Auctions vs. Sequential Mechanisms When Resale is Allowed*

Xiaogang Che† Tilman Klumpp‡

October 16, 2018

Abstract

We examine the problem of selling an object to a stream of potential buyers with independent private values and participation costs. If the object can be resold in the future, and resellers can set reserve prices, the original seller may prefer to deal with potential buyers sequentially instead of holding an auction. In particular, with a sufficiently large set of potential future buyers, the auction yields a zero revenue while a very simple form of sequential mechanism becomes approximately optimal. This contrasts with the result that sellers usually prefer auctions when resale is not allowed (see Bulow and Klemperer 2009).

Keywords: Sequential mechanism, auctions, participation cost, sequential entry, resale.

JEL codes: D44.

*We thank Ian Jewitt, Todd Kaplan, Rene Kirkegaard, Birendra Rai, Jingfeng Lu, Kunal Sengupta, and two anonymous referees for their helpful comments. An earlier version of the paper benefited from discussions at the SAET conference (Cambridge), the International Conference on Game Theory (Stony Brook), and the Durham Micro-Workshop. We also thank Peter Lee for excellent research assistance.

†Durham University Business School, Millhill Lane, Durham DH1 3LB, United Kingdom. E-mail: xiaogang.che@durham.ac.uk.

‡Department of Economics, University of Alberta. 9-20 Tory Building, Edmonton, AB, Canada T6G 2H4. E-mail: klumpp@ualberta.ca.

1 Introduction

Consider the problem of a seller who faces a group of potential buyers. Assume the buyers arrive in sequence and that each buyer can learn his private valuation by paying a cost. The seller can run an auction in which buyers first decide to enter, and those who enter then submit their bids simultaneously. Alternatively, the seller can deal with each buyer sequentially, by giving each buyer the opportunity bid before inviting the next buyer to participate.

Bulow and Klemperer (2009, henceforth BK) showed that sellers usually prefer to use the auction. Simply put, auctions are "more competitive" than sequential mechanisms. In the latter, bidders can place jump bids (i.e., publicly observable price commitments) that signal the presence of a high-value competitor and deter entry by other bidders. Buyers cannot send such signals in auctions, because bidding commences only after entry has stopped. Thus, in expectation there is more entry in the auction, resulting in more aggressive bids and a higher seller revenue. Of course, the sequential mechanism can lead to a higher price *ex post*, if the bidders who enter in the auction have relatively low valuations. However, BK demonstrate that this potential advantage is unlikely to dominate the competitiveness of the auction *ex ante*, as this requires a large set of potential bidders and a carefully chosen value distribution and entry cost.

In this paper, we examine the same choice between auctions and sequential mechanisms, but allow the buyer who acquires the object from the original seller to resell it to other buyers. We show that the original seller may no longer prefer to use the auction. The reason is the following: The opportunity to resell introduces a common value element to both the auction and the sequential mechanisms. All buyers—even those with low private values—now have a minimum willingness to pay equal to the item's value in the resale market. This reduces the information rents that buyers can earn and, thus, reduces entry in the auction. If the resale value is high enough, only one buyer will find it profitable to enter the initial auction, in which case the original seller's revenue in the auction must be zero. We show that this happens when the set of potential buyers is large enough. Thus, it is precisely the competitiveness of the auction that works against it in the presence of resale opportunities.

However, even a sequential mechanism may be too competitive. In particular, the mechanism studied by BK has, at every stage, two bidders competing in an ascending auction before the survivor makes an additional jump bid. But if a standard second-price auction with no reserve price leaves insufficient rents for more than one buyer to compete profitably, then the same must be true for an ascending auction that has a non-negative starting price and whose winner may have to compete again in the future. Thus, the first buyer in the sequential mechanism can deter all future entry by placing a zero (or epsilon) jump bid, leaving the seller with no revenue once again.

To avoid this problem, we introduce a sequential selling mechanism in which any direct, head-to-head competition between buyers is removed. Each bidder is permitted to make exactly one publicly observable price offer after entry, which he commits to paying if accepted by the seller. If, later, another bidder enters, this bidder is permitted to make exactly one price offer, and so on. We call this process a *fully sequential mechanism.* In the fully sequential mechanism, a bidder does not care about the valuation of the previous bidder (he only cares about the previous bid), so jump bidding serves no signaling purpose. But because bids cannot be increased later, in order to deter entry a jump bid must be sufficiently high to begin with. We show that the fully sequential mechanism generates more revenue than both the auction and the conventional sequential mechanism if the set of potential buyers to whom the object can be resold is sufficiently large, and it becomes optimal in the limit (whereas the auction and the conventional sequential mechanism become worst).

To put our results in perspective, it will be helpful to consider the following question: Why would the original seller not simply use a revenue-optimal selling mechanism in the first place? For the case of a fixed finite set of potential bidders (which is a special case of the environment considered here), such a mechanism is described in Cremer *et al.* (2009) and involves the use of reserve prices and entry fees. In our model, on the other hand, only resellers (but not the original seller) can use such instruments.[1] This assumption reflects situations in which the original seller is unable or unwilling to commit to not sell below a certain price, or in which the original seller does not have enough information to compute reserve prices. Such constraints are realistic in certain applications. For example, consider a government agency that wants to privatize a public asset. This agency may have efficiency concerns that override revenue concerns, such as a concern for maximizing the overall economic benefits associated with the transaction. If the asset is the right to operate a railroad in a given area, for instance, an outcome in which rail service is suspended because a reserve price has not been met may be unacceptable. In addition,

[1] More precisely, we assume that resellers can, but do not necessarily have to, use take-it-or-leave-it offers to sell the object.

the agency may face corporate buyers that are better informed than the government about the commercial value of the asset and how it is distributed in an industry, enabling them (but not the government seller) to compute optimal reserve prices.

One way for the seller to deal with these constraints is to enlist the help of some buyers to sell the object to others. For such a scheme to be successful, there must be gains from trade after the initial sale—that is, the initial mechanism must be inefficient. This is part of the reason why the seller should not let all buyers compete in an auction without a reserve price, and why both the auction and the BK sequential mechanism may fail to generate enough entry to raise positive revenue in our environment. The original seller's goal, rather, is to construct an alternative mechanism that sells with a high probability but not always to the "correct" buyer, and still leverages competition among potential resellers in a way that allows the original seller to extract some of the resale profits that unconstrained sellers can generate. In the context of the privatization example, this implies that post-privatization trade among private parties does not necessarily imply that money has been "left on the table," provided the government seller can find a way to extract some of the post-privatization gains at the initial selling stage. This is precisely what our fully sequential mechanism accomplishes. In fact, we show that the fully sequential mechanism asymptotically extracts *all* resale profits, and does so without having to use any of the additional instruments available to resellers.[2]

We proceed as follows. In Section 2 we review the related literature on auctions with resale. In Section 3 we introduce our basic model (which is the same as in BK), and in Section 4 we introduce a resale market to the model. In Section 5 we analyze entry and bidding behavior in the auction and the (conventional) sequential mechanism, and show that both fail to generate revenue if there are too many buyers. In Section 6 we introduce the fully sequential mechanism and show that it is approximately optimal with a large number of bidders. Section 7 concludes with a discussion on efficiency and an example for the small-number-of-bidders case. All proofs are in the Appendix.

[2]The requirement that selling mechanisms not depend on the details of the selling environment, such as the distribution of buyers' valuations, is sometimes referred to as the *Wilson doctrine* (see, e.g., Krishna 2002, p. 75). Our paper applies to environments where some, but not all, sellers are constrained by the Wilson doctrine. For a constrained seller, the Wilson doctrine reflects the criterion that "practical mechanisms should be simple and designed without assuming that the designer has very precise knowledge about the economic environment in which the mechanism will operate" (Milgrom 2004, p. 23). The cost of this simplicity is that such mechanisms are not generally revenue-optimal. What our results show is that, if resale is possible and the number of unconstrained sellers is sufficiently large, an initial selling mechanism that is approximately revenue-optimal and which does comply with the Wilson doctrine exists.

2 Relation to Previous Work

This paper is related to two strands of previous work: The literature on resale in auctions, and the literature on endogenous participation in auctions (and other selling mechanisms). To the best of our knowledge, ours is the first model to intersect these two areas. In the following, we briefly review both literatures and then highlight the contributions of our paper to each.

Auctions with resale. If bidders in an auction draw their valuations from asymmetric value distributions, common auction formats may fail to allocate efficiently. In this case, the opportunity for post-auction trade arises naturally and has received much attention in the literature (e.g., Gupta and Lebrun 1999; Zheng 2002; Garratt and Tröger 2006; Pagnozzi 2007, 2010; Hafalir and Krishna 2008; Garratt *et al.* 2009; Mylovanov and Tröger 2009; Lebrun 2010a, 2010b, 2012; Cheng 2011; Che *et al.* 2013; Zhang and Wang 2013; Virág 2013, 2016). On the other hand, if buyers' valuations are symmetrically distributed, common auction formats allocate to the buyer with the highest valuation. Thus, additional assumptions on the economic environment are necessary to generate post-auction resale. Haile (2000, 2003) develops models in which bidders face residual uncertainty about their private values, which is resolved after the initial auction. In this case, the winning bidder may attempt to resell to buyers who turn out to have higher private valuations *ex post*. Bose and Deltas (1999) and Haile (2001) assume that some bidders cannot participate in the initial auction. In this case, the winning bidder may attempt to resell to one of the excluded bidders. In either case, the anticipation of future resale opportunities affects bidders' willingness to pay in the initial auction, and depending on the initial auction format, this feedback can increase or decrease the expected revenue of the original seller, relative to the no-resale case.

Endogenous participation in auctions. The literature on endogenous participation in auctions can be divided into two branches. The first branch originates with Samuelson (1985) and assumes that potential bidders know their private values before making their costly entry decisions (see also Stegeman 1996; Tan and Yilankaya 2006; Lu 2009; Cao and Tian 2010; Moreno and Wooders 2011; Shi 2012; Lu and Ye 2012, 2014). The entry cost paid by a bidder reflects either entry fees charged by the seller, or the cost for preparing and delivering a formal bid. The second branch originates with McAfee and McMillan (1987) and Levin and Smith (1996) and assumes that potential bidders learn their private values only after making their entry decision. Here, the participation cost can be interpreted as resources spent by a bidder to investigate the item and determine his willingness to pay. Within this branch, the outcome of a selling mechanisms depends crucially on what is assumed about the timing of entry and bidding decisions: If bidding is possible before entry is completed, bidders can deter the participation of future competitors either by preemptive jump bidding (an effect first demonstrated by Fishman 1988) or by coordinated bidding (see Che and Klumpp 2016). Bulow and Klemperer (2009, "BK") later showed that, because of the possibility to deter entry, sellers generally receive more revenue if they use auctions in which no bids can be submitted until all entry is complete.

Our model belongs to the "symmetric values" class of resale models and to the "pay to learn your value" class of endogenous participation models, and it contributes to each. As in Bose and Deltas (1999) and Haile (2001), the reason why resale occurs in our model is that some buyers do not participate in the initial sale. However, instead of being exogenously excluded, these buyers choose not to participate in the initial sale because participation is not optimal. As BK, we compare two types of mechanisms: Simultaneous auctions and sequential processes that permit placing of preemptive bids. Within the latter, we now introduce a distinction between fully sequential mechanisms in which *only* preemptive bids are permitted, and mechanisms in which an element of simultaneous competition remains. Regardless of the mechanism, only some buyers participate, and one should expect the buyer who entered and won to try to resell the object to those who did not. This is exactly what our model allows them to do.

We are aware of two other papers that show that sequential mechanisms can have an advantage over auctions. Roberts and Sweeting (2013) assume that potential buyers first receive noisy signals about their valuations, then decide whether to enter, and then learn their actual valuation if they enter. In this environment, a bidder's entry decision depends not only on the number of previous entrants and their bids, but also on each bidder's signal. Roberts and Sweeting (2013) show that this facilitates entry by high value bidders in both the auction and the sequential mechanism. However, the selection effect is stronger in the latter and may result in a larger expected revenue compared to the auction. Davis *et al.* (2013) test BK's result in a laboratory experiment and find that, contrary to the theoretical prediction, sequential mechanisms tend to perform better than auctions. The authors show that this could be explained by random shocks to each bidder's (perceived) entry cost. In contrast to these models, ours retains BK's assumptions about information or payoffs and instead modifies the trading possibilities that are available to agents.

3 Basic Framework

The owner of an item faces a (possibly random) queue of risk-neutral potential buyers, indexed $i = 1, 2, \ldots$. The probability that at least n buyers are present, conditional on $n-1$ buyers being present, is $\rho_n \in [0,1]$. We assume that $\rho_1 = \rho_2 = 1$, i.e., at least two buyers exist with certainty, that $\rho_3 \geq \rho_4 \geq \ldots$, and that $\rho_n < 1$ for some $n \geq 3$. We denote by

$$\mathcal{N} \;=\; \rho_1\big(1+\rho_2\big(1+\rho_3\big(1+\ldots\big)\big)\big) \;\in\; \big[2,\infty\big)$$

the expected number of buyers, which is our measure of the market's size. Note that this model of buyer arrival subsumes the case of a fixed number N of bidders, which corresponds to $\rho_1 = \ldots = \rho_N = 1$ and $\rho_{N+1} = \rho_{N+2} = \ldots = 0$. In this case, the arrival sequence represents merely an exogenous ordering in which buyers are treated by the seller.[3]

Buyer i, if he exists, must pay $c > 0$ in order to learn his private valuation v_i. This valuation is drawn from an atomless distribution F with support $[0, \overline{v}]$ and independent of v_j for all $j \neq i$. We assume that $c < \int_0^{\overline{v}} (1 - F(v))F(v)dv$, i.e., at least two buyers could profitably enter a standard second-price auction. It will also be useful to define the (unique) value $0 < \hat{r} < \overline{v}$ that satisfies the condition

$$\int_{\hat{r}}^{\overline{v}} (v - \hat{r})dF(v) - c \;=\; 0. \tag{1}$$

If the item was offered at a posted price, then $\hat{r}$ is the highest such price at which a buyer is willing to pay the entry cost, learn his valuation, and buy the item if and only if his valuation exceeds the posted price.[4]

The owner's valuation for the item is zero. The owner can choose among two types of initial selling mechanism: Auctions and sequential mechanisms. In both cases, bidders arrive in sequence and, when they arrive, must decide whether to enter or not. In

[3]Our model of buyer arrival is the same as BK, except that we assume that ρ_n decreases in n and that at least one bidder arrives with probability strictly less than one. The first assumption isn't strictly needed for our results. We make it only so that $\mathcal{N}$ is always finite, which allows us to frame our results in terms of a sufficiently large market size.

