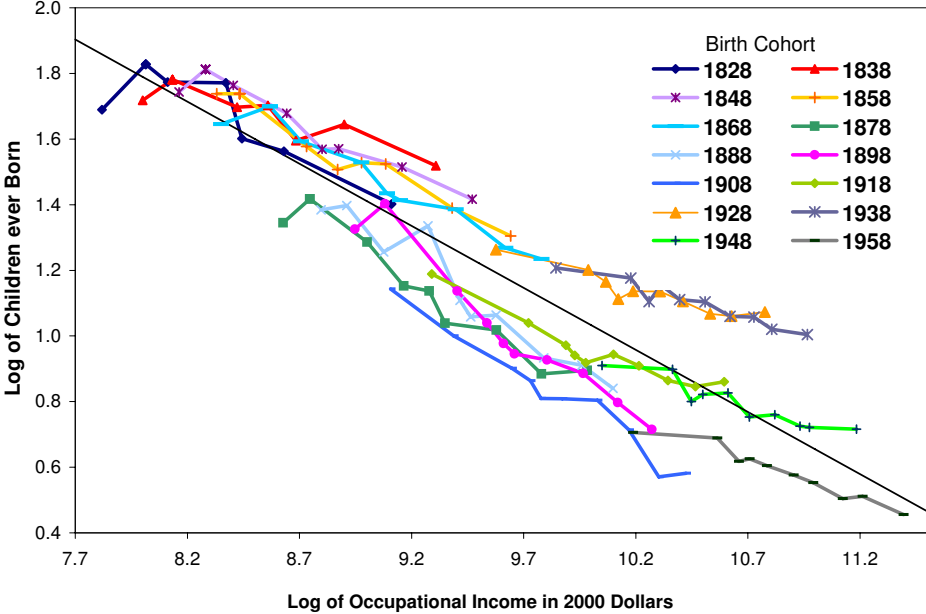
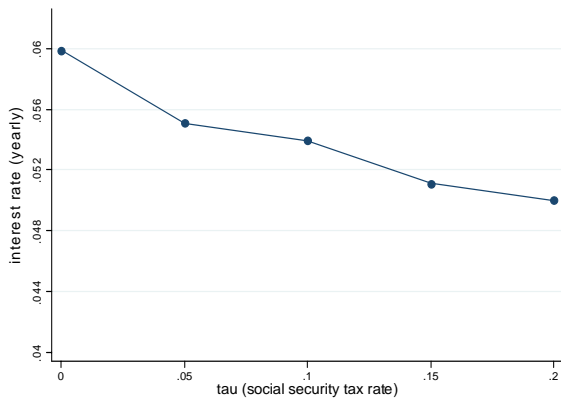
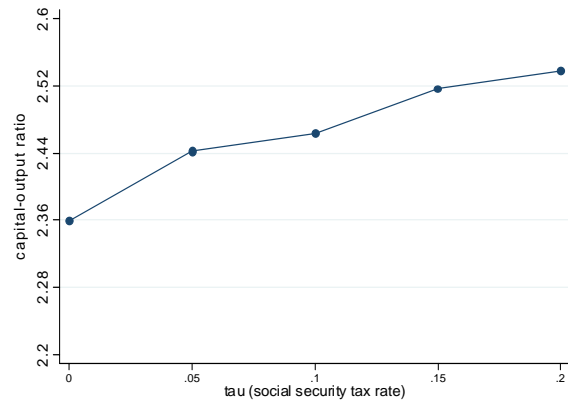


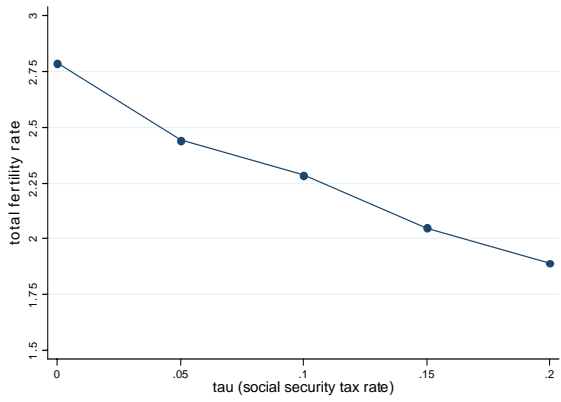
Figure 2: Aggregate variables and the SS tax