[4]Our term $\hat{r}$ is the same as the term V_K in BK.

auctions, bidding takes place only after the entry of all participants has stopped, whereas in sequential mechanisms, buyers can place bids on which later buyers can condition their entry decisions. BK compare a standard second-price auction to the sequential mechanism originally introduced by Fishman (1988), which they generalize to the case of more than two buyers.

Second-price auction. When bidder i arrives he observes the number of bidders who have already entered, and then decides whether to enter. If he enters, he pays c and learns his valuation v_i. Entry is complete if either the arrival of new buyers comes to a halt, or if a new buyer arrives and decides not to enter. We assume that previous entrants do not observe which of the two possibilities is the case.[5] At this time, all buyers who did enter simultaneously submit bids $b_i \geq 0$. The bidder who submitted the highest bid wins and pays the second-highest bid. If only a single bid is placed, the winner pays zero.

Fishman/BK sequential mechanism. The initial price is $p^0 = 0$. When bidder 1 arrives he decides whether to enter. If he enters, 1 pays c, learns v_1, and can place a bid of $p^1 \geq 0$. This is a *jump bid*, i.e., a public commitment to pay p^1 if 1 wins. When bidder 2 arrives, he observes p^1 and decides whether to enter. If he enters he pays c and learns v_2. Bidders 1 and 2 then simultaneously raise the price until one of them quits. The survivor is the new high bidder, and can may make an additional discrete jump bid. The price at the end of the second stage is p^2. Now bidder 3 arrives, observes p^2, and decides to enter. If he enters he pays c, learns v_3, and then competes with the survivor of the previous stage by simultaneously raising the price until one bidder quite. The survivor is the new high bidder and can place an additional jump bid, after which the price is p^3. This process continues until either the arrival of buyers stops, or until the first buyer arrives and decides not to enter. At this time, the current high bidder wins and pays the current price.

We compare the same two selling mechanisms, and then consider a third. The third mechanism is a sequential mechanism similar to the Fishman/BK mechanism, with one exception: The new entrant and the current high bidder do not simultaneously raise the price before the survivor makes a jump bid:

[5] This assumption matters for the bidding equilibrium of the auction insofar as, if entry stops due to a lack of potential buyers, there will be no resale market. If existing buyers knew that this was the case, their bidding behavior would be different from the case in which there was a further potential buyer who decided not to enter (in the latter case, existing buyers would know that there is at least one buyer to whom the object could be resold). However, our main result concerning entry into the auction would remain qualitatively unchanged if entrants were informed about why entry has stopped.

Fully sequential mechanism. The fully sequential mechanism proceeds as follows. The initial price is $p^0 = 0$. When bidder 1 arrives he decides whether to enter. If he enters, 1 pays c, learns v_1, and can make a jump bid of $p^1 \geq 0$. When bidder 2 arrives, he observes p^1 and decides whether to enter. If he enters he pays c and learns v_2. He can then either quit, or submit a jump bid $p^2 > p^1$ to become the new high bidder. Now bidder 3 arrives, observes p^2, and decides to enter. If he enters he pays c, learns v_3, and then either quits or bids $p^3 > p^2$ to become the new high bidder. This process continues until either the arrival of buyers stops, or until the first buyer arrives and decides not to enter. At this time, the current high bidder wins and pays the current price.

After the initial selling mechanism has concluded, the successful buyer may, if he wishes, attempt to resell the object. In the next section, we describe the resale stage and derive several results that will be used later when we examine the outcomes of both auctions and sequential mechanisms "in the shadow of resale."

4 The Resale Market

We begin by stating the formal assumptions underpinning our resale model; this is done in Section 4.1. We then develop a number of results concerning the item's resale value (and, hence, buyers' willingness to pay at the initial selling stage) in Section 4.2. Finally, in Section 4.3 we discuss the role of our assumptions for these results.

4.1 Assumptions and notation

Buyers who have already entered the initial mechanism are free to participate in the resale market. Any buyer who arrives later must pay the entry cost c, as he would in the initial mechanism. To keep our model tractable, we rule out repeated resale:

A1. The item can be resold at most once; that is, if a buyer acquires the item on the resale market he consumes it and cannot resell it again.

The reseller can choose among some class of available resale mechanisms. A resale mechanism is a complete description of the rules governing sale to any set of potential buyers. The winner of the initial mechanism announces a resale mechanism after the initial mechanism has ended, but before any subsequent buyers arrive. Regarding the set of mechanisms available to the reseller, we make the following assumptions:

A2. The set of available resale mechanisms does not depend on the original selling mechanism, on the identity of the current owner, or on how many bidders have already arrived or entered.

A3. The set of available resale mechanisms includes the option to not resell the object, and it includes resale via take-it-or-leave-it offers to individual buyers.

A4. All resale mechanisms are *ex ante* and *interim* individually rational. That is, a potential buyer cannot be forced to enter and participate in the mechanism, and if he enters and learns his valuation, he can walk away from the mechanism at no additional cost.

Once a resale mechanism is announced, buyers interact with each other through the mechanism by playing an (appropriately defined) equilibrium of the game induced by the mechanism. Instead of characterizing this equilibrium for every resale mechanism, we summarize its outcome as follows. Fix a resale mechanism, its associated equilibrium, the sequence $\{\rho_n\}_{n=1}^{\infty}$, the entry cost c, and the value distribution F, and suppose n bidders participated in the initial mechanism. Let $Q_n \in [0,1]$ be the probability that the resale mechanism results in a sale to some buyer $n' > n$. Let X_n be be the expected total payment made by buyers $n' > n$ in the resale mechanism, conditional on the mechanism resulting in a sale to some buyer $n' > n$. Finally, let X_n^0 be the expected total payment made by buyers $n' > n$ in the resale mechanism, conditional on the mechanism not resulting in a sale to some buyer $n' > n$.[6] The set of available resale mechanisms can then be summarized by the set of sequences $(Q, X, X^0) = (Q_n, X_n, X_n^0)_{n=1,2,\ldots}$ associated with each mechanism. We call this set Ω and make the following final assumption:

A5. The sets $\left\{(Q_n, X_n, X_n^0) : (Q, X, X^0) \in \Omega\right\}$ are compact for all n.

4.2 Resale revenue and effective valuations

Buyers' willingness to pay for the object at the initial selling stage will depend, in part, on the revenue they can expect to receive if they were to sell the object in the resale

[6]For example, suppose $v_i \sim U[0,1]$ and $\rho_i = \alpha$ for all $i \leq N$ and $\rho_i = 0$ for $i > N$. As a resale mechanism, consider a sequence of take-it-or-leave-it offers $p \in (0,1)$ that are made by the reseller to every buyer, until either the offer is accepted or the arrival of new buyers stops. For $n = 1, \ldots, N-1$ we have

$$Q_n = \alpha\big[p + (1-p)\alpha\big[p + (1-p)\alpha\big[\ \ldots\ \big]\big]\big] = \alpha p \frac{1 - \big[(1-p)\alpha\big]^{N-n}}{1-(1-p)\alpha}, \quad X_N = p, \quad \text{and} \quad X_n^0 = 0,$$

and for $n \geq N$ we have $Q_n = X_n = X_n^0 = 0$.

market. We now examine this relationship between expected resale revenue and the surplus a buyer obtains when he acquires the object in the initial mechanism. We begin by establishing an upper bound on resale revenue, which is driven entirely by the individual rationality assumption A4:

Lemma 1. *The expected revenue of any resale mechanism that satisfies assumption A4 and that sells with probability q is at most $q\hat{r}$, where $\hat{r}$ was defined in* (1).

Suppose that n buyers participate in the initial mechanism and that buyer $i \le n$ is the winner. Assuming that the initial mechanism always allocates to the buyer with the highest private valuation, Assumption A1 implies that if buyer i resells the object to buyer j, then $j > n$. We can then define

$$z_n(v_i) \ = \ \max_{(Q,X,X^0)\in\Omega} \left\{ Q_n X_n + (1 - Q_n)(v_i + X_n^0) \right\} \tag{2}$$

to be i's *effective valuation* in the initial selling mechanism with n participants, that is, the possible surplus that i can attain on expectation if he wins the object. Note that $z_n(0)$ is the "pure resale value" of the object to buyers $n' < n$. In the following result, we collect several properties of effective valuations to be used later:

Lemma 2. *Suppose the set of resale mechanisms satisfies assumption A1–A5. For given n, the effective valuation function z_n is well-defined and has the following properties:*

(i) z_n *is strictly increasing and weakly convex.*

(ii) $z_n(v_i) \ge \max\{z_n(0), v_i\}$.

(iii) $z_n(v_i) \le \max\{\hat{r}, v_i\}$, *where* $\hat{r} < \overline{v}$ *was defined in* (1).

(iv) $z_n(v_i) - z_n(v_i') \le v_i - v_i'$ *for all* $v_i > v_i'$.

Because the lowest effective valuation is $z_n(0)$, the difference $z_n(v_i) - z_n(0)$ determines the information rent buyer i can hope to earn in the initial mechanism. We will now show that this rent is small when the resale market is large.

To formalize the notion of a large resale market, recall that the number potential buyers to which the item can be resold to depends (stochastically) on the sequence (ρ_i) that specifies the conditional arrival probabilities of buyers. We endow the space of all such sequences with the product topology; that is, one sequence converges to another if it converges pointwise. Now consider a sequence of sequences $(\rho^t) = ((\rho_i)^t_{i=1,2,\ldots})^{t=1,2,\ldots}$, where t indexes the sequence and i indexes buyers in the sequence. Suppose that $\rho^t \to (1, 1, 1, \ldots)$. This includes two important cases: When there is a finite number of buyers and this number grows (e.g., $\rho_i^t = 1 \; \forall i \leq t$ and $\rho_i^t = 0 \; \forall i > t$); and when there is a potentially infinite number of bidders whose arrival probabilities increase (e.g., $\rho_i^t = \alpha^t < 1 \; \forall i, t$ and $\alpha^t \to 1$). Intuitively, in both cases the set of resale opportunities expands as $t \to \infty$. At the same time, if $\mathcal{N}^t$ is the expected number of buyers associated with ρ^t, then $\mathcal{N}^t \to \infty$ as $t \to \infty$.[7]

Thus, from now on when we say there is a "sufficiently large number of buyers" or "sufficiently large resale market," we mean that $\mathcal{N}$ is large (but still finite), which is equivalent to (ρ_i) being close, but not equal, to $(1, 1, 1, \ldots)$. The following result describes what happens to effective valuations in this case:

Lemma 3. *Suppose the set of resale mechanisms satisfies assumption A1–A5. Fix F and c. Take any sequence of arrival probability sequences (ρ^t), such that $\rho^t \to (1, 1, 1, \ldots)$ pointwise. Let $z_n^t : [0, \overline{v}] \to [0, \overline{v}]$ be the effective value function associated with sequence ρ^t, for given n. As $t \to \infty$,*

(i) $z_n^t(0) \to \hat{r}$;

(ii) $z_n^t(v_i) \to \max\{\hat{r}, v_i\}$ *uniformly;*

(iii) $E\big[z_n^t(v_i) - z_n(0)\big] \to c$.

4.3 Discussion of assumptions

Our model of the resale market encompasses a variety of ways in which resellers interact with buyers, subject to the requirements spelled out in assumptions A1–A5 above. Assumption A2 is made for convenience and can be relaxed at the expense of additional notation. Assumption A5 is a technical requirement that ensures that the reseller's choice of resale mechanism from the set Ω is well-defined. The substantive assumptions are A1, A3, and A4, and we now discuss the role of each for our results.

Assumption A1 is imposed in order to rule out Ponzi schemes in the resale market, that is, situations in which the object is sold indefinitely for an unbounded total profit. Suppose buyer i buys the object from the previous buyer $i-1$ for some price, say p^{i-1}, and then resells it to the next buyer $i+1$ for a higher price p^i. If $p^i > (c + p^{i-1})/\rho_{i+1}$,

[7] This follows from the assumption that the ρ-sequences are monotonic.

the scheme compensates buyer i for the entry cost c, the payment p^{i-1} to the previous buyer, and the risk that the next buyer does not arrive. As long as $\rho_i > 0 \ \forall i$, every buyer who arrives would earn a positive expected profit from trading, which is uncoupled from any buyer's consumption value of the traded item. In anticipation of such speculative bubbles in the resale market, buyers would have a potentially unlimited willingness to pay to acquire the object in the initial mechanism. We need to rule out this possibility, and the simplest way of doing so is by assuming the object can be resold only once.

The two remaining assumptions are A3, which serves to put a lower bound on a buyer's effective valuation, and A4, which serves to put an upper bound on on a buyer's effective valuation. With a large number of buyers these two bounds lie very close together, and our characterization of the outcome of the initial selling mechanism for large $\mathcal{N}$ all stem from this fact. Therefore, these two assumptions are central for our results.

The first part of assumption A3 is uncontroversial: It allows the winner of the initial mechanism to consume the object if expected resale revenue is below the initial winner's private valuation. The second part of A3—the ability to make take-it-or-leave-it offers—is the primary source of asymmetry between the original seller and resellers in our model. Note that we are not requiring resellers to actually sell via take-it-or-leave-it offers; we only assume that such offers *can* be made. With an infinite stream of potential buyers, an optimal individually rational resale mechanism is, in fact, a sequence of take-it-or-leave-it offers at price $\hat{r}$. However, this is not generally the case if $\mathcal{N} < \infty$, and resellers may well prefer to use other mechanisms, if available (e.g., auctions with buyer-specific or time-specific reserve prices). The ability to make take-it-or-leave-it offers only implies that any mechanism the reseller actually uses generates at least the same revenue in expectation as a sequence of take-it-or-leave-it offers would.

Assumption A4 imposes two individual rationality constraints on the reseller. The first is an *ex ante* constraint, which implies that a buyer who enters receives, on expectation, sufficient surplus from the mechanism to cover the entry cost c. This, in turn, limits the revenue the reseller can extract from each buyer—in particular, the proof of Lemma 1 depends only on the *ex ante* constraint. The second constraint is an interim individual rationality constraint, which means that the buyer can walk away from the mechanism at no cost (other than the entry cost c) after learning his valuation. Interim individual rationality precludes pre-entry contracts between resellers and potential buyers that provide the buyer with enough surplus on expectation to induce entry, but oblige the buyer to receive a negative surplus for some realizations of v_i.[8] The reason why we exclude such resale mechanisms from our model is purely technical: It implies that effective valuations are strictly monotone in private values (and not just weakly monotone; see the proof of Lemma 2). This guarantees that the auction and the BK/Fishman sequential mechanism allocate the object to the participating buyer with the highest private value, as they do in the no-resale case, which simplifies the analysis in Section 5.

5 Equilibrium in the Auction and Sequential Mechanism

In all environments we consider, agent i's strategy has three components: An entry decision in the initial mechanism, a bidding strategy in the initial mechanism, and a resale strategy. The latter includes a choice of resale mechanism if i wins the auction, and a choice on how to interact with any resale mechanism imposed by another reseller if i loses the auction. After having considered the resale stage in the previous section, we now focus on the entry and bidding strategies in the initial mechanism. In this section, we focus on the two mechanisms examined in BK: The second-price auction, and the Fishman/BK sequential mechanism.

5.1 Second-price auction

In the auction, a buyer's entry decision is conditioned on the number of buyers who have already entered when the buyer arrives, and his bid is conditioned on his private valuation and the total number of buyers who entered. Because bidding takes place only after all entry decisions have been made, we proceed by backward induction and consider the post-entry bidding stage first.

Suppose that, in the overall equilibrium, n^* buyers enter in the auction, if n^* or more buyers exist. Let n be the actual number of entrants. This number is observed by all buyers who enter. There are two cases to consider:

– If $n < n^*$ and $\rho_{n+1} < 1$, then each of the n participating buyers must infer that entry stopped because buyer $n+1$ does not exist. This means that all potential buyers who do exist participate in the auction, and as long as the auction allocates efficiently there can be no resale. It now follows that each buyer i should bid his private value v_i.

[8] For example, the mechanism examined in Cremer *et al.* (2009) does not satisfy interim individual rationality.

– If $n \geq n^*$, or if $n < n^*$ and $\rho_{n+1} = 1$, each participating buyer knows that, with probability ρ_{n+1} there is at least one additional buyer who has not entered.[9] Because all bidders now anticipate the possibility of being able to resell the object, their willingness to pay for the object will be equal to their effective valuations $z_n(\cdot)$. Because z_n is increasing, the auction still allocates to the bidder with the highest v_i. Thus, any future sale must be to some bidder $n' > n$, justifying that $z_n(v_i)$ is bidder i's willingness to pay in the initial mechanism.[10]

The above considerations lead to the following result:

Proposition 4. *Suppose that there is an overall equilibrium of the auction game in which the first n^* bidders enter, if they exist. Let n be the actual number of bidders who participate in the auction. Then, at the subgame in which bidding commences, the following is Bayesian Nash equilibrium in bidding strategies for $i = 1, \ldots, n$:*

$$b_i^{AU}(v_i) = \begin{cases} v_i & \text{if } n < n^* \text{ and } \rho_{n+1} < 1, \\ z_n(v_i) & \text{otherwise,} \end{cases}$$

where z_n is defined in (2).

Now turn to the entry stage. Again, focus on an equilibrium in which n^* bidders enter if they exist. If n is the actual number of entrants, and these entrants bid as described in Proposition 4, the expected surplus each of them receives is

$$\pi^{AU}(n, n^*) = \begin{cases} \displaystyle\int_0^{\overline{v}} \int_0^{v} v - w \, dF(w)^{n-1} dF(v) - c & \text{if } n < n^*, \\ \displaystyle\int_0^{\overline{v}} \int_0^{v} z_n(v) - z_n(w) dF(w)^{n-1} dF(v) - c & \text{if } n \geq n^*. \end{cases} \tag{3}$$

[9] If $n = n^*$ the equilibrium subgame has been reached. On the other hand, if $n > n^*$ or if $n < n^*$ and $\rho_{n+1} = 1$ an off-equilibrium subgame has been reached. However, in all cases the probability that an additional buyer exists who does not participate in the auction is ρ_{n+1}.

[10] The formal proof is slightly more complicated than this intuition, however, because bidding one's effective valuation is not a weakly dominant strategy. For example, suppose $\rho_i = 0$ ($i \geq 4$) and $n = 3$. In this case, $z_n(v_i) = v_i$. Assume further that buyers 1 and 2 bid as follows: $b_1(v_1) = 0$ and $b_2(v_2) = v_2$. Then bidder 3 maximizes his payoff by bidding strictly more than v_3. The reason is the following: If 3 wins the object he can make buyer 1 a take-it-or-leave it offer at some price $p \in (v_3, \overline{v})$. If the offer is rejected, 3 consumes the object and gets v_3. If it is accepted (which it will be with positive probability), 3 sells the object and gets $p > v_3$. Thus, given the bidding strategies by buyer 1, buyer 3's willingness to pay for the object is strictly larger than v_3. Given buyer 2's strategy and the fact that the auction is second-price, 3 should bid this willingness to pay. Therefore, for all $v_3 < \overline{v}$, buyer 3's best response to b_{-3} is $b_3(v_3) > v_3 = z_3(v_3)$.

When bidder n^* arrives, he compares $\pi^{AU}(n^*, n^*)$ to the expected "outside payoff" from not entering. Because z_n is bounded for all n, $\pi^{AU}(n^*, n^*)$ must be negative for all large enough n^*. On the other hand, the outside payoff is at least zero (but may be positive if there is a positive probability that a bidder who does not enter purchases the good on the resale market later). Thus, we choose n^* to be the largest integer such that $\pi^{AU}(n^*, n^*)$ is at least equal to the outside payoff. It is then an overall equilibrium of the auction game in which the first n^* bidders enter (if they exist) and then use the bidding strategy b_i^{AU} described in Proposition 4.[11]

The following result shows that, if the set of resale opportunities is large in the sense defined in Section 4, then only one bidder enters in the equilibrium.

Proposition 5. *Fix F and c. Assume the initial seller uses a second-price auction and entrants use the bidding strategies in Proposition 4. If $\mathcal{N}$ is sufficiently large, the equilibrium number of entrants in the auction is $n^* = 1$ and the initial seller receives a zero price.*

Thus, if the original seller uses the auction and there are a sufficiently many resale opportunities available to the winning bidder, the original seller will receive a zero price. In other words, the auction is the worst possible way to sell the object.

5.2 Fishman/BK sequential mechanism

In the sequential mechanism, entry and bidding decisions are intertwined: Buyer i's entry decision depends on the bid history prior to i's arrival (or, if the seller does not reveal the full history, it depends on the current price), and i's bidding strategy, including the decision of whether to place any discrete jump bids, will affect the entry decisions of future buyers.

It will be helpful to first review the equilibria of the sequential mechanism when there is no resale.

Recap of the equilibrium without resale. BK showed that the sequential mechanism has equilibria of the following form: Bidders enter at low enough prices. After entering,

[11] To confirm that bidder $n < n^*$ (if he exists) has an incentive to enter, note that if he enters and entry stops at n^* bidders, n receives $\pi^{AU}(n^*, n^*)$, which is at least as large as the payoff from not entering (by definition of n^*). If n enters and entry stops at n' bidders with $n \le n' < n^*$, there can be no resale market and the expected surplus from not entering is zero. In this event, n receives an expected surplus of $\pi^{AU}(n'|n^*)$ in the auction, which (using Lemma 2 (iv) and $n' < n^*$) is larger than $\pi^{AU}(n^*|n^*)$, which in turn is at least zero. Thus, if n^* has an incentive to enter, $n < n^*$ must have an incentive to enter as well.

a bidder whose valuation is below some cutoff v^* raises the price against his rivals (i.e., against the previous high bidder and, should he prevail, against future entrants) until there is no more entry and he wins, or until the price reaches his valuation and he leaves. Such bidders never place jump bids. A bidder with valuation v^* or higher bids in the same way first, but when he becomes the high bidder he places an additional jump bid that deters all further entry. Multiple equilibria of this form exist that differ in their threshold v^*. The largest cutoff value is $\hat{r}$ ($= V_K$ in BK), and the smallest is a value V_S that satisfies the condition

$$\frac{1}{1-F(V_S)} \int_{V_S}^{\overline{v}} \int_{V_S}^{v} v - w \, dF(w) dF(v) - c \ = \ 0. \tag{4}$$

To understand (4), note that if the current high-bidder's value was known to be at least V_S, then paying the entry cost to compete against this bidder yields exactly a zero surplus. A jump bid signals that the current bidder's valuation is at least v^*, and since $v^* \geq V_S$ entry is deterred. The precise value of the entry-deterring jump bid is determined by the requirement that only bidders above v^* want to use this costly signal.[12]

BK show that only the $v^* = V_S$ equilibrium satisfies a simple forward induction criterion—in the other equilibria, buyers with valuations in $[v^*, \overline{v}]$ can deviate and place lower jump bids that, if interpreted "correctly" by potential entrants, also deter entry and give the deviating player a strictly larger payoff. (We discuss this criterion in more detail at the end of Section 5.2.)

If resale is possible, the basic logic of entry deterrence through pooling on jump bids continues to hold, but the characterization of the lower-bound entry threshold (V_S) becomes tedious. The reason is that this threshold now depends on the period to which it applies. This is because, unlike the private value v_i, a bidder's effective valuation $z_n(v_i)$ is time-dependent.[13]

We steer clear of these complications by proving an asymptotic result: If the number of potential buyers is large, then every $v^* \in [0, \hat{r}]$ is an entry-deterring threshold in the initial stage of the mechanism. As in BK, a bidder with valuation above v^* signals this fact via an appropriately chosen jump bid. However, since every bidder's valuation is

[12]Because the expected payoff from *not* using the signal (i.e., continuing to compete in the mechanism) depends on how high the current auction price is, the value of the entry-deterring jump bid changes as the sequential mechanism progresses, even if the threshold v^* stays constant.

[13]That is, the function z_n is generally not the same as the function $z_{n'}$. The exception is the case in which new buyers arrive with a constant probability in every period (i.e., $\rho_3 = \rho_4 = \ldots$). In this case, the z_n-functions are the same for all n, and the lower bound V_S becomes stationary.

already known to be at least zero, in the $v^* = 0$ equilibrium the first bidder deters all further entry by making a zero jump bid. We show that this is the unique equilibrium that survives BK's refinement.

We begin by establishing a condition under which the sequential mechanism ends, which is independent of the equilibrium being played:

Lemma 6. *Fix F and c. Assume the initial seller uses the Fishman/BK sequential mechanism. Consider any stage n of the mechanism. If $\mathcal{N}$ is sufficiently large, the following holds in every equilibrium of the mechanism: If at the end of stage n of the mechanism the price is $p_n \geq z_n(0)$, and the distribution of the current incumbent's private valuation is $F(v|v \geq y)$ for some $y \in [0, \overline{v}]$, then potential entrant $n+1$ does not enter.*

Lemma 6 can now be used to prove that, for large $\mathcal{N}$, an equilibrium exists in which the entry-deterrence threshold $v^* = 0$. Suppose bidder 1 enters the sequential mechanism but does not submit a jump bid (or, equivalently, submits a jump bid of zero). Then, if potential buyer 2 enters at stage 2, he competes against bidder 1 in an ascending auction. Assume, for a moment, that buyer 3 does *not* enter at the third stage. This means that either bidder 1 or bidder 2 wins the object and can then resell it. Thus, bidders 1 and 2 should raise the price until it reaches $z_2(v_1)$ or $z_2(v_2)$, whichever comes first. If there is no further jump bid, the price at the end of stage 2 equals $p_2 = \min\{z_2(v_1), z_2(v_2)\} \geq z_2(0)$, and the new incumbent's valuation is at least $y = z_2^{-1}(p_2)$. Provided $\mathcal{N}$ is sufficiently large, Lemma 6 implies that bidder 3 does not enter, as was hypothesized. Therefore, the expected surplus that new entrant 2 achieves by entering against incumbent 1

$$\pi^{SM}(2) \;=\; \int_0^{\overline{v}} \int_0^{v} z_2(v_2) - z_2(v_1)\, dF(v_2) dF(v_1) - c \;=\; \pi^{AU}(2,2). \qquad (5)$$

Also for $\mathcal{N}$ sufficiently large, Proposition 5 implies that $\pi^{AU}(2,2) < 0$. In this case, an equilibrium of the sequential mechanism exists in which bidder 1 enters, does not bid, and no further bidders enter. Finally, if bidder 1 were to submit a positive jump bid, then as long as bidder 2 believes that all types of bidder 1 are equally likely to have made that mistakes, 2 expected payoff from entering would be even smaller and he would still prefer not to enter.