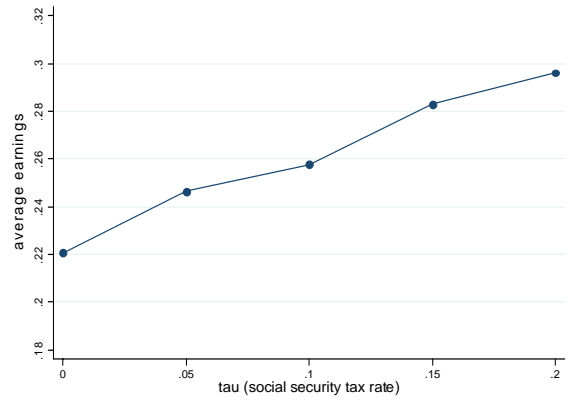
(a)



(b)



(c)



(d)

Figure 3: Differential fertility and the SS tax

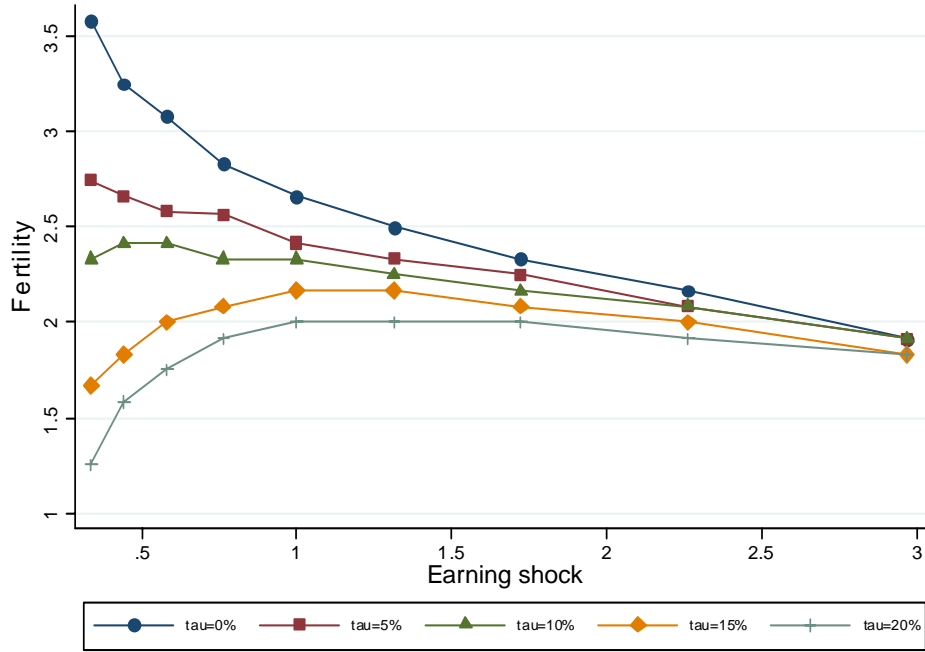


Figure 4: Earnings distribution and the SS tax

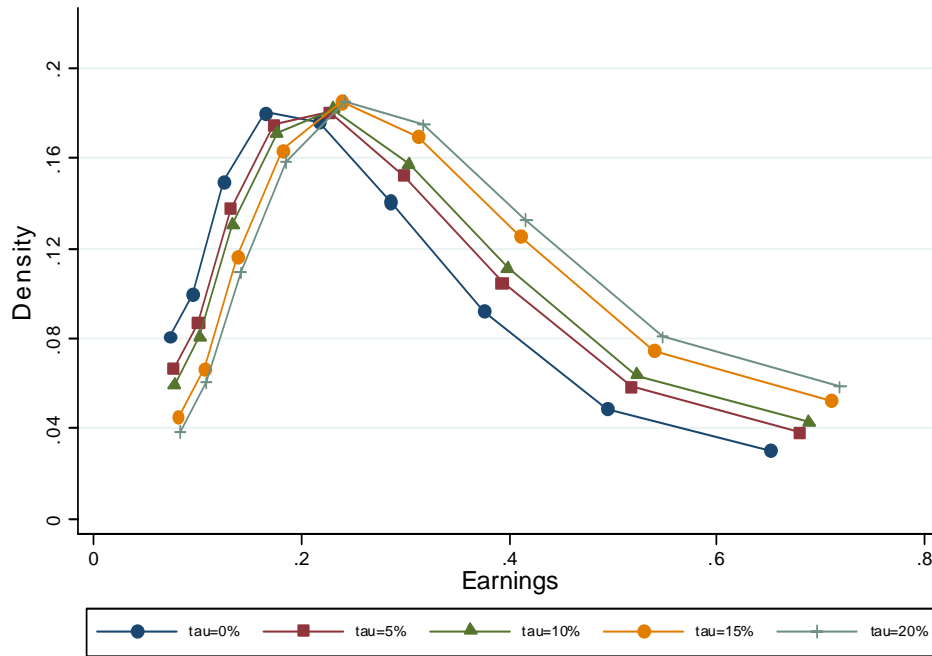


Figure 5: Fertility by top and bottom half of income distribution in the U.S.: 1826-1960.
 (From Jones and Tertilt (2007))

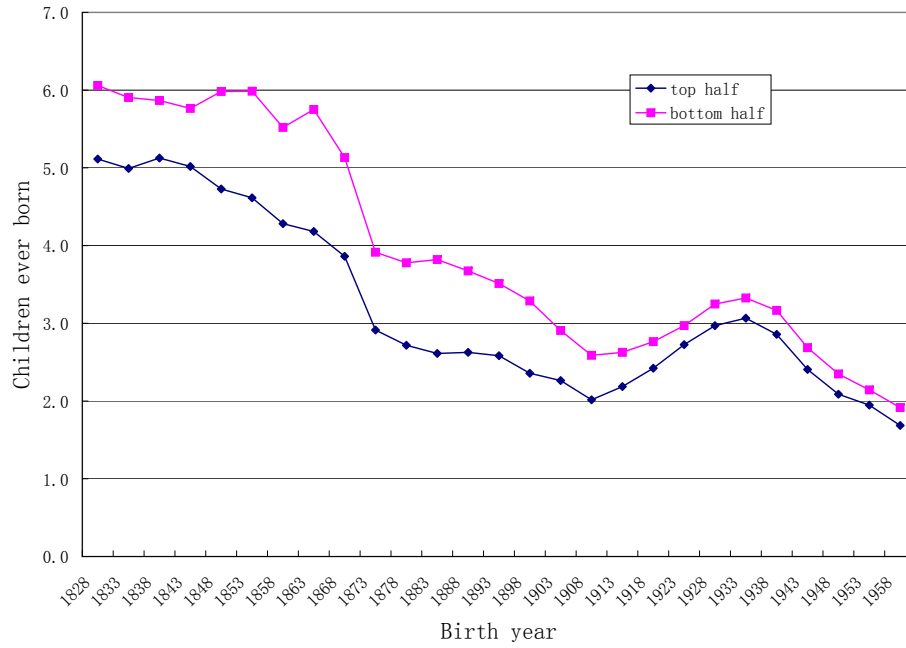


Figure 6: Fertility gap over time in the U.S.
 (Data source: Jones and Tertilt (2007))