This establishes the $v^* = 0$ equilibrium when the number of potential buyers is large enough. It can be shown that, if $v^* = 0$ is an equilibrium cutoff, then all $0 < v^* \leq \hat{r}$ are *also* equilibrium cutoffs. The following Proposition 7 contains this result.

Proposition 7. *Fix F and c. Assume the initial seller uses the Fishman/BK sequential mechanism. If $\mathcal{N}$ is sufficiently large, the following holds: For every $v^* \in [0, \hat{r}]$ there exists an equilibrium of the sequential mechanism in which bidder 1 deters entry by bidder 2 if and only if $v_1 \geq v^*$. The initial seller's expected revenue is positive in all equilibria except the $v^* = 0$ equilibrium. This equilibrium is the only one that is perfect sequential in the sense used by BK.*

Equilibria with $v^* > 0$ involve positive jump bids and, thus, positive seller revenue. Moreover, as is shown in the proof of Proposition 7 (see footnote 21), these jump bids can be very high: For equilibria with a small but positive v^*, expected seller revenue is approximately $\hat{r}$ when $\mathcal{N}$ is large. A discontinuity occurs as one passes to the case $v^* = 0$, where revenue is zero. Thus, when resale is possible and the expected number of potential buyers is large, the initial seller may want to use the sequential mechanism in the hope that one of the positive-revenue equilibria will be played.

However, only the $v^* = 0$ equilibrium involves reasonable beliefs after certain deviations, using the same forward induction criterion (*perfect sequentiality*) that selects the V_S-equilibrium in the no-resale case that is examined in BK. To understand this refinement, suppose that $\mathcal{N}$ is large enough to generate a range of equilibria with entry deterrence thresholds $v^* \in [0, \hat{r}]$, and consider an equilibrium with threshold $v^* > 0$. In this equilibrium, buyer 1 deters entry if $v_1 \geq v^*$ and accommodates entry otherwise. In order to deter entry, buyer 1 places jump bid $j_1^* > 0$, which is chosen to make the marginal type $v_1 = v^*$ indifferent between accommodating and deterring entry. To support this strategy as part of an equilibrium, buyer 2 must enter if buyer 1 were to place some positive jump bid $j_1' < j_1^*$ (otherwise, buyer 1 would not bid j_1^* to deter entry). Buyer 2's entry, in turn, can only be part of an equilibrium if, after observing j_1', buyer 2's off-equilibrium beliefs place enough weight on low values for v_1 to make entry optimal. We will argue that such beliefs are not forward induction-proof.

Imagine that bidder 1 actually did bid $j_1' \in (0, j_1^*)$. Given the equilibrium response by buyer 2 explained above, this bid is clearly a mistake for all types of buyer 1. Put differently, for j_1' not to be a mistake, buyer 1 must believe that a bid of j_1' would deter entry. The types of buyer 1 for whom a bid of j_1' would be preferable to their equilibrium bid *if it deterred entry* are buyers whose valuations are above some threshold, say, $v_1(j_1') < v^*$. Thus, if player 2 were to attempt to explain the unexpected bid j_1' as optimal behavior on part of buyer 1, he must conclude that $v_1 \geq v_1(j_1')$. Would he then still want to enter? The answer is no: We know that buyer 2 does not want to enter if buyer 1 made a zero jump bid and is believed to have value $v_1 \geq 0$ (this follows from the fact that the $v^* = 0$ equilibrium exists), so 2 should not prefer to enter if 1 made a positive jump bid j_1' and is believed to have value $v_1 \geq v_1(j_1') \geq 0$. Thus, all types of buyer 1 with $v_1 \geq v^*$ (as well as some with $v_1 < v^*$) should deviate and place a jump bid that is positive but smaller than the entry-deterring jump bid j_1^*. If the deviation is interpreted "correctly" by buyer 2, it would also deter entry, and would do so at a lower cost than the equilibrium bid j_1^*.

The only equilibrium in which such a deviation does not exist is the $v^* = 0$ equilibrium. If the $v^* = 0$ equilibrium is selected, the sequential mechanism fails to generate any revenue.

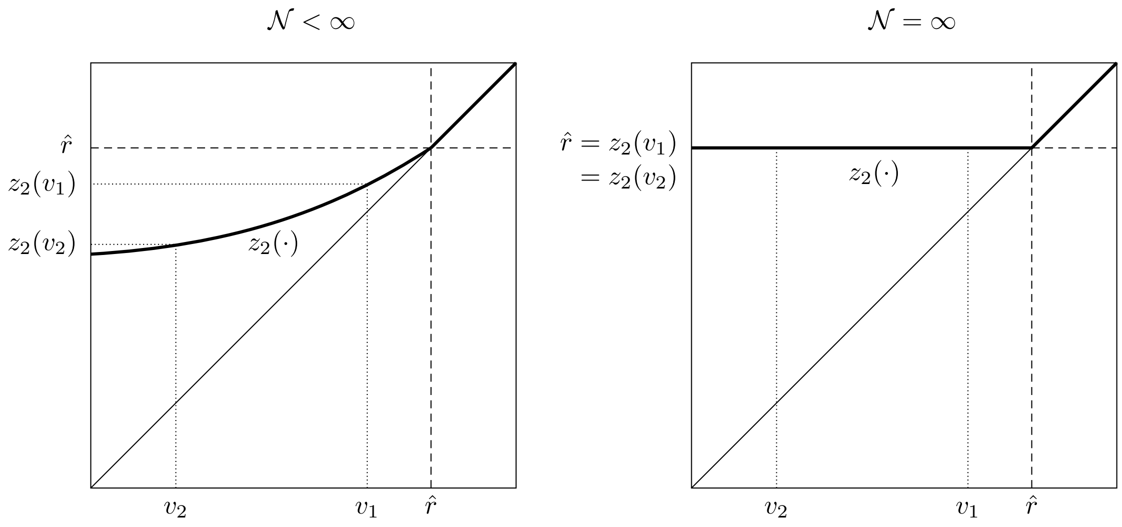
5.3 Intuition and numerical example

We showed that only one bidder enters in the equilibrium of both the auction and the sequential mechanism if resale is allowed and the expected number of bidders in the market is sufficiently large. To understand why, note that the introduction of a resale market has two consequences.

The first consequence is that even if a buyer draws a low private value, he can still resell the item if he wins it. Thus, every buyer's willingness to pay for the object at the initial selling stage (i.e., the buyer's effective valuation) increases. This does not necessarily translate into increased revenue for the initial seller, however, because buyers enter the initial mechanism only if they expect to earn a sufficiently high surplus from participating. The surplus the winning buyer receives in both the auction and the sequential mechanism is an information rent. A buyer's expected information rent may be small even if his expected willingness to pay is large, which happens, in particular, when *every* buyer's willingness to pay is large. The second consequence of having a resale market is precisely that it compresses the distribution of buyers' effective valuations: Every buyer has access to the same resale market; therefore, the resale profit one buyer can achieve is the same as the resale profit another buyer can achieve.

Figure 1 illustrates this effect. Suppose two buyers participate in the initial mechanism and buyers 1 and 2 are the buyers with the highest and second highest private valuation, respectively. Without resale, buyer 1's information rent in the initial mechanism would be the difference $v_1 - v_2$. The left diagram in Figure 1 shows the effective valuation function $z_n(\cdot)$ when resale is allowed: For private values less than $\hat{r}$, the effective valuation function is above the 45°-line (the first effect of resale) and the slope of the effective

Figure 1: Resale and compression of effective valuations.



valuation function is strictly less than one (the second effect of resale). Thus, the winner's information rent $z_n(v_1) - z_n(v_2)$ is strictly less than what it would be without resale. If $\mathcal{N}$ increases, effective valuations increase and compress more, and as $\mathcal{N} \to \infty$ the z_2-function approaches the limit shown in the right diagram (this is formally stated in Lemma 3). For $\mathcal{N}$ large enough, a buyer's expected information rent will fall below the entry cost c. When this happens only a single buyer will enter, and the initial seller's revenue drops to zero.

Our results did not answer the question *how* large the resale market needs to be for only one buyer to enter at the initial selling stage. We now compute this threshold in an example. We demonstrate that even for moderate $\mathcal{N}$, entry by all but a single buyer can be deterred in the equilibrium of the initial mechanism.

Example 1. We assume that $\rho_n = \alpha < 1$ for $n \geq 3$. The set of available resale mechanisms consists of take-it-or-leave-it offers. We consider three different distributions of private values on the unit interval: The uniform distribution ($F(v) = v$), and two "triangular" distributions where low values are more likely ($F(v) = 2v - v^2$) or high values are more likely ($F(v) = v^2$). Moreover, we consider four different values for the entry cost: $c = 0.1$, $c = 0.075$, $c = 0.05$, and $c = 0.025$.

Note that, once the first two buyers have arrived, the resale environment becomes stationary: If r is an optimal posted price offer in period $n \geq 3$ and the offer is not accepted by buyer n, the reseller makes the same posted price offer r in the next period to buyer $n+1$. If a reseller with private valuation v posts price r in every period $n \geq 3$, he obtains the following expected payoff (not counting his own entry cost and payment to the original seller):

$$(1-\alpha)v \;+\; \alpha\Big[\big(1-F(r)\big)r \;+\; F(r)\Big[(1-\alpha)v \;+\; \alpha\Big[\big(1-F(r)\big)r \;+\; \ldots$$
$$= \frac{(1-\alpha)v \;+\; \alpha\big(1-F(r)\big)r}{1-\alpha F(r)},$$

so that the effective valuation function for $n \geq 3$ is

$$z_n(v) \;=\; \begin{cases} \max\limits_r \dfrac{(1-\alpha)v \;+\; \alpha\big(1-F(r)\big)r}{1-\alpha F(r)} & \text{if } v < \hat{r}, \\ v & \text{if } v \geq \hat{r}. \end{cases} \tag{6}$$

If only one buyer enters the initial mechanism, he can resell the object in period $n=2$ already, and he knows that buyer 2 exists with probability 1. Thus, the effective valuation for $n=2$ is

$$z_2(v) \;=\; \begin{cases} \max\limits_r \big(1-F(r)\big)r + F(r)z_3(v) & \text{if } v < \hat{r}, \\ v & \text{if } v \geq \hat{r}. \end{cases} \tag{7}$$

Given F, α, and c, one can obtain a numerical solution for the effective valuation functions as well as the optimal posted prices in each period. We use these solutions to compute the second buyer's expected payoff from entering the initial mechanism, as well as the expected payoff from not entering the initial mechanism and instead entering the resale market if and only if the first buyer offers the item for resale. For all twelve possible combinations of F and c, Table 1 below shows the values of α (and thus $\mathcal{N}$) for which the payoff from not entering just exceeds the payoff from entering. Note that in all cases shown in the table, at least two bidders would enter in the auction if resale was not allowed.

For example, consider the uniform values case $F(v) = v$ and assume an entry cost of $c = 0.1$. If $\alpha > .8325$—or if at least 6.97 are in the market on average—only one bidder enters in the auction or sequential mechanism. If resale was not allowed, one can show that three bidders would have entered in the auction (assuming that the third bidder exists). For the two non-uniform distributions, the thresholds for α and $\mathcal{N}$ are (with two exceptions) lower than for the uniform distributions. The reason is that the uniform

Table 1: Values for α and $\mathcal{N}$ such that only one bidder enters the initial auction/sequential mechanism in Example 1.

		Entry cost: $c =$			
Distribution:		0.100	0.075	0.050	0.025
$F(v) = v$	$\alpha >$	.8325	.9240	.9720	.9942
	$\mathcal{N} >$	6.97	14.16	36.71	173.41
$F(v) = 2v - v^2$	$\alpha >$	.7470	.9075	.9725	.9958
	$\mathcal{N} >$	4.95	11.81	37.50	239.10
$F(v) = v^2$	$\alpha >$	.6079	.8262	.9363	.9866
	$\mathcal{N} >$	3.55	6.75	16.70	75.63

distribution is the maximum entropy distribution over a given interval. This means that a buyer's private information is, on expectation, most valuable, and the resulting expected information rent highest, in the uniform case. Thus, it is generally less difficult to deter entry in the non-uniform case, where information rents are lower to begin with due to the prior distribution being more informative.[14]

6 The Fully Sequential Mechanism

We will now show that a simple variant of the sequential mechanism, the *fully sequential mechanism*, can generate positive revenue for the original seller in cases where both the auction and the sequential mechanism fail to attract more than one entering bidder. Moreover, when the number of potential buyers is large, the fully sequential mechanism is approximately optimal. At the same time, the fully sequential mechanism does not use a reserve price, minimum bid, posted price, entry subsidy, or the like. It relies solely on competition between the buyers to generate revenue, but organizes this competition in a

[14]There is, however, a countervailing effect that depends critically on the possibility of resale. When buyers are likely to have low private valuations (e.g., if $F(v) = 2v - v^2$), the resale market is less profitable, and the winner of the initial mechanism becomes more inclined to consume the object instead of reselling it. This effect slows down the rate at which effective valuations become compressed when α increases. It is then possible that entry is deterred for larger α-values than in the uniform case, even though information rents are higher in the uniform case without resale. In Table 1, this happens in the last two columns for $F(v) = 2v - v^2$.

way that leaves a positive (if small) expected information rent to every buyer who does enter.

For simplicity, we consider here a very simple mechanism in which the original seller involves only two buyers. The seller first invites the first buyer to make an offer, j_1. If this buyer does not enter, the seller retains the object. If buyer 1 enters and bids j_1, the offer is publicly announced and the second buyer is invited to make an offer, j_2. If the second buyer enters and submits offer $j_2 \geq j_1$, he wins the object and pays j_2.[15] If the second buyer does not enter, or if he enters and submits offer $j_2 < j_1$, the first buyer wins and pays j_1. However the first buyer has no opportunity to come back and react to the second buyer's offer (if one is made), or to compete with the second buyer head-to-head, as would be the case in the Fishman/BK mechanism.

6.1 Equilibrium entry and bidding strategies

Strategies are simple: Bidder 1 must make an entry decision and, after entry, a bidding decision $j_1(v_1)$ that is a function of his valuation. Buyer 2 must make an entry decision that can be conditioned on buyer 1's bid, and a bidding decision $j_2(j_1, v_2)$ that is conditioned on buyer 1's bid and buyer 2's valuation.

One complication, however, is that the fully sequential mechanism does not always allocate efficiently among the first two bidders. In particular, buyer 2 may win the object even though he has a lesser private valuation than buyer 1. To see this, suppose that the allocation was efficient, so that any resale can only be to buyers $i \geq 3$. The effective valuations of the first two buyers, therefore, are given by $z_2(v_1)$ and $z_2(v_2)$. Since buyer 1 must pay his bid if he wins, he will generally want to bid $j_1 < z_2(v_1)$. If 2 enters and draws a valuation such that $z_2(v_2) \in (j_1, z_2(v_1))$, 2 can win with bid $j_2 = j_1$ (or $j_2 = j_1 + \varepsilon$ for small $\varepsilon > 0$). In this case, the object is awarded to the buyer with the smaller private and effective valuation, a contradiction.

To deal with this complication, define $\widetilde{z}_2(v_2, j_1)$ as the effective valuation of buyer 2 whose private value is v_2 and who can resell to potential buyers $3, 4, \ldots$, who have yet to arrive, *as well as* to buyer 1, who already entered and submitted bid j_1. Since buyer 2 can choose to not deal with buyer 1, $\widetilde{z}_2(v_2, j_1) \geq z_2(v_2)$. We then have the following result:

[15] The only reason we assume that buyer 2 wins if he offers at least the amount buyer 1 offered (instead of strictly more) is that in this case buyer 2 always has a well-defined best response to 1's offer. The assumption could easily be relaxed, either by permitting almost-best responses in equilibrium or by requiring offers to be integer multiples of some small currency unit.

Proposition 8. *Assume the initial seller uses the fully sequential mechanism in which only the first two buyers get to participate. The following holds in every subgame perfect equilibrium of the initial mechanism (regardless of F, c, and $\mathcal{N}$):*

(i) Buyer 1 enters and bids some amount $j_1(v_1) \in (z_2(0), \hat{r}]$ whenever $v_1 > 0$.

(ii) Buyer 2 enters if and only if $j_1 < \hat{r}$, and if he enters he bids

$$j_2(j_1, v_2) = \begin{cases} j_1 & \text{if } \widetilde{z}_2(v_2, j_1) > j_1, \\ 0 & \text{otherwise.} \end{cases}$$

The initial seller's revenue is at least $z_2(0)$, and is approximately $\hat{r}$ for large $\mathcal{N}$.

To prove the result, consider buyer 2's decision first. Suppose that $j_1 < \hat{r}$ and suppose 2 enters. If $\widetilde{z}_2(v_2, j_1) > j_1$, buyer 2 has two options:

– If he wants to buy the object in the initial mechanism, he can do so by bidding $j_2 = j_1$.

– If he wants to wait and see if buyer 1 attempts to resell the object, he can do so by bidding zero. But buyer 1 would not have made an offer to buy the object for j_1 if his resale strategy was to resell the object for an expected price of j_1 or less. Thus, conditional on buying the object on the resale market from buyer 1, buyer 2 must expect to pay strictly more than j_1.

Thus, if $\widetilde{z}_2(v_2, j_1) > j_1$, the first option is better and buyer 2 buys the object by submitting bid $j_2 = j_1$. Similarly, if $\widetilde{z}_2(v_2, j_1) \leq j_1$, the best option for buyer 2 is to not bid. Since $j_1 < \hat{r}$ and $\widetilde{z}_2(v_2, j_1) \geq z_2(v_2) \geq v_2$, by definition of $\hat{r}$ the first case ($\widetilde{z}_2(v_2, j_1) > j_1$) is sufficiently likely for the expected net surplus from entering to be positive. It follows buyer 2 enters if $j_1 < \hat{r}$. On the other hand, if $j_1 \geq \hat{r}$, then by definition of $\hat{r}$ buyer 2 cannot profitably enter.

Now consider 1's decision, anticipating 2's strategy. Note first that buyer 1 enters. If he does not enter, there is no resale market and 1's payoff is zero. If he enters, he can obtain a positive net surplus with the following bidding strategy:

$$j_1 = \begin{cases} \hat{r} & \text{if } v_1 \geq \hat{r}, \\ \dfrac{z_2(0) + z_2(v_1)}{2} & \text{otherwise.} \end{cases} \tag{8}$$

If $v_1 \geq \hat{r}$ and buyer 1 bids $\hat{r}$, then buyer 2 does not enter and 1 wins for a payment of $\hat{r}$. By definition of $\hat{r}$, this possibility is sufficient to yield exactly a zero net profit from entering. If $v_1 < \hat{r}$, then the offer $[z_2(0) + z_2(v_2)]/2$ induces entry by buyer 2; however, buyer 1 still wins with positive probability and, if he wins, pays a price that is strictly less than his effective valuation. Thus, strategy (8) generates an expected positive net surplus from entering, and it follows that buyer 1 enters.

Finally, conditional on buyer 1 entering, we observe that buyer 1 never bids $j_1 < z_2(0)$, for any v_1. The reason is that buyer 2 will enter and, with probability one, will win. Buyer 1 can improve his payoff in this case if he offered instead to pay $j_1' = [z_2(0) + z_2(v_1)]/2$, in which case buyer 2 would enters but 1 would still win with positive probability and, if he wins, pays a price that is strictly less than his effective valuation. At the same time, buyer 1 also never bids $j_1 > \hat{r}$. The reason is that any bid of at least $\hat{r}$ deters entry by bidder 2, and since buyer 1 must pay his bid, any offer $j_1 > \hat{r}$ is strictly dominated by $j_1' = \hat{r}$. □

6.2 Remarks

We showed that the fully sequential mechanism generates higher expected revenue than either the auction and sequential mechanism, provided the resale market is sufficiently large. This result should not be confused with BK's finding (for the no-resale case) that the sequential mechanism may generate more revenue than the auction if the set of potential buyers is large. Without resale, the potential advantage of dealing with buyers sequentially is that it permits the flexibility to generate additional entry if previous buyers have low valuations. This effect is mechanical and translates into an *expected* revenue advantage over the auction only under contrived assumptions about the value distribution and the entry cost (see BK, p. 1558). If resale is possible and there is a large stream of potential buyers, the fully sequential mechanism beats the auction for strategic reasons, and for all value distributions and all entry costs.[16]

The sequential mechanism requires that the initial seller commit to not give any buyer the opportunity to revise an offer once a better offer by a competing buyer is received. There are several ways in which such commitment can be achieved in practice. The seller may try to develop a reputation for not renegotiating previous offers. If the seller is a

[16]Moreover, when the set of potential buyers is small, the auction is always better if resale is not possible. If resale is possible, the sequential mechanism may be preferred; however, this now requires special assumptions about the value distribution and the entry cost (see Section 7 for an example).

large organization, it may explicitly instruct its agents to only deal with one buyer at a time; or it could limit the number of agents authorized to negotiate with buyers, which makes repeated bargaining with the same buyers less likely.

Notwithstanding the commitment requirement, the fully sequential mechanism has several practical advantages over alternative selling mechanisms. Note that the initial seller could decide to use either the auction or the sequential mechanism, but subsidize entry to ensure that at least two buyers participate. However, in order to compute the entry subsidy the initial seller would have to know at least the value of the entry cost c, and possibly also the distribution of buyer valuations and their arrival probabilities. Thus, the resulting mechanism would not satisfy the Wilson doctrine (see footnote 2). The fully sequential mechanism, on the other hand, does not require knowledge of these parameters. The fully sequential mechanism is also robust in another sense: It yields at least revenue $z_2(0)$, which is the pure resale value of the item. Thus, unless the number of buyers is exactly two, the mechanism generates positive revenue for the initial seller regardless of $\mathcal{N}$. While the auction and the sequential mechanism may generate higher revenue, they run the risk of not resulting in any revenue.

7 Conclusion

When entry into an auction is costly and the winning buyer can resell the item later, the resulting decrease in buyers' information rents reduces entry and can reduce the initial seller's revenue to zero. The same is true if the initial seller uses the sequential mechanism described in BK. However, we showed that the seller can extract the resale market profits asymptotically if she uses a sequential mechanism that allows buyers to make jump bids but in which any direct head-to-head competition between buyers is removed. We emphasize again that this surplus extraction does not require the use of reserve prices, take-it-or-leave-it-offers, etc. by the initial seller, as long as resellers can use these tools.

We conclude this paper with two comments on social efficiency in the presence of resale, and on the performance of auctions and sequential mechanisms with resale and a small number of bidders.

7.1 Efficiency

When resale is not allowed, BK (2009) showed that entry in both the auction and the sequential mechanism is socially efficient, conditional on the information available in each entry period. In other words, a social planner who had the same information as a potential buyer would have the buyer enter if and only if entry is privately optimal in the respective mechanism. This follows from the fact that, in both mechanisms, an additional buyer's marginal contribution to social welfare is the expected amount by which the buyer's valuation exceeds the highest valuation of existing buyers, minus the buyer's cost of entry. This is exactly equal to the buyer's private net gain from entering a mechanism in which the winner pays the second-highest valuation. However, the sequential mechanism yields higher expected social welfare than the auction, because it provides more information on which to condition entry.

When resale is allowed, these considerations change. In both mechanisms, prospective buyers base their entry decision on the expected amount that their effective valuation exceeds the highest effective valuation of the other buyers. Because effective valuations are more compressed than the buyers' private values, there is generally less entry in the initial mechanism than if resale was not possible. This, in itself, does not mean that entry is inefficiently low, because entry that is prevented at the initial selling stage may still occur at the resale stage. However, differences in effective valuations do not necessarily equal marginal contributions to social welfare, because part of each buyer's effective valuation is the expected revenue the buyer hopes to earn in the resale market, using a generally inefficient resale mechanism. Thus, the private decision to enter will not generally be socially optimal.

If the resale market is large, however, the fact that resellers may use inefficient resale mechanisms becomes unimportant. Suppose there is an infinite stream of potential buyers, so that only buyer 1 participates at the initial selling stage in all three mechanisms we examine. Lemma 1 then implies that it is optimal for this buyer to consume the object if $v_1 \geq \hat{r}$, or, if $v_1 < \hat{r}$, resell it by making each subsequent buyer a take-it-or-leave-it offer $\hat{r}$. Expected social welfare in this case is

$$\begin{aligned} & -c + \int_{\hat{r}}^{\overline{v}} v_1 dF(v_1) + F(\hat{r}) \Bigg[-c + \int_{\hat{r}}^{\overline{v}} v_2 dF(v_2) + F(\hat{r}) \Bigg[-c + \int_{\hat{r}}^{\overline{v}} v_3 dF(v_3) + F(\hat{r}) \Bigg[\ \ldots \\ & = \frac{1}{1 - F(\hat{r})} \Bigg[-c + \int_{\hat{r}}^{\overline{v}} v dF(v) \Bigg] \end{aligned}$$

$$= \frac{1}{1-F(\hat{r})}\Bigg[\underbrace{-c+\int_{\hat{r}}^{\overline{v}}(v-\hat{r})dF(v)}_{=0 \text{ due to (1)}}+\big(1-F(\hat{r})\big)\hat{r}\Bigg] = \hat{r}.$$

This is, in fact, equal to the maximal social welfare our environment permits:

Lemma 9. *The social welfare that can be achieved, on expectation, by a planner who has the ability to dictate allocations as well as entry decisions for each arriving bidder, and who learns each entering buyer's private value immediately after the buyer enters, is at most $\hat{r}$.*

In other words, while selling an object using take-it-or-leave-it offers (or some other form of reserve price mechanism) is not generally efficient, it is efficient as well as privately optimal if there are infinitely many buyers and if only one offer is made at a time.

Lemma 9 is a corollary to Lemma 1, which states that no selling mechanism can generate more than $\hat{r}$ in expected revenue.[17] Note that the same bound $\hat{r}$ is also the (approximate) payment the winning bidder makes in the fully sequential mechanism, assuming $\mathcal{N}$ is large. Thus, with a large resale market, the auction, sequential mechanism, and fully sequential mechanism differ only in their distributional implications: While the auction and the sequential mechanism allocate the entire social surplus to their respective winners, the fully sequential mechanism transfers the entire social surplus to the mechanism owner.

7.2 Small number of bidders

Our results apply to the case of a sufficiently large set of potential bidders, in the sense formalized in Section 4. Depending on the parameter values, the threshold on resale market size at which both the auction and the BK/Fishman sequential mechanism fail to generate entry by more than one buyer need not be very high: In Example 1, the expected number of bidders above which entry by the second buyer is prevented was as low as 3.55.[18]

[17] In the proof of Lemma 1, we showed that any *ex ante* individually rational (but not necessarily incentive compatible) direct selling mechanism can generate at most expected revenue $\hat{r}$. The planner's problem of maximizing expected social welfare is mathematically equivalent to that of maximizing the expected revenue of such a mechanism.

[18] We also computed the entry deterrence thresholds for the case where repeated resale is possible (which we assumed away in our theoretical analysis for tractability). The entry deterrence thresholds are then slightly lower than those reported in Table 1.