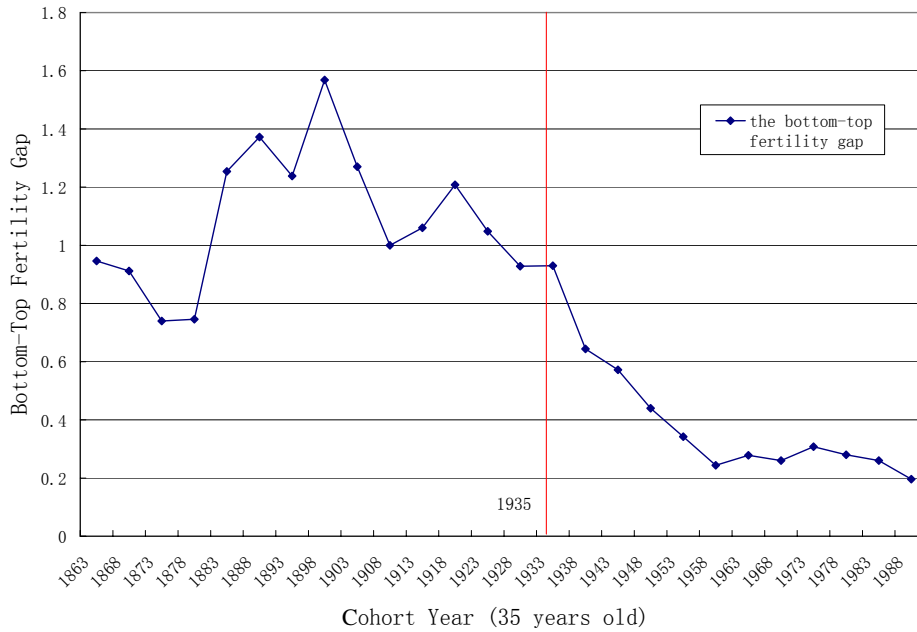


Figure 7: Income elasticities of fertility over time in the U.S.
 (Data source: Jones and Tertilt (2007))



Figure 8: Differential fertility and the SS tax (discrete fertility choice)

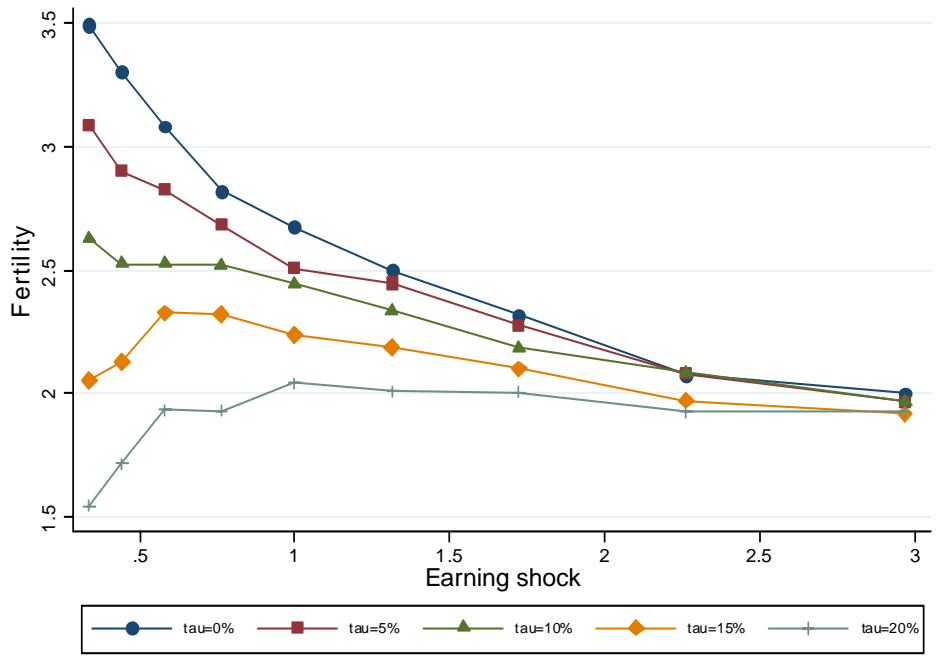


Figure 9: Earnings distribution and the SS tax (discrete fertility choice)