If the parameters are such that more than one bidder enters in the auction or the Fishman/BK sequential mechanism, the initial seller's revenue will be positive in both mechanisms. This will often make the auction more profitable for the initial seller, for the same reasons as those explored in BK. Nevertheless, if resale is allowed it is possible that the Fishman/BK sequential mechanism dominates the auction in situations where the auction does better if resale is not possible. Consider the following example:

Example 2. Suppose that there are exactly two bidders with values in $\{0, 1\}$ and that $Pr[v_i = 1] = \tau = 13/25$ for $i = 1, 2$. (Alternatively, assume a continuous approximation of this two-point distribution.) For bidders $i > 2$, assume that $\rho_i = 0$. The entry cost is $c = 1/6$ and the set of available resale mechanisms consists of take-it-or-leave-it offers.

Consider first the case of no resale. In the auction, both bidders enter and bid their private values, and the seller's expected revenue is $\tau^2 = 0.2704$. In the Fishman/BK sequential mechanism, the following is an ε-equilibrium for $\varepsilon \approx 0$ and positive: Bidder 1 enters and bids zero if $v_1 = 0$ and ε if $v_1 = 1$. If bidder 2 arrives and sees $p^1 = \varepsilon$, he knows that $v_1 = 1$ and therefore does not enter. If bidder 2 arrives and sees $p^1 = 0$, he know that $v_1 = 0$; bidder 2 then enters and bids ε to win the object. The seller's expected revenue in this scenario is approximately zero. Thus, the seller prefers the auction.

Now suppose resale is allowed. This is irrelevant in the auction, as both bidders enter and the auction allocates efficiently. Thus, expected revenue from the auction is still 0.2704. In the sequential mechanism, the following is now an equilibrium: Bidder 1 places a jump bid of $1 - c/[\tau(1 - \tau)] \approx 0.3323$, regardless of his value, and bidder 2 stays out. Bidder 1 wins and pays his bid. If $v_1 = 1$, bidder 1 keeps the object. If $v_1 = 0$, bidder 1 makes bidder 2 a take-it-or-leave-it offer to purchase at price $1 - c/\tau \approx 0.6795$. If bidder 2 receives such an offer, he enters the resale market, learns v_2, and buys the object if and only if $v_2 = 1$. The initial seller's revenue in the sequential mechanism is buyer 1's jump bid of 0.3323, which is higher than the expected revenue from the auction.

Appendix

Proof of Lemma 1

The part of Assumption 4 that matters for this result is *ex ante* individual rationality (EA-IR). Take any EA-IR resale mechanism that results in resale with probability q and has expected revenue R. By the revelation principle, without loss of generality we can assume that the mechanism is an EA-IR and incentive compatible (IC) direct revelation mechanism. We first show that a direct revelation mechanism exists that is EA-IR but not necessarily IC, is such that losing bidders pay exactly zero, sells with probability q, and generates revenue R if buyers report their valuations truthfully (Step 1 below). Second, we show that any mechanism that is EA-IR, is such that losing bidders pay exactly zero, and sells with probability q generates at most $q\hat{r}$ in expected revenue if buyers report their valuations truthfully (Step 2 below). The result that $R \leq q\hat{r}$ then follows.

Step 1. Consider a buyer j who arrives, observes information set I_j and then enters and learns valuation v_j. Denote by $q_j(I_j, v_j)$ the probability with which j wins the object conditional on his post-entry information (I_j, v_j), and let $m_j(I_j, v_j)$ be the expected payment j makes conditional on (I_j, v_j) (net of any entry fees charged, or entry subsidies paid, by the reseller). Since the mechanism is EA-IR, we have for all j and I_j

$$\int_0^{\overline{v}} q_j(I_j, v_j)v_j - m_j(I_j, v_j)dF(v_j) \;\geq\; c.$$

We will assume that $\int_0^{\overline{v}} q_j(I_j, v_j)dF(v_j) > 0$ for every buyer j who enters. This is without loss of generality: A buyer who expects a zero probability of winning would only enter if he paid $-c$ or less on expectation. We could then replace the mechanism with one in which this buyer pays zero and does not enter,[19] which would increase the reseller's expected revenue. Since our goal is to establish an upper bound on the reseller's revenue, we may assume that every buyer who does enter has a strictly positive probability of winning the object (conditional on the buyer's pre-entry information).

We now construct a new direct revelation mechanism with the same allocation rule as the first mechanism. The payment that bidder j makes in the new mechanism depends on (I_j, v_j) and whether j wins or loses, but not on any other variables, and is given by

$$x_j(I_j, v_j) \;=\; \begin{cases} 0 & \text{if } j \text{ loses,} \\ \displaystyle\int_0^{\overline{v}} m_j(I_j, t)dF(t) \Big/ \int_0^{\overline{v}} q_j(I_j, t)dF(t) & \text{if } j \text{ wins.} \end{cases}$$

[19]If it is necessary for the operation of the mechanism, the reseller can always "simulate" the presence of buyer j by drawing a random value v_j from F.

(This is well-defined because $\int_0^{\overline{v}} q_j(I_j, v_j)dF(v_j) > 0$ for every buyer j who enters.) Conditional on the pre-entry information I_j, a bidders expected payoff from entering is

$$\begin{aligned}\int_0^{\overline{v}} q_j(I_j, v_j)\left[v_j - x_j(I_j, v_j)\right] dF(v_j) &= \int_0^{\overline{v}} q_j(I_j, v_j)\left[v_j - \frac{\int_0^{\overline{v}} m_j(I_j, t)dF(t)}{\int_0^{\overline{v}} q_j(I_j, t)dF(t)}\right] dF(v_j) \\ &= \int_0^{\overline{v}} q_j(I_j, v_j)v_j - m_j(I_j, v_j)dF(v_j),\end{aligned}$$

which is the same as in the first mechanism. Hence, the new mechanism is still EA-IR and results in the same entry decisions as the first mechanism; and because the allocation rule is the same it results in selling probability q. Since the expected payments entering bidders make are the same as well, this new mechanism will have expected revenue R if buyers report their valuations truthfully.[20]

It follows that for every EA-IR and IC direct revelation resale mechanism that sells with probability q and yields expected revenue R, there exists an EA-IR (but not necessarily IC) direct revelation resale mechanism which sells with probability q, generates expected revenue R if buyers report their valuations truthfully, and in which losing bidder pay exactly zero.

Step 2. Next, we show that every EA-IR direct revelation mechanism that sells with probability q and in which losing bidders pay exactly zero can at most have revenue $q\hat{r}$ if buyers report their valuations truthfully. Consider buyer j's entry decision, having information I_j but not knowing his valuation v_j. Let λ be the probability with which j expects to win if he enters, and let τ be the expected payment j makes if he enters and wins. By EA-IR, buyer j enters if and only if

$$\lambda\big(E\big[v_j\big|I_j, j \text{ wins}\big] - \tau\big) \;\geq\; c. \tag{9}$$

For given λ, $E\big[v_j\big|I_j, j \text{ wins}\big]$ is maximized if the allocation rule of the mechanism is such that j wins the object whenever $v_j \geq F^{-1}(1-\lambda)$. In this case

$$\lambda E\big[v_j\big|I_j, j \text{ wins}\big] \;=\; \int_{F^{-1}(1-\lambda)}^{\overline{v}} v dF(v),$$

[20]Since the mechanism is not IC, this revenue is hypothetical: If the mechanism was actually used, buyers would misrepresent their information strategically and thereby reduce the revenue earned by the seller.

and it follows that, for given λ, the maximum τ that satisfies condition (9) is

$$\tau(\lambda) \;=\; \frac{1}{\lambda}\left[\int_{F^{-1}(1-\lambda)}^{\overline{v}} v dF(v) - c\right]. \tag{10}$$

Now maximize the expression on the right-hand side of (10) with respect to λ. The first-order condition is

$$\begin{aligned}
\tau'(\lambda) \;&=\; -\frac{1}{\lambda^2}\left[\int_{F^{-1}(1-\lambda)}^{\overline{v}} v dF(v) - c\right] - \frac{1}{\lambda}\left[F^{-1}(1-\lambda) f\big(F^{-1}(1-\lambda)\big)\frac{d}{d\lambda}F^{-1}(1-\lambda)\right] \\
&=\; -\frac{1}{\lambda^2}\left[\int_{F^{-1}(1-\lambda)}^{\overline{v}} v dF(v) - c\right] + \frac{1}{\lambda}F^{-1}(1-\lambda) \\
&=\; 0,
\end{aligned}$$

which can be rearranged to

$$\int_{F^{-1}(1-\lambda)}^{\overline{v}} v - F^{-1}(1-\lambda)\; dF(v) - c \;=\; 0. \tag{11}$$

Using (1), (11) implies $F^{-1}(1-\lambda) = \hat{r}$, or $\lambda = 1 - F(\hat{r})$. Plugging this back into (10), and using (1) again, we have

$$\tau(1 - F(\hat{r})) \;=\; \frac{1}{1 - F(\hat{r})}\left[\int_{\hat{r}}^{\overline{v}} v dF(v) - c\right] \;=\; \frac{1}{1 - F(\hat{r})}\int_{\hat{r}}^{\overline{v}} \hat{r} dF(v) \;=\; \hat{r}.$$

Thus, conditional on winning, the expected payment made by a buyer in this mechanism is at most $\hat{r}$. Since losing buyers do not pay, the expected revenue of this mechanism is at most $q\hat{r}$. □

Proof of Lemma 2

We begin by translating the definition of effective valuations (2) into a more convenient form.

Note that for any $(Q, X, X^0) \in \Omega$ and all n, if $Q_n = 0$ then $X_n^0 \leq 0$. This follows from the EA-IR constraint in assumption A4: Buyers would not pay the entry cost c to participate in a mechanism in which they had a zero chance of winning, unless $X_n^0 < 0$. However, since $(0, 0, 0) \in \Omega$ by assumption A3, if the reseller chooses a mechanism with $Q_n = 0$ he would never set $X_n^0 < 0$. Hence we can assume, without loss of generality, that $Q_n = 0$ implies $X_n^0 = 0$. Now define

$$Y_n \equiv \begin{cases} X_n + \dfrac{1 - Q_n}{Q_n} X_n^0 & \text{if } Q_n > 0, \\ \quad 0 & \text{if } Q_n = 0, \end{cases}$$

and let $\widetilde{\Omega} \equiv \{(Q, Y) : (Q, X, X^0) \in \Omega\}$. We can then restate (2) as follows:

$$\begin{aligned} z_n(v_i) &= \max_{(Q,X,X^0)\in\Omega} \{Q_n X_n + (1 - Q_n)(v_i + X_n^0))\} && \text{original definition (2)} \\ &= \max_{(Q,Y)\in\widetilde{\Omega}} \{Q_n Y_n + (1 - Q_n)v_i)\}. && (12) \end{aligned}$$

In other words, we can focus (without loss of generality) on resale mechanisms that generate non-zero revenue only in the event that the object is resold. Assumption A5 guarantees that $\max_{(Q,X,X^0)\in\Omega}\{Q_n X_n + (1 - Q_n)(v_i + X_n^0)\}$ exists for all n; hence $z_n(v_i)$ is well-defined.

With this simplification, we now proceed to establish properties (i)–(iv) of the result.

Property (i). Take some $v_i' \in [0, \overline{v}]$ and let

$$(Q', Y') = \arg\max_{(Q,Y)} \{Q_n Y_n + (1 - Q_n)v_i'\}.$$

If v_i is another valuation, with $v_i > v_i'$, then we have

$$\begin{aligned} z_n(v_i) &= \max_{(Q,Y)} \{Q_n Y_n + (1 - Q_n)v_i\} \geq Q_n' Y_n' + (1 - Q_n')v_i \\ &\geq Q_n' Y_n' + (1 - Q_n')v_i' = \max_{(Q,Y)} \{Q_n Y_n + (1 - Q_n)v_i'\} = z_n(v_i'), \quad (13) \end{aligned}$$

which shows that z_n is weakly increasing. To show that it is strictly increasing, we will argue that $Q' < 1$ for all $v_i' > 0$. (This implies that the second inequality in (13) is strict whenever $v_i > v_i' > 0$; since we already know that z_n is weakly increasing over $[0, \overline{v}]$, strict monotonicity follows.)

Since $\rho_j < 1$ for some j, there is a strictly positive probability that the arrival of buyers breaks down, and hence a strictly positive probability that $v_j < v_i'$ for all existing buyers $j > n$. Consider this event. By the interim individual rationality (I-IR) constraint in assumption A4, conditional on a buyer's entry decision and private valuation, the buyer's expected payment cannot exceed this valuation times the buyer's probability of winning. Thus, expected resale revenue must be strictly less than v_i'. However, a reseller with valuation v_i' would never choose a mechanism that yielded expected revenue strictly below v_i': He would prefer to either not sell the object or make a series of take-it-or-leave it offers at price v_i'; by assumption A3 both are possible. Thus, for a reseller with private valuation $v_i' > 0$, it cannot be optimal to choose a resale mechanism that results in resale with (unconditional) probability one.

To show convexity of z_n, take $\alpha \in (0,1)$ and observe that

$$\begin{aligned} z_n(\alpha v_i + (1-\alpha)v_i') &= \max_{(Q,Y)} \{Q_n Y_n + (1-Q_n)(\alpha v_i + (1-\alpha)v_i')\} \\ &\leq \alpha \max_{(Q,Y)} \{Q_n Y_n + (1-Q_n)v_i\} \\ &\qquad + (1-\alpha) \max_{(Q,Y)} \{Q_n Y_n + (1-Q_n)v_i')\} \\ &= \alpha z_n(v_i) + (1-\alpha) z_n(v_i'). \end{aligned}$$

Therefore, z_n is weakly convex.

Property (ii). Note that $Q_n = Y_n = 0$ is a feasible choice (i.e., by assumption A3, a buyer who wins the object has the option to not resell it), which implies $z_n(v_i) \geq v_i$. Since z_n is strictly increasing, we have $z_n(v_i) > z_n(0)$. Thus, $z_n(v_i) \geq \max\{z_n(0), v_i\}$.

Property (iii). Note that Lemma 1 implies $Q_n X_n + (1-Q_n)X_n^0 = Q_n Y_n \leq Q_n \hat{r}$ for all n. Thus $Y_n \leq \hat{r}$, which implies that for $v_i > \hat{r}$ the optimal reselling mechanism is such that $Q_n = Y_n = 0$; by assumption A2 such a mechanism is available. It then follows that $z_n(v_i) \leq \max\{\hat{r}, v_i\}$.

Property (iv). Recall the following properties: $z_n(v_i) \geq v_i$ (from (ii)), $z_n(\overline{v}) = \overline{v}$ (from (ii) and (iii)), and z_n increasing and convex (from (i)). Together, these properties imply that z_n is a (weak) contraction: $z_n(v_i) - z_n(v_i') \leq v_i - v_i'$ for $v_i > v_i'$. □

Proof of Lemma 3

Suppose $\rho^t \to (1,1,1,\ldots)$ pointwise. For every $\alpha \in (0,1)$ and every integer $k > n$, there exists $T(\alpha,k) < \infty$ such that $\rho_i^t > \alpha$ for all $i < k$ and $t > T(\alpha,k)$. Fix any such $t > T(\alpha,k)$. If the reseller can make a take-it-or-leave-it offer at posted price $\hat{r}$ to every $i > n$ (which is possible by assumption A3), his expected revenue is at least

$$\overbrace{\alpha\big(1-F(\hat r)\big)\hat r + F(\hat r)\Big[\alpha\big(1-F(\hat r)\big)\hat r + F(\hat r)\Big[\alpha\big(1-F(\hat r)\big)\hat r + \ \dots \ \Big]\Big]}^{k-n \text{ iterations}}$$

$$= \ \hat r\alpha\big(1-F(\hat r)\big)\frac{1-F(\hat r)^{k-n}}{1-F(\hat r)} \ = \ \hat r\alpha\big(1-F(\hat r)^{k-n}\big).$$

At the same time, by Lemma 1 the reseller's expected revenue can never be more than $\hat r$. Thus, for $t > T(\alpha,k)$, $\hat r\alpha\big(1-F(\hat r)^k\big) \le z_n^t(0) \le \hat r$. The term on the left-hand side becomes arbitrarily close to $\hat r$ as $\alpha \to 1$ and $k \to \infty$. It follows that $z_n^t(0) \to \hat r$ as $t \to \infty$; this establishes (i). Statement (ii) now follows from (i) and the fact that $\max\{z_n(0), v_i\} \le z_n(v_i) \le \max\{\hat r, v_i\}$. Statement (iii) follows from (i), (ii), and the definition of $\hat r$ in (1). □

Proof of Proposition 4

Let E be the set of bidders who entered the auction, with $|E| = n$. If $n < n^*$ and $\rho_{n+1} < 1$, it becomes commonly known that no potential buyers exist who have not entered. In this case, the auction is a standard second price auction with symmetrically distributed independent private values, and it is an equilibrium to bid these values. This gives $b_i^{AU}(v_i) = v_i$.

Now suppose that $n \ge n^*$, or $n < n^*$ and $\rho_{n+1} = 1$. Suppose, for a moment, that no resale takes place among agents in E. Fix some $i \le n$, let $\bar b_{-i}$ be the highest bid submitted by bidders $j \ne i$, and let G be the distribution of $\bar b_{-i}$. Bidder i's expected payoff from bidding b_i when his valuation is v_i can be expressed as

$$U_i(b_i|v_i) \ = \ \int_0^{b_i} z_n(v_i) - \bar b_{-i} \, dG(\bar b_{-i}),$$

and this is maximized at $b_i = z_n(v_i)$. Now consider the profile $b_i(v_i) = z_n(v_i)$ $\forall i \in E$. Since z_n is strictly increasing, the object is allocated to the bidder with the highest private valuation, which implies that there can be no resale among the buyers in E, as was posited initially.

Buyer i may still deviate from this bidding strategy *and* change his resale strategy, in a way that involves the bidders in E. We show that this cannot benefit i. If buyer i deviates from $b_i(v_i) = z_n(v_i)$ and, as a result of the deviation, wins even though he does not have the highest valuation, he pays $\max_{j\in E\setminus i} z_n(v_j)$ to the original seller. But this is the most any bidder in $E \backslash i$ would be willing to pay to acquire the object from i in the aftermarket, so i cannot strictly benefit from the deviation. Similarly, if buyer i deviates and does not win even though he has the highest valuation, and then tries to acquire the object in the aftermarket, he would have to pay at least $\max_{j \in E \backslash i} z_n(v_j)$ to the winner of the auction. But this is the same amount i would have had to pay to the original seller if he had won in the auction, so again i cannot strictly benefit from the deviation. It follows that the profile $b_i^{AU}(v_i) = z_n(v_i)\ \forall i \in E$ is a BNE in the auction with $n \geq n^*$ entrants. $\square$

Proof of Proposition 5

Take a sequence $\rho^t \to (1, 1, 1, \ldots)$ pointwise. For a given n, let z_n^t, π_n^t, and $\overline{\pi}_n^t$ be a buyer's effective valuation, expected payoff from participating in the auction with n bidders, and expected payoff from not participating in the auction, associated with the t^{th} sequence. By Lemma 3 (ii), $\lim_t z_n^t(v_i) = \max\{\hat{r}, v_i\}$ uniformly. Since (3) is continuous in z_n, we have

$$\begin{aligned}
\lim_{t\to\infty} \pi_n^t &= \int_0^{\overline{v}} \int_0^v \max\{\hat{r}, v_i\} - \max\{\hat{r}, w\}\, dF(w)^{n-1} dF(v) - c \\
&= \int_{\hat{r}}^{\overline{v}} \left(F(\hat{r})^{n-1}(v - \hat{r}) + \int_{\hat{r}}^{v} v - w\, dF(w)^{n-1} \right) dF(v) - c,
\end{aligned}$$

which is strictly decreasing in n. For $n = 2$ we have

$$\begin{aligned}
\lim_{t\to\infty} \pi_2^t &= \int_{\hat{r}}^{\overline{v}} \left(F(\hat{r})(v - \hat{r}) + \int_{\hat{r}}^{v} v - w\, dF(w) \right) dF(v) - c \\
&< \int_{\hat{r}}^{\overline{v}} \big(F(\hat{r})(v - \hat{r}) + (1 - F(\hat{r}))(v - \hat{r})\big) dF(v) - c \\
&= \int_{\hat{r}}^{\overline{v}} v - \hat{r}\, dF(v) - c \;=\; 0,
\end{aligned}$$

where the final equality follows from the definition of $\hat{r}$ in (1). This implies that for t sufficiently large, $\pi_n^t < 0 \leq \overline{\pi}_n^t$ for all $n > 1$. On the other hand, $\pi_1^t \geq \overline{\pi}_1^t = 0$ for all t ($\pi_1^t > 0$ because bidder 1 wins and pays a zero price; $\overline{\pi}_1^t = 0$ because there is no resale market if 1 does not enter). Stated in equivalent terms: For $\mathcal{N}$ sufficiently large,

$$\pi^{AU}(1,1) \;>\; 0 \;>\; \pi^{AU}(2,2) \;>\; \pi^{AU}(3,3) \;>\; \ldots,$$

so $n^* = 1$, which means the seller receives a zero price. $\square$

Proof of Lemma 6

Without loss of generality let bidder n be the current high-bidder after stage n of the mechanism is completed. If bidder $n+1$ enters, then he and incumbent n compete in an ascending auction. The value that bidder $n+1$ gains if he wins against n in this ascending auction is at most $z_{n+1}(v_{n+1})$. Furthermore, since bidder n will raise the price at least to v_n, if $n+1$ wins he must pay at least $\max\{p_n, v_n\} \geq \max\{z_n(0), v_n\}$. Thus, an upper bound on the expected surplus that $n+1$ achieves if he enters is

$$\frac{1}{1-F(y)} \int_{z_{n+1}^{-1}(p_n)}^{\overline{v}} \int_{y}^{z_{n+1}(v_{n+1})} \Big(z_{n+1}(v_{n+1}) - \max\big\{ z_n(0), v_n \big\} \Big) dF(v_n) dF(v_{n+1}) \; - \; c$$

$$\leq \int_0^{\overline{v}} \int_0^{z_{n+1}(v_{n+1})} \Big(z_{n+1}(v_{n+1}) - \max\big\{ z_n(0), v_n \big\} \Big) dF(v_n) dF(v_{n+1}) \; - \; c$$

$$\leq \int_0^{\hat{r}} \int_0^{z_{n+1}(v_{n+1})} \big(\hat{r} - z_n(0) \big) dF(v_n) dF(v_{n+1}) + \int_{\hat{r}}^{\overline{v}} \int_0^{\hat{r}} \big(v_{n+1} - z_n(0) \big) dF(v_n) dF(v_{n+1}) + \int_{\hat{r}}^{\overline{v}} \int_{\hat{r}}^{v_{n+1}} \big(v_{n+1} - v_n \big) dF(v_n) dF(v_{n+1}) \; - \; c$$

$$< \int_0^{\hat{r}} \int_0^{\hat{r}} \big(\hat{r} - z_n(0) \big) dF(v_n) dF(v_{n+1}) + \int_{\hat{r}}^{\overline{v}} \int_0^{\hat{r}} \big(v_{n+1} - z_n(0) \big) dF(v_n) dF(v_{n+1}) + \int_{\hat{r}}^{\overline{v}} \int_{\hat{r}}^{\overline{v}} \big(v_{n+1} - \hat{r} \big) dF(v_n) dF(v_{n+1}) + \int_{\hat{r}}^{\overline{v}} \int_{\hat{r}}^{\overline{v}} \big(\hat{r} - v_n \big) dF(v_n) dF(v_{n+1}) \; - \; c. \tag{14}$$

If $\mathcal{N} \to \infty$ then $\rho \to (1, 1, 1, \ldots)$, and Lemma 3 implies that (14) converges to

$$F(\hat{r})^2 (\hat{r} - \hat{r}) \; + \; F(\hat{r}) \int_{\hat{r}}^{\overline{v}} (v_{n+1} - \hat{r}) dF(v_{n+1}) \; + \; (1 - F(\hat{r})) \int_{\hat{r}}^{\overline{v}} (v_{n+1} - \hat{r}) dF(v_{n+1}) + \int_{\hat{r}}^{\overline{v}} \int_{\hat{r}}^{\overline{v}} \big(\hat{r} - v_n \big) dF(v_n) dF(v_{n+1}) \; - \; c$$

$$= \quad 0 \;+\; F(\hat{r})c \;+\; (1-F(\hat{r}))c \;+\; \int_{\hat{r}}^{\overline{v}}\int_{\hat{r}}^{\overline{v}} (\hat{r}-v_n)dF(v_n)dF(v_{n+1}) \;-\; c$$

$$= \quad \int_{\hat{r}}^{\overline{v}}\int_{\hat{r}}^{\overline{v}} (\hat{r}-v_n)dF(v_n)dF(v_{n+1}) \;<\; 0.$$

It follows that bidder $n+1$ should not enter for $\mathcal{N}$ sufficiently large. □

Proof of Proposition 7

For large enough $\mathcal{N}$, the existence of the equilibrium with $v^* = 0$ (the "0-equilibrium" for short) was established in the main text. For the other equilibria, we proceed in three steps. First, we show that, regardless of $\mathcal{N}$, an equilibrium exists in which $v^* = \hat{r}$. Second, assuming the 0-equilibrium exists, we construct equilibria with cutoffs $v^* \in (0, \hat{r})$. To do so, we use the $\hat{r}$-equilibrium as a post-entry continuation equilibrium. Third, we show that all equilibria with cutoffs $v^* > 0$ fail BK's selection criterion.

Step 1. The $\hat{r}$-equilibrium consists of the same strategies as the V_K-equilibrium in BK. It can be described as follows: After bidder 1 enters, he bids zero if $v_1 < \hat{r}$ and places jump bid $j_1 \in (0, \hat{r})$ if $v_1 \geq \hat{r}$. In every period n thereafter, bidder n enters as long as no entry-deterring jump bid occurred in period $n-1$. Bidder n and the previous incumbent, say i, then raise the price to the second-largest of $\{p_{n-1}, v_n, v_i\}$. If n becomes the new incumbent, he places a jump bid j_n if and only if $v_n \geq \hat{r}$. At each stage n, the value of the jump bid j_n is set to make a marginal bidder with valuation $\hat{r}$ indifferent between (a) deterring entry, and thus winning the object with probability one for a payment of j_n; and (b) accommodating entry by all future bidders who arrive, and thus winning the object with some probability less than one (if the arrival of new buyers breaks down and the bidder is still the incumbent) for a payment equal to the second-highest valuation among all entrants.

To see that this strategy profile is also an equilibrium if resale is possible, consider bidder n with private value $v_n \geq \hat{r}$. Note that $z_n(v_n) = v_n$ by Lemma 2. Thus, if deterring entry is optimal for this bidder in the no-resale case, it must be optimal in the resale case as well. Next, consider bidder n with private value $v_n \leq \hat{r}$. Note that $z_n(v_n) < \hat{r}$ by Lemma 2. Since a bidder with effective valuation equal to $\hat{r}$ is indifferent between deterring and accommodating entry, bidder n strictly prefers to accommodate. But in this case, bidder n wins only if the arrival of new buyers breaks down, and since there is no resale in this case, n should be willing to pay up to his private value v_n for the object.

He should therefore stay in the sequential mechanism until the price reaches v_n, which is the same entry accommodating strategy as in the no-resale case.

Finally, we need to consider the possibility that after period-n bidding is completed, the incumbent bidder places an additional jump bid $j_n' \neq j_n$ (the equilibrium value of the jump bid in period n). Such a jump bid is an out-of equilibrium event, and the out-of-equilibrium believes that we assign to the next bidder, $n+1$, in this event are as follows: If $j_n' > j_n$ then bidder $n+1$ believes that the buyer who placed the jump bid has valuation $v_i \geq \hat{r}$; if $j_n' < j_n$ then bidder $n+1$ believes that the buyer who placed the jump bid has valuation $v_i < \hat{r}$. With these beliefs, the entry strategy described above remains optimal; hence they support the equilibrium.

Step 2. Assume the 0-equilibrium exists, and fix any $v^* \in (0, \hat{r})$. Consider the following strategy profile: After bidder 1 enters, he bids as follows: If $v_1 < v^*$, he bids zero, and if $v_1 \geq v^*$, he places jump bid $j_1^* > 0$. (We will state in a moment how j_1^* is computed.) Bidder 2 does not enter if he observes a jump bid j_1^* or higher; in this case the game ends with bidder 1 winning. If bidder 2 observes no jump bid, or a jump bid less than j_1^*, he enters. At this point, all buyers (1, 2, and any future entrants) switch their strategies to the $\hat{r}$-equilibrium.

To see that this profile is an equilibrium, suppose bidder 2 observes jump bid j_1. Bidder 2's beliefs will be that $v_1 < v^*$ if $j_1 < j_1^*$, and $v_1 \geq v^*$ if $j_1 \geq j_1^*$. (As before, only $j_1 = 0$ and $j_1 = j_1^*$ are observed in equilibrium, in which case 2's beliefs are Bayesian. The other values of j_1 are out-of-equilibrium events, and the corresponding beliefs are out-of-equilibrium beliefs.) With these beliefs, the same arguments we made in the main text to construct the 0-equilibrium imply that, if $j_1 \geq j_1^*$, the expected payoff to bidder 2 from entering is negative, and 2 should therefore not enter. Now suppose bidder 2 observes jump bid $j_1 < j_1^*$, and believes that $v_1 < v^*$. As long as $j_1^* < \hat{r}$, bidder 2 should enter, as he would in the $\hat{r}$-equilibrium. The value of j_1^* is now chosen to make bidder 1 with private value $v_1 = v^*$ indifferent between winning the object for a payment of j_1^* (i.e., receiving $z_1(v^*) - j_1^*$ with certainty), and receiving the expected payoff of the $\hat{r}$-equilibrium. Such a j_1^* exists: Since the $\hat{r}$-equilibrium payoff to a bidder with valuation $v^* \in (0, \hat{r})$ is positive, $j_1^* < z_1(v^*)$; and since such a bidder does not win with certainty in the $\hat{r}$-equilibrium, $j_1^* > 0$.[21]

[21] Note that if $v^* \approx 0$, a buyer with valuation v^* has an almost zero chance of winning in the $\hat{r}$-equilibrium; thus the associated j_1^* must be very close to $z_1(v^*)$, which in turn is very close to $\hat{r}$ if $\mathcal{N}$ is large. Meanwhile, the smaller is v^*, the more likely is it that the first bidder deters entry in the v^*-equilibrium. Thus, when comparing the equilibria across different thresholds v^*, there is a revenue

Step 3. Finally, assume the full range of equilibria $0 \leq v^* \leq \hat{r}$ exists. Take any equilibrium where $v^* > 0$. In this equilibrium, bidder 1 with $v_1 \geq v^*$ places jump bid $j_1^* > 0$ and deters entry, and bidder 1 with $v_1 < v^*$ places jump bid zero and accommodates entry. We make the same forward induction argument ("perfect sequentiality") as BK to break this equilibrium.

Suppose that bidder 1 deviated and placed jump bid $0 < j_1^d < j_1^*$. In equilibrium, this deviation is interpreted by bidder 2 in a way that induces bidder 2 to enter. But if j_1^d leads to entry, while a zero jump bid also leads to entry, then placing bid j_1^d is a mistake for *all* types of bidder 1. In order to explain the deviation, bidder 2 must believe that bidder 1 believed that jump bid j_1^d would deter entry. The types of bidder 1 that would benefit (relative to the equilibrium) from making jump bid j_1^d, if such a bid deterred entry, are types with private values above some threshold $\underline{v}_1(j_1^d) < v^*$. To see this, not that, in the v^*-equilibrium, all types $v_1 \geq v^*$ deter entry with jump bid j_1^*. These types would clearly benefit if they could deter entry with a jump bid strictly less than j_1^*. On the other hand, types v_1 just below v^* are "almost indifferent" between accommodating entry and deterring entry with jump bid j_1^*, but have a slight preference for accommodating. If it was possible to deter entry with a jump bid strictly less than j_1^*, such types would also benefit if they could deter entry with the lower jump bid. It follows that the set of types of bidder 1 that would weakly benefit from placing jump bid $j_1^d < j_1^*$, if such a bid deterred entry, are types in some interval $[v_1(j_1^d), \overline{v}]$, where $0 \leq v_1(j_1^d) < v^*$.

Since the 0-equilibrium exists, entering is not profitable for bidder 2 conditional on knowing that $v_1 \geq 0$. Thus, entering is also not profitable conditional on knowing that $v_1 \geq \underline{v}_1(j_1^d)$. This implies that a jump bid of j_1^d would, in fact, deter entry by bidder 2 as long as it is interpreted by bidder 2 as having been placed by a type of bidder 1 who would prefer this jump bid over the equilibrium if it deterred entry. But in this case all types of bidder 1 with valuations above v^*, who deter entry in the equilibrium with jump bid j_1^*, should deviate and bid j_1^d instead of j_1^* (as this would still deter entry, but at a smaller cost). The only equilibrium for which such a deviation does not exist is the equilibrium with $v^* = 0$. □

discontinuity at $v^* = 0$: While the 0-equilibrium yields no revenue, an equilibrium with a positive but very small cutoff v^* yields a very large expected revenue.

Proof of Lemma 9

Buyer j's expected contribution to social welfare is $W_j = \lambda E\big[v_j \big| \, j \text{ wins}\big] - c$, where λ is the probability with buyer j wins the item. As long as $\lambda > 0$ we can express j's expected welfare contribution equivalently as $W_j = \lambda\tau$, where $\tau = E\big[v_j \big| \, j \text{ wins}\big] - c/\lambda$. For given λ, $E\big[v_j \big| \, j \text{ wins}\big]$ is maximized if the planner allocates the item to j whenever $v_j \geq F^{-1}(1-\lambda)$. In this case

$$E\big[v_j \big| \, j \text{ wins}\big] \;=\; \frac{1}{\lambda}\int_{F^{-1}(1-\lambda)}^{\overline{v}} v dF(v) \quad \text{and} \quad \tau \;=\; \frac{1}{\lambda}\left[\int_{F^{-1}(1-\lambda)}^{\overline{v}} v dF(v) - c\right].$$

By following the same steps as in Step 2 the proof of Lemma 1, one can show that $\tau \leq \hat{r}$. Thus, the expected contribution to social welfare made by a buyer who wins with probability λ is at most $\lambda\hat{r}$. Since at most one buyer can win the object, the expected social welfare the planner can achieve is at most $\hat{r}$. $\square$

References

[1] Athey, Susan, Jonathan Levin, and Enrique Seira (2011): "Comparing open and Sealed Bid Auctions: Evidence from Timber Auctions." *Quarterly Journal of Economics* 126 (1), 207–257.

[2] Bose, Subir and George Deltas (1999): "Welcoming the Middlemen: Restricting Competition in Auctions by Excluding the Consumers." In: *Studies in Economic Theory* 8, 119–131, Springer.

[3] Bulow, Jeremy and Paul Klemperer (2009): "Why Do Sellers (Usually) Prefer Auctions?" *American Economic Review* 99 (4), 1544–1575.

[4] Cao, Xiaoyong and Guoqiang Tian (2010): "Equilibria in First Price Auctions With Participation Costs." *Games and Economic Behavior* 69 (2), 258–273.

[5] Che, Xiaogang and Tilman Klumpp (2016): "Entry Deterrence in Dynamic Second-Price Auctions." *American Economic Journal: Microeconomics* 8, 168–201.

[6] Che, Xiaogang, Peter Lee, and Yibai Yang (2013): "The Impact of Resale on Entry in Second Price Auctions." *Mathematical Social Sciences* 66 (2), 163–168.

[7] Cheng, Harrison (2011): "Auctions With Resale and Bargaining Power." *Journal of Mathematical Economics* 47 (3), 300–308.

[8] Davis, Andrew Elena Katok, and Anthony Kwasnica (2013): "Should Sellers Prefer Auctions? A Laboratory Comparison of Auctions and Sequential Mechanisms." *Management Science* 60 (4), 990–1008.

[9] Fishman, Michael (1988): "A Theory of Preemptive Takeover Bidding." *RAND Journal of Economics* 19 (1), 88–101.

[10] Garratt, Rod and Thomas Tröger (2006): "Speculation in Standard Auctions With Resale." *Econometrica* 74 (3), 753–769.

[11] Garratt, Rod, Thomas Tröger, and Charles Zheng (2009): "Collusion via Resale." *Econometrica* 77 (4), 1095–1136.

[12] Gupta, Madhurima and Bernard Lebrun (1999): "First Price Auctions With Resale." *Economics Letters* 64 (2), 181–185.

[13] Hafalir, Isa and Vijay Krishna (2008): "Asymmetric Auctions with Resale." *American Economic Review* 98 (1), 87–112.

[14] Haile, Philip (2000): "Partial Pooling at the Reserve Price in Auctions With Resale Opportunities." *Games and Economic Behavior* 33 (2), 231–248.

[15] Haile, Philip (2001): "Auctions With Resale Markets: An Application to U.S. Forest Service Timber Sales." *American Economic Review* , 399–427.

[16] Haile, Philip (2003): "Auctions With Private Uncertainty and Resale Opportunities." *Journal of Economic Theory* 108 (1), 72–110.

[17] Krishna, Vijay (2002): *Auction Theory* (1st edition). London, UK: Academic Press.

[18] Lebrun, Bernard (2010a): "First-Price Auctions With Resale and With Outcomes Robust to Bid Disclosure." *RAND Journal of Economics* 41 (1), 165–178.

[19] Lebrun, Bernard (2010b): "Revenue Ranking of First-Price Auctions With Resale." *Journal of Economic Theory* 145 (5), 2037–2043.

[20] Lebrun, Bernard (2012): "Optimality and the English and Second-Price Auctions With Resale." *Games and Economic Behavior* 75 (2), 731–751.

[21] Levin, Dan and James Smith (1994): "Equilibrium in Auctions with Entry." *American Economic Review* 84 (3), 585–599.

[22] Li, Tong and Xiaoyong Zheng (2009): "Entry and Competition Effects in First-Price Auctions: Theory and Evidence From Procurement Auctions." *Review of Economic Studies* 76 (4), 1397–1429.

[23] Lu, Jingfeng (2009): "Auction Design With Opportunity Cost." *Economic Theory* 38 (1), 73–103.

[24] Lu, Jingfeng and Lixin Ye (2013): "Efficient and Optimal Mechanisms With Private Information Acquisition Costs." *Journal of Economic Theory* 148 (1), 393–408.

[25] Lu, Jingfeng and Lixin Ye (2014): "Optimal Two-Stage Auctions with Costly Information Acquisition." Working paper.

[26] McAfee, Preston and John McMillan (1987): "Auctions With Entry." *Economics Letters* 23, 343–347.

[27] Milgrom, Paul (2004): *Putting Auction Theory to Work*. Cambridge, UK: Cambridge University Press.

[28] Moreno, Diego and John Wooders (2011): "Auctions with Heterogeneous Entry Costs." *RAND Journal of Economics* 42 (2), 313–336.

[29] Mylovanov, Timofiy and Thomas Tröger (2009): "Optimal Auction With Resale—A Characterization of the Conditions." *Economic Theory* 40 (3), 509–528.

[30] Pagnozzi, Marco (2007): "Bidding to Lose? Auctions With Resale." *RAND Journal of Economics* 38 (4), 1090–1112.

[31] Pagnozzi, Marco (2010): "Are Speculators Unwelcome in Multiobject Auctions?" *American Economic Journal: Microeconomics* 2 (2), 97–131.

[32] Roberts, James and Andrew Sweeting (2013): "When Should Sellers Use Auctions?" *American Economic Review* 103 (5), 1830–1861.

[33] Samuelson, William (1985): "Competitive Bidding With Entry Costs." *Economics Letters* 17, 53–57.

[34] Shi, Xianwen (2012): "Optimal Auctions With Information Acquisition." *Games and Economic Behavior* 74 (2), 666–686.

[35] Stegeman, Mark (1996): "Participation Costs and Efficient Auctions." *Journal of Economic Theory* 71 (1), 228–259.

[36] Tan, Guofu and Okan Yilankaya (2006): "Equilibria in Second Price Auctions With Participation Costs." *Journal of Economic Theory* 130, 205–219.

[37] Virág, Gábor (2013): "First-Price Auctions with Resale: The Case of Many Bidders." *Economic Theory* 52 (1), 129–163.

[38] Virág, Gábor (2016): "Auctions with Resale: Reserve Prices and Revenues." *Games and Economic Behavior* 99, 239–249.

[39] Zhang, Jun and Ruqu Wang (2013): "Optimal Mechanism Design With Resale via Bargaining." *Journal of Economic Theory* 148 (5), 2096–2123.

[40] Zheng, Charles (2002): "Optimal Auction With Resale." *Econometrica* 70 (6), 2197–2224.